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Global attractivity of a discrete Lotka-Volterra competition system with infinite delays and feedback controls

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Abstract

In this paper, we propose a discrete Lotka-Volterra competition system with infinite delays and feedback controls. Sufficient conditions which ensure the global attractivity of the system are obtained. An example together with its numerical simulation shows the feasibility of the main results.

Keywords: competition system; feedback control; discrete; global attractivity

1 Introduction

For the last decades, the ecological competition systems governed by differential equations of Lotka-Volterra type have been investigated extensively. Many interesting results concerned with the global existence and attractivity of periodic solution, persistence and extinction of the population, *etc.* have been obtained; we refer to [1–4] and the references therein. Already, many authors [5–21] have argued that the discrete time models governed by difference equations are more appropriate than the continuous ones when the populations have nonoverlapping generations. Particularly, the persistence, permanence, extinction, local and global stability and the existence of positive periodic solutions, *etc.*, for discrete competitive systems are studied in [5, 9, 12, 13, 15–18, 21]. Chen and Zhou [9] discussed the following discrete Lotka-Volterra competition system:

$$\begin{cases} x_1(n+1) = x_1(n)\exp[r_1(n)(1 - \frac{x_1(n)}{K_1(n)} - \mu_1(n)x_2(n))], \\ x_2(n+1) = x_2(n)\exp[r_2(n)(1 - \frac{x_2(n)}{K_2(n)} - \mu_2(n)x_1(n))]. \end{cases} \tag{1.1}$$

They obtained sufficient conditions which guarantee the persistence of system (1.1). Also, for the periodic case, they obtained sufficient conditions for the existence of a globally stable periodic solution.

Chen [12] studied the following nonautonomous two-species discrete competitive systems with deviating arguments:

$$\begin{cases} x_1(n+1) = x_1(n)\exp[r_1(n)(1 - \frac{x_1(n)}{K_1(n)} - \mu_1(n)\sum_{s=-\infty}^{n} H_1(n-s)x_2(s))], \\ x_2(n+1) = x_2(n)\exp[r_2(n)(1 - \frac{x_2(n)}{K_2(n)} - \mu_2(n)\sum_{s=-\infty}^{n} H_2(n-s)x_1(s))]. \end{cases} \tag{1.2}$$

They obtained sufficient conditions for the permanence of system (1.2).

On the other hand, feedback control is the basic mechanism by which systems, whether mechanical, electrical or biological, maintain their equilibrium or homeostasis. In the higher life forms, the conditions under which life can continue are quite narrow. A change in body temperature of half a degree is generally a sign of illness. The homeostasis of the body is maintained through the use of feedback control [22]. A primary contribution of C.R. Darwin during the last century was the theory that feedback over long time periods is responsible for the evolution of species. In 1931 Volterra [23] explained the balance between two populations of fish in a closed pond using the theory of feedback. Later, a series of mathematical models have been established to describe the dynamics of feedback control systems; see [14, 16–20, 23–25] and the references therein.

The purpose of this paper is to study the global attractivity of the following discrete Lotka-Volterra competition system with infinite delays and feedback controls:

$$\begin{cases} x_1(n+1) = x_1(n)\exp[r_1 - a_{11}x_1(n) - a_{12}\sum_{s=-\infty}^{n} H_1(n-s)x_2(s) \\ \qquad\qquad - c_1\sum_{s=-\infty}^{n} H_2(n-s)\mu_1(s)], \\ x_2(n+1) = x_2(n)\exp[r_2 - a_{22}x_2(n) - a_{21}\sum_{s=-\infty}^{n} H_3(n-s)x_1(s) \\ \qquad\qquad - c_2\sum_{s=-\infty}^{n} H_4(n-s)\mu_2(s)], \\ \mu_1(n+1) = (1-a_1)\mu_1(n) + b_1\sum_{s=-\infty}^{n} H_5(n-s)x_1(s), \\ \mu_2(n+1) = (1-a_2)\mu_2(n) + b_2\sum_{s=-\infty}^{n} H_6(n-s)x_2(s), \end{cases} \tag{1.3}$$

where $0 < a_i < 1$, $r_i, b_i, c_i, a_{ij} \in (0,\infty)$, $i = 1,2$, $j = 1,2$, $x_i(n)$ $(i = 1,2)$ are the density of the i species at time n and $\mu_i(n)$ $(i = 1,2)$ are the control variables at time n. $H_i(n)$ $(i = 1,2,\dots,6)$ are bounded nonnegative sequences such that $\sum_{n=0}^{\infty} H_i(n) = 1$.

By the biological meaning, we focus our discussion on the positive solutions of (1.3). So, it is assumed that the initial conditions of (1.3) are of the form

$$x_i(s) = \Phi_i(s) \geq 0, \quad \Phi_i(0) > 0, \qquad \mu_i(s) = \Psi_i(s) \geq 0, \quad \Psi_i(0) > 0, i = 1,2, \tag{1.4}$$

where $s = \dots,-n,-n+1,\dots,-1,0$. One can easily show that the solutions of (1.3) with (1.4) remain positive for all $n \in Z_+$, where $Z_+ = \{0,1,2,\dots\}$.

Further, assume

$$(r_1a_{21} - r_2d_1)(r_2a_{12} - r_1d_2) > 0, \tag{1.5}$$

where $d_1 = \frac{1}{a_1}(a_1a_{11} + b_1c_1)$, $d_2 = \frac{1}{a_2}(a_2a_{22} + b_2c_2)$. Then system (1.3) has a unique positive equilibrium $(x_1^*, x_2^*, \mu_1^*, \mu_2^*)$ with

$$x_1^* = \frac{r_2a_{12} - r_1d_2}{a_{12}a_{21} - d_1d_2}, \qquad x_2^* = \frac{r_1a_{21} - r_2d_1}{a_{12}a_{21} - d_1d_2}, \qquad \mu_1^* = \frac{a_1}{b_1}x_1^*, \qquad \mu_2^* = \frac{a_2}{b_2}x_2^*.$$

The aim of this paper is, by developing the analysis technique of Chen [12], Liao and Yu [14], Chen and Teng [15], to obtain a set of sufficient conditions for the global attractivity of system (1.3). The paper is organized as follows. In Section 2, as preliminaries, some useful lemmas are given. In Section 3, we study the global attractivity of positive equilibrium of system (1.3). In Section 4, the numerical simulations on the global attractivity of equilibrium are given.

2 Preliminaries

In this section, we introduce some auxiliary lemmas which will be useful in the following.

Lemma 1 (see [15]) *Let the function $f(u) = u\exp(\alpha - \beta u)$, where α and β are positive constants. Then $f(u)$ is nondecreasing on $u \in (0, \frac{1}{\beta}]$.*

Lemma 2 (see [15]) *Assume that the sequence $\{u(n)\}$ satisfies*

$$u(n+1) = u(n)\exp\bigl(\alpha - \beta u(n)\bigr), \quad n = 1, 2, \ldots,$$

where α and β are positive constants and $u(0) > 0$. We have

(i) *if $\alpha < 2$, then $\lim_{n\to\infty} u(n) = \frac{\alpha}{\beta}$.*
(ii) *if $\alpha \leq 1$, then $u(n) \leq \frac{1}{\beta}$ for all $n = 2, 3, \ldots$.*

Lemma 3 (see [8]) *Suppose that functions $f, g : Z_+ \times [0,\infty) \to [0,\infty)$ satisfy $f(n,x) \leq g(n,x)$ ($f(n,x) \geq g(n,x)$) for $n \in Z_+$ and $x \in [0,\infty)$ and $g(n,x)$ is nondecreasing with respect to $x > 0$. If sequences $\{x(n)\}$ and $\{u(n)\}$ are the nonnegative solutions of the following difference equations:*

$$x(n+1) = f\bigl(n, x(n)\bigr), \qquad u(n+1) = g\bigl(n, u(n)\bigr), \quad n = 0, 1, 2, \ldots,$$

respectively, and $x(0) \leq u(0)$ ($x(0) \geq u(0)$), then for all $n \geq 0$, we have

$$x(n) \leq u(n)\bigl(x(n) \geq u(n)\bigr).$$

Lemma 4 (see [12]) *Let $x : Z \to R$ be a nonnegative bounded sequence, and let $H : Z_+ \to R$ be a nonnegative sequence such that $\sum_{n=0}^{\infty} H(n) = 1$, where $Z = \{0, \pm1, \pm2, \ldots\}$, $R = (-\infty, \infty)$. Then*

$$\liminf_{n\to+\infty} x(n) \leq \liminf_{n\to+\infty} \sum_{s=-\infty}^{n} H(n-s)x(s) \leq \limsup_{n\to+\infty} \sum_{s=-\infty}^{n} H(n-s)x(s) \leq \limsup_{n\to+\infty} x(n).$$

We further consider the following discrete linear equation:

$$u(n+1) = (1-\gamma_1)u(n) + \gamma_2 \sum_{s=-\infty}^{n} H(n-s)x(s), \quad n \in Z_+, \tag{2.1}$$

where $0 < \gamma_1 < 1$, $\gamma_2 \in (0,\infty)$. $H(n)$ is a nonnegative sequence defined on Z_+ such that $\sum_{n=0}^{\infty} H(n) = 1$ and $x(n)$ is a nonnegative bounded sequence defined on Z with

$$x_* \leq \liminf_{n\to\infty} x(n) \leq \limsup_{n\to\infty} x(n) \leq x^*,$$

where x_*, x^* are nonnegative constants.

Lemma 5 *Any solution of system* (2.1) *with $u(0) > 0$ satisfies*

$$\frac{\gamma_2}{\gamma_1} x_* \leq \liminf_{n\to\infty} u(n) \leq \limsup_{n\to\infty} u(n) \leq \frac{\gamma_2}{\gamma_1} x^*.$$

Proof From Lemma 4,

$$\limsup_{n\to+\infty}\sum_{s=-\infty}^{n}H(n-s)x(s)\le\limsup_{n\to+\infty}x(n)\le x^*.$$

Hence, for each $\varepsilon>0$, there exists an enough large integer n_0 such that for $n\ge n_0$,

$$\sum_{s=-\infty}^{n}H(n-s)x(n)\le x^*+\varepsilon.$$

By system (2.1), we can obtain

$$\begin{aligned}u(n) &= u(n_0)(1-\gamma_1)^{n-n_0}+\gamma_2\sum_{i=n_0}^{n-1}\left[(1-\gamma_1)^{n-i-1}\sum_{s=-\infty}^{i}H(i-s)x(s)\right]\\ &\le u_0(1-\gamma_1)^{n-n_0}+\gamma_2\bigl(x^*+\varepsilon\bigr)\sum_{i=n_0}^{n-1}(1-\gamma_1)^{n-i-1}\\ &= u_0(1-\gamma_1)^{n-n_0}+\gamma_2\bigl(x^*+\varepsilon\bigr)\frac{1-(1-\gamma_1)^{n-n_0}}{\gamma_1}\to\frac{\gamma_2(x^*+\varepsilon)}{\gamma_1},\quad n\to\infty.\end{aligned}$$

Thus,

$$\limsup_{n\to\infty}u(n)\le\frac{\gamma_2(x^*+\varepsilon)}{\gamma_1}.$$

By the arbitrariness of ε, we can obtain

$$\limsup_{n\to\infty}u(n)\le\frac{\gamma_2}{\gamma_1}x^*.$$

We can prove $\liminf_{n\to\infty}u(n)\ge\frac{\gamma_2}{\gamma_1}x_*$ in a similar way. Thus, we complete the proof. □

Lemma 6 *Assume* $\lim_{n\to\infty}x(n)=\bar{x}$. *For every solution* $u(n)$ *of equation* (2.1), *we have*

$$\lim_{n\to\infty}u(n)=\frac{\gamma_2}{\gamma_1}\bar{x}.$$

By Lemma 5, the proof of Lemma 6 is obtained easily. Hence, we omit it here.

3 Global attractivity

In this section, we derive sufficient conditions which guarantee that the positive equilibrium of system (1.3) is globally attractive. The technique of proofs is to use an iteration scheme.

Theorem 1 *Assume*

$$(r_1a_{21}-r_2d_1)(r_2a_{12}-r_1d_2)>0$$

and

$$\frac{r_2a_{12}}{a_{22}}+\frac{b_1c_1r_1}{a_1a_{11}}<r_1\le1,\qquad\frac{r_1a_{21}}{a_{11}}+\frac{b_2c_2r_2}{a_2a_{22}}<r_2\le1.$$

Then equilibrium $(x_1^*,x_2^*,\mu_1^*,\mu_2^*)$ *of system* (1.3) *with* (1.4) *is globally attractive.*

Proof Let $(x_1(n), x_2(n), \mu_1(n), \mu_2(n))$ be any solution of system (1.3) with (1.4). Denote

$$U_i = \limsup_{n\to\infty} x_i(n), \qquad V_i = \liminf_{n\to\infty} x_i(n), \quad i = 1, 2,$$

and

$$P_i = \limsup_{n\to\infty} \mu_i(n), \qquad Q_i = \liminf_{n\to\infty} \mu_i(n), \quad i = 1, 2.$$

We now claim that $U_i = V_i = x_i^*$, $P_i = Q_i = \mu_i^*$, $i = 1, 2$.

From the first equation of system (1.3), we obtain

$$x_1(n+1) \le x_1(n) \exp\bigl[r_1 - a_{11}x_1(n)\bigr], \quad n = 0, 1, 2, \ldots.$$

Consider the auxiliary equation

$$p(n+1) = p(n) \exp\bigl[r_1 - a_{11}p(n)\bigr]. \tag{3.1}$$

From $r_1 \le 1$, by the conclusion (ii) of Lemma 2, we have that $p(n) \le \frac{1}{a_{11}}$ for all $n \ge 2$, where $p(n)$ is any solution of equation (3.1) with initial value $p(0) > 0$. From Lemma 1, we have $f(p) = p\exp(r_1 - a_{11}p)$ is nondecreasing for $p \in (0, \frac{1}{a_{11}}]$.

Hence, from Lemma 3, we obtain $x_1(n) \le p(n)$ for all $n \ge 2$, where $p(n)$ is the solution of equation (3.1) with $p(2) = x_1(2)$. Further, combining it with the conclusion (i) of Lemma 2, we obtain

$$U_1 = \limsup_{n\to\infty} x_1(n) \le \lim_{n\to\infty} p(n) = \frac{r_1}{a_{11}} := M_1^1.$$

From the second equation of system (1.3), we obtain

$$x_2(n+1) \le x_2(n) \exp\bigl[r_2 - a_{22}x_2(n)\bigr], \quad n = 0, 1, 2, \ldots.$$

By a similar argument as that above, we have

$$U_2 = \limsup_{n\to\infty} x_2(n) \le \frac{r_2}{a_{22}} := M_1^2.$$

By Lemma 4 and Lemma 5, we obtain

$$P_i = \limsup_{n\to\infty} \mu_i(n) \le \frac{b_i}{a_i} M_1^i, \quad i = 1, 2.$$

Then, for any constant $\varepsilon > 0$ sufficiently small, there is an integer $n_1 > 2$ such that if $n \ge n_1$, then

$$x_i(n) \le M_1^i + \varepsilon, \qquad \mu_i(n) \le \frac{b_i}{a_i} M_1^i + \varepsilon, \quad i = 1, 2.$$

Further, from Lemma 4 and the first equation of system (1.3), we have

$$x_1(n+1) \ge x_1(n) \exp\left[r_1 - a_{11}x_1(n) - a_{12}\bigl(M_1^2 + \varepsilon\bigr) - c_1\left(\frac{b_1}{a_1} M_1^1 + \varepsilon\right)\right], \quad n \ge n_1.$$

Consider the auxiliary equation

$$p(n+1) = p(n)\exp\left[r_1 - a_{11}p(n) - a_{12}\left(M_1^2 + \varepsilon\right) - c_1\left(\frac{b_1}{a_1}M_1^1 + \varepsilon\right)\right], \quad n \geq n_1. \tag{3.2}$$

From $\frac{r_2 a_{12}}{a_{22}} + \frac{b_1 c_1 r_1}{a_1 a_{11}} < r_1 \leq 1$ and the arbitrariness of $\varepsilon > 0$, we have

$$0 < r_1 - a_{12}\left(M_1^2 + \varepsilon\right) - c_1\left(\frac{b_1}{a_1}M_1^1 + \varepsilon\right) < 1.$$

By the conclusion (ii) of Lemma 2, we have that $p(n) \leq \frac{1}{a_{11}}$ for all $n \geq n_1$, where $p(n)$ is any solution of equation (3.2) with initial value $p(n_1) > 0$. From Lemma 1, we have

$$f(p) = \exp\left[r_1 - a_{11}p - a_{12}\left(M_1^2 + \varepsilon\right) - c_1\left(\frac{b_1}{a_1}M_1^1 + \varepsilon\right)\right]$$

is nondecreasing for $p \in (0, \frac{1}{a_{11}}]$.

Hence, from Lemma 3, we have $x_1(n) \geq p(n)$ for all $n \geq n_1$, where $p(n)$ is the solution of equation (3.2) with $p(n_1) = x_1(n_1)$. Combining it with the conclusion (i) of Lemma 2, we obtain

$$V_1 = \liminf_{n\to\infty} x_1(n) \geq \lim_{n\to\infty} p(n) = \frac{1}{a_{11}}\left[r_1 - a_{12}\left(M_1^2 + \varepsilon\right) - c_1\left(\frac{b_1}{a_1}M_1^1 + \varepsilon\right)\right].$$

From the arbitrariness of $\varepsilon > 0$, we conclude $V_1 \geq m_1^1$, where

$$m_1^1 = \frac{1}{a_{11}}\left[r_1 - a_{12}M_1^2 - c_1\frac{b_1}{a_1}M_1^1\right].$$

From Lemma 4 and the second equation of system (1.3), we further have

$$x_2(n+1) \geq x_2(n)\exp\left[r_2 - a_{22}x_2(n) - a_{21}\left(M_1^1 + \varepsilon\right) - c_2\left(\frac{b_2}{a_2}M_1^2 + \varepsilon\right)\right], \quad n \geq n_1.$$

By a similar argument as that above, we can obtain

$$V_2 = \liminf_{n\to\infty} x_2(n) \geq \frac{1}{a_{22}}\left[r_2 - a_{21}M_1^2 - c_1\frac{b_1}{a_1}M_1^1\right] := m_1^2.$$

By Lemma 4 and Lemma 5, we further obtain

$$Q_i = \liminf_{n\to\infty} \mu_i(n) \geq \frac{b_i}{a_i}m_1^i, \quad i = 1, 2.$$

Hence, for $\varepsilon > 0$ sufficiently small, there is an $n_2 > n_1$ such that if $n \geq n_2$, then

$$x_i(n) \geq m_1^i - \varepsilon, \qquad \mu_i(n) \geq \frac{b_i}{a_i}m_1^i - \varepsilon, \quad i = 1, 2.$$

From Lemma 4 and the first equation of system (1.3), we further have

$$x_1(n+1) \leq x_1(n)\exp\left[r_1 - a_{11}x_1(n) - a_{12}\left(m_1^2 - \varepsilon\right) - c_1\left(\frac{b_1}{a_1}m_1^1 - \varepsilon\right)\right], \quad n \geq n_2.$$

Consider the auxiliary equation

$$p(n+1) \le p(n)\exp\left[r_1 - a_{11}p(n) - a_{12}\left(m_1^2 - \varepsilon\right) - c_1\left(\frac{b_1}{a_1}m_1^1 - \varepsilon\right)\right], \quad n \ge n_2.$$

From $\frac{r_2 a_{12}}{a_{22}} + \frac{b_1 c_1 r_1}{a_1 a_{11}} < r_1 \le 1$ and the arbitrariness of $\varepsilon > 0$, we have

$$0 < r_1 - a_{12}\left(m_1^2 - \varepsilon\right) - c_1\left(\frac{b_1}{a_1}m_1^1 - \varepsilon\right) < 1.$$

Similarly to the above discussion, we can obtain

$$U_1 = \liminf_{n\to\infty} x_1(n) \le \lim_{n\to\infty} p(n) = \frac{1}{a_{11}}\left[r_1 - a_{12}\left(m_1^2 - \varepsilon\right) - c_1\left(\frac{b_1}{a_1}m_1^1 - \varepsilon\right)\right].$$

From the arbitrariness of $\varepsilon > 0$, we conclude $U_1 \le M_2^1$, where

$$M_2^1 = \frac{1}{a_{11}}\left[r_1 - a_{12}m_1^2 - c_1\frac{b_1}{a_1}m_1^1\right].$$

From Lemma 4 and the second equation of system (1.3), we further have

$$x_2(n+1) \le x_2(n)\exp\left[r_2 - a_{22}x_2(n) - a_{21}\left(m_1^1 - \varepsilon\right) - c_2\left(\frac{b_2}{a_2}m_1^2 - \varepsilon\right)\right], \quad n \ge n_2.$$

By a similar argument as that above, we can obtain

$$U_2 = \liminf_{n\to\infty} x_2(n) \le \frac{1}{a_{22}}\left[r_2 - a_{21}m_1^1 - c_2\frac{b_2}{a_2}m_1^2\right] := M_2^2.$$

By Lemma 4 and Lemma 5, we obtain

$$P_i = \limsup_{n\to\infty} \mu_i(n) \le \frac{b_i}{a_i}M_2^i, \quad i = 1, 2.$$

Hence, for $\varepsilon > 0$ sufficiently small, there is an n_3 such that if $n \ge n_3$,

$$x_i(n) \le M_2^i + \varepsilon, \qquad \mu_i(n) \le \frac{b_i}{a_i}M_2^i + \varepsilon, \quad i = 1, 2.$$

From Lemma 4 and the first equation of system (1.3), we have

$$x_1(n+1) \ge x_1(n)\exp\left[r_1 - a_{11}x_1(n) - a_{12}\left(M_2^2 + \varepsilon\right) - c_1\left(\frac{b_1}{a_1}M_2^1 + \varepsilon\right)\right], \quad n \ge n_3.$$

Consider the auxiliary equation

$$p(n+1) = p(n)\exp\left[r_1 - a_{11}p(n) - a_{12}\left(M_2^2 + \varepsilon\right) - c_1\left(\frac{b_1}{a_1}M_2^1 + \varepsilon\right)\right], \quad n \ge n_3.$$

Since

$$0 < r_1 - a_{12}\left(M_2^2 + \varepsilon\right) - c_1\left(\frac{b_1}{a_1}M_2^1 + \varepsilon\right) < 1,$$

similarly to the above discussion, we obtain

$$V_1 = \liminf_{n\to\infty} x_1(n) \geq \lim_{n\to\infty} p(n) = \frac{1}{a_{11}}\left[r_1 - a_{12}\left(M_2^2 + \varepsilon\right) - c_1\left(\frac{b_1}{a_1}M_2^1 + \varepsilon\right)\right].$$

From the arbitrariness of $\varepsilon > 0$, we conclude $V_1 \geq m_2^1$, where

$$m_2^1 = \frac{1}{a_{11}}\left[r_1 - a_{12}M_2^2 - c_1\frac{b_1}{a_1}M_2^1\right].$$

From Lemma 4 and the second equation of system (1.3), we further have

$$x_2(n+1) \geq x_2(n)\exp\left[r_2 - a_{22}x_2(n) - a_{21}\left(M_2^1 + \varepsilon\right) - c_2\left(\frac{b_2}{a_2}M_2^2 + \varepsilon\right)\right], \quad n \geq n_3.$$

By a similar argument as that above, we can obtain

$$V_2 = \liminf_{n\to\infty} x_2(n) \geq \frac{1}{a_{22}}\left[r_2 - a_{21}M_2^1 - c_2\frac{b_2}{a_2}M_2^2\right] := m_2^2.$$

Continuing the above process, we can obtain four sequences $\{M_n^i\}$, $\{m_n^i\}$, $i = 1,2$ such that

$$M_{n+1}^1 = \frac{1}{a_{11}}\left[r_1 - a_{12}m_n^2 - c_1\frac{b_1}{a_1}m_n^1\right], \qquad M_{n+1}^2 = \frac{1}{a_{22}}\left[r_2 - a_{21}m_n^1 - c_2\frac{b_2}{a_2}m_n^2\right] \tag{3.3}$$

and

$$m_n^1 = \frac{1}{a_{11}}\left[r_1 - a_{12}M_n^2 - c_1\frac{b_1}{a_1}M_n^1\right], \qquad m_n^2 = \frac{1}{a_{22}}\left[r_2 - a_{21}M_n^1 - c_2\frac{b_2}{a_2}M_n^2\right]. \tag{3.4}$$

Clearly, we have

$$m_n^i \leq V_i \leq U_i \leq M_n^i, \quad i = 1,2.$$

Now, by means of the inductive method, we prove $\{M_n^i\}$ is monotonically decreasing, $\{m_n^i\}$ is monotonically increasing, $i = 1,2$.

Firstly, it is clear that $M_2^i \leq M_1^i$, $m_2^i \geq m_1^i$, $i = 1,2$. For $n = k$ ($k \geq 2$), we assume $M_k^i \leq M_{k-1}^i$ and $m_k^i \geq m_{k-1}^i$, $i = 1,2$, then we have

$$M_{k+1}^1 = \frac{1}{a_{11}}\left[r_1 - a_{12}m_k^2 - c_1\frac{b_1}{a_1}m_k^1\right] \leq \frac{1}{a_{11}}\left[r_1 - a_{12}m_{k-1}^2 - c_1\frac{b_1}{a_1}m_{k-1}^1\right] = M_k^1,$$

$$M_{k+1}^2 = \frac{1}{a_{22}}\left[r_2 - a_{21}m_k^1 - c_2\frac{b_2}{a_2}m_k^2\right] \leq \frac{1}{a_{22}}\left[r_2 - a_{21}m_{k-1}^1 - c_2\frac{b_2}{a_2}m_{k-1}^2\right] = M_k^2$$

and

$$m_{k+1}^1 = \frac{1}{a_{11}}\left[r_1 - a_{12}M_{k+1}^2 - c_1\frac{b_1}{a_1}M_{k+1}^1\right] \geq \frac{1}{a_{11}}\left[r_1 - a_{12}M_k^2 - c_1\frac{b_1}{a_1}M_k^1\right] = m_k^1,$$

$$m_{k+1}^2 = \frac{1}{a_{22}}\left[r_2 - a_{21}M_{k+1}^1 - c_2\frac{b_2}{a_2}M_{k+1}^2\right] \geq \frac{1}{a_{22}}\left[r_2 - a_{21}M_k^1 - c_2\frac{b_2}{a_2}M_k^2\right] = m_k^2.$$

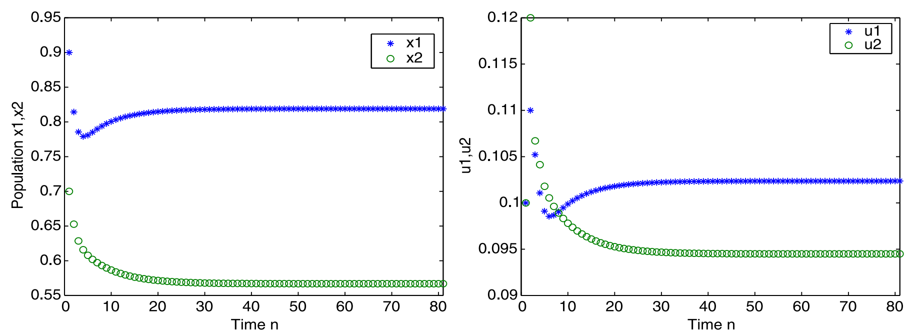

Figure 1 Dynamical behaviors of system (1.3) with $r_1 = 0.6$, $r_2 = 0.4$, $a_{12} = 0.3$, $a_{21} = 0.2$, $a_{11} = 0.5$, $a_{22} = 0.4$, $a_1 = 0.8$, $a_2 = 1.2$, $b_1 = 0.1$, $b_2 = 0.2$, $c_1 = 0.2$, $c_2 = 0.1$, $H_1(n) = H_4(n) = \frac{e-1}{e}e^{-n}$, $H_2(n) = H_3(n) = \frac{e^2-1}{e^2}e^{-2n}$, $H_5(n) = H_6(n) = \frac{e^4-1}{e^4}e^{-4n}$.

Therefore, $\{M_n^i\}$ is monotonically decreasing, $\{m_n^i\}$ is monotonically increasing, $i = 1, 2$. Consequently, $\lim_{n\to\infty} M_n^i$ and $\lim_{n\to\infty} m_n^i$ both exist, $i = 1, 2$. Let

$$\lim_{n\to\infty} M_n^i = \bar{x}_i, \qquad \lim_{n\to\infty} m_n^i = \bar{y}_i, \quad i = 1, 2.$$

From (3.3) and (3.4), we obtain

$$\begin{cases} a_{11}\bar{x}_1 + a_{12}\bar{y}_2 + \frac{b_1c_1}{a_1}\bar{y}_1 = r_1, \\ a_{22}\bar{x}_2 + a_{21}\bar{y}_1 + \frac{b_2c_2}{a_2}\bar{y}_2 = r_2, \\ a_{11}\bar{y}_1 + a_{12}\bar{x}_2 + \frac{b_1c_1}{a_1}\bar{x}_1 = r_1, \\ a_{22}\bar{y}_2 + a_{21}\bar{x}_1 + \frac{b_2c_2}{a_2}\bar{x}_2 = r_2. \end{cases} \tag{3.5}$$

It is clear that $(x_1^*, x_2^*, \mu_1^*, \mu_2^*)$ is a unique solution of equations (3.5). Therefore,

$$U_i = V_i = \lim_{n\to\infty} x_i(n) = x_i^*, \quad i = 1, 2.$$

Further, by Lemma 6, we can obtain $\lim_{n\to\infty} \mu_i(n) = \mu_i^*$, $i = 1, 2$. Thus, we complete the proof of Theorem 1. □

4 Example

The following example shows the feasibility of the main results.

Example 1 Choose $r_1 = 0.6$, $r_2 = 0.4$, $a_{12} = 0.3$, $a_{21} = 0.2$, $a_{11} = 0.5$, $a_{22} = 0.4$, $a_1 = 0.8$, $a_2 = 1.2$, $b_1 = 0.1$, $b_2 = 0.2$, $c_1 = 0.2$, $c_2 = 0.1$, $H_1(n) = H_4(n) = \frac{e-1}{e}e^{-n}$, $H_2(n) = H_3(n) = \frac{e^2-1}{e^2}e^{-2n}$, $H_5(n) = H_6(n) = \frac{e^4-1}{e^4}e^{-4n}$ in system (1.3). By calculating, we have that positive equilibrium $(0.8189, 0.5669, 0.1024, 0.0945)$, $(r_1a_{21} - r_2d_1)(r_2a_{12} - r_1d_2) = 0.0117$, $r_1 - (\frac{r_2a_{12}}{a_{22}} + \frac{b_1c_1r_1}{a_1a_{11}}) = 0.27$, $r_2 - (\frac{r_1a_{21}}{a_{11}} + \frac{b_2c_2r_2}{a_2a_{22}}) = 0.1433$. Since

$$(r_1a_{21} - r_2d_1)(r_2a_{12} - r_1d_2) > 0, \qquad \left(\frac{r_2a_{12}}{a_{22}} + \frac{b_1c_1r_1}{a_1a_{11}}\right) < r_1 \le 1,$$
$$\left(\frac{r_1a_{21}}{a_{11}} + \frac{b_2c_2r_2}{a_2a_{22}}\right) < r_2 \le 1,$$

the conditions of Theorem 1 hold. So, equilibrium $(0.8189, 0.5669, 0.1024, 0.0945)$ is globally attractive.

Choose initial values $(x_1(s), x_2(s), \mu_1(s), \mu_2(s)) = (0.9, 0.7, 0.2, 0.1)$, $s = \dots, -n, -n+1, \dots, -1, 0$.

By the numerical simulation (see Figure 1), we find that the solution $(x_1(n), x_2(n), \mu_1(n), \mu_2(n))$ turns to equilibrium $(0.8189, 0.5669, 0.1024, 0.0945)$ as $n \to \infty$.

Competing interests
The author declares that he has no competing interests.

Author's contributions
The author carried out the proof of the theorem and approved the final manuscript.

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References
1. Ahmad, S: On the nonautonomous Lotka-Volterra competition equation. Proc. Am. Math. Soc. **117**, 199-204 (1993)
2. Ahmad, S, Lazer, AC: Average growth and extinction in a Lotka-Volterra system. Nonlinear Anal. **62**, 545-557 (2005)
3. Ahmad, S, Lazer, AC: Average conditions for global asymptotic stability in a nonautonomous Lotka-Volterra system. Nonlinear Anal. **40**, 37-49 (2000)
4. Tang, X, Zou, X: On positive periodic solutions of Lotka-Volterra competition systems with deviating arguments. Proc. Am. Math. Soc. **134**, 2967-2974 (2006)
5. Liao, X, Zhou, S, Chen, Y: On permanence and global stability in a general Gilpin-Ayala competition predator-prey discrete system. Appl. Math. Comput. **190**, 500-509 (2007)
6. Zhou, Z, Zou, X: Stable periodic solutions in a discrete periodic logistic equation. Appl. Math. Lett. **16**(2), 165-171 (2003)
7. Liu, X: A note on the existence of periodic solution in discrete predator-prey models. Appl. Math. Model. **34**, 2477-2483 (2010)
8. Wang, L, Wang, M: Ordinary Difference Equation. Xinjing Univ. Press, Xinjiang (1989)
9. Chen, YM, Zhou, Z: Stable periodic solution of a discrete periodic Lotka-Volterra competition system. J. Math. Appl. **277**(1), 358-366 (2003)
10. Yang, XT: Uniform persistence and periodic solutions for a discrete predator-prey system with delays. J. Math. Anal. Appl. **316**(1), 161-177 (2006)
11. Wang, K: Permanence and global asymptotical stability of a predator-prey model with mutual interference. Nonlinear Anal., Real World Appl. **12**, 1062-1071 (2011)
12. Chen, FD: Permanence in a discrete Lotka-Volterra competition model with deviating arguments. Nonlinear Anal., Real World Appl. **9**, 2150-2155 (2008)
13. Xiu, JB, Teng, ZD, Jiang, HJ: Permanence and global attractivity for discrete nonautonomous two-species Lotka-Volterra competitive system with delays and feedback controls. Period. Math. Hung. **63**(1), 19-45 (2011)
14. Liao, LS, Yu, JS, Wang, L: Global attractivity in a logistic difference model with a feedback control. Comput. Math. Appl. **44**, 1403-1411 (2002)
15. Chen, GY, Teng, ZD: On the stability in a discrete two-species competition system. J. Appl. Math. Comput. **38**, 25-36 (2012)
16. Kong, XZ, Chen, LP, Yang, WS: Note on the persistent property of a discrete Lotka-Volterra competition system with delays and feedback controls. Adv. Differ. Equ. **2**, 9 (2010)
17. Liao, X, Zhou, S, Chen, Y: Permanence and global stability in a discrete *n*-species competition system with feedback controls. Nonlinear Anal., Real World Appl. **9**, 1661-1671 (2008)
18. Liao, X, Ouyang, Z, Zhou, S: Permanence of species in nonautonomous discrete Lotka-Volterra competition system with delays and feedback controls. J. Comput. Appl. Math. **211**, 1-10 (2008)
19. Li, YK, Zhang, TW: Permanence and almost periodic sequence solution for a discrete delay logistic equation with feedback control. Nonlinear Anal., Real World Appl. **12**, 1850-1864 (2011)
20. Li, QY, Liu, HW, Zhang, FQ: The permanence and extinction of a discrete predator-prey system with time delay and feedback controls. Adv. Differ. Equ. **2**, 20 (2010)
21. Yu, ZX, Li, Z: Permanence and global attractivity of a discrete two-prey one-predator model with infinite delay. Discrete Dyn. Nat. Soc. (2009). doi:10.1155/2009/732510
22. Wiener, N: Cybernetics or Control and Communication in the Animal and the Machine. MIT Press, Cambridge (1948)
23. Gopalsamy, K: Stability and Oscillations in Delay Differential Equations of Population Dynamics. Kluwer Academic, Boston (1992)
24. Gopalsamy, K, Weng, PX: Feedback regulation of logistic growth. Int. J. Math. Math. Sci. **16**, 177-192 (1993)
25. Liao, LS: Feedback regulation of a logistic growth variable coefficients. J. Math. Appl. **259**(2), 489-500 (2001)

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