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Coupling Ishikawa algorithms with hybrid techniques for pseudocontractive mappings

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Abstract

It is well-known that Mann's algorithm fails to converge for Lipschitzian pseudocontractions and strong convergence of Ishikawa's algorithm for Lipschitzian pseudocontractions have not been achieved without compactness assumption on pseudocontractive mapping T or underlying space C. A new algorithm, which couples Ishikawa algorithms with hybrid techniques for finding the fixed points of a Lipschitzian pseudocontractive mapping, is constructed in this paper. Strong convergence of the presented algorithm is shown without any compactness assumption.

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Keywords: pseudocontractive mappings; Ishikawa algorithm; hybrid algorithms; fixed point; strong convergence

1 Introduction

In the present article, we are devoted to finding the fixed points of pseudocontractive mappings. Interest in pseudocontractive mappings stems mainly from their firm connection with the class of nonlinear accretive operators. It is a classical result, see Deimling [1], that if T is an accretive operator, then the solutions of the equations $Tx = 0$ correspond to the equilibrium points of some evolution systems. This explains why a considerable research effort has been devoted to iterative methods for approximating solutions of the equation above, when T is accretive or corresponding to the iterative approximation of fixed points of pseudocontractions. Results of this kind have been obtained firstly in Hilbert spaces, but only for Lipschitz operators, and then they have been extended to more general Banach spaces (thanks to several geometric inequalities for general Banach spaces developed) and to more general classes of operators. There are still no results for the case of arbitrary Lipschitzian and pseudocontractive operators, even when the domain of the operator is a compact and convex subset of a Hilbert space. It is now well known that Mann's algorithm [2] fails to converge for Lipschitzian pseudocontractions. This explains the importance, from this point of view, of the improvement brought by the Ishikawa iteration, which was introduced by Ishikawa [3] in 1974.

The original result of Ishikawa involves a Lipschitzian pseudocontractive self-mapping T on a convex and compact subset C of a Hilbert space. It establishes sufficient conditions such that Ishikawa iteration converges strongly to a fixed point of T.

However, a strong convergence has not been achieved without a compactness assumption on T or C. Consequently, considerable research efforts, especially within the past 40 years or so, have been devoted to iterative methods for approximating fixed points of T, when T is pseudocontractive (see, for example, [4–17] and the references therein). On the other hand, some convergence results are obtained by using the hybrid method in mathematical programming, see, for example, [14, 18–20]. Especially, Zegeye *et al.* [21] assumed that the interior of $\mathrm{Fix}(T)$ is nonempty ($\mathrm{int}\,\mathrm{Fix}(T) \neq \emptyset$) to achieve a strong convergence, when T is a self-mapping of a nonempty closed convex subset of a real Hilbert space. This appears very restrictive, since even in $\mathbb{R}$ with the usual norm, Lipschitz pseudocontractive maps with finite number of fixed points do not enjoy this condition that $\mathrm{int}\,\mathrm{Fix}(T) \neq \emptyset$.

The purpose of this article is to construct a new algorithm, which couples Ishikawa algorithms with hybrid techniques for finding the fixed points of a Lipschitzian pseudocontractive mapping. Strong convergence of the presented algorithm is given without any compactness assumption.

2 Preliminaries

Let H be a real Hilbert space with the inner product $\langle \cdot, \cdot \rangle$ and the norm $\| \cdot \|$, respectively. Let C be a nonempty closed convex subset of H. Recall that a mapping $T : C \to C$ is called pseudocontractive (or a pseudocontraction) if

$$\langle Tx - Ty, x - y \rangle \leq \|x - y\|^2$$

for all $x, y \in C$.

It is easily seen that T is pseudocontractive if and only if T satisfies the condition

$$\|Tx - Ty\|^2 \leq \|x - y\|^2 + \big\|(I - T)x - (I - T)y\big\|^2 \tag{2.1}$$

for all $x, y \in C$.

A mapping $T : C \to C$ is called L-Lipschitzian if there exists $L > 0$ such that

$$\|Tx - Ty\| \leq L\|x - y\|$$

for all $x, y \in C$.

We will use $\mathrm{Fix}(T)$ to denote the set of fixed points of T, that is,

$$\mathrm{Fix}(T) = \{x \in C : x = Tx\}.$$

The original result of Ishikawa is stated in the following.

Theorem 2.1 *Let C be a convex and compact subset of a Hilbert space H, and let $T : C \to C$ be a Lipschitzian pseudocontractive mapping. Given $x_1 \in C$, then the Ishikawa iteration $\{x_n\}$ defined by*

$$\begin{cases} y_n = (1 - \beta_n)x_n + \beta_n T x_n, \\ x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T y_n, \end{cases} \tag{2.2}$$

for all $n \in \mathbb{N}$, where $\{\alpha_n\}$, $\{\beta_n\}$ are sequences of positive numbers satisfying

(a) $0 \leq \alpha_n \leq \beta_n \leq 1$,

(b) $\lim_{n\to\infty}\beta_n = 0$,
(c) $\sum_{n=1}^{\infty}\alpha_n\beta_n = \infty$,
converges strongly to a fixed point of T.

To make our exposition self-contained, we have to recall that the (nearest point or metric) projection from H onto C, denoted by P_C, assigns to each $x \in H$ the unique point $P_C(x) \in C$ with the property

$$\|x - P_C(x)\| = \inf\{\|x - y\| : y \in C\}.$$

It is well known that the metric projection P_C of H onto C is characterized by

$$\langle x - P_C(x), y - P_C(x)\rangle \le 0 \tag{2.3}$$

for all $x \in H$, $y \in C$. Also, it is well known that in a real Hilbert space H, the following equality holds

$$\|tx + (1-t)y\|^2 = t\|x\|^2 + (1-t)\|y\|^2 - t(1-t)\|x-y\|^2 \tag{2.4}$$

for all $x, y \in H$ and $t \in [0,1]$.

Lemma 2.1 [7] *Let H be a real Hilbert space, let C be a closed convex subset of H. Let $T: C \to C$ be a continuous pseudocontractive mapping. Then*
(i) $\mathrm{Fix}(T)$ *is a closed convex subset of* C.
(ii) $(I - T)$ *is demiclosed at zero.*

In the sequel, we shall use the following notations:
- $\omega_w(x_n) = \{x : \exists x_{n_j} \to x \text{ weakly}\}$ denote the weak ω-limit set of $\{x_n\}$;
- $x_n \rightharpoonup x$ stands for the weak convergence of $\{x_n\}$ to x;
- $x_n \to x$ stands for the strong convergence of $\{x_n\}$ to x.

Lemma 2.2 [18] *Let C be a closed convex subset of H. Let $\{x_n\}$ be a sequence in H, and let $u \in H$. Let $q = P_C u$. If $\{x_n\}$ is such that $\omega_w(x_n) \subset C$ and satisfies the condition*

$$\|x_n - u\| \le \|u - q\| \quad \textit{for all } n \in \mathbb{N},$$

then $x_n \to q$.

3 Main results

In this section, we state our main results.

Let C be a nonempty, closed and convex subset of a real Hilbert space H. Let $T: C \to C$ be an L-Lipschitzian pseudocontractive mapping with $\mathrm{Fix}(T) \neq \emptyset$.

Firstly, we present our new algorithm, which couples Ishikawa's algorithm (2.2) with the hybrid projection algorithm.

Algorithm 3.1 Let $x_0 \in H$. For $C_1 = C$ and $x_1 = P_{C_1}(x_0)$, define a sequence $\{x_n\}$ of C as follows:

$$\begin{cases} y_n = (1-\alpha_n)x_n + \alpha_n Tx_n, \\ z_n = \beta_n x_n + (1-\beta_n)Ty_n, \\ C_{n+1} = \{x^* \in C_n, \|z_n - x^*\| \le \|x_n - x^*\|\}, \\ x_{n+1} = P_{C_{n+1}}(x_0), \end{cases} \tag{3.1}$$

for all $n \ge 1$, where $\{\alpha_n\}$ and $\{\beta_n\}$ are two sequences in $[0,1]$.

In the sequel, we assume that the sequences $\{\alpha_n\}$ and $\{\beta_n\}$ satisfy the following conditions

$$0 < k \le 1 - \beta_n \le \alpha_n < \frac{1}{\sqrt{1+L^2}+1}$$

for all $n \in \mathbb{N}$.

Remark 3.1 Without loss of generality, we can assume that the Lipschitz constant $L > 1$. If not, then T is nonexpansive. In this case, algorithm (3.1) is trivial. So, in this article, we assume $L > 1$. It is obvious that $\frac{1}{\sqrt{1+L^2}+1} < \frac{1}{L}$ for all $n \ge 1$.

We prove the following several lemmas, which will support our main theorem below.

Lemma 3.1 $\mathrm{Fix}(T) \subset C_n$ *for* $n \ge 1$ *and* $\{x_n\}$ *is well defined.*

Proof We use mathematical induction to prove $\mathrm{Fix}(T) \subset C_n$ for all $n \in \mathbb{N}$.

(i) $\mathrm{Fix}(T) \subset C_1 = C$ is obvious.

(ii) Suppose that $\mathrm{Fix}(T) \subset C_k$ for some $k \in \mathbb{N}$. Take $u \in \mathrm{Fix}(T) \subset C_k$. From (3.1), by using (2.4), we have

$$\begin{aligned} \|z_n - u\|^2 &= \big\|\beta_n(x_n - u) + (1-\beta_n)\big(T\big((1-\alpha_n)x_n + \alpha_n Tx_n\big) - u\big)\big\|^2 \\ &= \beta_n\|x_n - u\|^2 + (1-\beta_n)\big\|T\big((1-\alpha_n)x_n + \alpha_n Tx_n\big) - u\big\|^2 \\ &\quad - \beta_n(1-\beta_n)\big\|x_n - T\big((1-\alpha_n)x_n + \alpha_n Tx_n\big)\big\|^2. \end{aligned} \tag{3.2}$$

Since $u \in \mathrm{Fix}(T)$, from (2.1), we have

$$\|Tx - u\|^2 \le \|x - u\|^2 + \|x - Tx\|^2 \tag{3.3}$$

for all $x \in C$.

From (2.4) and (3.3), we obtain

$$\begin{aligned} &\big\|T\big((1-\alpha_n)x_n + \alpha_n Tx_n\big) - u\big\|^2 \\ &\quad \le \big\|(1-\alpha_n)x_n + \alpha_n Tx_n - T\big((1-\alpha_n)x_n + \alpha_n Tx_n\big)\big\|^2 + \big\|(1-\alpha_n)x_n + \alpha_n Tx_n - u\big\|^2 \\ &\quad = \big\|(1-\alpha_n)\big(x_n - T\big((1-\alpha_n)x_n + \alpha_n Tx_n\big)\big) + \alpha_n\big(Tx_n - T\big((1-\alpha_n)x_n + \alpha_n Tx_n\big)\big)\big\|^2 \end{aligned}$$

$$
\begin{aligned}
&+ \big\|(1-\alpha_n)(x_n-u)+\alpha_n(Tx_n-u)\big\|^2 \\
&= (1-\alpha_n)\big\|x_n - T\big((1-\alpha_n)x_n+\alpha_n Tx_n\big)\big\|^2 + \alpha_n\big\|Tx_n - T\big((1-\alpha_n)x_n+\alpha_n Tx_n\big)\big\|^2 \\
&\quad - \alpha_n(1-\alpha_n)\|x_n-Tx_n\|^2 + (1-\alpha_n)\|x_n-u\|^2 + \alpha_n\|Tx_n-u\|^2 \\
&\quad - \alpha_n(1-\alpha_n)\|x_n-Tx_n\|^2 \\
&\le (1-\alpha_n)\|x_n-u\|^2 + \alpha_n\big(\|x_n-u\|^2+\|x_n-Tx_n\|^2\big) - \alpha_n(1-\alpha_n)\|x_n-Tx_n\|^2 \\
&\quad + (1-\alpha_n)\big\|x_n - T\big((1-\alpha_n)x_n+\alpha_n Tx_n\big)\big\|^2 + \alpha_n\big\|Tx_n - T\big((1-\alpha_n)x_n+\alpha_n Tx_n\big)\big\|^2 \\
&\quad - \alpha_n(1-\alpha_n)\|x_n-Tx_n\|^2.
\end{aligned}
$$

Note that T is L-Lipschitzian. It follows that

$$
\begin{aligned}
&\big\|T\big((1-\alpha_n)x_n+\alpha_n Tx_n\big)-u\big\|^2 \\
&\quad\le (1-\alpha_n)\|x_n-u\|^2 + \alpha_n\big(\|x_n-u\|^2+\|x_n-Tx_n\|^2\big) - \alpha_n(1-\alpha_n)\|x_n-Tx_n\|^2 \\
&\qquad + (1-\alpha_n)\big\|x_n - T\big((1-\alpha_n)x_n+\alpha_n Tx_n\big)\big\|^2 + \alpha_n^3L^2\|x_n-Tx_n\|^2 \\
&\qquad - \alpha_n(1-\alpha_n)\|x_n-Tx_n\|^2 \\
&\quad= \|x_n-u\|^2 + (1-\alpha_n)\big\|x_n - T\big((1-\alpha_n)x_n+\alpha_n Tx_n\big)\big\|^2 \\
&\qquad - \alpha_n\big(1-2\alpha_n-\alpha_n^2L^2\big)\|x_n-Tx_n\|^2. \qquad (3.4)
\end{aligned}
$$

By condition $\alpha_n < \frac{1}{\sqrt{1+L^2}+1}$, we have $1-2\alpha_n-\alpha_n^2L^2 > 0$. Substituting (3.4) to (3.2), we obtain

$$
\begin{aligned}
\|z_n-u\|^2 &= \beta_n\|x_n-u\|^2 + (1-\beta_n)\big\|T\big((1-\alpha_n)x_n+\alpha_n Tx_n\big)-u\big\|^2 \\
&\quad - \beta_n(1-\beta_n)\big\|x_n - T\big((1-\alpha_n)x_n+\alpha_n Tx_n\big)\big\|^2 \\
&\le \beta_n\|x_n-u\|^2 + (1-\beta_n)\big[\|x_n-u\|^2 \\
&\quad + (1-\alpha_n)\big\|x_n - T\big((1-\alpha_n)x_n+\alpha_n Tx_n\big)\big\|^2\big] \\
&\quad - \beta_n(1-\beta_n)\big\|x_n - T\big((1-\alpha_n)x_n+\alpha_n Tx_n\big)\big\|^2 \\
&= \|x_n-u\|^2 + (1-\beta_n)(1-\alpha_n-\beta_n)\big\|x_n - T\big((1-\alpha_n)x_n+\alpha_n Tx_n\big)\big\|^2.
\end{aligned}
$$

Since $\alpha_n+\beta_n \ge 1$, we deduce

$$\|z_n-u\| \le \|x_n-u\|. \qquad (3.5)$$

Hence $u \in C_{k+1}$. This implies that

$$\mathrm{Fix}(T) \subset C_n$$

for all $n \in \mathbb{N}$.

Next, we show that C_n is closed and convex for all $n \in \mathbb{N}$.

It is obvious that $C_1 = C$ is closed and convex.

Suppose that C_k is closed and convex for some $k \in \mathbb{N}$. For $u \in C_k$, it is obvious that $\|z_k-u\| \le \|x_k-u\|$ is equivalent to $\|z_k-x_k\|^2 + 2\langle z_k-x_k, x_k-u\rangle \le 0$. So, C_{k+1} is closed and convex. Then, for any $n \in \mathbb{N}$, the set C_n is closed and convex. This implies that $\{x_n\}$ is well defined. □

Lemma 3.2 *The sequence* $\{x_n\}$ *is bounded.*

Proof Using the characterization inequality (2.3) of metric projection, from $x_n = P_{C_n}(x_0)$, we have

$$\langle x_0 - x_n, x_n - y\rangle \ge 0 \quad \text{for all } y \in C_n.$$

Since $\operatorname{Fix}(T) \subset C_n$, we also have

$$\langle x_0 - x_n, x_n - u\rangle \ge 0 \quad \text{for all } u \in \operatorname{Fix}(T).$$

So, for $u \in \operatorname{Fix}(T)$, we obtain

$$\begin{aligned} 0 &\le \langle x_0 - x_n, x_n - u\rangle \\ &= \langle x_0 - x_n, x_n - x_0 + x_0 - u\rangle \\ &= -\|x_0 - x_n\|^2 + \langle x_0 - x_n, x_0 - u\rangle \\ &\le -\|x_0 - x_n\|^2 + \|x_0 - x_n\|\|x_0 - u\|. \end{aligned}$$

Hence,

$$\|x_0 - x_n\| \le \|x_0 - u\| \quad \text{for all } u \in \operatorname{Fix}(T). \tag{3.6}$$

This implies that the sequence $\{x_n\}$ is bounded. □

Lemma 3.3 $\lim_{n\to\infty} \|x_{n+1} - x_n\| = 0$.

Proof From $x_n = P_{C_n}(x_0)$ and $x_{n+1} = P_{C_{n+1}}(x_0) \in C_{n+1} \subset C_n$, we have

$$\langle x_0 - x_n, x_n - x_{n+1}\rangle \ge 0.$$

Hence,

$$\begin{aligned} 0 &\le \langle x_0 - x_n, x_n - x_{n+1}\rangle \\ &= \langle x_0 - x_n, x_n - x_0 + x_0 - x_{n+1}\rangle \\ &= -\|x_0 - x_n\|^2 + \langle x_0 - x_n, x_0 - x_{n+1}\rangle \\ &\le -\|x_0 - x_n\|^2 + \|x_0 - x_n\|\|x_0 - x_{n+1}\|, \end{aligned}$$

and, therefore,

$$\|x_0 - x_n\| \le \|x_0 - x_{n+1}\|,$$

which implies that $\lim_{n\to\infty} \|x_n - x_0\|$ exists. Thus,

$$\begin{aligned}
\|x_{n+1} - x_n\|^2 &= \big\|(x_{n+1} - x_0) - (x_n - x_0)\big\|^2 \\
&= \|x_{n+1} - x_0\|^2 - \|x_n - x_0\|^2 - 2\langle x_{n+1} - x_n, x_n - x_0\rangle \\
&\le \|x_{n+1} - x_0\|^2 - \|x_n - x_0\|^2 \\
&\to 0. \qquad \square
\end{aligned}$$

Theorem 3.2 *The sequence* $\{x_n\}$ *defined by* (3.1) *converges strongly to* $P_{\mathrm{Fix}(T)}(x_0)$.

Remark 3.3 Note that $\mathrm{Fix}(T)$ is closed and convex. Thus, the projection $P_{\mathrm{Fix}(T)}$ is well defined.

Proof Since $x_{n+1} \in C_{n+1} \subset C_n$, we have

$$\|z_n - x_{n+1}\| \le \|x_n - x_{n+1}\| \to 0.$$

Further, we obtain

$$\|z_n - x_n\| \le \|z_n - x_{n+1}\| + \|x_{n+1} - x_n\| \to 0.$$

From (3.1), we get

$$\begin{aligned}
\|x_n - Tx_n\| &\le \|x_n - z_n\| + \|z_n - Tx_n\| \\
&\le \|x_n - z_n\| + \beta_n\|x_n - Tx_n\| + (1-\beta_n)\|Ty_n - Tx_n\| \\
&\le \|x_n - z_n\| + \beta_n\|x_n - Tx_n\| + (1-\beta_n)L\alpha_n\|x_n - Tx_n\| \\
&= \|x_n - z_n\| + \big[\beta_n + (1-\beta_n)L\alpha_n\big]\|x_n - Tx_n\|.
\end{aligned}$$

Since $0 < k \le 1 - \beta_n \le \alpha_n < \frac{1}{\sqrt{1+L^2}+1}$ and $1 - [\beta_n + (1-\beta_n)L\alpha_n] > k(1 - \frac{L}{\sqrt{1+L^2}+1}) > 0$, it follows that

$$\begin{aligned}
\|x_n - Tx_n\| &\le \frac{1}{1 - [\beta_n + (1-\beta_n)L\alpha_n]}\|x_n - z_n\| \\
&\le \frac{1}{k(1 - \frac{L}{\sqrt{1+L^2}+1})}\|x_n - z_n\| \to 0.
\end{aligned} \tag{3.7}$$

Now, (3.7) and Lemma 2.1 guarantee that every weak limit point of $\{x_n\}$ is a fixed point of T. That is, $\omega_w(x_n) \subset \mathrm{Fix}(T)$. This fact, inequality (3.6) and Lemma 2.2 ensure the strong convergence of $\{x_n\}$ to $P_{\mathrm{Fix}(T)}(x_0)$. This completes the proof. $\square$

Remark 3.4 It is easily seen that all of the results above hold for nonexpansive mappings.

Remark 3.5 It is nowadays quite clear that, for large classes of contractive type operators, it suffices to consider the simpler Mann iteration, even if the Ishikawa iteration, which is more general but also computationally more complicated than the Mann iteration, could always be used. But if T is only a pseudocontractive mapping, then generally, the Mann iterative process does not converge to the fixed point, and strong convergence of the Ishikawa iteration has not been achieved without the compactness assumption on T or C. However, our algorithm (3.1) has a strong convergence without the compactness assumption.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors completed the paper together. All authors read and approved the final manuscript.

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References
1. Deimling, K: Zeros of accretive operators. Manuscr. Math. **13**, 365-374 (1974)
2. Mann, WR: Mean value methods in iteration. Proc. Am. Math. Soc. **4**, 506-510 (1953)
3. Ishikawa, S: Fixed points by a new iteration method. Proc. Am. Math. Soc. **44**, 147-150 (1974)
4. Chidume, CE, Mutangadura, SA: An example on the Mann iteration method for Lipschitz pseudo-contractions. Proc. Am. Math. Soc. **129**, 2359-2363 (2001)
5. Chidume, CE, Zegeye, H: Approximate fixed point sequences and convergence theorems for Lipschitz pseudo-contractive maps. Proc. Am. Math. Soc. **132**, 831-840 (2003)
6. Zhang, QB, Cheng, CZ: Strong convergence theorem for a family of Lipschitz pseudocontractive mappings in a Hilbert space. Math. Comput. Model. **48**, 480-485 (2008)
7. Zhou, H: Strong convergence of an explicit iterative algorithm for continuous pseudo-contractions in Banach spaces. Nonlinear Anal. **70**, 4039-4046 (2009)
8. Zegeye, H, Shahzad, N, Mekonen, T: Viscosity approximation methods for pseudocontractive mappings in Banach spaces. Appl. Math. Comput. **185**, 538-546 (2007)
9. Zegeye, H, Shahzad, N, Alghamdi, MA: Minimum-norm fixed point of pseudocontractive mappings. Abstr. Appl. Anal. **2012**, Article ID 926017 (2012)
10. Ofoedu, EU, Zegeye, H: Iterative algorithm for multi-valued pseudocontractive mappings in Banach spaces. J. Math. Anal. Appl. **372**, 68-76 (2010)
11. Ofoedu, EU, Zegeye, H: Further investigation on iteration processes for pseudocontractive mappings with application. Nonlinear Anal. **75**, 153-162 (2012)
12. Udomene, A: Path convergence, approximation of fixed points and variational solutions of Lipschitz pseudo-contractions in Banach spaces. Nonlinear Anal. **67**, 2403-2414 (2007)
13. Yao, Y, Liou, YC, Chen, R: Strong convergence of an iterative algorithm for pseudo-contractive mapping in Banach spaces. Nonlinear Anal. **67**, 3311-3317 (2007)
14. Yao, Y, Liou, YC, Marino, G: A hybrid algorithm for pseudo-contractive mappings. Nonlinear Anal. **71**, 4997-5002 (2009)
15. Morales, CH, Jung, JS: Convergence of paths for pseudo-contractive mappings in Banach spaces. Proc. Am. Math. Soc. **128**, 3411-3419 (2000)
16. Ceng, LC, Petrusel, A, Yao, JC: Strong convergence of modified implicit iterative algorithms with perturbed mappings for continuous pseudocontractive mappings. Appl. Math. Comput. **209**, 162-176 (2009)
17. Berinde, V: Iterative Approximation of Fixed Points. Lecture Notes in Mathematics. Springer, Berlin (2007)
18. Martinez-Yanes, C, Xu, HK: Strong convergence of the CQ method for fixed point processes. Nonlinear Anal. **64**, 2400-2411 (2006)
19. Takahashi, W, Takeuchi, Y, Kubota, R: Strong convergence theorems by hybrid methods for families of nonexpansive mappings in Hilbert spaces. J. Math. Anal. Appl. **341**, 276-286 (2008)
20. Nadezhkina, N, Takahashi, W: Strong convergence theorem by a hybrid method for nonexpansive mappings and Lipschitz-continuous monotone mappings. SIAM J. Optim. **16**, 1230-1241 (2006)
21. Zegeye, H, Shahzad, N, Alghamdi, MA: Convergence of Ishikawas iteration method for pseudocontractive mappings. Nonlinear Anal. **74**, 7304-7311 (2011)

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