Fuzzy Sliding Mode Control for Hyper Chaotic Chen System

Morteza SARAILOO, Zahra RAHMANI, Behrooz REZAIE
Intelligent System Research Group, Electrical and Computer Engineering Department, Babol University of Technology, Babol, 4714871167, Iran
m.serailoo@gmail.com, zrahmani@nit.ac.ir, brezaie@nit.ac.ir

***Abstract*—In this paper, a fuzzy sliding mode control method is proposed for stabilizing hyper chaotic Chen system. The main objective of the control scheme is to stabilize unstable equilibrium point of the system by controlling the states of the system so that they converge to a pre-defined sliding surface and remain on it. A fuzzy control technique is also utilized in order to overcome the main disadvantage of sliding mode control methods, i.e. chattering problem. It is shown that the equilibrium point of the system is stabilized by using the proposed method. A stability analysis is also performed to prove that the states of the system converge to the sliding surface and remain on it. Simulations show that the control method can be effectively applied to Chen system when it performs hyper chaotic behavior.**

***Index Terms*—nonlinear systems, chaos, fuzzy control, Lyapunov method, sliding mode control.**

I. INTRODUCTION

In 1963, Lorenz found the first chaotic attractor in a 3-dimensional autonomous system when he studied atmospheric convection. In 1979, Rossler reported the first hyper chaotic system with two positive Lyapunov exponents [1]. In 1999, Chen produced a 3-dimensional autonomous chaotic system based on Lorenz System. Chen et al. introduced a new 4-dimensional hyper chaotic system which had larger Lyapunov exponents in comparison with the previous ones [2].

In recent years, the study of chaotic and hyper chaotic systems has grown up in many fields such as laser [3-4], nonlinear circuits [5], communication [6], oscillators [7], power system [8], and photovoltaic system [9]. Hyper chaotic behavior can appear only in high dimensional systems (e.g., more than 4 dimensions for continuous- time autonomous systems). Furthermore at least two terms must be existed in the equations that cause instability, and one of them must be a nonlinear function [10].

The control of hyper chaos is also of interesting among the researchers [11-16]. Recently, researchers have studied the control of hyper chaos by using various approaches: for instance delayed feedback methods [13], sliding mode control [10], [12], fuzzy control [11].

The control objectives for such systems can be defined as eliminating the chaotic behavior and drive the system to its equilibrium point [12], [15], stabilizing unstable trajectories toward a stable limit cycle [18-19], synchronizing two chaotic systems, e.g. [20-22]. The existing control methods for such systems can be classified in two groups:

The first group is the controllers such as delayed feedback methods [13] and impulsive control [14], which use inherent properties of chaos for controlling it. Another group includes the controllers designed based on common methods, in which the inherent properties of chaotic systems is not taken into account. These systems must be nonlinear systems, and general control methods can be considered for it, such as linear feedback control [15], [23], sliding mode control [12], [17], [24–26], optimal control [16], fuzzy control [11].

For many of the control methods, there exist some problems such as complicated design, lack of robustness and costly implementation. Variable structure control with sliding mode control is a powerful control method for overcoming such problems. In this method, switching state feedback control causes states or errors of system converge to sliding surface. The system controlled by the sliding mode control method is insensitive to parametric uncertainties and/or external disturbances.

Due to the simplicity of its calculation and implementation, there are many works addressing the sliding mode control for hyper chaotic systems [11], [25–29]:

In [27], a sliding mode controller has been designed for Rossler hyper chaotic system. In this paper the state variables of this system converged to the surface in the presence of unstructured external disturbance. For this purpose, the proportional-integral sliding surface was utilized. Although the advantages of the method, the proposed controller was implemented for Rossler hyper chaotic system, and it cannot be easily applied on other hyper chaotic systems, since the proposed control signal is discrete and the control input was applied linearly.

In [28], Rossler hyper chaotic system was stabilized using sliding mode control method in the presence of disturbance and nonlinear control inputs. Defining proportional-integral sliding surface, and moreover, H_∞ norm of the transformation function representing the ratio of disturbance to the system input made it to be decreased to a certain value. But this value never reduces to zero. A continuous control signal was also proposed as well, for eliminating the chattering phenomenon.

Chang et al. made the state variables of Rossler low dimensions chaotic system to converge to the equilibrium point using a sliding mode control. An important point of this study is to define a proportional-integral sliding surface for sliding mode control. But no uncertainty and disturbance was considered for the chaotic system. Moreover, the existence of sign function in control signal without any solution for eliminating it causes high frequency switching,

Digital Object Identifier 10.4316/AECE.2012.01014

which affect the system performance [25].

In [29], a hybrid control scheme consisting of a sliding mode control and an adaptive control method was used to control a unified chaotic system. In this work, it was taken into consideration that there are uncertainty, disturbance and even nonlinear control input for chaotic system. Comparing with nonlinear control inputs of similar studies, the control inputs have a wide comprehensiveness. The main disadvantage of this work is the discreteness of control signal due to the presence of sign function.

Finally, the idea of combination of sliding mode control and fuzzy control for controlling a class of chaotic systems in the presence of disturbance and uncertainty has been utilized in [11]. The authors [11] eliminated the discreteness of sliding control signal and removed the chattering by replacing sign function with fuzzy functions. This work has two main disadvantages. First, the proposed method is deployable only for a certain form of chaotic systems which can includes a limited range of chaotic systems. Another disadvantage is that no attention was paid to the nonlinearity of the control inputs.

In order to overcome such disadvantages, in the present paper we propose a fuzzy sliding mode variable structure approach for controlling a well-known hyper chaotic system namely Chen system. The Chen system captures many of the features of chaotic dynamics. This model describes unpredictable behaviors associated with the weather. The unpredictable chaotic mode in Chen system can be very destructive, and therefore it is important to control it. The main control aim in our study is to stabilize the unstable equilibrium point of the system by using a sliding mode controller. Furthermore, a fuzzy control technique is used to overcome the chattering problem. A stability analysis is also presented to show the stability and convergence of the system controlled by the proposed method.

The organization of this paper is as follows: In the next section, we explain the model of Chen hyper chaotic system and its behavior. Section 3 presents the design procedure for high order sliding mode controller using an adaptive sliding surface. In section 4, by using a fuzzy method, we improve the control scheme in order to enhance the performance of the closed loop system. Simulation results show the applicability and effectiveness of the proposed method for controlling hyper chaotic systems. Finally, conclusion remarks are given in the last section.

II. Chen Hyper Chaotic System

Now consider Chen hyper chaotic system described by the following equations:

$$\begin{cases} \dot{x} = a(y-x) + eyz \\ \dot{y} = cx - dxz + y + u \\ \dot{z} = xy - bz \\ \dot{u} = -ky \end{cases} \tag{1}$$

where x, y, z, u are the system states, and a, b, c, d, e, and k are the parameters of the system. The system has an equilibrium point at [0 0 0 0] which is unstable. The system can exhibit some basic properties of hyper chaotic systems such as periodic, pseudo-periodic, chaotic and hyper chaotic behavior based on the values of its parameters. To obtain a hyper chaotic regime, the parameters are considered as a=35, b=4.9, c=25, d=5, e=35, k=22 [10]. Subsequently, Chen system has been plotted in Fig. 1 for the above values and initial condition as $u_0 = 0$, $z_0 = 5$, $y_0 = 0$, $x_0 = 5$.

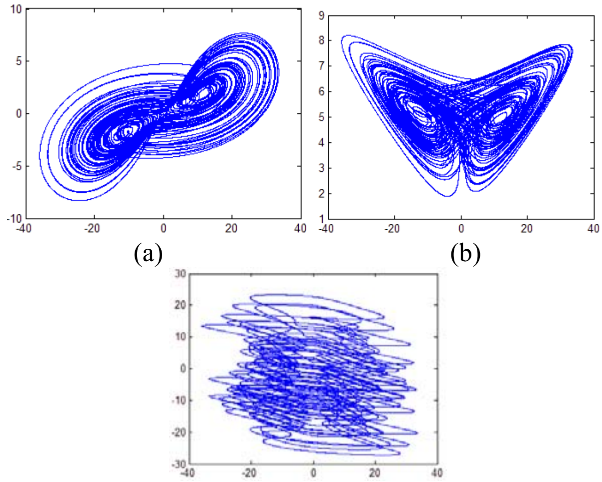


Figure 1. Chen's hyper chaotic system: (a) x-y (b) x-z (c) x-u

In the next section, we propose a control strategy for stabilizing the unstable equilibrium point of the Chen hyper chaotic system (1).

III. Sliding Mode Control Method

In this section, we propose a high sliding mode controller to stabilize equilibrium point of Chen hyper chaotic system. For this purpose, we extend the idea of [12], [30] for hyper chaotic systems. Same as [12], [30] consider Chen's hyper chaotic system:

$$\begin{cases} \dot{x} = a(y-x) + eyz \\ \dot{y} = f(x,y,z) + y + u + \Delta f + v \\ \dot{z} = gxy - bz \\ \dot{u} = -ky \end{cases} \tag{2}$$

where $a, b, e, g, k \in R^+$ and v represents control input signal. Δf shows uncertainty or disturbance where $|\Delta f| < M$, and $M > 0$. The general controller law can be written as follow:

$$v = V_{eq} + V_r \tag{3}$$

Now, we design the control law in order to the states converge to the common of the sliding surface described by $S(t)$ and $\dot{S}(t)$ within a limited time and stay on it. For this purpose we define the sliding surface as follows:

$$S(t) = y(t) + n(t) \tag{4}$$

where $n(t)$ is an adaptive nonlinear function which can be obtained by solving the following differential equation:

$$\dot{n}(t) = (e+g)xz + ax - ku + k'yu^2, \quad k' > 0 \tag{5}$$

Thus:

$$\dot{S}(t) = \dot{y}(t) + \dot{n}(t) \tag{6}$$

We can obtain V_r and V_{eq} as follow:

$$V_{eq} = -\dot{n}(t) - f(x,y,z) - y - u \tag{7}$$

$$V_r = K_S \operatorname{sgn}(S) \tag{8}$$

In equation (8), K_S can be obtained as follow:

$$\dot{K}_S = -\gamma|S|, \quad K_S(0) = K \quad K < -M \tag{9}$$

Next we show that states of the system (2) converge to the surface and stay on it. In order to show convergence states to the sliding surface, we determine Lyapunov function as follow:

$$V = \frac{1}{2}S^2 + \frac{1}{2\gamma}(K - K_S)^2, \quad V(0) = 0 \tag{10}$$

Derivative of (10) with respect to time is:

$$\dot{V} = S\dot{S} - \frac{1}{\gamma}(K - K_S)\dot{K}_S \tag{11}$$

Thus, we have:

$$\dot{V} = S(\dot{y} + \dot{n}) - \frac{1}{\gamma}(K - K_S)\dot{K}_S \tag{12}$$

Substituting equations (5), (7) and (8) into (12), we have:

$$\dot{V} = S[K_S \operatorname{sgn}(S) + \Delta f] - \frac{1}{\gamma}K\dot{K}_S + \frac{1}{\gamma}K\dot{K}_S \tag{13}$$

Substituting equations (9) into (13), gives:

$$\dot{V} = K_S S[\operatorname{sgn}(S)] + (\Delta f)S + K|S| - K_S|S| = (\Delta f)S + K|S| \tag{14}$$

Considering $|\Delta f| < M$ and $K > -M$ we have:

$$\dot{V} \le (K + M)|S| \Rightarrow \dot{V} \le 0 \tag{15}$$

Therefore the states will converge to the sliding surface.

Now, Consider $S(t) = 0$ and $\dot{S}(t) = 0$. We can write that:

$$\dot{S}(t) = \dot{y}(t) + \dot{n}(t) = 0 \Rightarrow \dot{y}(t) = -\dot{n}(t) \tag{16}$$

Substituting $\dot{y}(t) = -\dot{n}(t)$ in equation (2) we have:

$$\begin{cases} \dot{x} = a(y - x) + eyz \\ \dot{y} = -[(e+g)xz + ax - ku + k'yu^2] \quad k' > 0 \\ \dot{z} = gxy - bz \\ \dot{u} = -ky \end{cases} \tag{17}$$

Now, we prove that the system (17) is stable. We consider the following Lyapunov function:

$$V = \frac{1}{2}(x^2 + y^2 + z^2 + u^2) \tag{18}$$

For equilibrium point at [0 0 0 0], we have $V(0) = 0$

Hence:

$$\dot{V} = x\dot{x} + y\dot{y} + z\dot{z} + u\dot{u} = -ax^2 - bz^2 - k'y^2u^2 \Rightarrow \dot{V} < 0 \tag{19}$$

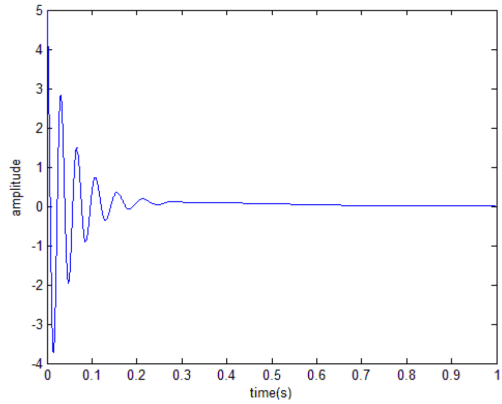
Thus, the state will be remained on the sliding surface ($S(t) = 0$), by using Barbalat's lemma (refer to [31], [32]) it can be shown that:

$$\lim_{t\to\infty} S(t) \to 0 \Rightarrow \lim_{t\to\infty} \dot{S}(t) \to 0 \tag{20}$$

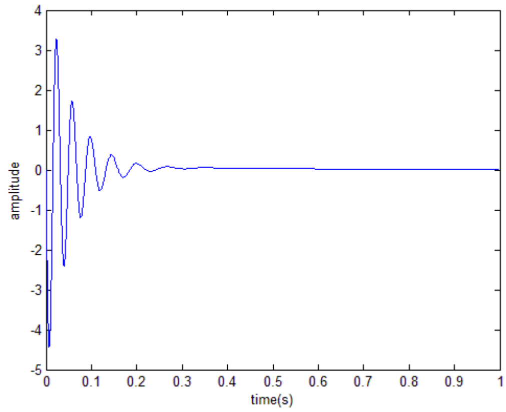
Therefore, the states of the system (17) will be remained in sliding surface.

Theorem 1: if we consider system (2), sliding surface like (4) and control law as (3) then the states will converge to the sliding surface and remain on it.

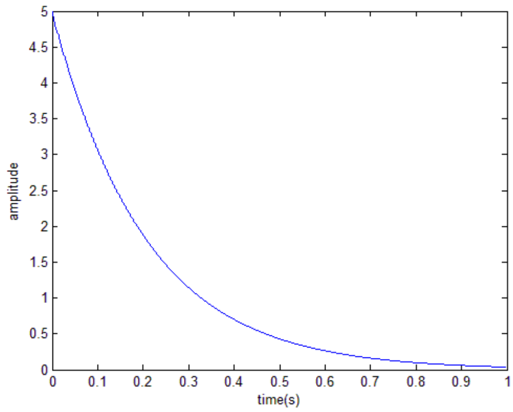
Now by applying controller (3) to Chen's system (2) with the mentioned initial value in section 2, the states of the system are shown in Fig. 2 and Fig. 3 contains the control signal.



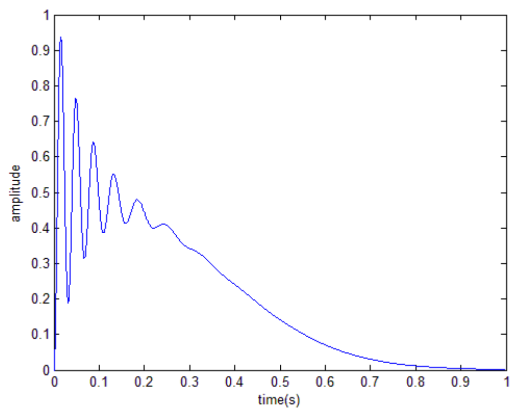
(a) x(t)



(b) y(t)



(c) z(t)



(d) u(t)

Figure 2. System states variable

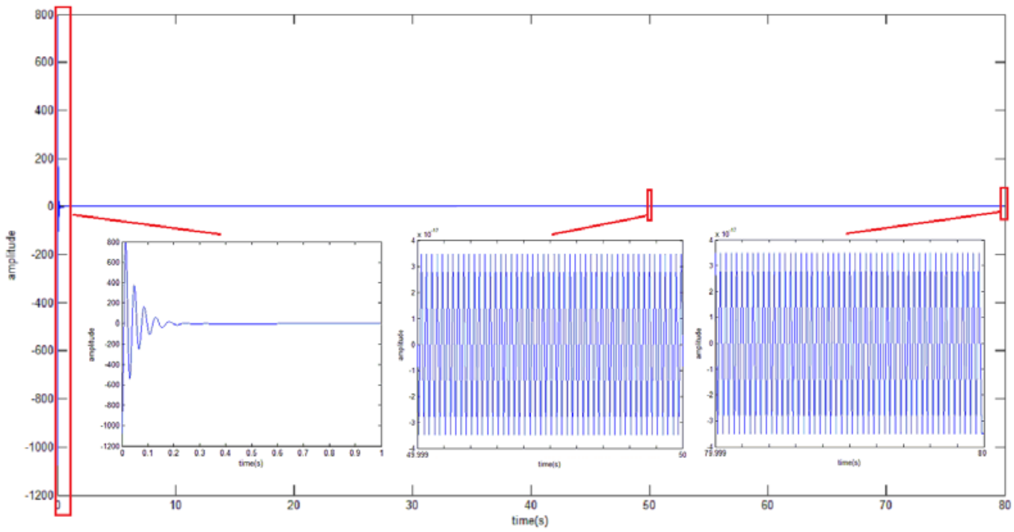


Figure 3. Control signal

The problem of applying controller (3) to the Chen hyper chaotic system (2) is the states chattering on the sliding surface, that can be obtain from input controller which is shown in Fig. 3.

IV. Fuzzy Sliding Mode Control Method

In this section by considering [11], we propose a high sliding mode controller which mixed with fuzzy method to removing chattering and stabilize equilibrium point of Chen hyper chaotic system. The first part of the controller, the equivalent control law (V_{eq}) (7), is as before and the second part (V_r), is as:

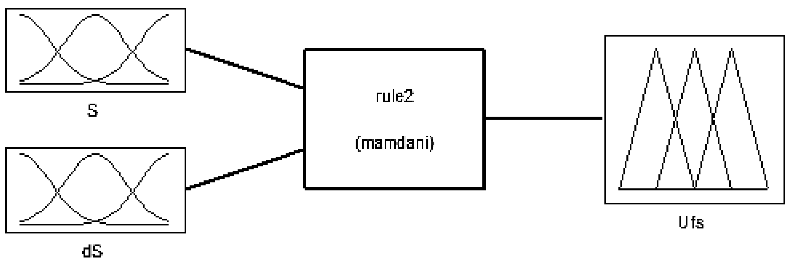
$$V_r = K_{fs} U_{fs} \qquad (21)$$

where $K_{fs} > M > 0$ is the normalizing factor and U_{fs} is a fuzzy function which is:

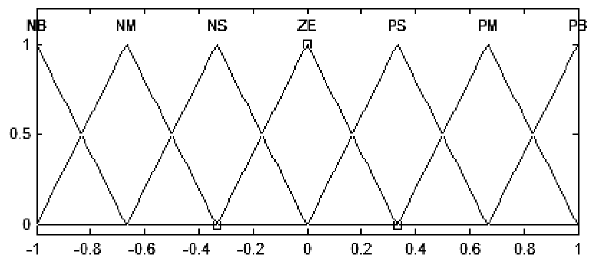
$$U_{fs} = FSMC(S, \dot{S}) \qquad (22)$$

In the Fig. 4, we depict the fuzzy system model which is including a Mamdani fuzzy inference system, system's inputs (S, $\dot{S}$) and output (U_{fs}) membership functions and law database which has a structure like follow:

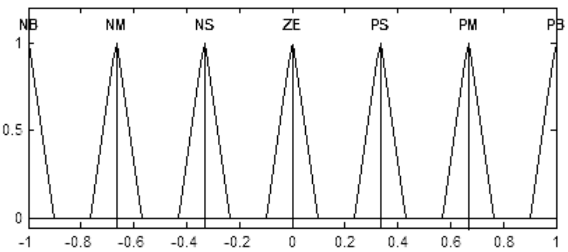
"If S is … and $\dot{S}$ is … Then U_{fs} is …"



(a) Fuzzy system



(b) S and $\dot{S}$ membership function



(c) Output of fuzzy system U_{fs}

U_{fs}		s						
		PB	PM	PS	ZE	NS	NM	NB
$\dot{s}$	PB	NB	NB	NB	NB	NM	NS	ZE
	PM	NB	NB	NB	NM	NS	ZE	PS
	PS	NB	NB	NM	NS	ZE	PS	PM
	ZE	NB	NM	NS	ZE	PS	PM	PB
	NS	NM	NS	ZE	PS	PM	PB	PB
	NM	NS	ZE	PS	PM	PB	PB	PB
	NB	ZE	PS	PM	PB	PB	PB	PB

(d) Law database

Figure 4. Structure of Fuzzy system

Where NB, NM, NS, ZE, PS, PM, PB respectively represent large negative, negative, small negative, zero, small positive, positive, large positive.

Therefore, the control law can be written as:

$$v = V_{eq} + K_{fs} U_{fs} \tag{23}$$

Next we prove the states converge to the sliding surface by applying (23) to Chen's system (2). Therefore, we consider a definite positive Lyapunov function as follow:

$$V = \frac{1}{2} S^2 \tag{24}$$

Now by substituting equations (4), $\dot{y}$ from (2) and controller (23) in derivative of (24) with respect to time, we have:

$$\dot{V} = S\dot{S} = S(\Delta f + K_{fs} U_{fs}) \leq M\|S\|_1 + S(K_{fs} U_{fs}) \tag{25}$$

where $\|.\|_1$ indicate 1-norm.

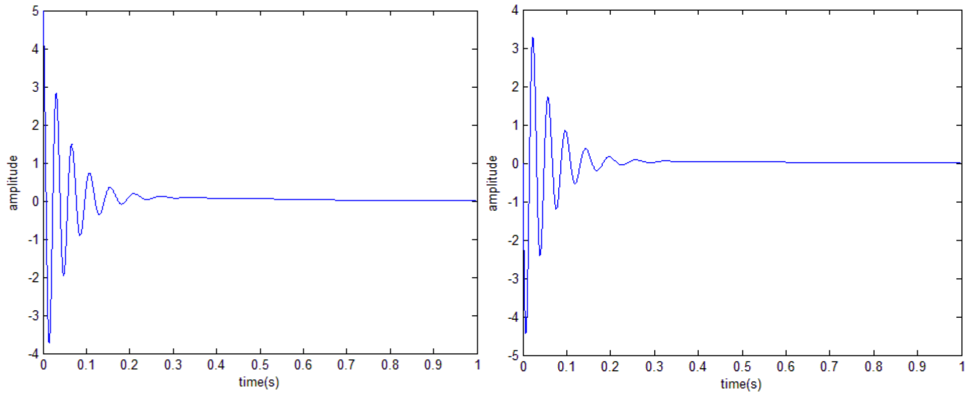
According to the fuzzy rule database (Fig. 4-d), we can write:

$$\dot{V} \leq M\|S\|_1 + K_{fs}(-|S|) \leq (M - K_{fs})\|S\|_1 \tag{26}$$

As we consider K_{fs} is a positive constant and greater than M, the equation (26) is always negative. Therefore, the states converge to the sliding surface. For prove that the states remain on sliding surface see previous proof in section 3.

Now, the controller is applying to hyper chaotic Chen's system. Fig. 5 contains the system output and control signal is shown in Fig. 6.

Regarding to the results, it is clear that chattering is eliminated on sliding surface as proof attend to the controller signal (Fig. 6).



(a) x(t) (b) y(t)

(c) z(t) (d) u(t)

Figure 5. System states variable

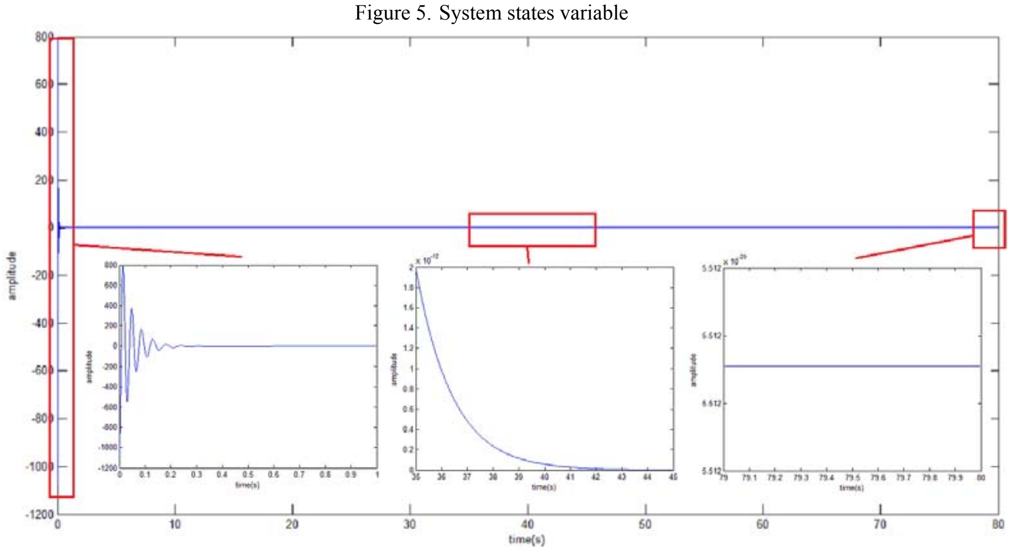


Figure 6. Control signal

V. Conclusion

In this paper, a fuzzy sliding mode control method was proposed for stabilizing hyper chaotic Chen system. The controller was designed based on sliding mode control, and a fuzzy control technique was used in order to eliminate chattering phenomena of designing sliding mode control. It is shown that the equilibrium point of the system is stabilized using the proposed control method. The proposed method is robust against the uncertainty and has good performance.

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