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Module-valued functors preserving the covering dimension

Comment.Math.Univ.Carolin. 56,3 (2015) 377–399.

Abstract: We prove a general theorem about preservation of the covering dimension \dim by certain covariant functors that implies, among others, the following concrete results.

- (i) If G is a pathwise connected separable metric NSS abelian group and X, Y are Tychonoff spaces such that the group-valued function spaces $C_p(X, G)$ and $C_p(Y, G)$ are topologically isomorphic as topological groups, then $\dim X = \dim Y$.
- (ii) If free precompact abelian groups of Tychonoff spaces X and Y are topologically isomorphic, then $\dim X = \dim Y$.
- (iii) If R is a topological ring with a countable network and the free topological R -modules of Tychonoff spaces X and Y are topologically isomorphic, then $\dim X = \dim Y$.

The classical result of Pestov [*The coincidence of the dimensions \dim of l -equivalent spaces*, Soviet Math. Dokl. **26** (1982), no. 2, 380–383] about preservation of the covering dimension by l -equivalence immediately follows from item (i) by taking the topological group of real numbers as G .

Keywords: covering dimension; topological group; function space; topology of pointwise convergence; free topological module; l -equivalence; G -equivalence

AMS Subject Classification: 54H11, 54H13

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