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Local/global uniform approximation of real-valued continuous functions

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Abstract: For a Tychonoff space X, $C(X)$ is the lattice-ordered group (l-group) of real-valued continuous functions on X, and $C^*(X)$ is the sub-l-group of bounded functions. A property that X might have is (AP) whenever G is a divisible sub-l-group of $C^*(X)$, containing the constant function 1, and separating points from closed sets in X, then any function in $C(X)$ can be approximated uniformly over X by functions which are locally in G. The vector lattice version of the Stone-Weierstrass Theorem is more-or-less equivalent to: Every compact space has AP. It is shown here that the class of spaces with AP contains all Lindelöf spaces and is closed under formation of topological sums. Thus, any locally compact paracompact space has AP. A paracompact space failing AP is Roy's completely metrizable space Δ.

Keywords: real-valued function, Stone-Weierstrass, uniform approximation, Lindelöf space, locally in
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