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Stability of solitonic solutions of Super KdV equations under susy breaking conditions

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Abstract

A supersymmetric breaking procedure for $N = 1$ Super Korteweg-de Vries (SKdV), preserving the positivity of the hamiltonian as well as the existence of solitonic solutions, is implemented. The resulting solitonic system is shown to have nice stability properties.

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1 Introduction

The breaking of supersymmetry in physical systems is always an interesting aspect to analyze. We consider a solitonic system [1] arising from the breaking of supersymmetry in the $N = 1$ Super KdV system [2, 3]. In the latter, there is only one hamiltonian structure in distinction to the bi-hamiltonian one in the Korteweg-de Vries (KdV) system. The $N = 1$ SKdV hamiltonian, however, is not manifestly positive. Nevertheless, the quantum formulation of the theory yields a manifestly positive definite self-adjoint operator. The stability of the ground state of the system is then assured from it. We considered in [1] the supersymmetry breaking of the SKdV system by changing the Grassmann algebra structure of the SKdV formulation to a Clifford algebra one. This susy breaking mechanism has already appeared in several works, see for example [4]. One may then obtain a solitonic system with the same evolution equation for the new Clifford algebra-valued field as one had for the odd Grassmann algebra-valued one in the SKdV system, and what is more important, with a bounded from below hamiltonian. The system presents solitonic solutions, although the infinite set of local conserved quantities of SKdV breaks down. We will show in this work that this solitonic system has nice stability properties.

The stability in the sense of Liapunov of the one-soliton solution of KdV equation was first proven by Benjamin [5] and Bona [6]. The proof makes use of the first few conserved quantities of the KdV equation. In particular, the fact that one of them is the square of L_2 norm is relevant in their argument. The use of conserved quantity was also considered in a stability argument by Boussinesq [7]. In this paper, we make use of this main idea to prove stability of the one-soliton solution of the coupled equation, with fields valued on a Clifford algebra, derived from the supersymmetric breaking of the $N = 1$ SKdV equation.

2 SKdV and the breaking of supersymmetry

The fields $u(x, t)$ and $\xi(x, t)$ describing $N = 1$ SKdV equations [2] take values on the even and odd part of a Grassmann algebra, respectively. The $N = 1$ SKdV equations are

$$
\begin{aligned}
u_t &= -u''' + 6uu' - 3\xi\xi'', \\
\xi_t &= -\xi''' + 3(\xi u)'.
\end{aligned}
\tag{1}
$$

This system of partial differential equations has infinite local conserved charges as well as infinite non-local conserved charges [2, 8–11]. The first few local ones are

$$
\begin{aligned}
H_{\frac{1}{2}} &= \int_{-\infty}^{+\infty} \xi \, dx, \qquad H_1 = \int_{-\infty}^{+\infty} u \, dx, \\
H_3 &= \int_{-\infty}^{+\infty} \left(u^2 - \xi\xi'\right) dx, \qquad H_5 = \int_{-\infty}^{+\infty} \left(2u^3 + \left(u'\right)^2 - \xi'\xi'' - 4u\xi\xi'\right) dx.
\end{aligned}
\tag{2}
$$

To break supersymmetry, we consider the fields u and ξ taking values on a Clifford algebra instead of being Grassmann algebra-valued. We thus take u to be a real-valued field, while ξ to be an expansion in terms of the generators e_i, $i = 1, \dots$ of the Clifford algebra:

$$
\xi = \sum_i \varphi_i e_i + \sum_{ij} \varphi_{ij} e_i e_j + \sum_{ijk} \varphi_{ijk} e_i e_j e_k + \cdots,
\tag{3}
$$

where

$$
e_i e_j + e_j e_i = -2\delta_{ij},
\tag{4}
$$

and $\varphi_i, \varphi_{ij}, \varphi_{ijk}, \dots$ are real-valued functions. We define $\bar{\xi} = \sum_{i=1}^{\infty} \varphi_i \bar{e}_i + \sum_{ij} \varphi_{ij} \bar{e}_j \bar{e}_i + \sum_{ijk} \varphi_{ijk} \times \bar{e}_k \bar{e}_j \bar{e}_i + \cdots$, where $\bar{e}_i = -e_i$. We denote as in superfield notation the body of the expansion those terms associated with the identity generator and the soul the remaining ones. Consequently, the body of $\xi\bar{\xi}$, denoted by $\mathcal{P}(\xi\bar{\xi})$, is equal to $\sum_i \varphi_i^2 + \sum_{ij} \varphi_{ij}^2 + \sum_{ijk} \varphi_{ijk}^2 + \cdots$. In what follows, without loss of generality, we rewrite $\mathcal{P}(\xi\bar{\xi}) = \sum_i \varphi_i^2 + \sum_{ij} \varphi_{ij}^2 + \sum_{ijk} \varphi_{ijk}^2 + \cdots$ simply as $\mathcal{P}(\xi\bar{\xi}) = \sum_i \varphi_i^2$.

The system of partial differential equations arising from the breaking of supersymmetry, which has the required properties as discussed in [1] is

$$
\begin{aligned}
u_t &= -u''' - uu' - \frac{1}{4}\left(\mathcal{P}(\xi\bar{\xi})\right)', \\
\xi_t &= -\xi''' - \frac{1}{2}(\xi u)'.
\end{aligned}
\tag{5}
$$

System (5) has the following conserved charges:

$$
\begin{aligned}
\hat{H}_{\frac{1}{2}} &= \int_{-\infty}^{+\infty} \xi \, dx, \\
\hat{H}_1 &= \int_{-\infty}^{+\infty} u \, dx, \\
V \equiv \hat{H}_3 &= \int_{-\infty}^{+\infty} \left(u^2 + \mathcal{P}(\xi\bar{\xi})\right) dx, \\
M \equiv \hat{H}_5 &= \int_{-\infty}^{+\infty} \left(-\frac{1}{3}u^3 - \frac{1}{2}u\mathcal{P}(\xi\bar{\xi}) + \left(u'\right)^2 + \mathcal{P}\left(\xi'\bar{\xi}'\right)\right) dx.
\end{aligned}
\tag{6}
$$

The system has multi-solitonic solutions, for example, $u(x,t) \equiv \phi(x,t) = 3C \frac{1}{\cosh^2(z)}$, $z \equiv \frac{1}{2} C^{\frac{1}{2}}(x - (1 + C)t + a)$, $\xi(x,t) = 0$, where $\phi(x,t)$ is the one-soliton solution of KdV equation, a is an arbitrary real number, and $C > 0$.

This system with a change of sign in the third term of the right-hand member of the first equation of system (5) was considered in [12, 13].

It will be important in the following stability argument that

$$\int_{-\infty}^{\infty} \mathcal{P}(\xi\bar{\xi})\,dx = \|\xi\|_{L_2}^2, \tag{7}$$

hence V is L_2 norm of (u, ξ). This property is absent for the system with a positive sign on the third term of the first equation in (5).

3 Stability properties of the system

We may now analyze the stability of the ground state as well as the stability of the one-soliton solution of system (5).

We start the analysis by considering an *a priori* bound for the solutions of system (5). We denote by $\| \ \|_{H_1}$ the Sobolev norm

$$\|(u,\xi)\|_{H_1}^2 = \int_{-\infty}^{+\infty} \left[\left(u^2 + \sum_{i=0}^{\infty} \varphi_i^2 \right) + \left(u'^2 + \sum_{i=0}^{\infty} \varphi_i'^2 \right) \right] dx. \tag{8}$$

We obtain

$$\|(u,\xi)\|_{H_1}^2 \leq V + M + \frac{1}{2} \int_{-\infty}^{+\infty} |u| \left(u^2 + \mathcal{P}(\xi\bar{\xi}) \right) dx, \tag{9}$$

we now use

$$\sup |u| \leq \frac{1}{\sqrt{2}} \|u\|_{H_1} \leq \frac{1}{\sqrt{2}} \|(u,\xi)\|_{H_1}; \tag{10}$$

it yields

$$\|(u,\xi)\|_{H_1}^2 \leq V + M + \frac{1}{2\sqrt{2}} V \|(u,\xi)\|_{H_1}. \tag{11}$$

From (11), it follows that

$$\|(u,\xi)\|_{H_1} \leq \frac{d + \sqrt{d^2 + 4e}}{2}, \tag{12}$$

where $d = \frac{1}{2\sqrt{2}} V$ and $e = V + M$. We notice that $d^2 + 4e \geq 0$.

Consequently, given V and M, from the initial data and a solution satisfying those initial conditions, the $\|(u,\xi)\|_{H_1}$ is bounded by (12). The *a priori* bound is a strong evidence of the existence of the solution for $0 \leq t$. We will consider this existence problem elsewhere. In this work, we assume the existence of the solution and its continuous dependence on the initial data under smooth enough assumptions on the initial perturbation.

We consider the stability in the sense of Liapunov. In particular, we take the same definition as in [5]: $(\hat{u}, \hat{\xi})$, a solution of (5), is stable if given ϵ, there exists δ such that for any solution (u, ξ) of (5), satisfying at $t = 0$

$$d_I\left[(u, \xi), (\hat{u}, \hat{\xi})\right] < \delta, \tag{13}$$

then

$$d_{II}\left[(u, \xi), (\hat{u}, \hat{\xi})\right] < \epsilon \tag{14}$$

for all $t \geq 0$.

d_I and d_{II} denote two distances to be defined.

We consider now the stability problem of the ground state solution $\hat{u} = 0$, $\hat{\xi} = 0$.

We take d_I and d_{II} to be the Sobolev norm $\|(u - \hat{u}, \xi - \hat{\xi})\|_{H_1}$.

We get

$$V \leq \left\|(u, \xi)\right\|_{H_1}^2$$

and

$$M \leq \int_{-\infty}^{+\infty} \left(\frac{1}{2}|u|\left(u^2 + \mathcal{P}(\xi\bar{\xi})\right) + u'^2 + \mathcal{P}\left(\xi'\bar{\xi}'\right)\right) dx \leq \frac{1}{2\sqrt{2}}\left\|(u, \xi)\right\|_{H_1}^3 + \left\|(u, \xi)\right\|_{H_1}^2.$$

At $t = 0$, we then have, using (13),

$$V < \delta^2,$$

$$M < \frac{1}{2\sqrt{2}}\delta^3 + \delta^2.$$

Consequently, from the *a priori* bound (12), we obtain for any $t \geq 0$

$$\left\|(u, \xi)\right\|_{H_1} < \epsilon$$

for any given ϵ, provided δ is conveniently chosen.

This argument proves the stability of the ground state solution.

We now consider the stability of the one-soliton solution $\hat{u} = \phi$, $\hat{\xi} = 0$. The proof of stability is based on estimates for the u field, which are analogous to the one presented in [5, 6] while a new argument will be given for the ξ field. The distances we will use are

$$d_I\left[(u_1, \xi_1), (u_2, \xi_2)\right] = \left\|(u_1 - u_2, \xi_1 - \xi_2)\right\|_{H_1}, \tag{15}$$

$$d_{II}\left[(u_1, \xi_1), (u_2, \xi_2)\right] = \inf_{\tau}\left\|(\tau u_1 - u_2, \xi_1 - \xi_2)\right\|_{H_1}, \tag{16}$$

where τu_1 denotes the group of translations along the x-axis. d_{II} is a distance on a metric space obtained by identifying the translations of each $u \in H_1(\mathbb{R})$ [5]. d_{II} is related to a stability in the sense that a solution u remains close to $\hat{u} = \phi$ only in shape but not necessarily in position.

We first assume that

$$V(u, \xi) = V(\hat{u}, \hat{\xi}) = V(\phi, 0) \tag{17}$$

and

$$\int_{-\infty}^{+\infty} \xi \, dx = \int_{-\infty}^{+\infty} \hat{\xi} \, dx = 0, \tag{18}$$

and then we will relax this conditions to get the most general formulation of the stability problem.

Following [5], we define

$$h(x, t) = u(x - a, t) - \phi(x + Ct),$$

where a is defined, for each $t \geq 0$, by

$$\int_{-\infty}^{+\infty} \left[u(x, t) - \phi(x + a) \right]^2 dx = \inf_{y \in \mathbb{R}} \int_{-\infty}^{+\infty} \left[u(x, t) - \phi(x + y) \right]^2 dx.$$

In [6], it was proven that the infimum is taken on finite values of y.

We obtain

$$\Delta M \equiv M(u, \xi) - M(\phi, 0)$$

$$= \int_{-\infty}^{+\infty} \left[h'^2 + (C - \phi) h^2 + \sum_{i=0}^{\infty} \varphi_i'^2 \right.$$

$$\left. + \left(C - \frac{1}{2}\phi \right) \sum_i \varphi_i^2 - \left(\frac{1}{3} h^3 - \frac{1}{2} \sum_{i=0}^{\infty} \varphi_i^2 h \right) \right] dx, \tag{19}$$

where we have used $C(V(u, \xi) - V(\phi, 0)) = 0$ and that ϕ is the soliton solution of KdV equation, and hence it satisfies

$$\phi'' + \frac{1}{2}\phi^2 = C\phi. \tag{20}$$

We denote $\delta^3 M \equiv \int_{-\infty}^{+\infty} (-\frac{1}{3} h^3 + \frac{1}{2} \sum_{i=0}^{\infty} \varphi_i^2 h) \, dx$. We then have

$$\left| \delta^3 M \right| \leq \frac{1}{2} \sup |h| \left\| (h, \xi) \right\|_{H_1}^2 \leq \frac{1}{2\sqrt{2}} \left\| (h, \xi) \right\|_{H_1}^3, \tag{21}$$

where we have used $\sup |h| \leq \frac{\sqrt{2}}{3} \|h\|_{H_1}$.

Coming back to ΔM, we get

$$|\Delta M| \leq \int_{-\infty}^{+\infty} \left(h'^2 + C h^2 + \sum_{i=0}^{\infty} \varphi_i'^2 + C \sum_{i=0}^{\infty} \varphi_i^2 \right) dx + \left| \delta^3 M \right|$$

$$\leq \max(1, C) \left\| (h, \xi) \right\|_{H_1}^2 + \frac{1}{3\sqrt{2}} \left\| (h, \xi) \right\|_{H_1}^3. \tag{22}$$

At $t = 0$, we will assume that

$$d_I\big[(u,\xi),(\phi,0)\big] = \big\|(h,\xi)\big\|_{H_1} < \delta, \tag{23}$$

hence

$$|\Delta M| \le \left[\max(1,\mathcal{C}) + \frac{1}{3\sqrt{2}}\delta\right]\big\|(h,\xi)\big\|_{H_1}^2. \tag{24}$$

ΔM is a conserved quantity on the space of solutions of system (5). By taking δ small enough in (23), we can make ΔM as small as we wish. The second step in the proof of stability is to argue that at any $t \ge 0$, ΔM satisfies the bound

$$\Delta M \ge K d_{II}\big[(u,\xi),(\phi,0)\big]. \tag{25}$$

This bound completes the proof, in fact, we can make, at $t = 0$, $|\Delta M|$ as small as we wish, and hence, we can always satisfy (14) for any given ϵ.

We decompose ΔM into

$$\Delta M = \delta_h^2 M + \delta_\xi^2 M + \delta^3 M, \tag{26}$$

where

$$\begin{aligned}
\delta_h^2 M &= \int_{-\infty}^{+\infty}\big[h'^2 + (\mathcal{C} - \phi)h^2\big]\,dx, \\
\delta_\xi^2 M &= \int_{-\infty}^{+\infty}\left[\sum_{i=0}^{\infty}\varphi_i'^2 + \left(\mathcal{C} - \frac{1}{2}\phi\right)\sum_{i=0}^{\infty}\varphi_i^2\right]dx.
\end{aligned} \tag{27}$$

Using the same argument as in [5, 6], we obtain

$$\delta_h^2 M \ge \frac{1}{4}\int_{-\infty}^{+\infty}\big(h'^2 + \mathcal{C}h^2\big)\,dx - \frac{2}{5}\mathcal{C}^{\frac{1}{4}}\big\|(h,\xi)\big\|_{H_1}^3 \tag{28}$$

and

$$\|h\|_{H_1} \ge d_{II}(u,h).$$

We will now obtain a lower bound for $\delta_\xi^2 M$.

We consider the operator $H = -\frac{d^2}{dx^2} - \frac{1}{2}\phi$ with domain in the Hilbert space $L_2(\mathbb{R})$. It is a symmetric essentially self-adjoint operator. We denote with the same letter H its self-adjoint extension. It has two eigenvalues $\lambda_1 = -\mathcal{C}$ and $\lambda_2 = -\frac{\mathcal{C}}{4}$ and a continuous spectrum $[0,\infty)$. The eigenfunctions are proportional to $\frac{1}{\cosh^2 z}$ and $\frac{\sinh z}{\cosh^2 z}$, respectively, where $z = \frac{1}{2}\mathcal{C}^{\frac{1}{2}}$. The spectral theorem for self-adjoint operators ensures the existence of a unitary transformation from the domain $D(H)$ in the Hilbert space $\mathcal{H}$ to $L_2(\mathbb{R}, d\rho)$. In the case in which $\mathcal{H} = L_2(\mathbb{R})$, this unitary transformation may be realized in terms of the eigenfunctions $\psi_1, \psi_2 \in L_2(\mathbb{R})$ and the hypergeometric functions $\psi_\lambda(x)$ which satisfy point to point

$$H\psi_\lambda = \lambda\psi_\lambda$$

for $\lambda > 0$, but do not belong to $L_2(\mathbb{R})$. Under the unitary map $f(x) \in L_2(\mathbb{R})$ can be expressed

$$f(x) = F_1\psi_1(x) + F_2\psi_2(x) + \int_0^\infty F(\lambda)\psi_\lambda(x)\,d\rho(\lambda),$$

where $F(\lambda)$ belongs to $L_2(\mathbb{R}^+, d\rho)$. The action of H in $\mathcal{H}$ corresponds to the multiplication by λ in $L_2(\mathbb{R}^+, d\rho)$:

$$Hf = \lambda_1 F_1\psi_1 + \lambda_2 F_2\psi_2 + \int_0^\infty \lambda F(\lambda)\psi_\lambda\,d\rho(\lambda).$$

We notice that for $f \in D(H)$, the third term on the right-hand member belongs to $L_2(\mathbb{R})$. ψ_λ are normalized in order to have $\|f\|_{\mathcal{H}}^2 = F_1^2 + F_2^2 + \int_0^\infty F^2(\lambda)\,d\rho(\lambda)$. ψ_1, ψ_2, ψ_λ are pairwise orthogonal with the internal product in $L_2(\mathbb{R})$.

If we denote $g(x) = \int_0^\infty F(\lambda)\psi_\lambda(x)\,d\rho(\lambda)$, $g \in L_2(\mathbb{R})$, it follows that

$$\int_{-\infty}^{+\infty} Hg\,dx = \int_{-\infty}^{+\infty}\left[\int_0^\infty \lambda F(\lambda)\psi_\lambda\,d\rho(\lambda)\right]dx,$$

its left-hand member is zero, hence

$$\int_{-\infty}^{+\infty}\left[\int_0^\infty \lambda F(\lambda)\psi_\lambda\,d\rho(\lambda)\right]dx = 0 \tag{29}$$

for any $F(\lambda)$ on the space $L_2(\mathbb{R}^+, d\rho)$.

We consider first $F(\lambda)$ to have support in the complement of a neighborhood of $\lambda = 0$. We may then consider $\lambda F(\lambda) = \chi(\lambda)$, where $\chi(\lambda)$ is the characteristic function with value one in the interval $(\hat{\lambda} - \epsilon, \hat{\lambda} + \epsilon)$ and 0 otherwise. We have

$$\int_{-\infty}^\infty \int_{\hat{\lambda}-\epsilon}^{\hat{\lambda}+\epsilon} \psi_\lambda\,d\rho(\lambda)\,dx = 0 \tag{30}$$

for any $\hat{\lambda}$ in the support of $F(\lambda)$.

It then follows, by decomposing $F(\lambda)$, that

$$\int_{-\infty}^{+\infty}\left[\int_0^\infty F(\lambda)\psi_\lambda\,d\rho(\lambda)\right]dx = 0. \tag{31}$$

Since the $F(\lambda)$ considered is dense in $L_2(\mathbb{R}^+, d\rho)$, we conclude that (31) is valid for any $F(\lambda) \in L_2(\mathbb{R}^+, d\rho)$.

We may always decompose $\xi \in L_2(\mathbb{R})$ as

$$\xi = F_1\psi_1 + F_2\psi_2 + \int_0^\infty F(\lambda)\psi_\lambda\,d\rho(\lambda)$$

and use (18) together with (31) to obtain

$$F_1 = 0. \tag{32}$$

We now consider the Rayleigh quotient of any ξ in the domain of H

$$R(\xi) \equiv \frac{\langle -\xi, H\xi \rangle}{\langle -\xi, \xi \rangle}.$$

$\langle\,,\,\rangle$ denotes the internal product in the $L_2(\mathbb{R})$ space.

The eigenvalues satisfy (using the min-max theorem)

$$\lambda_1 = \inf\{R(\xi) : \xi \in \text{domain}\,H\},$$

$$\lambda_2 = \inf\{R(\xi) : \xi \perp \text{span}(\psi_1)\}.$$

We thus get

$$\sum_i \langle \varphi_i, H\varphi_i \rangle \geq \lambda_2 \sum_i \langle \varphi_i, \varphi_i \rangle.$$

From (27), and using $\lambda_2 = -\frac{C}{4}$, we obtain

$$\delta_\xi^2 M \geq \frac{3}{4} C \sum_i \langle \varphi_i, \varphi_i \rangle$$

and introducing a parameter $0 \leq \beta \leq 1$

$$
\begin{aligned}
\delta_\xi^2 M &= (1-\beta)\delta_\varphi^2 M + \beta \delta_\varphi^2 M \\
&\geq (1-\beta) \sum_{i=0}^{\infty} \int_{-\infty}^{+\infty} \left(\varphi_i'^2 + C\varphi_i^2 \right) dx \\
&\quad + (1-\beta) \sum_{i=0}^{\infty} \int_{-\infty}^{+\infty} \left(-\frac{1}{2}\phi \right) \varphi_i^2 \, dx + \frac{3}{4}\beta C \sum_{i=0}^{\infty} \varphi_i^2.
\end{aligned}
$$

Consequently, using $\phi \leq 3C$, we have

$$\delta_\xi^2 M \geq (1-\beta) \min(1, C) \|\xi\|_{H_1}^2$$

for any β satisfying $\frac{2}{3} < \beta < 1$. In particular, for $\beta = \frac{3}{4}$,

$$\delta_\xi^2 M \geq \frac{1}{4} \min(1, C) \|\xi\|_{H_1}^2. \tag{33}$$

From (28) and (33), we get

$$\delta_h^2 M + \delta_\xi^2 M \geq \frac{1}{4} \min(1, C) \left\| (h, \xi) \right\|_{H_1}^2 - \frac{2}{5} C^{\frac{1}{4}} \left\| (h, \xi) \right\|_{H_1}^2, \tag{34}$$

and from (21),

$$\delta^3 M \geq -\frac{1}{2\sqrt{2}} \left\| (h, \xi) \right\|_{H_1}^2.$$

Finally,

$$\Delta M \geq \frac{1}{4}l\|(h,\xi)\|_{H_1}^2 - b\|(h,\xi)\|_{H_1}^3, \tag{35}$$

where

$$l = \min(1,\mathcal{C}), \qquad b = \frac{1}{2\sqrt{2}} + \frac{2}{5}\mathcal{C}^{\frac{1}{4}}.$$

We now use an argument in [5, 6] to show, for δ small enough, that

$$\Delta M \geq \frac{1}{6}l\|(h,\xi)\|_{H_1}^2.$$

Since

$$\|(h,\xi)\|_{H_1} \geq d_{II}[(u,\xi),(\phi,0)],$$

we then obtain

$$\Delta M \geq \frac{1}{6}l\{d_{II}[(u,\xi),(\phi,0)]\}^2$$

for δ small enough.

The stability proof has then been completed.

We may now relax the assumptions (17) and (18). The assumption (17) may be removed by an application of the triangle inequality as is [5]. While (18) may be relaxed by considering

$$\tilde{\xi} = \xi - F_1\psi_1,$$

which satisfies $\int_{-\infty}^{\infty} \tilde{\xi}\,dx = 0$.

Using again the triangle inequality for the distance d_{II}, we get

$$d_{II}[(u,\xi),(\phi,0)] \leq d_{II}[(u,\tilde{\xi}),(\phi,0)] + d_{II}[(u,\xi),(u,\tilde{\xi})],$$

where $d_{II}[(u,\xi),(u,\tilde{\xi})] = \|F_1\psi_1\|_{H_1}$ is conserved by $\hat{H}_{\frac{1}{2}}$ and bounded by $\|(u,\xi)\|_{H_1}$ at $t = 0$. The solitonic solution $(\phi,0)$ is then stable in the sense (13), (14).

4 Conclusions

Following [1], we considered the breaking of the supersymmetry in the $N = 1$ Super KdV system and analyzed a solitonic model in terms of a Clifford algebra-valued field. We showed the stability in the Liapunov extended sense of the ground state and the one-soliton solution of the integrable model. The approach introduced in [5] to prove the stability of the solitonic solutions of KdV equation and extended here for Clifford-valued fields may also be used in the stability analysis of supersymmetric solitons in the bosonization scheme in [14, 15].

Independently of the original motivation, the breaking of supersymmetry, the system we analyzed in this paper is interesting, because it contains the same soliton solutions as

the KdV equation but is more realistic in the sense that the symmetry associated to the infinite number of conserved charges of KdV is broken.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

Both authors contributed equally and read and approved the final manuscript.

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