

# Consensus of Mobile Robots Under Markovian Communication

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**Abstract** For mobile robots moving in a plane, the mean square consensus problem is investigated under Markovian communication of partly known transition probabilities. Based on linear matrix inequalities, bisection search and numerical optimization, a design method is presented of feedback gains guaranteeing mean square consensus.

**Keywords** Mean Square Consensus, Mobile Robot, Markov Chain, Partly Known Transition Probabilities

## 1. Introduction

The consensus problem of mobile robots (or multi-agents) by information exchange is very important in engineering, and therefore, over the past few decades, it has attracted much research attention. In [1,2], Olfati-Saber and Murray introduced theoretical framework for solving the consensus problem. In [3,4], the consensus problems of the first-order integrator system with communication of undirected switching topology were proposed. In [5], for the single integrator system with communication of directed switching topology, it has been demonstrated that consensus is reached when the graph has the spanning tree. Hatano and Mesbahi [6]

proposed the agreement problem above the random information exchange network. In [7], Porfiri and Stilwell provided sufficient conditions for reaching consensus almost certainly in the case of a linear system where the communication flow is given by a directed graph derived from a random graph process. Under a similar model of communication topology, Zhang [8] presented the necessary and sufficient conditions of the mean square consensus of double-integrator agents with stochastic switching communication topology.

For systems whose dynamics change in various possible scenarios, the Markovian jump system is an effective description. Recently, some interesting results with regard to Markovian jump systems have been presented on uncertain transition probabilities [10-12]. Noticing that the Markov chain can be used to describe the stochastic switching of communication among mobile robots, this paper addresses the mean square consensus problem of mobile robots under the Markovian communication of partly known transition probabilities. An sufficient condition of this problem is provided and the corresponding design algorithm is given.

In this paper,  $\mathbb{Z}^+$  is used to denote the set of all nonnegative integers. The  $n \times n$  real identity matrix is

denoted by  $I_n$ . The Euclidean norm is denoted by  $\|\cdot\|$ . If a matrix  $P$  is positive (negative) definite, it is denoted by  $P>0(<0)$ . The Kronecker product is represented by  $\otimes$  and the expected value is represented by  $E[\cdot]$ .

The remainder of the paper is organized as follows. Section 2 describes a mobile robot system and its consensus problem. The condition and algorithm for the mean square consensus problem is derived in Section 3. Section 4 provides the numerical simulation results and Section 5 draws conclusions.

## 2. Mobile robots system and its consensus problem

Consider  $n$  mobile robots moving in a plane. At time  $t \in [0, \infty)$ , the state of the  $i$ th ( $i \in \{1, \dots, n\}$ ) robot is

$$z_{ci}(t) = \begin{bmatrix} p_{xi}(t) \\ \dot{p}_{xi}(t) \\ p_{yi}(t) \\ \dot{p}_{yi}(t) \end{bmatrix} \in \mathbb{R}^4$$

where  $(p_{xi}(t), p_{yi}(t))$  and  $(\dot{p}_{xi}(t), \dot{p}_{yi}(t))$  represent position and velocity, respectively. Accordingly, the dynamics of the  $i$ th robot is modelled as

$$\begin{aligned} \dot{z}_{ci}(t) &= \begin{bmatrix} \dot{p}_{xi}(t) \\ \ddot{p}_{xi}(t) \\ \dot{p}_{yi}(t) \\ \ddot{p}_{yi}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_{xi}(t) \\ \dot{p}_{xi}(t) \\ p_{yi}(t) \\ \dot{p}_{yi}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{p}_{xi}(t) \\ \ddot{p}_{yi}(t) \end{bmatrix} \\ &= A_{ci} z_{ci}(t) + B_{ci} u_{ci}(t) \end{aligned} \quad (1)$$

where acceleration  $(\ddot{p}_{xi}(t), \ddot{p}_{yi}(t))$  acts as the control input  $u_{ci}(t)$ . Suppose that these mobile robots are controlled in the sampled-data system framework [16] of zero-order holders and a given sampling period  $h>0$ . Thus, the discrete time model of the  $i$ th robot is obtained as

$$z_i(k+1) = A_i z_i(k) + B_i u_i(k), \quad k \in \mathbb{Z}^+ \quad (2)$$

with

$$\begin{aligned} z_i(k) &= z_{ci}(kh) \\ u_i(k) &= u_{ci}(kh) \\ A_i &= e^{A_{ci}h} = \begin{bmatrix} 1 & h & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ B_i &= \int_0^h e^{A_{ci}(h-\tau)} B_{ci} d\tau = \begin{bmatrix} \frac{h^2}{2} & 0 \\ h & 0 \\ 0 & \frac{h^2}{2} \\ 0 & h \end{bmatrix} \end{aligned}$$

The communication situation among these  $n$  robots is described by matrix

$$D(k) = \begin{bmatrix} d_{11}(k) & \dots & d_{1n}(k) \\ \vdots & \ddots & \vdots \\ d_{n1}(k) & \dots & d_{nn}(k) \end{bmatrix} \in \mathbb{R}^{n \times n}, \quad k \in \mathbb{Z}^+ \quad (3)$$

where  $d_{ij}(k) \in \{0, 1\}$ :  $d_{ij}(k)=1$  means that  $z_j(k)$  is known by the  $i$ th robot at  $kh$ , while  $d_{ij}(k)=0$  means that  $z_j(k)$  is unknown by the  $i$ th robot at  $kh$ . Given  $m$  constant matrix  $D_1, \dots, D_m \in \mathbb{R}^{n \times n}$ . The entries of these matrices take value from  $\{0, 1\}$ . It is assumed that  $D(k)$  randomly varies in  $\{D_1, \dots, D_m\}$  as  $k$  increases, i.e.,  $D(k)$  can be expressed as  $D_{r_k}$  with a stochastic process  $r_k$  whose state space is  $\{1, \dots, m\}$ . In this paper,  $r_k$  is typically assumed to be a discrete time homogeneous Markovian chain of partly known transition probabilities. That is to say, some entries are inaccessible for the transition probabilities matrix  $\Gamma = (\gamma_{ls}) \in \mathbb{R}^{m \times m}$  with  $\gamma_{ls} = \Pr(r_{k+1} = s | r_k = l)$ . For example, when  $m=4$ ,  $\Gamma$  may be

$$\begin{bmatrix} ? & \gamma_{12} & \gamma_{13} & ? \\ ? & \gamma_{22} & ? & \gamma_{24} \\ \gamma_{31} & ? & \gamma_{33} & ? \\ \gamma_{41} & ? & ? & \gamma_{44} \end{bmatrix}$$

where '?' represents the inaccessible entries. For  $l = \{1, \dots, m\}$ , define

$$U_l \triangleq \{s : \gamma_{ls} \text{ is known}\} \subset \{1, \dots, m\}$$

$$W_l = \{1, \dots, m\} \setminus U_l$$

The task of these  $n$  mobile robots is to go to a prescribed target point  $(p_x^*, p_y^*)$ . Among the  $n$  robots, only the 1st robot knows  $(p_x^*, p_y^*)$ . As the leading robot, the 1st robot adopts the following control law

$$u_1(k) = -G \left( \sum_{j=1}^n d_{1j}(k) (z_j(k) - z_1(k)) - (z_1(k) - z^*) \right) \quad (4)$$

with  $z^* = \begin{bmatrix} p_x^* & 0 & p_y^* & 0 \end{bmatrix}^T \in \mathbb{R}^4$  and the feedback gain matrix

$$G = \begin{bmatrix} k_1 & k_2 & 0 & 0 \\ 0 & 0 & k_1 & k_2 \end{bmatrix} \in \mathbb{R}^{2 \times 4} \quad (5)$$

The other robots unaware of  $(p_x^*, p_y^*)$  have to utilize a control law without  $z^*$

$$u_i(k) = -G \sum_{j=1}^n d_{ij}(k) (z_j(k) - z_i(k)), \quad \forall i \in \{2, \dots, n\} \quad (6)$$

The mobile robots system is said to reach mean square consensus if  $\forall i \in \{1, \dots, n\}, \forall z_i(0) \in \mathbb{R}^4, \forall r_0 \in \{1, \dots, m\}, \lim_{k \rightarrow \infty} E \|z_i(k) - z^*\|^2 = 0$ . This paper aims to design  $k_1$  and

$k_2$  such that the  $n$  robots reach mean square consensus.

### 3. Main result

#### 3.1 A Sufficient Condition of Mean Square Consensus

For the mobile robot system, from (2), (4), (5) and (6), we can easily see that its discrete time dynamics and control in  $x$ -direction are the same as that in the  $y$ -direction. Therefore an investigation in  $x$ -direction is enough. For  $i \in \{1, \dots, n\}$ , denote

$$e_i(k) = \begin{bmatrix} p_{xi}(kh) \\ \dot{p}_{xi}(kh) \end{bmatrix} - \begin{bmatrix} p_x^* \\ 0 \end{bmatrix} \in \mathbb{R}^2 \quad (7)$$

From (2), (4) and (7), we can obtain

$$e_1(k+1) = \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} e_1(k) - \begin{bmatrix} \frac{h^2}{2} \\ h \end{bmatrix} \left[ k_1 \quad k_2 \right] \left( \sum_{j=1}^n d_{1j}(r_k) (e_j(k) - e_1(k)) - e_1(k) \right) \quad (8)$$

From (2), (6) and (7), we can obtain  $\forall i \in \{2, \dots, n\}$

$$e_i(k+1) = \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} e_i(k) - \begin{bmatrix} \frac{h^2}{2} \\ h \end{bmatrix} \left[ k_1 \quad k_2 \right] \sum_{j=1}^n d_{ij}(r_k) (e_j(k) - e_i(k)) \quad (9)$$

Now, consider all the  $n$  robots together. Define

$$e(k) = \begin{bmatrix} e_1^T(k) & e_2^T(k) & \dots & e_n^T(k) \end{bmatrix}^T \in \mathbb{R}^{2n} \quad (10)$$

A combination of (8), (9) and (10) leads to

$$e(k+1) = (A - BKC_{r_k})e(k) \triangleq F(k_1, k_2, r_k)e(k) \quad (11)$$

where

$$A = I_n \otimes \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} \in \mathbb{R}^{2n \times 2n}$$

$$B = I_n \otimes \begin{bmatrix} \frac{h^2}{2} \\ h \end{bmatrix} \in \mathbb{R}^{2n \times n}$$

$$K = I_n \otimes \begin{bmatrix} k_1 & k_2 \end{bmatrix} \in \mathbb{R}^{n \times 2n}$$

$$C_{r_k} = D_{r_k} \otimes I_2$$

$$- \text{diag} \left( 1 + \sum_{j=1}^n d_{1j}(k), \sum_{j=1}^n d_{2j}(k), \dots, \sum_{j=1}^n d_{nj}(k) \right) \otimes I_2$$

It is noticed that  $C_{r_k}$  driven by  $r_k$  varies in the set  $\{C_1, \dots, C_m\}$  which consists of  $m$  constant matrices. For any  $l \in \{1, \dots, m\}$ ,  $C_l \in \mathbb{R}^{2n \times 2n}$  can be calculated from  $D_l$ .

Theorem 1: The robot system described in (2), (4), (5) and (6) reaches mean square consensus under the feedback gains  $k_1$  and  $k_2$  if  $m$  positive definite matrices  $P_1, \dots, P_m \in \mathbb{R}^{2n \times 2n}$  and a real number  $\beta \geq 1$  exist such that  $\forall l \in \{1, \dots, m\}$ ,

$$\beta F^T(k_1, k_2, l) \sum_{s \in U_l} \gamma_{ls} P_s F(k_1, k_2, l) - \sum_{s \in U_l} \gamma_{ls} P_l < 0, \quad (12)$$

$$\beta F^T(k_1, k_2, l) P_s F(k_1, k_2, l) - P_l < 0, \quad s \in W_l \quad (13)$$

Proof: Conditions (12) and (13) imply  $\forall l \in \{1, \dots, m\}$ ,

$$F^T(k_1, k_2, l) \sum_{s \in U_l} \gamma_{ls} P_s F(k_1, k_2, l) - \sum_{s \in U_l} \gamma_{ls} P_l < 0$$

$$F^T(k_1, k_2, l) P_s F(k_1, k_2, l) - P_l < 0, \quad s \in W_l$$

and hence

$$\eta_l = \lambda_{\max} \left[ F^T(k_1, k_2, l) \sum_{s \in U_l} \gamma_{ls} P_s F(k_1, k_2, l) - \sum_{s \in U_l} \gamma_{ls} P_l \right] < 0$$

$$\xi_l = \max_{s \in W_l} \left( \lambda_{\max} \left[ F^T(k_1, k_2, l) P_s F(k_1, k_2, l) - P_l \right] \right) < 0$$

Consider the stochastic Lyapunov function

$$V(e(k), r_k) = e^T(k) P_{r_k} e(k)$$

Then  $\forall r_k = l \in \{1, \dots, m\}$ , we have

$$\begin{aligned} & E[V(e(k+1), r_{k+1}) | e(k), r_k] - V(e(k), r_k) \\ &= e^T(k) \left( F^T(k_1, k_2, l) \sum_{s \in \{1, \dots, m\}} \gamma_{ls} P_s F(k_1, k_2, l) - P_l \right) e(k) \\ &= e^T(k) \left( F^T(k_1, k_2, l) \sum_{s \in U_l} \gamma_{ls} P_s F(k_1, k_2, l) - \sum_{s \in U_l} \gamma_{ls} P_l \right) e(k) \\ &\quad + e^T(k) \left( F^T(k_1, k_2, l) \sum_{s \in W_l} \gamma_{ls} P_s F(k_1, k_2, l) - \sum_{s \in W_l} \gamma_{ls} P_l \right) e(k) \\ &\leq \eta_l \|e(k)\|^2 + \sum_{s \in W_l} \gamma_{ls} \xi_l \|e(k)\|^2 \\ &\leq -\rho \|e(k)\|^2 \end{aligned}$$

where

$$\rho = \left| \max_{l \in \{1, \dots, m\}} \left( \eta_l + \left( 1 - \sum_{s \in U_l} \gamma_{ls} \right) \xi_l \right) \right| > 0.$$

Therefore,  $\forall N \in \{1, 2, 3, \dots\}$ ,  $\forall e(0) \in \mathbb{R}^2$ ,  $\forall r_0 \in \{1, \dots, m\}$

$$\begin{aligned} & \sum_{k=0}^N E[\|e(k)\|^2] \\ &\leq \frac{1}{\rho} \sum_{k=0}^N (E[V(e(k), r_k)] - E[V(e(k+1), r_{k+1})]) \\ &\leq \frac{1}{\rho} (V(e(0), r_0) - E[V(e(N+1), r_{N+1})]) \\ &\leq \frac{1}{\rho} V(e(0), r_0) \end{aligned}$$

which means

$$\lim_{k \rightarrow \infty} E \left[ \|e(k)\|^2 \right] = 0$$

Since the  $y$ -direction model is the same as the  $x$ -direction model, the motion in  $y$ -direction is also convergent. Thus we conclude

$$\lim_{k \rightarrow \infty} E \left[ \left\| \begin{bmatrix} z_1(k) - z^* \\ \vdots \\ z_n(k) - z^* \end{bmatrix} \right\|^2 \right] = 0.$$

### 3.2 Design Method

Given a pair  $(k_1, k_2)$ , let the conditions in Theorem 1

$$\beta F^T(k_1, k_2, l) \sum_{s \in U_l} \gamma_{ls} P_s F(k_1, k_2, l) - \sum_{s \in U_l} \gamma_{ls} P_l < 0$$

$$\beta F^T(k_1, k_2, l) P_s F(k_1, k_2, l) - P_l < 0, \quad s \in W_l$$

$$0 \leq \beta \in \mathbb{R}, \quad 0 < P_i \in \mathbb{R}^{2n \times 2n}, \quad i \in \{1, \dots, m\}, \quad l \in \{1, \dots, m\}$$

be represented by  $L(k_1, k_2, \beta) < 0$ . Obviously, when  $\beta$  is fixed,  $L(k_1, k_2, \beta) < 0$  is a linear matrix inequality (LMI) which can be solved efficiently. For any a pair of  $(k_1, k_2)$ , define

$$\bar{\beta}(k_1, k_2) = \sup_{L(k_1, k_2, \beta) < 0} \beta$$

Based on the LMI technique and bisection search, we provide a calculation method of  $\bar{\beta}(k_1, k_2)$  as follows.

- Step a) Set a precision  $\varepsilon > 0$ , a sufficiently large  $\beta_{\max}$  such that  $L(k_1, k_2, \beta_{\max}) < 0$  has no solution and a sufficiently small  $\beta_{\min} > 0$  such that  $L(k_1, k_2, \beta_{\min}) < 0$  has solutions.
- Step b) Let  $\beta_{\text{temp}} = (\beta_{\min} + \beta_{\max}) / 2$ , solve  $L(k_1, k_2, \beta_{\text{temp}}) < 0$ .
- Step c) If  $L(k_1, k_2, \beta_{\text{temp}}) < 0$  has solutions,  $\beta_{\min} = \beta_{\text{temp}}$ ; if  $L(k_1, k_2, \beta) < 0$  has no solution,  $\beta_{\max} = \beta_{\text{temp}}$ .
- Step d) If  $\beta_{\max} - \beta_{\min} < \varepsilon$ ,  $\bar{\beta}(k_1, k_2) = \beta_{\min}$  and terminate the algorithm; if  $\beta_{\max} - \beta_{\min} \geq \varepsilon$ , go to Step b).

From Theorem 1, it is known that those  $(k_1, k_2)$ s whose  $\bar{\beta}(k_1, k_2) > 1$  can guarantee the mean square consensus of the mobile robot system. Roughly speaking, in these  $(k_1, k_2)$ s whose  $\bar{\beta}(k_1, k_2) > 1$ , a pair of  $(k_1, k_2)$  with a large  $\bar{\beta}$  produce better robustness which is a capability of the system of retaining mean square consensus under uncertainties such as parameter perturbances and noises. Therefore, we want to find the pair of  $(k_1, k_2)$  which have the largest  $\bar{\beta}$ , i.e., to solve the optimization problem

$$\mu = \sup_{\substack{k_1 \in \mathbb{R} \\ k_2 \in \mathbb{R}}} \bar{\beta}(k_1, k_2). \quad (14)$$

Problem (14) is an unconstrained nonlinear optimization problem of only two variables. It is solved in this paper by the following method. At first, in the  $k_1$ - $k_2$  plane, the designer gives a rectangle  $[k_1, \bar{k}_1] \times [k_2, \bar{k}_2]$  in which the optimal solution lies. Subsequently, give a positive integer  $N$  (e.g.,  $N = 500$ ) and determine the set

$$X = \left\{ (k_1, k_2) \begin{cases} k_1 = \underline{k}_1 + i(\bar{k}_1 - \underline{k}_1) / N, i \in \{0, 1, \dots, N\} \\ k_2 = \underline{k}_2 + j(\bar{k}_2 - \underline{k}_2) / N, j \in \{0, 1, \dots, N\} \end{cases} \right\}$$

containing  $(N+1)^2$  points. Then, calculate  $\bar{\beta}$  for every point of  $X$  and obtain

$$(k_1^*, k_2^*) = \arg \max_{(k_1, k_2) \in X} \bar{\beta}(k_1, k_2).$$

Finally, with  $(k_1^*, k_2^*)$  as the initial guess, solve (14) using the BFGS Quasi-Newton algorithm [19] and acquire the optimal solution  $(k_{1\text{opt}}, k_{2\text{opt}})$ .

### 4. A numerical example

In the numerical example,  $n=6$ ,  $h=0.1s$ ,  $m=4$ ,

$$D_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad D_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Figure 1 illustrates the four communication situations corresponding to  $D_1 \sim D_4$ . An arrow from the  $i$ th to the  $j$ th one in Figure 1 represents  $d_{ij} = 1$ . The partly unknown transition probabilities of  $r_k$  are

$$\Gamma = \begin{bmatrix} 0.3 & ? & 0.04 & ? \\ 0.52 & 0.4 & ? & ? \\ ? & ? & 0.03 & 0.5 \\ 0.3 & ? & ? & 0.64 \end{bmatrix}.$$

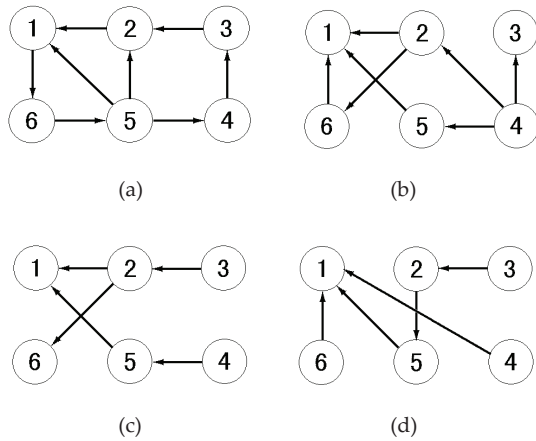


Figure 1. Four communication situations

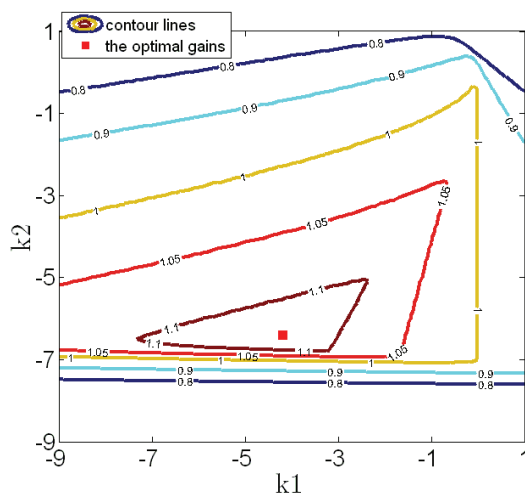


Figure 2. Contour lines of  $\bar{J}$  in  $k_1$ - $k_2$  plane

Using the calculation steps in Section 3,  $\bar{J}$  s of different  $(k_1, k_2)$  are obtained and displayed in Figure 2. The optimization problem (14) is solved. The result is  $\mu = 1.1270 > 1$  and

$$(k_{1opt}, k_{2opt}) = (-4.18, -6.43)$$

which is also displayed in Figure 2. With target  $(p_x^*, p_y^*) = (183, 152)$  and initial states selected randomly, the motion of six robots is simulated under transition probabilities

$$\Gamma_1 = \begin{bmatrix} 0.3 & 0.25 & 0.04 & 0.41 \\ 0.52 & 0.4 & 0.05 & 0.03 \\ 0.4 & 0.07 & 0.03 & 0.5 \\ 0.3 & 0.05 & 0.01 & 0.64 \end{bmatrix},$$

$$\Gamma_2 = \begin{bmatrix} 0.3 & 0.36 & 0.04 & 0.3 \\ 0.52 & 0.4 & 0.01 & 0.07 \\ 0.2 & 0.27 & 0.03 & 0.5 \\ 0.3 & 0.04 & 0.02 & 0.64 \end{bmatrix},$$

respectively. Figure 3 and Figure 4 show the motion. It can be seen that these robots move to the target point under either  $\Gamma_1$  or  $\Gamma_2$ .

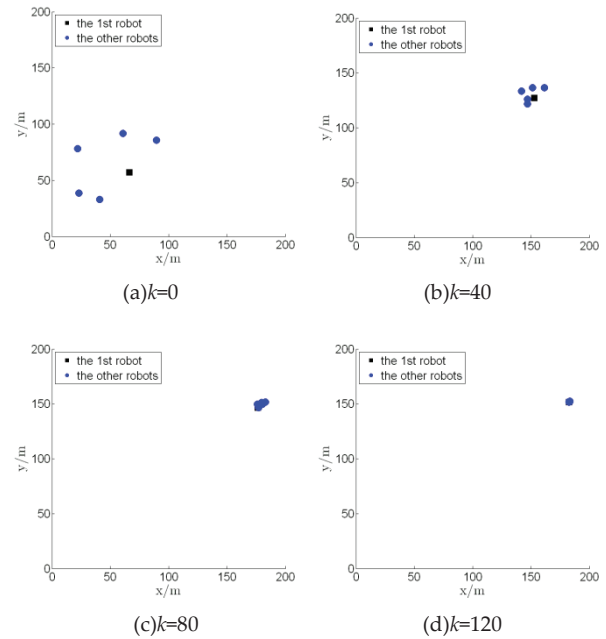


Figure 3. Motion of six robots under  $\Gamma_1$

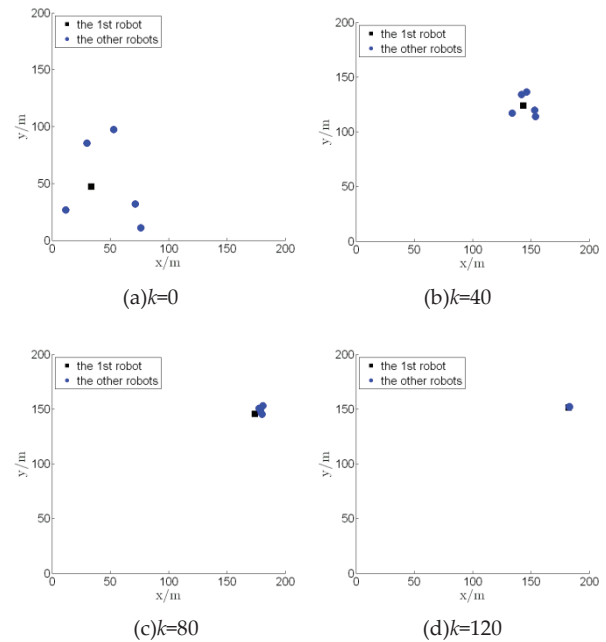


Figure 4. Motion of six robots under  $\Gamma_2$

## 5. Conclusion

The consensus problem of a mobile robot system has been studied under a Markovian switching communication topology of partly known transition probabilities. The control input of each robot depends on the information exchange among robots. A stochastic Lyapunov function has been employed to investigate the mean square consensus of mobile robots, and the controller design

problem has been solved by using numerical optimization techniques.

## 6. Acknowledgments

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