

# A New Filter Design Method for Disturbed Multilayer Hopfield Neural Networks

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**Abstract**—This paper investigates the passivity based filtering problem for multilayer Hopfield neural networks with external disturbance. A new passivity based filter design method for multilayer Hopfield neural networks is developed to ensure that the filtering error system is exponentially stable and passive from the external disturbance vector to the output error vector. The unknown gain matrix is obtained by solving a linear matrix inequality (LMI), which can be easily facilitated by using some standard numerical packages. An illustrative example is given to demonstrate the effectiveness of the proposed filter.

**Index Terms**—passive filtering, multilayer Hopfield neural networks, linear matrix inequality (LMI), external disturbance

## I. INTRODUCTION

Hopfield neural networks are a class of recurrent artificial neural networks invented by John Hopfield [1]. Hopfield neural networks serve as content-addressable memory systems with binary threshold units. In recent years, Hopfield neural networks have been given a lot of attention of many scientists including physicists and computer scientists because of their potential for the tasks of image processing, pattern recognition, and associative memories [2].

In large-scaled neural networks, the partial states of neural networks are often measured in the outputs of neural networks. In order to use neural networks in several applications, we often need to estimate the states of neural networks using measurements and then use the estimated states of neural networks to achieve design objectives [3, 4]. A possible scheme in control application [5] is to use neural networks to build mathematical models from experimental data and then design a nonlinear controller using the estimated states of these neural networks. Learning methods which guarantee exponential error convergence for neural network identifiers already exist. Therefore, from the point of view of control, the state estimation for neural networks is important for several applications. Recently, the state estimation problem for neural networks has received some research interest [3, 4, 6, 7, 8].

The passivity theory [9, 10] plays an important role in electric circuits and nonlinear control systems, provides a useful tool for analyzing the stability of nonlinear systems. The passivity theory was first introduced in the circuit theory and has been applied in several areas such as signal processing, fuzzy control, and chaos synchronization. In [11], a passivity based filter was proposed for delayed neural networks. Now the following question arises: Can we

obtain a passivity based state estimation filter for multilayer neural networks? We now attempt to answer this question. To the authors' best knowledge, the passivity based filtering of multilayer neural networks has not been previously reported.

In this paper, we present a new passivity based state estimation filter for multilayer Hopfield neural networks. This estimation filter is a new contribution in the field of state estimation for neural networks. First, we present a sufficient linear matrix inequality (LMI) condition for the passivity based filtering of multilayer Hopfield neural networks. Then, this filter is shown to ensure that the filtering error system is passive from the external disturbance vector to the output error vector. When there is no the external disturbance vector, this filter guarantees the exponential filtering. The unknown filter gain matrix is determined by solving the LMI, which is facilitated readily by existing numerical algorithms [12, 13].

This paper is organized as follows. In Section II, we present an LMI problem for the passivity based filtering of multilayer Hopfield neural networks. In Section III, a numerical example is given, and finally, conclusions are presented in Section IV.

## II. PASSIVITY BASED FILTER DESIGN FOR MULTILAYER HOPFIELD NEURAL NETWORKS

Consider the following multilayer Hopfield neural network with external disturbance:

$$\dot{x}(t) = Ax(t) + W_2\phi(W_1x(t)) + J(t) + Gw(t), \quad (1)$$

$$y(t) = Cx(t) + Fw(t), \quad (2)$$

where  $x(t) = [x_1(t) \dots x_n(t)]^T \in R^n$  is the state vector,  $y(t) = [y_1(t) \dots y_m(t)]^T \in R^m$  is the output vector,  $w(t) = [w_1(t) \dots w_m(t)]^T \in R^m$  is the external disturbance,

$$A = \text{diag}\{-a_1, \dots, -a_n\} \in R^{n \times n}$$

( $a_k > 0, k = 1, \dots, n$ ) is the self-feedback matrix,

$W_1 \in R^{p \times n}$  and  $W_2 \in R^{n \times p}$  are the connection weight matrices,  $\phi(\cdot) = [\phi_1(\cdot) \dots \phi_p(\cdot)]^T : R^p \rightarrow R^p$  is the nonlinear function vector satisfying the global Lipschitz condition with Lipschitz constant  $L_\phi > 0$ ,  $G \in R^{n \times m}$ ,

$C \in R^{m \times n}$ , and  $F \in R^{m \times m}$  are known constant matrices, and  $J(t) \in R^n$  is an external input vector. For the

multilayer Hopfield neural network (1)-(2), we propose the following filter:

$$\begin{aligned}\dot{\hat{x}}(t) &= A\hat{x}(t) + W_2\phi(W_1\hat{x}(t)) + J(t) \\ &\quad + L(y(t) - \hat{y}(t)),\end{aligned}\quad (3)$$

$$\hat{y}(t) = C\hat{x}(t), \quad (4)$$

where  $\hat{x}(t) = [\hat{x}_1(t) \dots \hat{x}_n(t)]^T \in R^n$  is the state vector of the filter,  $\hat{y}(t) = [\hat{y}_1(t) \dots \hat{y}_m(t)]^T \in R^m$  is the output vector of the filter, and  $L \in R^{n \times m}$  is the gain matrix of the filter to be designed. If we define the filtering errors  $e(t) = x(t) - \hat{x}(t)$  and  $\tilde{y}(t) = y(t) - \hat{y}(t)$ , then the filtering error system can be represented as follows:

$$\begin{aligned}\dot{e}(t) &= (A - LC)e(t) + W_2(\phi(W_1x(t)) - \phi(W_1\hat{x}(t))) \\ &\quad + (G - LF)w(t),\end{aligned}\quad (5)$$

$$\tilde{y}(t) = Ce(t) + Fw(t). \quad (6)$$

The purpose of this paper is to design a suitable filter of the form (3)-(4) for the multilayer Hopfield neural network (1)-(2) for the estimation of the state vector  $x(t)$  in the passivity framework. In other words, find a proper filter such that the filtering error system (5)-(6) with  $w(t) = 0$  is exponentially stable and

$$\int_0^t w^T(\tau) \tilde{y}(\tau) d\tau + \beta \geq \int_0^t \Phi(e(\tau)) d\tau, \forall t \geq 0, \quad (7)$$

where  $\beta$  is a nonnegative constant,  $\tilde{y}(t) = \exp(\kappa t) \tilde{y}(t)$ ,  $\kappa$  is a positive constant, and  $\Phi(e(t))$  is a positive semi-definite storage function.

The following theorem presents an LMI-based criterion for the passivity based filtering.

**Theorem 1.** Assume that there exist matrices  $P = P^T > 0$ ,  $S = S^T > 0$ , and  $M$  such that

$$\begin{bmatrix} PA - MC + (PA - MC)^T + \kappa P + L_\phi^2 W_1^T W_1 + S & (PG - MF)^T - \frac{1}{2}C & W_2^T P \\ & PG - MF - \frac{1}{2}C^T & PW_2 \\ & -F & 0 \\ & 0 & -I \end{bmatrix} < 0. \quad (8)$$

Then the filtering error system (5)-(6) is passive from the external disturbance  $w(t)$  to the output error  $\tilde{y}(t)$  and the gain matrix of the filter (3)-(4) is given by

$$L = P^{-1}M. \quad (9)$$

**Proof.** Consider the following Lyapunov function:

$$V(t) = \exp(\kappa t) e^T(t) P e(t). \quad (10)$$

Calculating the time derivative of  $V(t)$  along the trajectory of the filtering error system (5)-(6), we have

$$\begin{aligned}\dot{V}(t) &= \exp(\kappa t) \dot{e}^T(t) P e(t) + \exp(\kappa t) e^T(t) P \dot{e}(t) \\ &\quad + \kappa \exp(\kappa t) e^T(t) P e(t)\end{aligned}$$

$$\begin{aligned}&= \exp(\kappa t) e^T(t) [(A - LC)^T P + P(A - LC) \\ &\quad + \kappa P] e(t) + \exp(\kappa t) e^T(t) P W_2 (\phi(W_1 x(t)) \\ &\quad - \phi(W_1 \hat{x}(t))) + \exp(\kappa t) (\phi(W_1 x(t)) - \phi(W_1 \hat{x}(t)))^T \\ &\quad \times W_2^T P e(t) + \exp(\kappa t) e^T(t) P (G - LF) w(t) \\ &\quad + \exp(\kappa t) w^T(t) (G - LF)^T P e(t).\end{aligned}\quad (11)$$

Adding and subtracting  $\exp(\kappa t) w^T(t) [Ce(t) + Fw(t)]$ , we obtain

$$\begin{aligned}\dot{V}(t) &= \exp(\kappa t) e^T(t) [(A - LC)^T P + P(A - LC) \\ &\quad + \kappa P] e(t) + \exp(\kappa t) e^T(t) P W_2 (\phi(W_1 x(t)) \\ &\quad - \phi(W_1 \hat{x}(t))) + \exp(\kappa t) (\phi(W_1 x(t)) - \phi(W_1 \hat{x}(t)))^T \\ &\quad \times W_2^T P e(t) + \exp(\kappa t) e^T(t) \left[ P(G - LF) - \frac{1}{2}C^T \right] \\ &\quad \times w(t) + \exp(\kappa t) w^T(t) \left[ (G - LF)^T P - \frac{1}{2}C \right] \\ &\quad \times e(t) - \exp(\kappa t) w^T(t) Fw(t) + \exp(\kappa t) w^T(t) \\ &\quad \times [Ce(t) + Fw(t)].\end{aligned}\quad (12)$$

If we use the inequality  $X^T Y + Y^T X \leq X^T \Lambda X + Y^T \Lambda^{-1} Y$ , which is valid for any matrices  $X \in R^{n \times m}$ ,  $Y \in R^{n \times m}$ ,  $\Lambda = \Lambda^T > 0$ ,  $\Lambda \in R^{n \times n}$ , we have

$$\begin{aligned}&e^T(t) P W_2 (\phi(W_1 x(t)) - \phi(W_1 \hat{x}(t))) + (\phi(W_1 x(t)) \\ &\quad - \phi(W_1 \hat{x}(t)))^T W_2^T P e(t) \\ &\leq (\phi(W_1 x(t)) - \phi(W_1 \hat{x}(t)))^T (\phi(W_1 x(t)) - \phi(W_1 \hat{x}(t))) \\ &\quad + e^T(t) P W_2 W_2^T P e(t) \\ &\leq L_\phi^2 (x(t) - \hat{x}(t))^T W_1^T W_1 (x(t) - \hat{x}(t)) + e^T(t) P W_2 W_2^T \\ &\quad \times P e(t) \\ &= L_\phi^2 e^T(t) W_1^T W_1 e(t) + e^T(t) P W_2 W_2^T P e(t).\end{aligned}\quad (13)$$

Using (13), we obtain

$$\begin{aligned}\dot{V}(t) &\leq \exp(\kappa t) e^T(t) [(A - LC)^T P + P(A - LC) \\ &\quad + \kappa P + P W_2 W_2^T P + L_\phi^2 W_1^T W_1] e(t) \\ &\quad + \exp(\kappa t) e^T(t) \left[ P(G - LF) - \frac{1}{2}C^T \right] w(t) \\ &\quad + \exp(\kappa t) w^T(t) \left[ (G - LF)^T P - \frac{1}{2}C \right] e(t) \\ &\quad - \exp(\kappa t) w^T(t) Fw(t) + \exp(\kappa t) w^T(t) [Ce(t) \\ &\quad + Fw(t)] = \exp(\kappa t) \begin{bmatrix} e(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} (1, 1) \\ (G - LF)^T P - \frac{1}{2}C \end{bmatrix} \\ &\quad \times \begin{bmatrix} P(G - LF) - \frac{1}{2}C^T \\ -F \end{bmatrix} \begin{bmatrix} e(t) \\ w(t) \end{bmatrix} - \exp(\kappa t) \\ &\quad \times e^T(t) S e(t) + \exp(\kappa t) w^T(t) [Ce(t) + Fw(t)],\end{aligned}\quad (14)$$

where

$$(1,1) = (A - LC)^T P + P(A - LC) + \kappa P + PW_2 W_2^T P + L_\phi^2 W_1^T W_1 + S.$$

If the following matrix inequality is satisfied:

$$\begin{bmatrix} (1,1) & P(G - LF) - \frac{1}{2}C^T \\ (G - LF)^T P - \frac{1}{2}C & -F \end{bmatrix} < 0, \quad (15)$$

we have

$$\begin{aligned} \dot{V}(t) &< -\exp(\kappa t)e^T(t)Se(t) + \exp(\kappa t)w^T(t)[Ce(t) + Fw(t)] \\ &= -\exp(\kappa t)e^T(t)Se(t) + w^T(t)\bar{y}(t). \end{aligned} \quad (16)$$

Integrating both sides of (16) from 0 to  $t$  gives

$$\begin{aligned} V(t) - V(0) &< -\int_0^t \exp(\kappa \sigma)e^T(\sigma)Se(\sigma)d\sigma \\ &\quad + \int_0^t w^T(\sigma)\bar{y}(\sigma)d\sigma. \end{aligned} \quad (17)$$

Let  $\beta = V(0)$ . Since  $V(t) \geq 0$ ,

$$\begin{aligned} &\int_0^t w^T(\sigma)\bar{y}(\sigma)d\sigma + \beta \\ &> \int_0^t \exp(\kappa \sigma)e^T(\sigma)Se(\sigma)d\sigma + V(t) \\ &\geq \int_0^t \exp(\kappa \sigma)e^T(\sigma)Se(\sigma)d\sigma. \end{aligned} \quad (18)$$

The relation (18) satisfies (7). Therefore, the filtering error system (5)-(6) is rendered to be passive from the external disturbance  $w(t)$  to the output error  $\bar{y}(t)$  under the filter (3)-(4). From Schur complement, the matrix inequality (15) is equivalent to

$$\begin{bmatrix} \{1,1\} & P(G - LF) - \frac{1}{2}C^T & PW_2 \\ (G - LF)^T P - \frac{1}{2}C & -F & 0 \\ W_2^T P & 0 & -I \end{bmatrix} < 0, \quad (19)$$

where  $\{1,1\} = (A - LC)^T P + P(A - LC) + \kappa P + S + L_\phi^2 W_1^T W_1$ . If we let  $M = PL$ , (19) is equivalently changed into the LMI (8). Then the filter gain matrix is given by  $L = P^{-1}M$ . This completes the proof.

**Corollary 1.** Without the external disturbance, the filtering error system (5)-(6) is exponentially stable.

**Proof.** When  $w(t) = 0$ , we obtain

$$\dot{V}(t) < -\exp(\kappa t)e^T(t)Se(t) \quad (20)$$

from (16). That is,  $\dot{V}(t) < 0$  for all  $e(t) \neq 0$ . Thus, it implies that  $V(t) < V(0)$  for any  $t \geq 0$ . In addition, from (10), one has

$$\begin{aligned} V(t) &< V(0) \\ &= e^T(0)Pe(0). \end{aligned} \quad (21)$$

Also, we have

$$V(t) \geq \lambda_{\min}(P)\exp(\kappa t)\|e(t)\|^2, \quad (22)$$

where  $\lambda_{\min}(P)$  is the minimum eigenvalue of the matrix  $P$ . It follows immediately from (21) and (22) that

$$\begin{aligned} \|e(t)\| &< \sqrt{\frac{e^T(0)Pe(0)}{\lambda_{\min}(P)\exp(\kappa t)}} \\ &= \sqrt{\frac{e^T(0)Pe(0)}{\lambda_{\min}(P)}} \exp\left(-\frac{\kappa}{2}t\right). \end{aligned} \quad (23)$$

Since

$$\sqrt{\frac{e^T(0)Pe(0)}{\lambda_{\min}(P)}} > 0, \quad \frac{\kappa}{2} > 0,$$

the exponential stability of the filtering error system (5)-(6) is guaranteed. This completes the proof.

**Remark 1.** Various efficient convex optimization algorithms can be used to check whether the LMI (8) is feasible. In this paper, in order to solve the LMI, we utilize MATLAB LMI Control Toolbox [13], which implements state-of-the-art interior-point algorithms.

### III. APPLICATION TO STATE ESTIMATION PROBLEM

Consider the following nonlinear system:

$$\begin{aligned} \dot{x}_1(t) &= -1.2x_1(t) + x_2(t) - \sin(x_1(t) + x_2(t)) + w(t), \\ \dot{x}_2(t) &= -4.5x_2(t) + 1.5\cos(x_1(t))\sin(x_2(t)) + w(t), \\ y(t) &= x_1(t) + w(t), \end{aligned}$$

where  $w(t)$  is a Gaussian noise with mean 0 and variance 1. This system can be represented by the multilayer Hopfield neural network (1)-(2) with the following parameters:

$$\begin{aligned} x(t) &= \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \phi(W_1 x(t)) = \begin{bmatrix} \tanh(W_1 x(t)) \\ \tanh(W_1 x(t)) \\ \tanh(W_1 x(t)) \end{bmatrix}, \\ A &= \begin{bmatrix} -2.2 & 0 \\ 0 & -3.5 \end{bmatrix}, J(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, G = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}, F = 1. \end{aligned}$$

For the weights training, state vectors were replaced by uniformly distributed random numbers in  $[-1, 1]$ . After 5000 training steps using the backpropagation algorithm, the weights converge to

$$W_1 = \begin{bmatrix} 0.1676 & 0.0925 \\ 0.1339 & 0.7284 \\ 0.6588 & 0.4299 \end{bmatrix}, W_2 = \begin{bmatrix} 0.7359 & 0.1864 & 0.1392 \\ 0.4484 & -0.6187 & 0.0229 \end{bmatrix}.$$

By applying Theorem 1 via the Matlab LMI Control Toolbox [13], the gain matrix of the filter (3)-(4) is obtained as

$$L = \begin{bmatrix} 0.3260 \\ 1.0046 \end{bmatrix}.$$

When the initial conditions are given by

$$x(0) = \begin{bmatrix} -1.2 \\ 1.5 \end{bmatrix}, \quad \hat{x}(0) = \begin{bmatrix} 1.4 \\ -2.1 \end{bmatrix},$$

the simulation results for the passivity based filter design are shown in Figures 1-3. Figures 1 and 2 show the true

states  $x_1(t)$  and  $x_2(t)$  and their estimations  $\hat{x}_1(t)$  and  $\hat{x}_2(t)$ , respectively, and Figure 3 shows the responses of the filtering error  $e(t)$ . The simulation results confirm that the proposed filter reduces the effect of the external disturbance  $w(t)$  on the filtering error  $e(t)$ .

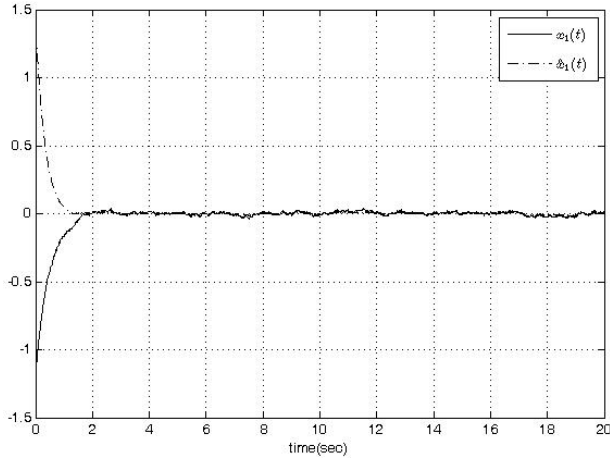


Figure 1: Responses of the state  $x_1(t)$  and its estimation  $\hat{x}_1(t)$

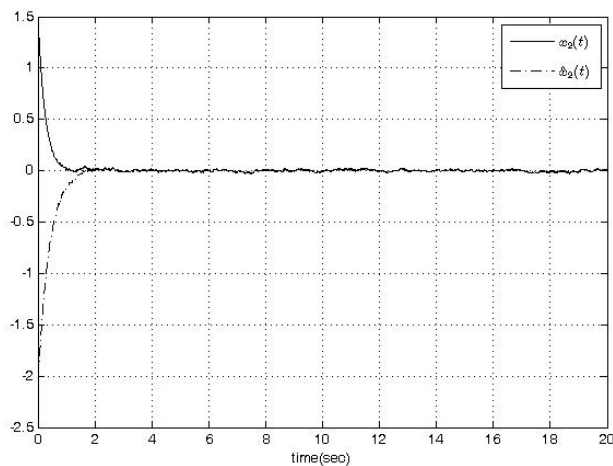


Figure 2: Responses of the state  $x_2(t)$  and its estimation  $\hat{x}_2(t)$

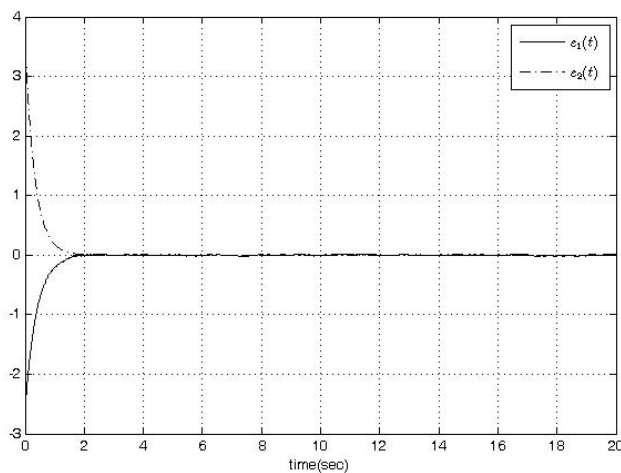


Figure 3: Responses of the filtering error  $e(t)$

#### IV. CONCLUSION

In this paper, we have proposed a new passivity based state estimation filter for multilayer Hopfield neural networks. This passivity based filter was shown to ensure that the filtering error system is passive from the external disturbance vector to the output error vector. The exponential filtering was guaranteed without the external disturbance vector. We obtained the gain matrix of the proposed state estimation filter solving the LMI. A numerical example was given to show the effectiveness of the proposed state estimation filter.

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