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Interval arithmetic-based fuzzy discrete-time crane control scheme design

J. SMOCZEK*

AGH University of Science and Technology, Faculty of Mechanical Engineering and Robotics,
30 Mickiewicza Ave., 30-059 Cracow, Poland

Abstract. In many manufacturing segments, container terminals and shipping yards the automation of material handling systems is an important element of enhancing productivity, safety and efficiency. The fast, precise and safe transfer of goods in crane operations requires a control application solving the problems, including non-collision trajectory planning and limitation of payload oscillations. The paper presents the interval arithmetic-based method of designing a discrete-time closed-loop anti-sway crane control system based on the fuzzy interpolation of linear controller parameters. The interval analysis of a closed-loop control system characteristic polynomial coefficients deviation from their nominal values is proposed to define a minimum number of fuzzy sets on the scheduling variables universe of discourse and to determine the distribution of triangular-shaped membership functions parameters, which satisfy the acceptable range of performances deterioration in the presence of the system's parameters variation. The effectiveness of this method was proved in experiments conducted using the PAC system on the laboratory scaled overhead crane.

Key words: interval arithmetic, discrete-time control, fuzzy control, pole placement, anti-sway control.

1. Introduction

Cranes, which can be classified into different types (e.g. container cranes, overhead cranes, tower cranes, jib cranes) are widely used for shifting all kinds of cargos in various areas. The automation of cranes operations is very important owing to necessity of ensuring the safety and efficiency of the transportation process, that is involved by requirements of enhancing the productivity of manufacturing processes [1, 2]. Those requirements motivate scientists and engineers to develop and implement solutions for the crane control system which faces up to the following problem: transfer a payload as fast as possible from point to point with avoidance of collision with obstacles, and precise positioning at a final point with reduction of sway of a payload suspended at the end of a rope.

The most attention in literature is focused on the problem of crane positioning and sway of a payload reduction [3, 4]. The best known industrial applications are the open-loop control systems applying mostly the input shaping techniques. Other approaches to crane control are based on the time-optimal control theory [5, 6] combining also the feedback control schemes for desired motion trajectory tracking [7, 8], sliding mode techniques [9], a direct or indirect adaptive control scheme, Lyapunov techniques employed for state-feedback controller design, gain-scheduling, and Linear Quadratic Gausian robust control [3, 4]. Furthermore, the soft computing techniques, especially fuzzy logic, are widely employed to the considered problem. The linguistic-rule-based fuzzy controllers are reported in [10–12], as well as proposed for PID gains tuning [13, 14], or sliding mode control [15]. TSK-type fuzzy controllers are proposed in [16–18].

Some researchers have adopted off-line or on-line techniques to design the fuzzy rule-based controller implemented in a crane control scheme. Membership functions tuning techniques are elaborated based on an inverse dynamic [19], gradient algorithm [20], genetic algorithm [21–23], fuzzy clustering methods [16, 17], and through applying artificial neural network [16, 24].

The most of the fuzzy logic-based approaches to an anti-sway crane control problem, which are described in the literature are linguistic rule-based strategies. The proposed techniques of fuzzy controller designing are mostly applied for only tuning the membership functions parameters for a fixed number of rules involving the training data examples. The robustness of the crane control system is also frequently analysed taking into account only the rope length variation. Furthermore, most of the control strategies are only proved using mathematical models or mechatronic laboratory models. Thus, there is still place for those researchers who are looking for efficient control laws, software-hardware solutions and measurement equipment of crane control systems that could be implemented in industrial practice.

In this paper, the interval arithmetic-based iterative method is proposed for synthesis a fuzzy logic-based interpolation control scheme adjusting the discrete-time controllers parameters in the presence of rope length and mass of a payload variation. The interval mathematics [25, 26] provides useful tools for robust control systems synthesis and stability analysis taking into consideration the system parameters uncertainty [27–29]. Numerous authors, studied the closed-loop control system considering an interval transfer function [30–32], or state space model with interval parameters [33–34].

*e-mail: smoczek@agh.edu.pl

Interval analysis is implemented for modeling interval systems and designing robust controller according to the iterative procedures [35–37] or through applying soft computing methods, e.g. genetic algorithms [38–39] and artificial neural network [40]. The interval analysis of closed-loop control system characteristic polynomial coefficients deviation from their nominal values is proposed in this paper to define minimum number of fuzzy sets on the scheduling variables universe of discourse and to determine the distribution of triangular-shaped membership functions parameters, which sutisfy the acceptable range of performances deterioration in the presence of system's parameters variation.

The paper is organized as follows. Section two describes a fuzzy logic-based discrete-time closed-loop control scheme for a planar model of a crane. In section three, the iterative procedure used to design a complete and coherent rule base (RB) of a fuzzy scheduler is proposed. Section four presents the experimental results obtained on the laboratory scaled overhead crane. Section five delivers the final conclusions.

2. Fuzzy interpolation-based control scheme

The system under consideration is the laboratory scaled overhead traveling crane with lifting capacity of 150 kg and motion mechanisms driven by DC motors. The planar model of a crane (Fig. 1) transferring a payload, which is assumed to be a point-mass suspended at the end of a massless rigid cable, is simplified to the first and second-order discrete-time transfer functions, which describe the relation between crane speed and input function (1), and sway angle of a payload and crane speed (2), where model's parameters vary in relation to the rope length l and mass of a payload m.

$$G_1(z,l,m) = \frac{\dot{X}(z,l,m)}{U(z,l,m)} = \frac{d_0(l,m)}{z + c_0(l,m)}, \tag{1}$$

$$\begin{aligned} G_2(z,l,m) &= \frac{\alpha(z,l,m)}{\dot{X}(z,l,m)} \\ &= \frac{b_1(l,m)z + b_0(l,m)}{z^2 + a_1(l,m)z + a_0(l,m)}. \end{aligned} \tag{2}$$

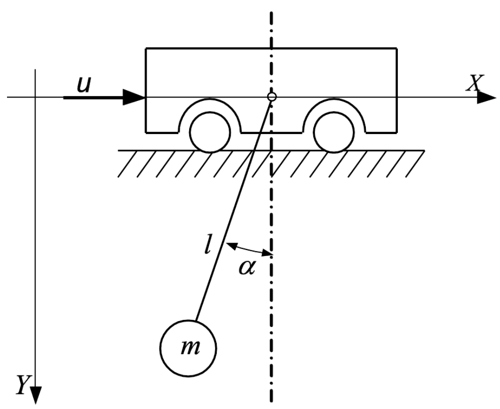

Fig. 1. Planar model of a crane, where m, l, u and α are, respectively, mass of a payload, rope length, controlling signal corresponding to control force acting on a crane, and sway angle of a payload

The adaptive control scheme can be based on a set of linear controllers for crane position, speed and first-order discrete-time controller of payload sway angle with parameters denoted k_1, k_2, q_0, q_1 and s_0 which are interpolated by a MIMO fuzzy system based on l and m input variables (Fig. 2). The fuzzy interpolation scheme is composed of a set of N rules (3) with the singleton type conclusions representing the controller parameters determined at operating point associating with the centre points of triangular-shaped functions (Fig. 3) used to evaluate the membership degree of l and m crisp input values to fuzzy sets specified in the antecedent of a rule.

$$\begin{aligned} R_k: \ \text{If } l \text{ is } \mathbf{L}_i \text{ and } m \text{ is } \mathbf{M}_j \\ \text{Then } \mathbf{y}_k \text{ is } \mathbf{K}_k, \end{aligned} \tag{3}$$

where $\mathbf{y}_k$ is the vector of rule output (where $k = 1, 2, ..., N$), $\mathbf{K}_k^T = [k_1, k_2, q_0, q_1, s_0]_k$ is the vector of controller parameters determined at operating point (l_i, m_j) associating with the centre points of triangular membership functions of fuzzy sets $\mathbf{L}_i$ and $\mathbf{M}_j$ on l and m input variables universe of discourse, where $i = 1, 2, ..., n$ and $j = 1, 2, ..., r$ (n and r are the numbers of fuzzy sets defined for l and m, respectively).

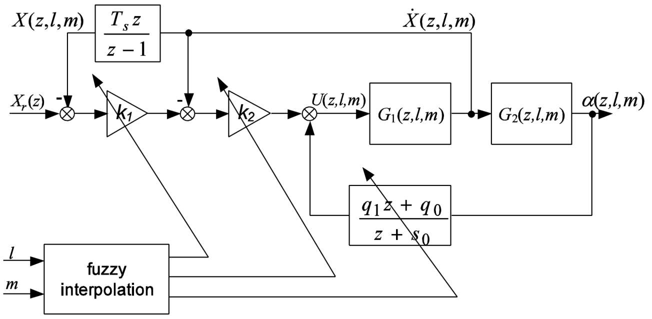

Fig. 2. Discrete-time control system with fuzzy logic-based interpolation of controller parameters

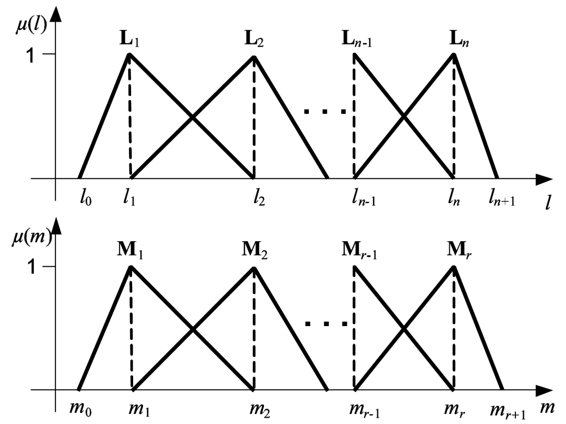

Fig. 3. Membership functions defined for fuzzy sets on the l and m input variables

The fuzzy sets $\mathbf{L}_i$ and $\mathbf{M}_j$ correspond to the triangular membership functions (4)–(5) with the centre points [l_1, l_2, ..., l_n] and [m_1, m_2, ..., m_r] distributed within the expected range of scheduling variables changes.

$$\mu_{\mathrm{L}_i}(l)=\max\left(\min\left(\frac{l-l_{i-1}}{l_i-l_{i-1}},\ \frac{l_{i+1}-l}{l_{i+1}-l_i}\right),0\right), \qquad (4)$$

where $l_{i-1}\le l_i\le l_{i+1}$, $i=1,2,n$.

$$\mu_{\mathrm{M}_j}(l)=\max\left(\min\left(\frac{m-m_{j-1}}{m_j-m_{j-1}},\ \frac{m_{j+1}-m}{m_{j+1}-m_j}\right),0\right), \qquad (5)$$

where $m_{j-1}\le m_j\le m_{j+1}$, $j=1,2,...,r$.

The output vector of fuzzy scheduler is calculated as the weighted average of all rules output:

$$\mathrm{y}=\left(\sum_{k=1}^{N} w_k\cdot \mathrm{K}_k\right)\cdot\left(\sum_{k=1}^{N} w_k\right)^{-1}, \qquad (6)$$

where a rule's activation degree (*firing strength*) is calculated as follows:

$$w_k=\mu_{\mathrm{L}_i}(l)\cdot\mu_{\mathrm{M}_j}(m). \qquad (7)$$

Considering the above assumptions, the closed-loop control system design involves to select minimum RB size corresponding to the midpoints $(l_i,\ m_j)$ of membership functions at which the linear controllers are determined. The number of fuzzy sets and distribution of the membership functions parameters should ensure the desired performances in the expected range of parameters variation.

3. Interval analysis-based control system design

3.1. Interval analysis-based fuzzy control scheme synthesis. In this section, the interval arithmetic-based synthesis of a fuzzy logic-based control system is described. The closed-loop control system (Fig. 2) transfer function can be presented in the following form:

$$\frac{\alpha(z,l,m)}{X_r(z)} = \frac{k_1(l,m)k_2(l,m)d_0(l,m)\begin{pmatrix} b_1(l,m)z^2\\ +(b_1(l,m)s_0+b_0(l,m))z\\ +b_0(l,m)s_0\end{pmatrix}}{z^5+\mathrm{z}\mathrm{S}(l,m)\mathrm{R}(l,m)}, \qquad (8)$$

where $\mathbf{z}=[z^4,z^3,z^2,z^1,z^0]$, $\mathbf{S}$ is a matrix of model's parameters and $\mathbf{R}$ is a vector containing of controller parameters interpolated by fuzzy system.

The RB of a fuzzy scheduler can be determined through assigning the closed-loop control system poles at the midpoints of desired poles intervals. The objective of fuzzy scheduler is to place all the closed-loop control system characteristic polynomial coefficients within desired intervals. Thus, a set of operating points associating with the centre points of membership functions, which satisfy the robust performances, can be obtained based on the objective function derived from the interval Diophantine equation. Considering the that all poles z_f (where $f=1,2,...,5$) of a closed-loop control system at each $k=1,2,...,N$ operating point $(l_i,\ m_j)$ are assigned at the midpoints of the real numbers intervals representing desired region of stable poles denoted as:

$$[z_f]_k=\left[z_f^-,\ z_f^+\right]_k=\left\{z_f\in\Re\mid z_f^-\le z_f\le z_f^+\right\}, \qquad (9)$$

the performances of fuzzy logic-based control system satisfy desired conditions if the coefficients of closed-loop system characteristic equation lie within the coefficients intervals of desired polynomial (10) determined using the arithmetic operations on intervals [24, 37]:

$$P_k(z)=\prod_{f=1}^{5}\left(z-\left[z_f^-,\ z_f^+\right]_k\right)=z^5+\mathrm{z}\left[\mathrm{P}_k\right], \qquad (10)$$

where $[\mathbf{P}_k]$ is an interval vector of desired characteristic equation coefficients:

$$[\mathrm{P}_k]=\left[[p_4]_k,\ [p_3]_k,\ [p_2]_k,\ [p_1]_k,\ [p_0]_k\right]^T. \qquad (11)$$

Hence, the controller parameters vectors $\mathbf{K}_k$ defined in rules conclusions can be determined based on the equations system:

$$\mathrm{S}_k\mathrm{R}_k=\mathrm{P}_k, \qquad (12)$$

where

$$\mathrm{S}_k=\begin{bmatrix} a_1+c_0-1 & \begin{pmatrix} a_0-a_1+\\ c_0(a_1-1)\end{pmatrix} & \begin{pmatrix} c_0(a_0-a_1)\\ -a_0\end{pmatrix} & -a_0c_0 & 0\\ d_0 & d_0(a_1-1) & d_0(a_0-a_1) & -d_0a_0 & 0\\ 0 & 0 & -d_0b_1 & d_0(b_1-b_0) & -d_0b_0\\ 0 & -d_0b_1 & d_0(b_1-b_0) & d_0b_0 & 0\\ 1 & a_1+c_0-1 & \begin{pmatrix} a_0-a_1+\\ c_0(a_1-1)\end{pmatrix} & \begin{pmatrix} c_0(a_0-a_1)\\ -a_0\end{pmatrix} & -a_0c_0\\ T_sd_0 & T_sd_0a_1 & T_sd_0a_0 & 0 & 0\\ 0 & d_0 & d_0(a_1-1) & d_0(a_0-a_1) & d_0a_0\\ 0 & T_sd_0 & T_sd_0a_1 & T_sd_0a_0 & 0 \end{bmatrix}^T,$$

$\mathrm{R}_k = [1,\, k_2,\, q_0,\, q_1,\, s_0,\, k_1k_2,\, k_2s_0,\, k_1k_2s_0]^T$, and $\mathbf{P}_k$ consists of interval vector $[\mathbf{P}_k]$ midpoints.

The robust performances objective function derived from the equations system (12) is defined as:

$$\mathrm{S}(l,m)\mathrm{R}(l,m) \in [\mathrm{P}_k]\,. \tag{13}$$

Therefore, the fuzzy logic-based control scheme satisfies the desired performances for the system's parameters varying within the expected ranges of rope length $l \in [l^-,\, l^+]$ and mass of a payload $m \in [m^-,\, m^+]$, if the condition (13) is not violated for the interval vectors (11) associating with the rules which has been activated to interpolate the controller parameters with the firing strength factor $w_k > 0$.

3.2. Iterative method used for fuzzy logic-based anti-sway control design. In this section, the iterative procedure applied to determine minimum number of fuzzy sets on l and m input variables universe of discourse is described. The proposed algorithm involves to identify the parametric models (1–2) of crane dynamic at operating points corresponding to the lower and upper bounds of scheduling variables intervals $[l^-,\, l^+]$ and $[m^-,\, m^+]$, and to assume the centre points of triangular membership functions as $l_1 = l^-$, $l_n = l^+$, $m_1 = m^-$ and $m_r = m^+$. Assuming the desired poles intervals (9) for each operating point $(l_1,\, m_1)$, $(l_1,\, m_r)$, $(l_n,\, m_1)$ and $(l_n,\, m_r)$, the vectors of controller parameters are derived from (12), and RB is formulated as:

$$\begin{aligned}
&R_1: \text{ If } l \text{ is } \mathbf{L}_1 \text{ and } m \text{ is } \mathbf{M}_1 \text{ Then } \mathbf{y}_1 \text{ is } \mathbf{K}_1,\\
&R_2: \text{ If } l \text{ is } \mathbf{L}_1 \text{ and } m \text{ is } \mathbf{M}_r \text{ Then } \mathbf{y}_2 \text{ is } \mathbf{K}_2,\\
&R_3: \text{ If } l \text{ is } \mathbf{L}_n \text{ and } m \text{ is } \mathbf{M}_1 \text{ Then } \mathbf{y}_3 \text{ is } \mathbf{K}_3,\\
&R_4: \text{ If } l \text{ is } \mathbf{L}_n \text{ and } m \text{ is } \mathbf{M}_r \text{ Then } \mathbf{y}_4 \text{ is } \mathbf{K}_4.
\end{aligned} \tag{14}$$

The iterative design of fuzzy interpolation control scheme is a two-stage process in which the intervals of input variables $[l^-,\, l^+]$ and $[m^-,\, m^+]$ are divided into small intervals to obtain N_l and N_m sample points. In the first stage (Algorithm 1), starting from the lower bound of interval $[l^-,\, l^+]$, the sample points are incremented from $s = 1$ until $s = N_l$. Each sample point l_s is assumed as the centre point l_{n-1} of a new membership function (4). Assuming the desired poles intervals (9), the controller parameters are calculated for the operating points

$$(l_{n-1},\, m_1)\,,\, (l_{n-1},\, m_r)\,, \tag{15}$$

according to the equations system (12). It leads to complete the RB by adding the two new rules:

$$\begin{aligned}
&R_{N+1}: \text{ If } l \text{ is } \mathbf{L}_{n-1} \text{ and } m \text{ is } \mathbf{M}_1 \text{ Then } \mathbf{y}_{N+1} \text{ is } \mathbf{K}_{N+1},\\
&R_{N+2}: \text{ If } l \text{ is } \mathbf{L}_{n-1} \text{ and } m \text{ is } \mathbf{M}_r \text{ Then } \mathbf{y}_{N+2} \text{ is } \mathbf{K}_{N+2},
\end{aligned} \tag{16}$$

where N is the number of fuzzy rules in the previous iteration.

In the next step, the condition (13) is tested for operating points corresponding to the crossover point of membership functions $\mu_{\mathbf{L}_{n-1}}(l)$ and $\mu_{\mathbf{L}_{n-2}}(l)$, and the centre points of functions $\mu_{\mathbf{M}_1}(m)$ and $\mu_{\mathbf{M}_r}(m)$:

$$((l_{n-1} + l_{n-2})\,/2,\, m_1)\,,\, ((l_{n-1} + l_{n-2})\,/2,\, m_r)\,. \tag{17}$$

If the condition (13) is satisfied, the temporary fuzzy set $\mathbf{A}_{n-1}$ and rules (16) are removed from the fuzzy scheduler. If it is violated, the new fuzzy set is created with the centre point of membership function at l_{s-1}, and the conclusions of rules (16) are determined for operating points which has been tested successfully in the previous iteration.

Algorithm 1. Selecting the minimum number of fuzzy sets for l

Input: $s := 1$; $n := 2$; $N = 4$;
1: *while* $s < N_l$ *do*
2: $n := n + 1$;
3: $l_{n-1} := l_s$;
4: define the fuzzy set $\mathbf{L}_{n-1} = \{l, \mu_{\mathbf{L}_{n-1}}(l);\, l \in [l^-,\, l^+]\}$, where $\mu_{\mathbf{L}_{n-1}}(l)$ is defined according to (4);
5: determine $[\mathbf{P}_{N+1}]$, $[\mathbf{P}_{N+2}]$, $\mathbf{K}_{N+1}$, $\mathbf{K}_{N+2}$, for operating points (15) according to (12);
6: add the rules (16) to the RB;
7: $N := N + 2$;
8: test the condition (13) for operating points (17);
9: *if* the condition (13) is satisfied
10: remove the fuzzy set $\mathbf{L}_{n-1}$;
11: remove the rules (16) from the RB;
12: $s := s + 1$; $n := n - 1$; $N := N - 2$;
13: *else*
14: $l_{n-1} := l_{s-1}$;
15: repeat step 5;
16: *end if*
17: *end while*

In the second stage, the Algorithm 2 leads to obtain the minimum number of fuzzy sets on the $[m^-,\, m^+]$ universe of discourse. At each iteration, the temporary fuzzy set $\mathbf{M}_{r-1}$ is determined, and fuzzy rules are added to the current RB:

$$\begin{aligned}
&R_{N+1}: \text{ If } l \text{ is } \mathbf{L}_1 \text{ and } m \text{ is } \mathbf{M}_{r-1} Then \mathbf{y}_{N+1} \text{ is } \mathbf{K}_{N+1},\\
&R_{N+2}: \text{ If } l \text{ is } \mathbf{L}_2 \text{ and } m \text{ is } \mathbf{M}_{r-1} \text{ Then } \mathbf{y}_{N+2} \text{ is } \mathbf{K}_{N+2},\\
&\vdots\\
&R_{N+n}: \text{ If } l \text{ is } \mathbf{L}_n \text{ and } m \text{ is } \mathbf{M}_{r-1} \text{ Then } \mathbf{y}_{N+n} \text{ is } \mathbf{K}_{N+n},
\end{aligned} \tag{18}$$

where the conclusions parameters are determined for operating points:

$$(l_i,\, m_{r-1}) \qquad \text{for} \qquad i = 1, 2, ..., n. \tag{19}$$

The condition (13) is tested for most hazardous operating points associating with the crossover points of membership functions:

$$\begin{aligned}
&(l_i,\, (m_{r-1} + m_{r-2})\,/2) \quad \text{for} \quad i = 1, 2, ..., n,\\
&((l_i + l_{i+1})\,/2,\, (m_{r-1} + m_{r-2})\,/2)\\
&\qquad\qquad\qquad\qquad \text{for} \quad i = 1, 2, ..., n-1,\\
&((l_i + l_{i+1})\,/2,\, m_{r-1}) \quad \text{for} \quad i = 1, 2, ..., n-1
\end{aligned} \tag{20}$$

Algorithm 2. Selecting the minimum number of fuzzy sets for m

Input: $s := 1$; $r := 2$; $N = 2n$;
1: *while* $s < N_m$ *do*
2: $r := r + 1$;
3: $m_{r-1} := m_s$;
4: define the fuzzy set
$\mathrm{M}_{r-1} = \left\{ m, \mu_{\mathrm{M}_{r-1}}(m);\ m \in \left[m^-,\ m^+ \right] \right\}$,
where $\mu_{\mathrm{M}_{r-1}}(m)$ is defined according to (5);
5: determine $[\mathbf{P}_{N+1}]$, $[\mathbf{P}_{N+2}]$, ..., $[\mathbf{P}_{N+n}]$,
and $\mathbf{K}_{N+1}$, $\mathbf{K}_{N+2}$, ..., $\mathbf{K}_{N+n}$
for operating points (19) according to (12);
6: add the rules (18) to the RB;
7: $N := N + n$;
8: test the condition (13) for operating points (20);
9: *if* the condition (13) is satisfied
10: remove the fuzzy set $\mathbf{M}_{r-1}$;
11: remove the rules (18) from the RB;
12: $s := s + 1$; $n := n - 1$; $N := N - n$;
13: *else*
14: $m_{r-1} := m_{s-1}$;
15: repeat step 5;
16: *end if*
17: *end while*

The Algorithms 1 and 2 involve interpolating the crane dynamic model parameters between operating points (l_1, m_1), (l_1, m_r), (l_n, m_1) and (l_n, m_r) at which the open-loop identification experiments were conducted.

Owing to the assumed conditions, the two-stage procedure results in selecting the minimum number of membership functions, their parameters distribution within the expected ranges of scheduling variables, as well as complete and coherent RB. Nevertheless, the parameters of the crane's model should be identified at new operating points determined through applying Algorithm 1 and 2, and the iterative procedure should be repeated to validate the robustness of closed-loop control system.

4. Results of experiments conducted on the laboratory stand

The proposed algorithm was employed to design the fuzzy logic-based control scheme of the DC motors driven motion mechanism of laboratory scaled overhead crane. The hardware-software equipment utilized during identification and control process was composed of PC with I/O board and Matlab software used in the open-loop identification experiments and control system design, and PAC system on which the control algorithm was implemented in the form of structured text and tested during experiments conducted on the lab-scaled material handling device. The objective of closed-loop system synthesis was to design a fuzzy scheduling control scheme taking into consideration the parameters variation within the intervals [0.8, 2.2] m and [10, 90] kg. The models of system's dynamic at operating points corresponding to the bounds of those intervals were identified using output error (OE) method with sample time $T_s = 0.1$ s. At each operating point considering as the centre point of membership function, the intervals of desired stable poles were assumed as:

$$[z_f]_k = [\exp\left(\left(-\omega_n \mp 0.1\omega_n\right) T_s\right)]_k\,, \tag{21}$$

where the natural not dumped pulsation of considered system can vary in the interval $\omega_n \in [2.06, 2.95]$ rad/s. For those assumptions, the expected range of step unit response settle time is [5.5, 6.5] s.

Through dividing the expected ranges of rope length and mass of a payload into $N_l = 14$ and $N_m = 8$ intervals, the iterative method, which has been described in Sec. 3, results in designing the fuzzy scheduler with 6 rules, and 3 and 2 fuzzy sets determined for l and m, respectively, with the centre points fixed at $l_1 = 0.8$ m, $l_2 = 1.5$ m, $l_3 = 2.2$ m, and $m_1 = 10$ kg, $m_1 = 90$ kg. The open-loop identification experiments were conducted for operating points (1.5 m, 10 kg) and (1.5 m, 90 kg) determined using iterative method, and Algorithm 1 and 2 were used again to confirm the performances robustness. Figure 4 depicts the natural pulsation of a dynamic model at the identified operating points and for operating points at which the model's parameters were linearly interpolated between bounds of the interval [0.8, 2.2] m (during designing the fuzzy scheduler), and between bounds of the intervals [0.8, 1.5] m and [1.5, 2.2] m (during closed-loop control system validation). The maximum interpolation error was 7.26% in designing process, and 0.9% during validation.

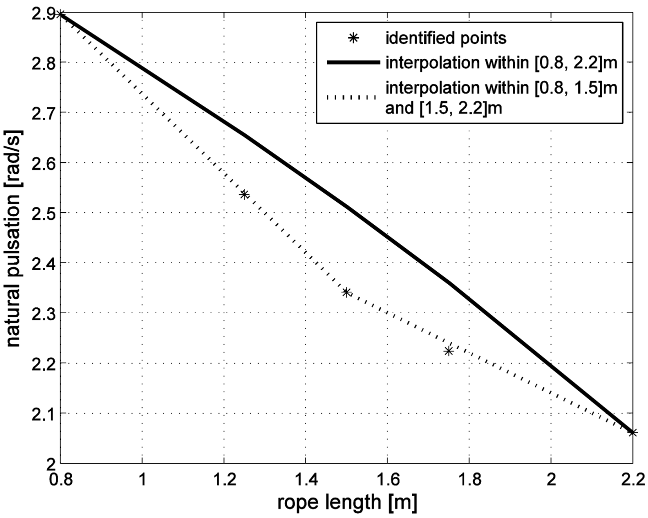

Fig. 4. The natural pulsation for operating points at which the parameters of a pendulum model were linearly interpolated between points at which the OE identification was conducted

Figures from 5 to 10 present the examples of unit step responses obtained during experiments conducted for operating points corresponding to the midpoints (Fig. 5–8) and crossover points (Fig. 9–10) of triangular membership functions. The experiments conducted for selected operating points confirmed the robust performances: settle time was in the desired interval [5.5, 6.5] s, while crane positioning and reduction of payload deviation were obtain with the expected tolerance ±0.02 m. Thus, the results of the experiments prove the effectiveness of the proposed method for designing a fuzzy logic-based scheduling control system.

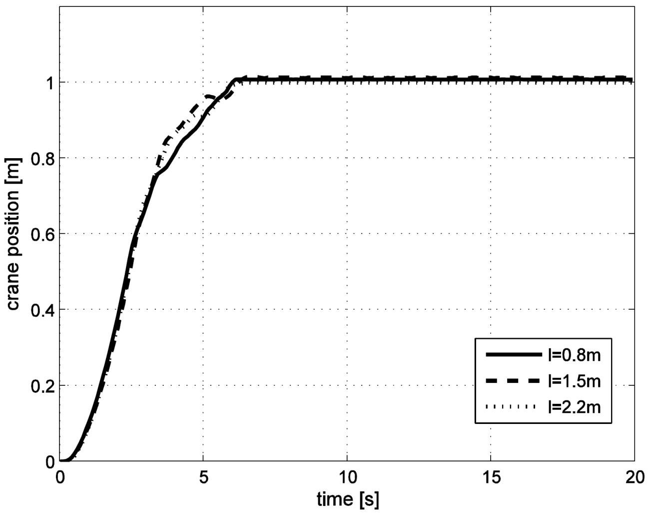

Fig. 5. Crane position – experiments for $m = 10$ kg

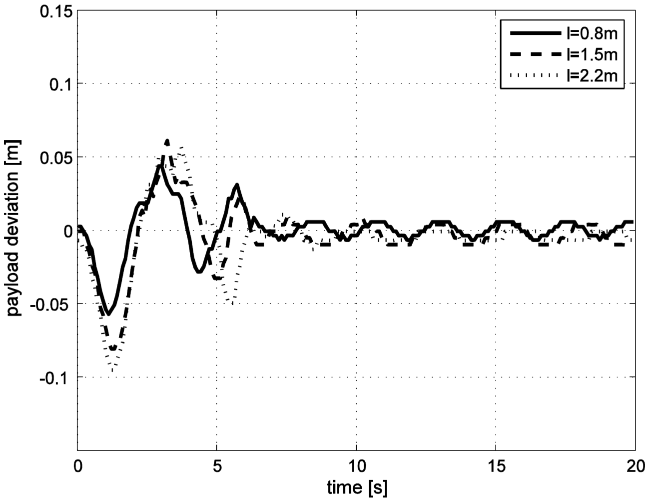

Fig. 6. Payload deviation – experiments for $m = 10$ kg

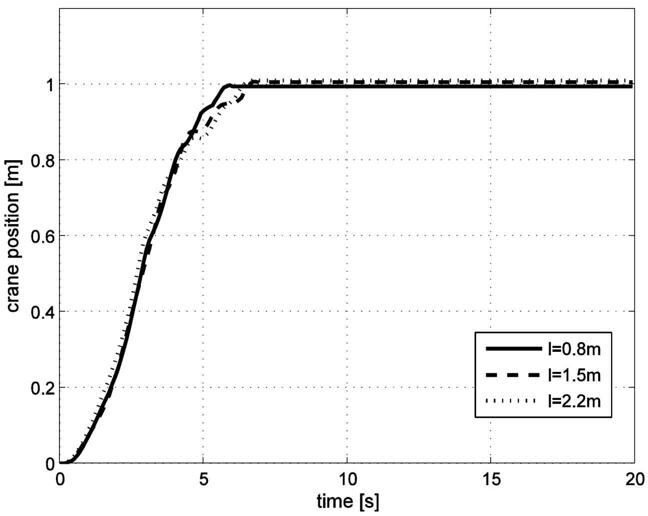

Fig. 7. Crane position – experiments for $m = 90$ kg

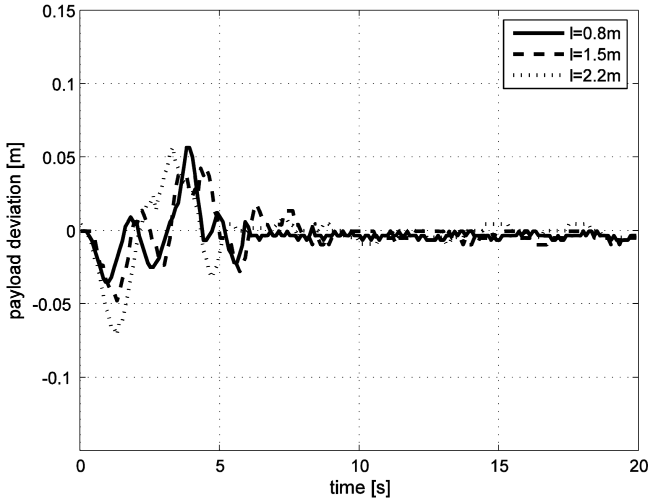

Fig. 8. Payload deviation – experiments for $m = 90$ kg

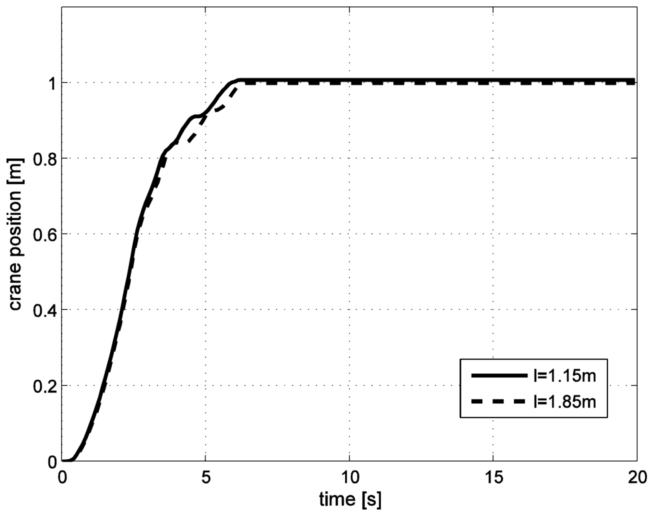

Fig. 9. Crane position – experiments for $m = 50$ kg

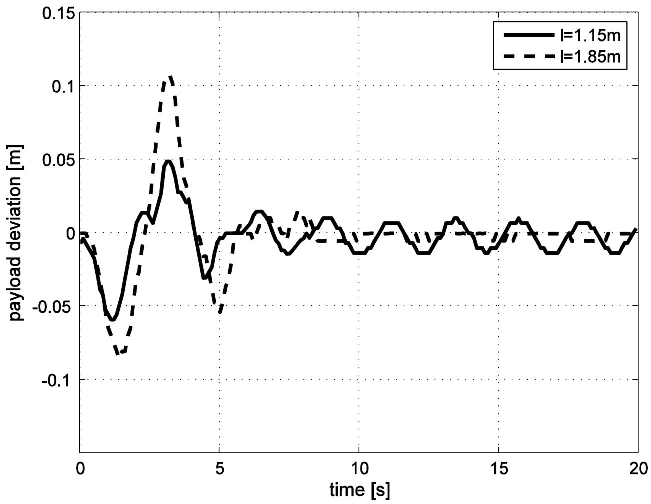

Fig. 10. Payload deviation – experiments for $m = 50$ kg

5. Conclusions

The fast, precise and safe transfer of goods in crane operations requires a control application to solve the problems of payload oscillations limitation. The interval arithmetic-based iterative method is proposed to design the robust fuzzy logic-based control scheme. The fuzzy rule-base system is applied to interpolate the controller parameters for adjusting control performances for the rope length and mass of a payload variation. The problem of fuzzy interpolation control scheme design is solved by interval analysis of closed-loop control system characteristic polynomial coefficients. The proposed method based on the interval Diophantine equation is applied to find minimum number of fuzzy sets on the scheduling variables universe of discourse and to determine the distribution of triangular-shaped membership functions parameters, which satisfy the acceptable range of performances deterioration in the presence of the system's parameters variation. The effectiveness of this method was proved in experiments conducted using the PAC system on the laboratory scaled overhead crane. The future challenge is the implementation of this method for design the control system of a large scale material handling device.

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REFERENCES

[1] J. Szpytko and D.A. Wozniak, "To keep operational potential of transport device e-based on reliability indicators", *Eur. Safety and Reliability Conf. ESREL* 1, 2377–2384 (2007).

[2] Z. Smalko and J. Szpytko, "Safety in engineering practice", *17th Eur. Safety and Reliability Conf. ESREL* 1, 1231–1237 (2009).

[3] E.M. Abdel-Rahman, A.H. Nayfeh, and Z.N. Masoud, "Dynamics and control of cranes: A review", *J. Vibration and Control* 9, 863–908 (2003).

[4] P. Hyla, "The crane control systems: a survey", *17th IFAC Int. Conf. on Methods and Models in Automation and Robotics MMAR* 1, 505–509 (2012).

[5] Y. Sakawa and Y. Shindo, "Optimal control of container cranes", *Automatica* 18 (3), 257–266 (1982).

[6] J.W. Auernig and H. Troger, "Time optimal control of overhead cranes with hoisting of the load", *Automatica* 23 (4), 437–447 (1987).

[7] Y. Fang, B. Ma, P. Wang, and X. Zhang, "A motion planning-based adaptive control method for an underactuated crane system", *IEEE Trans. on Control Systems Technology* 20 (1), 241–248 (2012).

[8] J. Neupert, E. Arnold, K. Schneider, and O. Sawodny, "Tracking and anti-sway control for boom cranes", *Control Engineering Practice* 18, 31–44 (2010).

[9] A. Nowacka-Leverton, M. Michałek, D. Pazderski, and A. Bartoszewicz, "Experimental verification of SMC with moving switching lines applied to hoisting crane vertical motion control", *ISA Transactions* 51 (6), 682–693 (2012).

[10] A. Benhidjeb and G.L. Gissinger, "Fuzzy control of an overhead crane performance comparison with classic control", *Control Engineering Practice* 3 (12), 1687–1696 (1995).

[11] M. Mahfouf, C.H. Kee, M.F. Abbod and D.A. Linkens, "Fuzzy logic-based anti-sway control design for overhead cranes", *Neural Computing and Applications* (9), 38–43 (2000).

[12] C.-Y. Chang, "The switching algorithm for the control of overhead crane", *Neural Computing and Applications* 15 (3–4), 350–358 (2006).

[13] X. Li and W. Yu, "Anti-swing control for an overhead crane with fuzzy compensation", *Intelligent Automation and Soft Computing* 17 (X), 1–11 (2004).

[14] M.I. Solihin, J. Wahyudi, and A. Legowo, "Fuzzy-tuned antiswing control of automatic gantry crane", *J. Vibration and Control* 16 (1), 127–145 (2010).

[15] D. Liu, J. Yi, D. Zhao, and W. Wang, "Adaptive sliding mode fuzzy control for a two-dimensional overhead crane", *Mechatronics* 15 (5), 505–522 (2005).

[16] S.-K. Oh, W. Pedrycz, S.-B. Rho, and T.-C. Ahn, "Parameter estimation of fuzzy controller and its application to inverted pendulum", *Engineering Applications of Artificial Intelligence* 17 (1), 37–60 (2004).

[17] N. Sadati and A. Hooshmand, "Design of a gain-scheduling anti-sway controller for tower cranes using fuzzy clustering techniques", *Int. Conf. on Computational Intelligence for Modeling, Control and Automation* 1, CD-ROM (2006).

[18] J. Smoczek and J. Szpytko, "A mechatronics approach in intelligent control systems of the overhead traveling cranes prototyping", *Information Technology and Control* 37 (2), 154–158 (2008).

[19] M.B. Trabia, J.M. Renno and K.A.F. Moustafa, "Generalized design of an anti-swing fuzzy logic controller for an overhead crane with hoist", *J. Vibration and Control* 14(3), 319–346 (2008).

[20] C.-Y. Chang, "Adaptive fuzzy controller of the overhead crane with nonlinear disturbances", *IEEE Trans. on Industrial Informatics* 3 (2), 164–172 (2007).

[21] B. Filipic, T. Urbancic, and V. Krizman, "A combined machine learning and genetic algorithm approach to controller design", *Engineering Applications of Artificial Intelligence* 12 (4), 401–409 (1999).

[22] D. Liu, J. Yi and M. Tan, "Proposal of GA-based two-stage fuzzy control of overhead crane", *IEEE Conf. on Computers, Communications, Control and Power Engineering* 3, 1721–1724 (2002).

[23] J. Smoczek and J. Szpytko, "Design of gain scheduling anti-sway controller using genetic fuzzy system", *17th IFAC Int. Conf. on Methods and Models in Automation and Robotics MMAR* 1, 573–578 (2012).

[24] W. Yu, A. Moreno-Wrmendariz, and F.O. Rodriguez, "Stable adaptive compensation with fuzzy CMAC for an overhead crane", *Information Sciences* 181 (21), 4895–4907 (2011).

[25] M. Warmus, "Calculus of approximations", *Bull. de l'Academie Polonaise des Sciences* IV (5), 253–259 (1956).

[26] R. Moore, *Interval Analysis*, Prentice-Hall, Englewood Cliffs, 1966.

[27] M. Dahleh, A. Tesi, and A. Vicino, "An overview of extremal properties for robust control of interval plants", *Automatica* 29 (3), 707–721 (1993).

[28] H. Chapellat, L.H. Keel, and S.P. Bhattacharyya, "External robustness properties of multilinear interval systems", *Automatica* 30 (6), 1037–1042 (1994).

[29] S. Mallan, M. Milanese, and M. Taragna, "Robust analysis and design of control systems using interval arithmetic", *Automatica* 33 (7), 1363–1372 (1997).

[30] Y.C. Soh, R.J. Evans, I.R. Petersen, and R.E. Betz, "Robust pole assignment", *Automatica* 23 (5), 601–610 (1987).

[31] L. Kolev and D. Penev, "Design of control systems under interval uncertainties", *Electrical and Electronics Engineering* 2–3, 63–68 (2004).

[32] A.D.S. Lordelo, E.A. Juzzo, and P.A.V. Ferreira, "Analysis and design of robust controllers using interval diophantine equation, *Reliable Computing* 12 (5), 371–388 (2006).

[33] M. Busłowicz, "Robust stability of the new general 2D model of a class of continuous-discrete linear systems", *Bull. Pol. Ac.: Tech.* 58 (4), 561–565 (2010).

[34] B.M. Patre and P.J. Deore, "Robust state feedback for interval systems: an interval analysis approach", *Reliable Computing* 14, 46–60 (2010).

[35] M.S. Fadali and G. Bebis, "Control system design for LTI systems using interval analysis", *ISCA Conf. on Computers and Applications* 1, CD-ROM (1998).

[36] M. Dyvak, P. Stakhiv, and A. Pukas, "Algorithms of parallel calculations in task of tolerance ellipsoidal estimation of interval model parameters", *Bull. Pol. Ac.: Tech.* 60 (1), 159–164 (2012).

[37] J. Xing, C. Chen and P. Wu, "Calculation of interval damping ratio under uncertain load in power system", *Bull. Pol. Ac.: Tech.* 60 (1), 151–158 (2012).

[38] M.D. Lorenzo del Casale, N. Femia, P. Lamberti, and V. Mainardi, "Selection of optimal closed-loop controllers for DC-DC regulators based on nominal and tolerance design", *IEEE Trans. on Industrial Electronics* 51 (4), 840–849 (2004).

[39] C.-C. Hsu, S.-C. Chang, and C.-Y. Yu, "Tolerance design of robust controllers for uncertain interval systems based on evolutionary algorithms", *IET Control Theory and Applications* 1 (1), 244–252 (2007).

[40] C.-H. Lee, Y.-H. Lee and C.-C. Teng, "A novel robust PID controllers design by fuzzy neural network", *American Control Conf.* 1, 1561–1566 (2002).