

# A novel parametric macro-modeling of S-parameter data by interpolating residues of root models

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**Abstract:** This paper presents a new technique for parametric macro-modeling of tabulated S parameter data in frequency domain. The traditional technique is modified in such a way that barycentric interpolation is performed at residues of root models, rather than the whole root models. It avoids increasing the number of poles which represents the model complexity and makes the parametric macro-model more compact. An illustrative example involving a low-pass filter is presented for validation of the proposed method.

**Keywords:** parametric macro-modeling, rational approximation, interpolation, vector fitting

**Classification:** Microwave and millimeter-wave devices, circuits, and modules

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## 1 Introduction

Parametric macro-modeling is becoming more and more important for the design, study and optimization of microwave circuits and systems. The parametric macro-model can approximate the frequency response of a system with high precision, that is parameterized by one or more design variables. Such parametric macro-models have been widely used for real-time design optimization and sensitivity analysis [1, 2, 3].

In the past, many parametric modeling techniques were proposed. Artificial neural networks (ANN) are frequently used to calculate parametric models [4, 5]. Although it has ability to nonlinear behavior, it is difficult to find a proper topology. At the same time, a large number of training data and long training time are always required. Lamecki et al. applied multidimensional Cauchy method to create a high quality parametric model of microwave circuit from electromagnetic simulation data by solving the ill-conditioned interpolation problem [6]. But in the process of modeling, the use of polynomials is required to avoid numerical ill-conditioning. Then Lehmensiek and Meyer avert the numerical instability by using a multivariate rational interpolation based on the Thiele-type branched continued fractions [7, 8]. But the method can not be applied to the data samples which are contaminated with noise. In 2008, Dirk Deschrijver et al. [9] extend the vector fitting (VF) [10] to multivariate case. The technique combines the use of an iterative least squares estimator [11] and orthonormal rational basis functions to create parametric macro-model. By the method, the data samples which are sparse or dense, and deterministic or noisy can be modeled. But the method does not guarantee passivity of the

parametric macro-model. In [12], Dirk Deschrijver et al. resolves the passivity problem by proposing a new parametric macro-model representation. The parametric macro-model is created by combining the univariate models, also called root models which are computed by VF for various combinations of design variables with barycentric Lagrange interpolation. The enforcement for the passivity of the parametric macro-model is carried out by perturbation of the barycentric weights. But for the method when the poles of root models are different respectively, the order of parametric model created will sharply grow up with the number of root models. It will result in high computation cost.

To avoid the problem presented above, a new technique for parametric macro-modeling of tabulated S parameter in frequency domain is proposed. By creating the root models with the same poles, the order of parametric macro-modeling which is created by interpolating residues of root models can remain the same as root models'. The modeling process of method proposed has two steps. In the first step, the root models with the same poles are created. In the second step, barycentric interpolation is performed at residues and constant terms of root models. It avoids increasing the number of poles which presents the model complexity (order) and saves the computation cost.

## 2 Traditional parametric macro-modeling method

The parametric macro-modeling method with barycentric interpolation of root models is first introduced in [12]. In [12], the root models are computed by VF [10] with simulated frequency responses at different values of the design variable. Next simply introduce the VF Method.

### 2.1 VF method

In [10], VF was introduced to build macro-model  $H(s)$ , based on frequency domain data samples.

$$H(s) = \sum_{n=1}^N \frac{r_n}{s - p_n} + d. \quad (1)$$

Where  $s = j\omega$ .  $p_n$  and  $r_n$  are the poles and residues respectively,  $\forall n = 1, \dots, N$ . And  $d$  is a constant term.

#### 2.1.1 Pole identification

Specify a set of starting poles  $\mathbf{P}_0 = (\bar{p}_1, \bar{p}_2, \dots, \bar{p}_N)$  in Eq. (1), and an unknown function  $\sigma(s)$  called scale factor is introduced.

$$\sigma(s) = \sum_{n=1}^N \frac{\tilde{r}_n}{s - \bar{p}_n} + 1. \quad (2)$$

Where  $\tilde{r}_n$  is the nth residue for scale factor  $\sigma(s)$ .

A new rational model is obtained by multiplying  $H(s)$  with  $\sigma(s)$ .

$$\sum_{n=1}^N \frac{\hat{r}_n}{s - \bar{p}_n} + \hat{d} = H(s) \left( \sum_{n=1}^N \frac{\tilde{r}_n}{s - \bar{p}_n} + 1 \right). \quad (3)$$

Where  $\hat{r}_n$  is the nth residue and  $\hat{d}$  is a constant term for the new rational model. Then a new equation is obtained as follow.

$$\left(\sum_{n=1}^N \frac{\hat{r}_n}{s - \bar{p}_n} + \hat{d}\right) - H(s) \left(\sum_{n=1}^N \frac{\tilde{r}_n}{s - \bar{p}_n}\right) = H(s). \quad (4)$$

$\hat{r}_n, \hat{d}, \tilde{r}_n$  are regarded as unknown variables in the discrete frequency points  $s_i = j\omega_i, i = 1, 2, \dots, K$ . Then  $H(s)$  is replaced by  $H_{Raw}(s)$  which represents the data samples simulated or measured at the discrete complex frequencies  $s_k = j\omega_k$  ( $k = 1, \dots, K$ ). Eq. (4) can be written as

$$A_p X = b. \quad (5)$$

Where

$$A_p = \begin{bmatrix} \frac{1}{j\omega_1 - \bar{p}_1} & \cdots & \frac{1}{j\omega_1 - \bar{p}_N} & 1 & \frac{-H_{Raw}(j\omega_1)}{j\omega_1 - \bar{p}_1} & \cdots & \frac{-H_{Raw}(j\omega_1)}{j\omega_1 - \bar{p}_N} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{1}{j\omega_K - \bar{p}_1} & \cdots & \frac{1}{j\omega_K - \bar{p}_N} & 1 & \frac{-H_{Raw}(j\omega_K)}{j\omega_K - \bar{p}_1} & \cdots & \frac{-H_{Raw}(j\omega_K)}{j\omega_K - \bar{p}_N} \end{bmatrix}_{(K \times 2N+1)}$$

$$X = (\hat{r}_1, \hat{r}_2, \dots, \hat{r}_N, \hat{d}, \tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_N)^T \in \mathbf{C}^{2N+1}$$

$$b = (H_{Raw}(j\omega_1), \dots, H_{Raw}(j\omega_K))^T \in \mathbf{C}^K$$

Eq. (5) which is always an over-determined linear equations is solved in a least-squares sense. Once the solution has been obtained, the new poles are found by forming the minimal state-space realization of  $\sigma(s)$  and solving an eigenvalue problem. All details about this procedure are reported in [11, 13]. Then these new poles replace the previous (starting) poles and the iteration is repeated until scale factor  $\sigma(s)$  in (2) tends to unity.

### 2.1.2 Residue identification

Once the poles  $p_n$  ( $\forall n = 0, \dots, N$ ) are obtained, the residues  $r_n$  ( $\forall n = 0, \dots, N$ ) and  $d$  of the Eq. (1) can be identified by solving the over-determined Eq. (6) in a least-squares sense.

$$A' x' = b'. \quad (6)$$

where

$$A' = \begin{bmatrix} \frac{1}{j\omega_1 - p_1} & \cdots & \frac{1}{j\omega_1 - p_N} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{1}{j\omega_K - p_1} & \cdots & \frac{1}{j\omega_K - p_N} & 1 \end{bmatrix}_{K \times (N+1)}$$

$$X' = (r_1, r_2, \dots, r_N, d)^T \in \mathbf{C}^{N+1}$$

$$b' = (H_{Raw}(j\omega_1), \dots, H_{Raw}(j\omega_K))^T \in \mathbf{C}^K$$

### 2.2 Traditional parametric modeling

The root models  $H(s, \alpha_v)|_{v=1}^V$  can be created by VF with simulated frequency responses at different values of the design variable  $\alpha$ . Then the parametric macro-model is created by interpolating these root models with barycentric interpolation [12].

$$H(s, \alpha) = \frac{\sum_{v=1}^V \frac{\omega_v}{\alpha - \alpha_v} H(s, \alpha_v)}{\sum_{v=1}^V \frac{\omega_v}{\alpha - \alpha_v}} \quad (7)$$

Where

$$\omega_v = \prod_{k=1, k \neq v}^V (\alpha_v - \alpha_k)^{-1}$$

$$H(s, \alpha_v) = \sum_{n=1}^N \frac{r_n^v}{s - p_n^v} + d^v$$

Where  $\alpha$  represents the design variable to be parameterized.  $\omega_v$  is the weight.  $p_n^v$  and  $r_n^v$  respectively represent the pole and residual of root model  $H(s, \alpha_v)$  created by VF. And  $d^v$  represents the constant term of root model  $H(s, \alpha_v)$ .

It can be seen that Eq. (7) is a rational interpolation of the root models as a function of  $\alpha$ , with non-zero weights  $\omega_v$ . And the parametric macro-model is in fact an  $\alpha$ -weighted sum of the root models. So the parametric macro-model will contain all the poles of all the root models. When the poles of root models created by traditional VF are different respectively, the parametric macro-model will yield a higher-order macro-model which results in high computational cost for simulation.

### 3 Method proposed

This letter resolves the problem mentioned above by applying barycentric interpolation to the residues at the numerator of the root models which have the same poles.

#### 3.1 Root models with the same poles

VF can be used to create a series of root models with the same poles. Next simply introduce the process of modeling.

$H(s, \alpha_v)$  ( $v = 1, \dots, V$ ) are root models to be created with the same poles  $p_n$  ( $\forall n = 1, \dots, N$ ).

$$H(s, \alpha_v) = \sum_{n=1}^N \frac{r_n^v}{s - p_n} + d^v \quad (8)$$

Eq. (8) can be further converted into

$$\begin{bmatrix} H_{Raw}(s, \alpha_1) \\ \vdots \\ H_{Raw}(s, \alpha_V) \end{bmatrix} \approx \begin{bmatrix} H(s, \alpha_1) \\ \vdots \\ H(s, \alpha_V) \end{bmatrix} = \begin{bmatrix} \sum_{n=1}^N \frac{r_n^1}{s - p_n} + d^1 \\ \vdots \\ \sum_{n=1}^N \frac{r_n^V}{s - p_n} + d^V \end{bmatrix} \quad (9)$$

Where  $H_{Raw}(s, \alpha_v)|_{v=1}^V$  represents the S-parameter data simulated at the discrete complex frequencies  $s_k = j\omega_k$  ( $k = 1, \dots, K$ ) with different values  $\alpha_v|_{v=1}^V$  of the design variable  $\alpha$ . Then  $p_n$  and  $r_n^v$  ( $\forall n = 1, \dots, N, v = 1, \dots, V$ ) are the poles and residues respectively for root models  $H(s, \alpha_v)$ . And  $d^v$  is a constant term.

Similarly specify a set of starting poles  $\mathbf{P}_0 = (\bar{p}_1, \bar{p}_2, \dots, \bar{p}_N)$ , and scale factor  $\sigma(s)$  is introduced as Eq. (2).

Eq. (4) can be changed into

$$\begin{bmatrix} \sum_{n=1}^N \frac{\hat{r}_n^1}{s - \bar{p}_n} \\ \vdots \\ \sum_{n=1}^N \frac{\hat{r}_n^V}{s - \bar{p}_n} \end{bmatrix} + \begin{bmatrix} \hat{d}^1 \\ \vdots \\ \hat{d}^V \end{bmatrix} - \begin{bmatrix} H_{Raw}(s, \alpha_1) \left( \sum_{n=1}^N \frac{\tilde{r}_n}{s - \bar{p}_n} \right) \\ \vdots \\ H_{Raw}(s, \alpha_V) \left( \sum_{n=1}^N \frac{\tilde{r}_n}{s - \bar{p}_n} \right) \end{bmatrix} = \begin{bmatrix} H_{Raw}(s, \alpha_1) \\ \vdots \\ H_{Raw}(s, \alpha_V) \end{bmatrix} \quad (10)$$

Where  $\hat{r}_n^v$  ( $\forall n = 1, \dots, N, v = 1, \dots, V$ ) represent the residues and  $\hat{d}^v|_{v=1}^V$  are the constant terms for the new rational models.  $\hat{r}_n^v, \hat{d}^v, \tilde{r}_n$  ( $\forall n = 1, \dots, N, v = 1, \dots, V$ ) are regarded as unknown variables. Then Eq. (10) can be written as

$$A_k X_p = b_k \quad (11)$$

Where

$$A_k = \begin{bmatrix} \frac{1}{s_k - \bar{p}_1}, \dots, \frac{1}{s_k - \bar{p}_N}, 1, & 0 & \dots, \frac{-H_{Raw}(s_k, \alpha_1)}{s_k - \bar{p}_1}, \dots, \frac{-H_{Raw}(s_k, \alpha_1)}{s_k - \bar{p}_N} \\ \vdots & \vdots & \vdots \\ 0 & \dots, \frac{1}{s_k - \bar{p}_1}, \dots, \frac{1}{s_k - \bar{p}_N}, 1, & \dots, \frac{-H_{Raw}(s_k, \alpha_V)}{s_k - \bar{p}_1}, \dots, \frac{-H_{Raw}(s_k, \alpha_V)}{s_k - \bar{p}_N} \end{bmatrix}$$

$$X_p = [\hat{r}_1^1, \dots, \hat{r}_N^1, \hat{d}^1, \dots, \hat{r}_1^V, \dots, \hat{r}_N^V, \hat{d}^V, \tilde{r}_1, \dots, \tilde{r}_N]^T$$

$$b_k = [H_{Raw}(s_k, \alpha_1), \dots, H_{Raw}(s_k, \alpha_V)]^T$$

Write Eq. (11) for discrete frequency points  $s_k = j\omega_k, k = 1, 2, \dots, K$  and an over-determined linear matrix equation is obtained and solved in a least-squares sense. Then the process of solving the poles  $p_n$  ( $\forall n = 1, \dots, N$ ) is the same to section 2.1.1.

After poles  $p_n$  ( $\forall n = 1, \dots, N$ ) are obtained, the residuals  $r_n^v$  and constant term  $d^v$  are solved the same as section 2.1.2.

Since the accuracy of each root model can be improved with the increase of pole number  $N$ ,  $N$  can be gradually increased by step one until all the accuracies of the root models satisfy the predefined precision  $\sigma_0$ . In such way, the pole number of  $N$  for root models can be determined by predefined precision  $\sigma_0$ .

### 3.2 Novel parametric macro-modeling

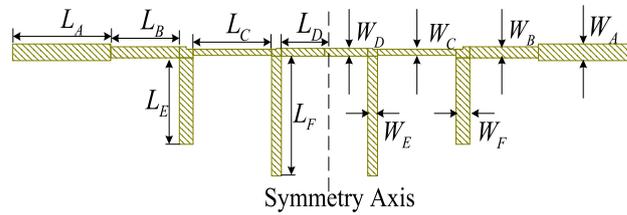
A novel parametric macro-model can be created by interpolating residues and constant terms of root models. It effectively voids increasing the number of poles which presents the model complexity and simplifies the process of parametric macro-modeling.

$$H(s, \alpha) = \sum_{n=1}^N \frac{\sum_{v=1}^V \left( \frac{\omega_v}{\alpha - \alpha_v} r_n^v \right)}{s - p_n} + \frac{\sum_{v=1}^V \left( \frac{\omega_v}{\alpha - \alpha_v} d^v \right)}{\sum_{v=1}^V \frac{\omega_v}{\alpha - \alpha_v}} \quad (12)$$

Where  $r_n^v$  and  $d^v$  respectively represent the residual and constant term of root model  $H(s, \alpha_v)$  ( $\forall n = 1, \dots, N, v = 1, \dots, V$ ).

#### 4 Numerical example

In order to illustrate the method proposed, an example is taken from micro-strip of low pass filter. The geometry of filter which is a symmetric structure is shown in Fig. 1. In Fig. 1, the dotted line presents symmetry axis. The specification of the filter is shown in Table I.



**Fig. 1.** Circuit schematic of microwave filter.

**Table I.** Parameters of the microwave filter (mil)

$L_A$	100
$L_B$	70
$L_C$	80
$L_D$	44
$L_E$	78
$L_F$	108
$W_A$	15
$W_B$	10
$W_C$	5.5
$W_D$	7
$W_E$	14
$W_F$	10

**Table II.** RMS for each root model

Root model	Model order	RMS
Root model ( $W_A = 8$ mil)	13	$6.823 \times 10^{-4}$
Root model ( $W_A = 9$ mil)	13	$3.041 \times 10^{-4}$
Root model ( $W_A = 10$ mil)	13	$7.138 \times 10^{-4}$
Root model ( $W_A = 11$ mil)	13	$6.102 \times 10^{-4}$

Its low pass-band is 0~8 GHz. Its S21 is modeled using the new method proposed.

A parametric macro-model is created as a function of the frequency  $f \in [6, 12]$  GHz and a design variable  $W_A \in [8, 11]$  mil. The S21 parameter is simulated by Microwave Office (MWO) for four different values of the design variable  $W_A$ , which are equidistantly spread over the design variable  $W_A$  range of interest. The predefined precision of RMS (root mean square) is set to be  $\sigma_0 = 1 \times 10^{-3}$ . The root models are finally created with 13 identical poles shown in Fig. 2. Table II shows the RMS for each root model. The maximum RMS of four root models is  $7.1383 \times 10^{-4}$ , which is satisfied with predefined precision  $\sigma_0 = 1 \times 10^{-3}$ .

Table III shows the same poles of four root models created, and Fig. 3 shows the residuals of four root models. Then by Eq. (12), a parametric macro-model can be created as a function of the design variable  $W_A \in [8, 11]$  mil.

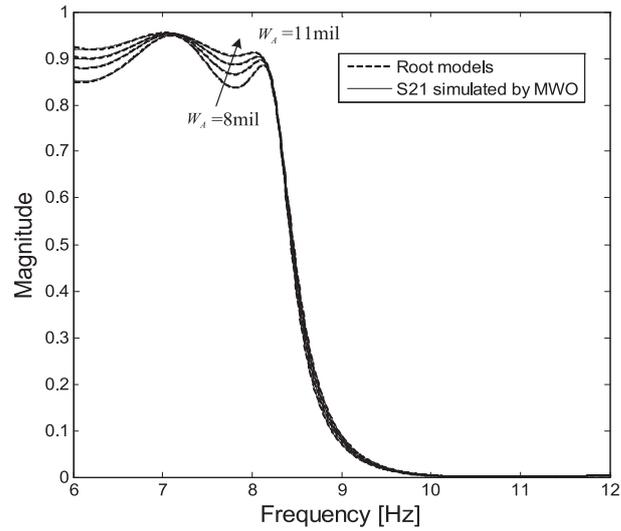


Fig. 2. Root models with the same poles

Table III. The same poles of four root models

1	-25849838078.7704
2	-6004778261.01633 + 31836905184.8372i
3	-6004778261.01633 - 31836905184.8372i
4	-5375365671.28718 + 45228966220.2241i
5	-5375365671.28718 - 45228966220.2241i
6	-4116327506.08379 + 46404325787.2425i
7	-4116327506.08379 - 46404325787.2425i
8	-1719972003.90041 + 52023436090.8872i
9	-1719972003.90041 - 52023436090.8872i
10	-1751099198.67606 + 52173010989.8319i
11	-1751099198.67606 - 52173010989.8319i
12	-19639514759.2882 + 79555455568.7807i
13	-19639514759.2882 - 79555455568.7807i

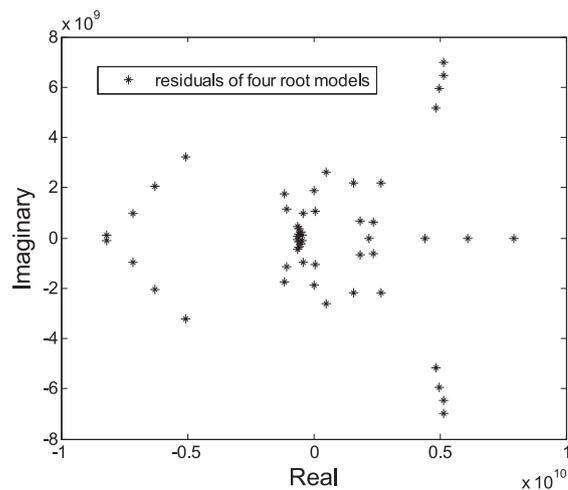
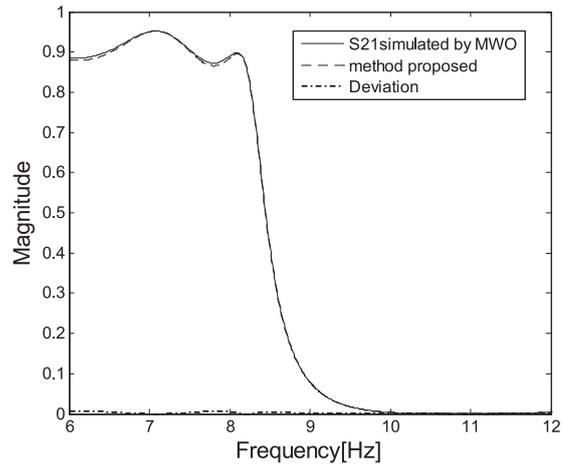


Fig. 3. Residuals of four root models

To test the precision, the value of S21 with  $W_A = 9.25$  mil is calculated by the parametric macro-model created. Fig. 4 shows the result of method proposed compared with S11 data simulated by MWO. In Fig. 4 the maximum deviation



**Fig. 4.** Comparison between the method proposed and data simulated by MWO ( $W_A = 9.25$  mil)

**Table IV.** The results of comparison ( $W_A = 9.25$  mil)

Method	Frequency band	Model order	RMS	Calculation time
Method proposed	6~12 GHz	13	$4.5973 \times 10^{-4}$	0.106842 s
Reference [12]	6~12 GHz	28	0.0028	0.182813 s

is 0.0012, and its RMS is  $4.5973 \times 10^{-4}$ . It shows that with the pole number unchanged, the parametric macro-model created also has high accuracy.

For comparison, the method of reference [12] is used to create the traditional parametric model with the same data simulated above. As reference [12] illustrated, 4 root models are first created by VF method. Here the pole number of 4 root models created is set to be 7. Then the traditional parametric model can be created by Eq. (7). Since each pole of 4 root models is different, so there are totally 28 different poles. When  $W_A$  is set to be 9.25 mil, the model of S21 with 28 model order can be created.

Table IV shows the results of comparison between the method proposed and method of reference [12]. The two models for  $W_A = 9.25$  mil are both calculated in 6~12 GHz. And compared with the method of reference [12], the method proposed with lower model order has higher precision and less calculation time. So the method proposed has lower computation cost than method of reference [12].

## 5 Conclusion

A modified parametric macro-modeling method is proposed. The method starts by computing several root models with the same poles at different values of a design variable. Then a new parametric macro-model is created by interpolating the residues and constant terms of the root models, rather than the whole root models. It is shown that with high precision, the method proposed avoids the number of poles dramatically increasing and keeps the number of poles the same as root model's. It dramatically saves the computation cost of model created.

## Acknowledgments

The research work was supported by National Natural Science Foundation of China (No. 61271045).