Research Article

INTERNATIONAL JOURNAL OF ADVANCED ROBOTIC SYSTEMS

International Journal of Advanced Robotic Systems
November-December 2017: 1–9
© The Author(s) 2017
DOI: 10.1177/1729881417748442
journals.sagepub.com/home/arx
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Smooth time-invariant control for leaderless consensus of networked nonholonomic systems

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Abstract

This article studies the leaderless consensus control problem of multiple nonholonomic chained systems. Two smooth time-invariant static distributed controllers are derived based on Lyapunov method, graph theory, and LaSalle invariance principle. Both of the proposed controllers guarantee that the states of the multiple nonholonomic systems globally asymptotically converge to a common vector, provided that the interconnection topology is undirected and connected. In particular, the second control scheme can reduce the size of the control inputs via saturated control and is more applicable in real engineering. Several numerical simulations are implemented for kinematic models of four nonholonomic unicycle mobile robots, demonstrating the effectiveness of the proposed control schemes.

Keywords

Nonholonomic chained systems, leaderless consensus control, saturated control, LaSalle invariance principle

Date received: 2 April 2017; accepted: 19 November 2017

Topic: Mobile Robots and Multi-Robot Systems
Topic Editor: Lino Marques
Associate Editor: Jesus Capitan Fernandez

Introduction

In the past few years, distributed cooperative control of multiagent systems has attracted a lot of attention due to its widespread applications, including flocking, swarming, formation control, and so forth.[1–6] As a classical nonlinear system, nonholonomic system has received increasing interest of researchers.[7–15] Two consensus control problems have been taken into account for nonholonomic systems. The first one is the leaderless consensus control problem, where controllers are derived to make the states of the multiple nonholonomic systems converge to a common configuration. The second one is the consensus tracking control problem with a leader, where controllers are designed to steer the multiple nonholonomic systems to track the leader.[16]

For any individual nonholonomic system, there exists no continuous time-invariant static controller that asymptotically stabilizes the state of system to a fixed point,[17] and hence, only the time-varying or discontinuous controller[18] can achieve asymptotic stabilization. For multiple leaderless consensus case, most of the published references still focus on the design of time-varying, discontinuous, or dynamic controller, see the literatures[19–29] which achieve the leaderless consensus/formation control of multiple nonholonomic systems. In addition, it was reported in the study by Zhai et al.[30] that the smooth time-invariant static distributed control law can realize the asymptotic leaderless

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consensus of the nonholonomic chained systems over undirected connected graph. However, the Lyapunov-like function candidate in the study by Zhai et al.[30] is only shown to be positive semidefinite, and hence, the system stability requires further analysis. Moreover, Zhai et al.[30] do not take into account the input saturation. Therefore, it is deserved to develop a new smooth time-invariant static distributed control law for leaderless consensus control of multiple nonholonomic systems subject to input saturation.

This article investigates the leaderless consensus control problem of networked nonholonomic chained systems. A Lyapunov function is firstly constructed based on the Laplacian matrix of the associated communication graph, which is proved to be nonnegative and equal to zero only at the point of zero consensus error in the case of undirected connected communication graph. With the aid of the carefully constructed Lyapunov function, the first smooth time-invariant distributed controller is designed ensuring that the Lyapunov function is non-increasing under the undirected connected communication graph. According to LaSalle invariance principle, the consensus error of the nonholonomic systems is shown to globally asymptotically decay to zero. Modifying the first control scheme, the second saturated smooth time-invariant static control law is obtained, which not only achieves the global asymptotic consensus control but also satisfies the input saturation condition. Comparing to the related works,[19–29] the proposed control strategies are smooth and time-invariant. Comparing to the work of Zhai et al.,[30] the second control law can realize the consensus goal in the presence of input saturation.

The rest of this article is arranged as follows. The problem formulation is firstly presented, and then, the two controllers are constructed. Effectiveness of the proposed saturated control strategy is illustrated in "Simulation" section. Finally, the conclusion is included.

Problem statement

Consider that there are N nonholonomic chained systems, and the nonlinear model of the i th system is given by

$$\begin{bmatrix} \dot{x}_{i1} \\ \dot{x}_{i2} \\ \dot{x}_{i3} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ x_{i2} & 0 \end{bmatrix}}_{\triangleq B_i} \underbrace{\begin{bmatrix} u_{i1} \\ u_{i2} \end{bmatrix}}_{\triangleq u_i} = B_i u_i \tag{1}$$

where $X_i = [x_{i1}, x_{i2}, x_{i3}]^T \in R^3$ and $u_i = [u_{i1}, u_{i2}]^T \in R^2$ are the state and control input of system i, respectively. Any kinematic completely nonholonomic systems with three states and two inputs can be converted to the form (1).[31]

Suppose that the N nonholonomic systems are interconnected via some equipments such that they can exchange information. The information flow between them can be expressed by a communication graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, A)$. To be more precise, the N systems are treated as N nodes with the index set denoted by $\mathcal{V} = \{1, 2, \ldots, N\}$, and if system i can access the state of system j, then we say the ordered pair $(i, j) \in \mathcal{E}$ and $a_{ij} = 1$, otherwise $a_{ij} = 0$, where a_{ij} is the entry of the adjacent matrix $A = [a_{ij}] \in R^{N \times N}$. In particular, $a_{ii} = 0$ for all $i \in \mathcal{V}$. Under the graph structure, the induced Laplacian matrix $L = [l_{ij}] \in R^{N \times N}$ plays a key role in analyzing the stability of cooperative control systems. The expression of L is $L = D - A$, where $D = \text{diag}\{d_1, d_2, \ldots, d_N\}$ is a diagonal matrix with $d_i = \sum_{j=1}^{N} a_{ij}$. Based on this definition, the sum of each row of L equals zero, that is, $L\overrightarrow{1} = 0$ with $\overrightarrow{1} = [1, 1, \ldots, 1]^T \in R^N$, implying that zero is an eigenvalue of L with eigenvector $\overrightarrow{1}$.

Denote the consensus error between systems i and j by

$$E_{ij} = X_i - X_j$$

Then, the consensus error of the whole N nonholonomic systems can be written as

$$E = [E_{1N}^T, E_{2N}^T, E_{3N}^T, \ldots, E_{N-1,N}^T]^T \in R^{3(N-1)} \tag{2}$$

The leaderless consensus control problem of networked nonholonomic systems is stated as: design a distributed control law $\left(u_{i1}(\cdot), u_{i2}(\cdot)\right)$ such that the states of the networked nonholonomic systems globally asymptotically converge to a common vector, that is, $\lim_{t \to \infty} \| E \| = 0$, where $\| \cdot \|$ denotes the Euclidean norm of vectors.

Remark 1. It is well known that the system (1) can not be asymptotically stabilized to a fixed configuration by any continuous time-invariant static state feedback control law. However, the asymptotical leaderless consensus control of equation (1) is not limited by Brockett's condition, and can be realized by smooth time-invariant static feedback, since the common vector that the system state vectors are required to converge to is not necessarily time-invariant.

Controller development

In this section, two smooth time-invariant distributed controllers are proposed to solve the leaderless consensus problem of networked nonholonomic systems by using LaSalle invariance principle.

Let $X = [X_1^T, X_2^T, \ldots, X_N^T]^T$. Taking the derivative of X, the collective model of the networked nonholonomic systems can be written as

$$\dot{X} = BU \tag{3}$$

where

$$B = \text{diag}\{B_1, B_2, \ldots, B_N\} \in R^{(3N) \times (2N)}$$
$$U = [u_1^T, u_2^T, \ldots, u_N^T]^T \in R^{2N}$$

Define the following function

$$V_1 = X^T (L \otimes P) X \tag{4}$$

where L is the Laplacian matrix of the communication graph and $P \in R^{3\times 3}$ is a constant matrix. Then, we have the following technical lemma.

Lemma 1. If the communication graph is undirected and connected, and the constant matrix P is positive definite and symmetric, then the function in equation (4) satisfies

$$\begin{cases} V_1 \geq 0, \forall E \in R^{3(N-1)} \\ V_1 = 0 \Leftrightarrow E = 0 \\ V_1 \to \infty, \text{if } \| E \| \to \infty. \end{cases} \tag{5}$$

Proof. Since $P = P^T$ and $L = L^T$ for undirected graph, one has

$$\begin{aligned} V_1 &= X^T(L\otimes P)X = [X^T(I_N \otimes P)][(L\otimes I_3)X] \\ &= X^T \begin{bmatrix} P & 0 & \cdots & 0 \\ 0 & P & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & P \end{bmatrix} \begin{bmatrix} l_{11}I_3 & l_{12}I_3 & \cdots & l_{1N}I_3 \\ l_{21}I_3 & l_{22}I_3 & \cdots & l_{2N}I_3 \\ \vdots & \vdots & \ddots & \vdots \\ l_{N1}I_3 & l_{N2}I_3 & \cdots & l_{NN}I_3 \end{bmatrix} X \\ &= \left[X_1^TP, X_2^TP, \ldots, X_N^TP\right] \begin{bmatrix} \sum_{j=1}^{N}(a_{1j}I_3) & -a_{12}I_3 & \cdots & -a_{1N}I_3 \\ -a_{21}I_3 & \sum_{j=1}^{N}(a_{2j}I_3) & \cdots & -a_{2N}I_3 \\ \vdots & \vdots & \ddots & \vdots \\ -a_{N1}I_3 & -a_{N2}I_3 & \cdots & \sum_{j=1}^{N}(a_{Nj}I_3) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{bmatrix} \\ &= [X_1^TP, X_2^TP, \ldots, X_N^TP] \begin{bmatrix} \sum_{j=1}^{N}[a_{1j}(X_1 - X_j)] \\ \sum_{j=1}^{N}[a_{2j}(X_2 - X_j)] \\ \vdots \\ \sum_{j=1}^{N}[a_{Nj}(X_N - X_j)] \end{bmatrix} \\ &= \sum_{i=1}^{N}\sum_{j=1}^{N}[a_{ij}X_i^TP(X_i - X_j)] \end{aligned} \tag{6}$$

where I_N and I_3 are the $N \times N$ and 3×3 identity matrices, respectively. As $P = P^T$, the following equality holds

$$\sum_{i=1}^{N}\sum_{j=1}^{N}(a_{ij}Z_i^TPZ_j) = \sum_{i=1}^{N}\sum_{j=1}^{N}(a_{ij}Z_j^TPZ_i) \tag{7}$$

Since $\sum_{j=1}^{N} a_{ij} = \sum_{j=1}^{N} a_{ji}$ for undirected graph, one gets

$$\begin{aligned} \sum_{i=1}^{N}\sum_{j=1}^{N}(a_{ij}X_i^TPX_i) &= \sum_{i=1}^{N}\left(X_i^TPX_i\sum_{j=1}^{N}a_{ij}\right) \\ &= \sum_{i=1}^{N}\left[X_i^TPX_i\sum_{j=1}^{N}a_{ji}\right] = \sum_{j=1}^{N}\left[X_j^TPX_j\sum_{i=1}^{N}a_{ij}\right] \\ &= \sum_{i=1}^{N}\sum_{j=1}^{N}(a_{ij}X_j^TPX_j) \end{aligned} \tag{8}$$

Substituting equations (7) and (8) into equation (6) results in

$$\begin{aligned} V_1 &= \sum_{i=1}^{N}\sum_{j=1}^{N}[0.5a_{ij}(X_i^T PX_i + X_j^T PX_j \\ &\quad -X_i^T PX_j - X_j^T PX_i)] \\ &= 0.5\sum_{i=1}^{N}\sum_{j=1}^{N}[a_{ij}(X_i - X_j)^T P(X_i - X_j)] \\ &= 0.5\sum_{i=1}^{N}\sum_{j=1}^{N}(a_{ij}E_{ij}^T PE_{ij}) \end{aligned} \tag{9}$$

As $P > 0$ (positive definite) and $a_{ij} \geq 0$, one has $a_{ij}E_{ij}^T PE_{ij} \geq 0$ and $V_1 \geq 0$ for any $E \in R^{3(N-1)}$. Moreover, since the communication graph is connected, V_1 in equation (9) is obviously radially unbounded with respect to E, and $V_1 = 0$ if and only if $E_{ij} = 0$ for any $i \neq j$. That is, $V_1 = 0$ if and only if $E = 0$. □

Lemma 1 indicates that the constructed function V_1 is positive definite and radially unbounded with respect to the consensus error E, and hence can be taken as a Lypaunov function for the leaderless consensus problem of multi-agent linear/nonlinear systems.

Now, based on Lemma 1, we take V_1 as the Lyapunov function, and compute its derivative as

$$\begin{aligned} \dot{V}_1 &= X^T(L\otimes P)BU + U^T B^T(L\otimes P)X \\ &= 2X^T(L\otimes P)BU \end{aligned} \tag{10}$$

where $L = L^T$ and $P = P^T$ are utilized. Design

$$U = -kB^T(L\otimes P)X \tag{11}$$

where k is a positive controller parameter. Under the control law in equation (11), equation (10) becomes

$$\begin{aligned} \dot{V}_1 &= -2kX^T(L\otimes P)BB^T(L\otimes P)X \\ &= -2kX^T(L^T\otimes P^T)BB^T(L\otimes P)X \leq 0 \end{aligned} \tag{12}$$

According to

$$U = -kB^T(L\otimes P)X = -kB^T(I_N\otimes P)(L\otimes I_3)X$$

the i th component of U can be easily written as

$$u_i = -k B_i^T P\sum_{j=1}^{N}[a_{ij}(X_i - X_j)] \tag{13}$$

which is only dependent on the states of system i and its neighbors. In this sense, the derived control law (equation (11) or equation (13)) is distributed.

As $\dot{V}_1$ is negative semidefinite, the rest work is to determine whether there exists an appropriate matrix P guaranteeing that only $E = 0$ can stay identically in the invariant set $\{X \in R^{3N} | \dot{V}_1 = 0\}$, and to determine the explicit form of P.

Let us define

$$P = \begin{bmatrix} p_1 & 0 & 0 \\ 0 & p_2 & p_{23} \\ 0 & p_{23} & p_3 \end{bmatrix} \tag{14}$$

where $p_1 > 0, p_2 > 0$, and $p_2p_3 - p_{23}^2 > 0$ should be guaranteed such that $P > 0$. Then, the following results can be obtained:

Theorem 1. The control law in equations (13) and (14) guarantees that the consensus error E globally asymptotically converges to zero, provided that $k > 0$, $p_1 > 0$, $p_2 > 0$, $p_2p_3 - p_{23}^2 > 0$, $p_{23} \neq 0$, and the communication graph between systems is undirected and connected.

Proof. Since $p_1 > 0, p_2 > 0$, and $p_2p_3 - p_{23}^2 > 0$, hence the matrix P is ensured to be positive definite. Moreover, as the communication graph is undirected and connected, one concludes from Lemma 1 that V_1 is positive definite and radially unbounded with respect to consensus error, and $\dot{V}_1 \leq 0$. Substituting equation (14) into equation (13), one has

$$u_i = -k\begin{bmatrix} p_1 & p_{23}x_{i2} & p_3x_{i2} \\ 0 & p_2 & p_{23} \end{bmatrix}\begin{bmatrix} \sum_{j=1}^{N}[a_{ij}(x_{i1} - x_{j1})] \\ \sum_{j=1}^{N}[a_{ij}(x_{i2} - x_{j2})] \\ \sum_{j=1}^{N}[a_{ij}(x_{i3} - x_{j3})] \end{bmatrix}$$

Let $\dot{V}_1 \equiv 0$, then $B^T(L\otimes P)X \equiv 0$, that is $u_i = [u_{i1}, u_{i2}]^T \equiv 0$. Hence

$$\begin{cases} f_i \triangleq p_1\sum_{j=1}^{N}[a_{ij}(x_{i1} - x_{j1})] + p_{23}x_{i2} \times \\ \sum_{j=1}^{N}[a_{ij}(x_{i2} - x_{j2})] + p_3x_{i2}\sum_{j=1}^{N}[a_{ij}(x_{i3} - x_{j3})] \equiv 0 \\ g_i \triangleq p_2\sum_{j=1}^{N}[a_{ij}(x_{i2} - x_{j2})] + p_{23}\sum_{j=1}^{N}[a_{ij}(x_{i3} - x_{j3})] \equiv 0 \end{cases} \tag{15}$$

Since $\sum_{i=1}^{N}\sum_{j=1}^{N}[a_{ij}(x_{i1} - x_{j1})] = \overrightarrow{1}^T L[x_{11}, x_{21}, \ldots, x_{N1}]^T = 0$, it follows

$$\begin{aligned} \sum_{i=1}^{N} f_i &= p_{23}\sum_{i=1}^{N}\left[x_{i2}\sum_{j=1}^{N}[a_{ij}(x_{i2} - x_{j2})]\right] \\ &\quad + p_3\sum_{i=1}^{N}\left[x_{i2}\sum_{j=1}^{N}[a_{ij}(x_{i3} - x_{j3})]\right] \equiv 0 \end{aligned} \tag{16}$$

From the second equation of equation (15) and $p_{23} \neq 0$, one has

$$\sum_{j=1}^{N}[a_{ij}(x_{i3} - x_{j3})] \equiv -\frac{p_2}{p_{23}}\sum_{j=1}^{N}[a_{ij}(x_{i2} - x_{j2})] \quad (17)$$

Substituting equation (17) into equation (16) results in

$$\left(p_{23} - \frac{p_2 p_3}{p_{23}}\right)\sum_{i=1}^{N}\left[x_{i2}\sum_{j=1}^{N}[a_{ij}(x_{i2} - x_{j2})]\right] \equiv 0 \quad (18)$$

Since $p_2 p_3 - p_{23}^2 > 0$ and $p_{23} \neq 0$, it follows that $p_{23} - \frac{p_2 p_3}{p_{23}} \neq 0$ and hence $\sum_{i=1}^{N}[x_{i2}\sum_{j=1}^{N}[a_{ij}(x_{i2} - x_{j2})]] \equiv 0$. Moreover

$$\sum_{i=1}^{N}\left[x_{i2}\sum_{j=1}^{N}[a_{ij}(x_{i2} - x_{j2})]\right] = 0.5\sum_{i=1}^{N}\sum_{j=1}^{N}\left[a_{ij}(x_{i2} - x_{j2})^2\right] \quad (19)$$

holds for undirected communication graph, which is equal to zero, thus $x_{i2} - x_{j2} \equiv 0$ for any (i,j) under the connected graph. Based on $x_{i2} - x_{j2} \equiv 0$, it follows from equation (17) that $L[x_{13}, x_{23}, \ldots, x_{N3}]^T = 0$, and hence, $x_{i3} - x_{j3} \equiv 0$ for any (i,j). Substituting both $x_{i2} - x_{j2} \equiv 0$ and $x_{i3} - x_{j3} \equiv 0$ into the first equation of equation (15), it can be further concluded that $x_{i1} - x_{j1} \equiv 0$ for any (i,j).

The above analysis shows that only $E = 0$ can stay in the invariant set $\{X \in R^{3N} | \dot{V}_1 = 0\}$. According to LaSalle invariance principle, the consensus error E is globally asymptotically convergent to zero. □

Remark 2. Different from the existing time-varying or discontinuous controllers in the literatures,[19–29] which addressed the leaderless consensus/formation control problem of nonholonomic systems, the distributed controller (equation (11) or (13)) is smooth, time-invariant, and static. The achievement of this relies heavily on the Lyapunov function V_1 constructed in equation (4). Firstly, V_1 is dependent on L, which allows us to design the distributed controller (equation (11)). Secondly, V_1 is dependent on the matrix P, not the identity matrix, which ensures $\dot{V}_1 = 0 \Rightarrow E = 0$.

Note that the size of controller (equation (11) or (13)) may become a little large for big x_{i2} or big consensus error, and the convergence rate may be slow for small consensus error. To deal with the two issues, we propose another saturated version for equation (11) or (13)

$$\begin{aligned} U &= -k\tanh[\varepsilon^{-1}B^T(L \otimes P)X] \\ u_i &= -k\tanh\left\{\varepsilon^{-1}B_i^T P\sum_{j=1}^{N}[a_{ij}(X_i - X_j)]\right\} \end{aligned} \quad (20)$$

where $\tanh([s_1, s_2, \ldots, s_N]^T) = [\tanh(s_1), \tanh(s_2), \ldots, \tanh(s_N)]^T$ and ε is a small positive constant. The usage of $\tanh(\cdot)$ ensures

$$|u_{i1}| \leq k, |u_{i2}| \leq k$$

Under the new saturated control law (equation (20)), equation (12) becomes

$$\dot{V}_1 = -2kX^T(L^T \otimes P^T)B \cdot \tanh[\varepsilon^{-1}B^T(L \otimes P)X] \leq 0$$

The small ε here results in a bigger $\dot{V}_1$ and hence a quicker convergence rate, which brings a better transient performance of the closed-loop nonholonomic consensus error system. According to the proof of Theorem 1, we have $\dot{V}_1 = 0 \Rightarrow B^T(L \otimes P)X = 0 \Rightarrow E = 0$, meaning that the global asymptotic convergence of E can be still guaranteed by equation (20). Thus, the following results can be obtained:

Theorem 2. The saturated control law (equation (20)) guarantees that the consensus error E globally asymptotically converges to zero, provided that that $\varepsilon > 0, k > 0, p_1 > 0, p_2 > 0, p_2 p_3 - p_{23}^2 > 0, p_{23} \neq 0$, and the communication graph between systems is undirected and connected.

Remark 3. For the second control scheme, the size of the control inputs u_{i1} and u_{i2} can be tuned artificially by adjusting k. Compared to the one in the study by Zhai et al.,[30] our second control scheme is effective in the case of input saturation and can achieve a quick convergence rate in the small neighborhood of zero consensus errors.

Simulation

In this section, simulation examples are provided to demonstrate the effectiveness of the proposed control schemes. Consider the multiple nonholonomic mobile robots that can be transformed into the nonholonomic chained form. The kinematic model of nonholonomic mobile robots is[27]

$$\dot{\bar{x}}_i = v_i \cos\theta_i, \dot{\bar{y}}_i = v_i \sin\theta_i, \dot{\theta}_i = \omega_i$$

where $(\bar{x}_i, \bar{y}_i)$ and θ_i are the position and angle of robot i, respectively, and (v_i, ω_i) are the linear and angular velocities. The following state and input transformations[30]

$$\begin{aligned} x_{i1} &= \theta_i, x_{i2} = -\bar{x}_i\cos\theta_i - \bar{y}_i\sin\theta_i \\ x_{i3} &= -\bar{x}_i\sin\theta_i + \bar{y}_i\cos\theta \\ u_{i1} &= \omega_i, u_{i2} = -v_i + (\bar{x}_i\sin\theta_i - \bar{y}_i\cos\theta_i)\omega_i \end{aligned}$$

convert the robot model to the form (1). Assume there are four robots ($N = 4$) connected by the topology depicted in figure 1 of the work of Zhai et al.[30] Under this topology, the Laplacian matrix is

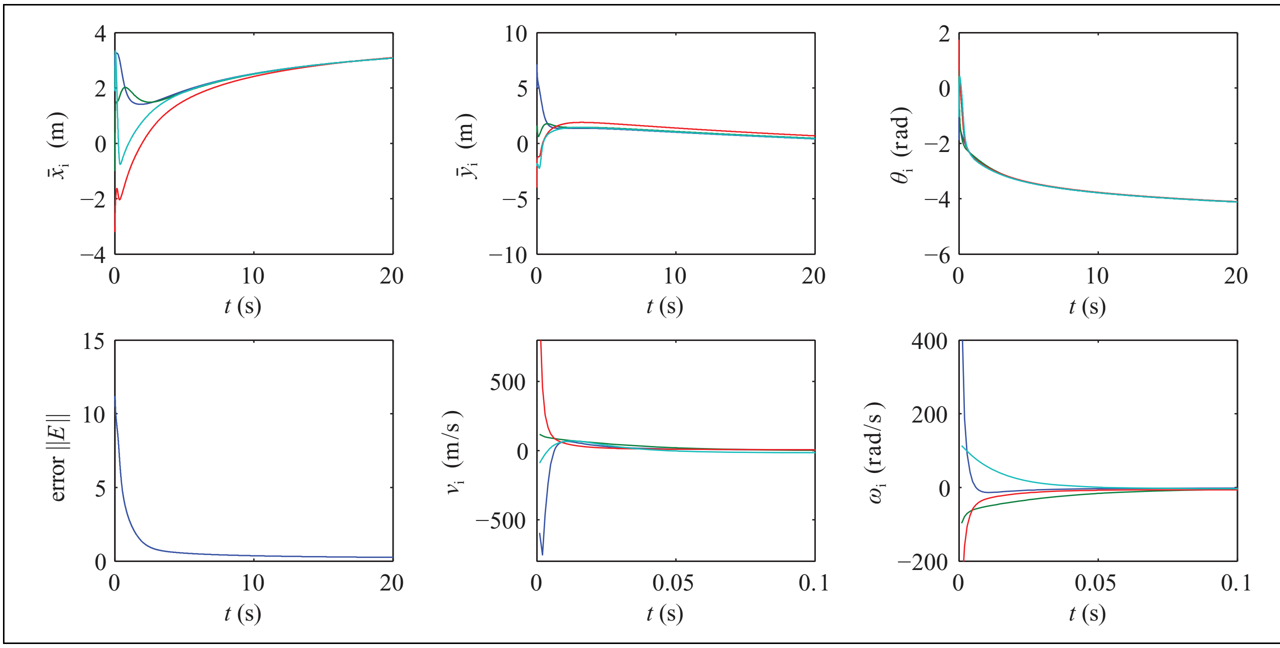

Figure 1. Case 1: Simulation results under the first controller (blue line: robot 1; green line: robot 2; red line: robot 3; and cyan line: robot 4).

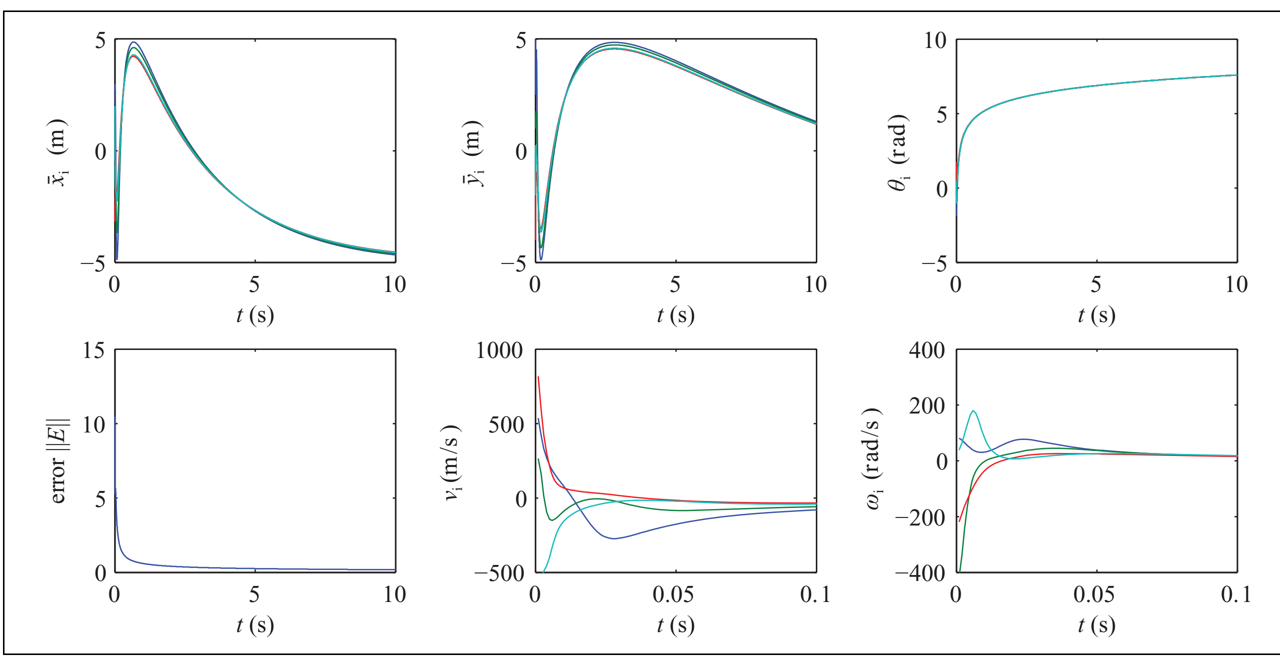

Figure 2. Case 2: Simulation results under the controller in the study by Zhai et al.[30] (blue line: robot 1; green line: robot 2; red line: robot 3; and cyan line: robot 4).

$$L = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

Set the initial states of robots the same as those in the study by Zhai et al.[30] Three cases are taken into account for the simulation.

Case 1: Control strategy equation (11) or (13) *with* $k = 3$, $p_1 = 12, p_2 = 1, p_3 = 2$, *and* $p_{23} = -1.2$.

Case 2: Control strategy in the study by Zhai et al.[30] *with the controller parameters unchanged.*

Case 3: Control strategy (equation (20)) with $\varepsilon = 0.3, k = 3$, $p_1 = 12, p_2 = 1, p_3 = 2$, *and* $p_{23} = -1.2$.

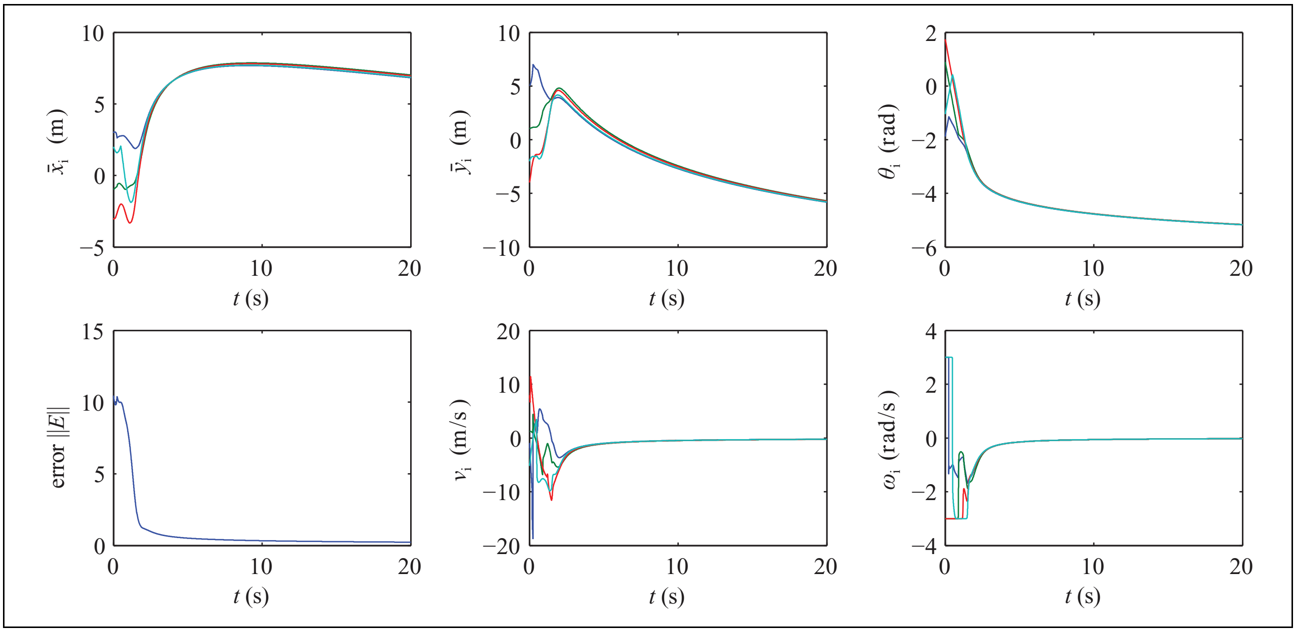

Figure 3. Case 3: Simulation results under the second controller (blue line: robot 1; green line: robot 2; red line: robot 3; and cyan line: robot 4).

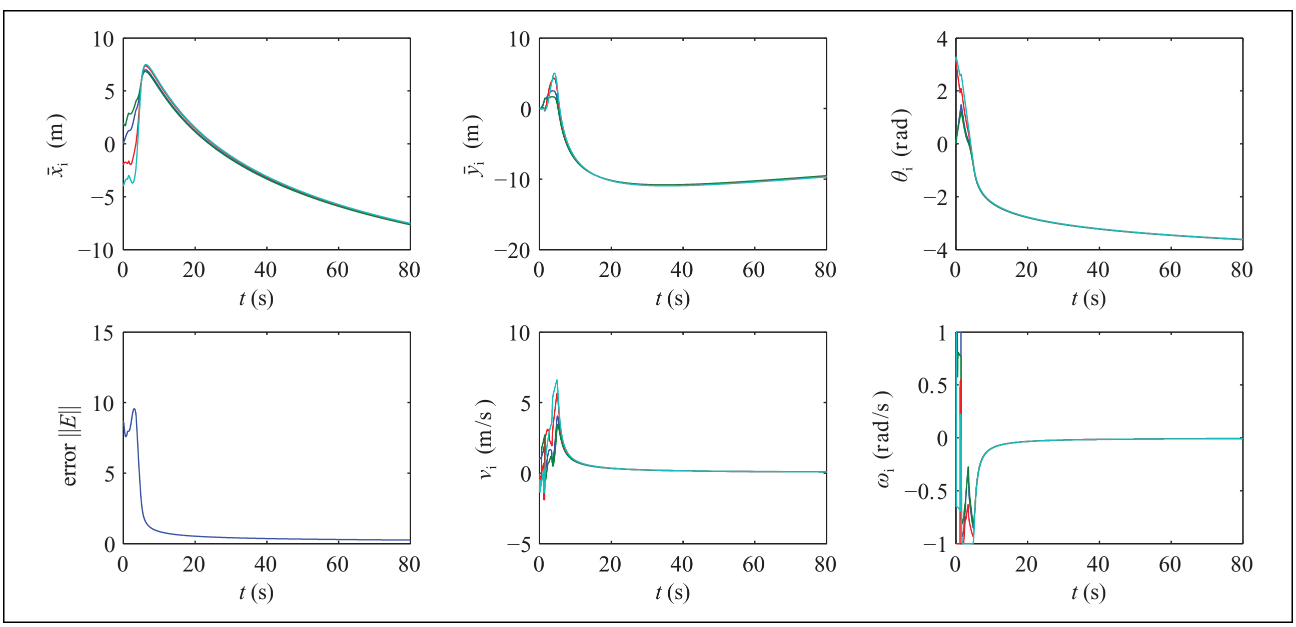

Figure 4. Special case: simulation results under the second controller with the initial condition $(\bar{x}_1, \bar{y}_1, \theta_1)(0) = (0, 0, 0)$, $(\bar{x}_2, \bar{y}_2, \theta_2)(0) = (2, 0, 0)$, $(\bar{x}_3, \bar{y}_3, \theta_3)(0) = (-2, 0, \pi)$, and $(\bar{x}_4, \bar{y}_4, \theta_4)(0) = (-4, 0, \pi)$ (blue line: robot 1; green line: robot 2; red line: robot 3; and cyan line: robot 4).

Simulation results for cases 1 to 3 are obtained in Figures 1 to 3, respectively. From Figures 1 and 2, it can be observed that the control scheme (equation (11)) and the one in the study by Zhai et al.[30] are both capable of forcing the state vector of the four robots to a common vector. To see the trajectories of control inputs in the transient process, only the change of control inputs in initial 0.1 s is depicted in Figures 1 and 2, showing that the size of control inputs is too large under the controller (equation (11)) or the one in the study by Zhai et al.[30] However, the actuators can not provide such large control signals in real engineering. So, we construct the saturated control law (equation (20)) and draw the simulation results in Figure 3. Obviously, Figure 3 illustrates that the proposed saturated control scheme (equation (20)) can greatly reduce the size of control inputs and simultaneously achieve the leaderless consensus control objective for nonholonomic chained systems. With the introduction of $\tanh(\cdot/\varepsilon)$, $\varepsilon > 0$, the second saturated law remains smooth and can be easily extended to dynamic case. The trajectory of $\tanh(\cdot/\varepsilon)$ is similar to that of $\text{sign}(\cdot)$ for a small ε, but still remains smooth. Comparing with the first proposed control strategy and the one in the study by Zhai et al.,[30] the second proposed control strategy has a saturated input signal (Figures 1 to 3).

What's more, we further consider the special case: the four robots initially locate in a straight line with opposite orientations, that is, $(\bar{x}_1, \bar{y}_1, \theta_1)(0) = (0, 0, 0)$, $(\bar{x}_2, \bar{y}_2, \theta_2)(0) = (2, 0, 0)$, $(\bar{x}_3, \bar{y}_3, \theta_3)(0) = (-2, 0, \pi)$, and $(\bar{x}_4, \bar{y}_4, \theta_4)(0) = (-4, 0, \pi)$. Under this initial condition, we implement the second controller with the control parameters $\varepsilon = 0.05$, $k = 1$, $p_1 = 12$, $p_2 = 1$, $p_3 = 2$, and $p_{23} = -1.2$, and get the simulation results in Figure 4. It can be seen from Figure 4 that the second controller can globally asymptotically stabilize the consensus error to zero in this special case.

Conclusion

In this article, two new simple smooth time-invariant distributed controllers are proposed to make the states of networked nonholonomic chained systems approach a common vector. Strict stability analysis is presented based on Lyapunov method and LaSalle invariance principle, proving that the consensus error globally asymptotically decays to zero. The obtained results clarify that the smooth time-invariant static controllers can achieve the asymptotic leaderless consensus of nonholonomic systems. Future research may lie on extending our method to the consensus control problem of higher-order nonholonomic chained systems.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work was supported by Fundamental and Frontier Research Project of Chongqing (No. cstc2016jcyjA0404), Fundamental Research Funds for the Central Universities (XDJK2016C038 and SWU115046), and National Nature Science Foundation of China (No. 61573034 and 61327807).

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