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Nonnormality of remainders of some topological groups

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Abstract: It is known that every remainder of a topological group is Lindelöf or pseudocompact. Motivated by this result, we study in this paper when a topological group G has a normal remainder. In a previous paper we showed that under mild conditions on G, the Continuum Hypothesis implies that if the Čech-Stone remainder G^* of G is normal, then it is Lindelöf. Here we continue this line of investigation, mainly for the case of precompact groups. We show that no pseudocompact group, whose weight is uncountable but less than $\mathfrak{c}$, has a normal remainder under $\mathsf{MA}+\neg\mathsf{CH}$. We also show that if a precompact group with a countable network has a normal remainder, then this group is metrizable. We finally show that if $C_p(X)$ has a normal remainder, then X is countable (Corollary 4.10) This result provides us with many natural examples of topological groups all remainders of which are nonnormal.

Keywords: remainder; compactification; topological group; normal space
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