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CRITICAL ASSESSMENT OF THE ISSUES IN THE APPLICATION OF HILBERT TRANSFORM TO COMPUTE THE LOGARITHMIC DECREMENT

KRYTYCZNA ANALIZA ZASTOSOWAŃ TRANSFORMATY HILBERTA DO OBLICZEŃ LOGARYTMICZNEGO DEKREMENTU TŁUMIENIA

The parametric OMI (Optimization in Multiple Intervals), the Yoshida-Magalas (YM) and a novel Hilbert-twin (H-twin) methods are advocated for computing the logarithmic decrement in the field of internal friction and mechanical spectroscopy of solids. It is shown that dispersion in experimental points results mainly from the selection of the computing methods, the number of oscillations, and noise. It is demonstrated that conventional Hilbert transform method suffers from high dispersion in internal friction values. It is unequivocally demonstrated that the Hilbert-twin method, which yields a 'true envelope' for exponentially damped harmonic oscillations is superior to conventional Hilbert transform method. The 'true envelope' of free decaying strain signals calculated from the Hilbert-twin method yields excellent estimation of the logarithmic decrement in metals, alloys, and solids.

Keywords: Logarithmic decrement, internal friction, mechanical spectroscopy, Hilbert transform, envelope, interpolated discrete Fourier transform DFT

Do estymacji logarytmicznego dekrementu tłumienia w spektroskopii mechanicznej i w badaniach tarcia wewnętrznego ciał stałych zarekomendowano w pracy następujące metody obliczeniowe: metodę parametryczną OMI (Optimization in Multiple Intervals), metodę Yoshida-Magalas (YM) i nową metodę Hilbert-twin (H-twin). Wykazano, że dyspersja punktów eksperymentalnych logarytmicznego dekrementu tłumienia i tarcia wewnętrznego zdeterminowana jest przede wszystkim wyborem metody obliczeniowej, liczbą oscylacji i obecnością szumu w dyskretnych sygnałach odkształceń sprężystych badanych materiałów. Zastosowanie do obliczeń logarytmicznego dekrementu tłumienia klasycznej transformaty Hilberta powoduje bardzo dużą dyspersję punktów eksperymentalnych. W pracy wykazano, że metoda Hilbert-twin (transformata Hilberta zbliźniakowanego dyskretnego sygnału odkształceń sprężystych) po raz pierwszy umożliwia uzyskanie „prawdziwej obwiedni" wykładniczo tłumionych drgań harmonicznych próbki. Nowa metoda obliczeń „prawdziwej obwiedni" umożliwia bardzo dokładną estymację logarytmicznego dekrementu tłumienia w materiałach metalicznych i w ciałach stałych.

1. Introduction

Practical applications of the Hilbert transform are rare in mechanical spectroscopy and other spectroscopic techniques. An interesting attempt to compute the internal friction, Q^{-1}, in a resonant mechanical analyzer using conventional Hilbert transform was reported by Bonetti *et al.* [1]. In this paper, we analyze the experimental results obtained from the conventional Hilbert transform [1] and compare to our results obtained from the following computing methods: (1) the parametric OMI (Optimization in Multiple Intervals) [2-7], (2) the Hilbert transform designed in this work, (3) the Hilbert-twin (H-twin) – a novel method presented in [2] for the first time, (4) the Yoshida-Magalas (YM) based on interpolated discrete Fourier transform (IpDFT) [2,3], and the Yoshida method (IpDFT) [8].

The OMI has been proved to be the most effective and computationally compact method to compute the logarithmic decrement for high – and extreme high-damping levels for exponentially damped harmonic oscillations containing noise of electrical or other origin [4-6]. This is why we analyze in the current paper dispersion of the internal friction values for $Q_1^{-1} = 0.014$, as reported in Ref. [1]. It is important to demonstrate that various computing methods generate different estimations of the internal friction Q_1^{-1} from 'free-elastic decays' [2-6] containing the same noise (the concept of the 'free-elastic decay' is introduced in [2]). This effect ultimately results from different computing methods and algorithms, and deserves more attention nowadays.

2. Experimental results and discussion

Figure 1 shows temperature variation of the internal friction in polycrystalline nickel [1]. The internal friction was computed by Bonetti *et al.* [1] using conventional fast Hilbert transform, which is available in the LabView package. The dis-

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persion of internal friction points reaches a maximum around the peak temperature, 475 K, and decreases around 630 K. An average value of the internal friction at the peak temperature is Q_1^{-1} = 0.014±0.002. The high value of the standard deviation is responsible for strong dispersion of experimental points, which is readily visualized by the high density of experimental points (Fig. 1a). The relative error in estimation of the internal friction, Q_1^{-1} = 0.014, is around 16.7%.

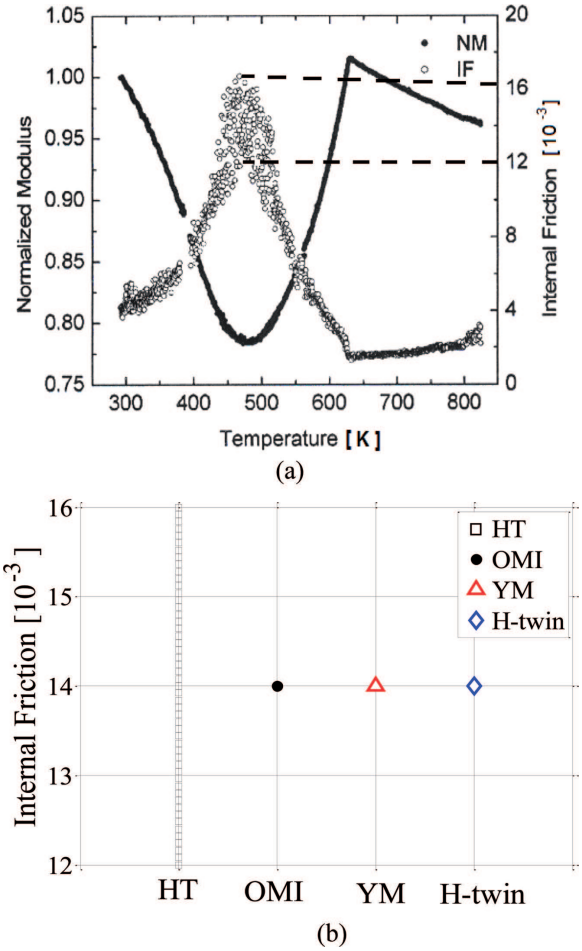

Fig. 1. (a) Dispersion in experimental values of internal friction (IF) points computed according to conventional Hilbert transform (◯ ; LabView) in polycrystalline nickel after *in situ* annealing at 1300K for 1h [1]. (b) Comparison between dispersion obtained from conventional Hilbert transform (□; the first vertical raw from left) and internal friction values obtained from the OMI (•), YM (Δ), and H-twin (◊) methods for the same set of 100 free decays. Q^{-1} = 0.014

High dispersion of Q^{-1} values reported by Bonetti *et al.* [1] is confirmed in this work by careful analysis of the intrinsic properties of conventional Hilbert transform. It is tacitly assumed that free-elastic decays, analyzed in this study, do not contain the zero-point drift (ZPD) [7] and the envelope is calculated without an advanced signal pre-processing, smoothing, low-pass filtering, or averaging [2]. Figure 1b and Table 1 indicate that dispersion observed for three analyzed Q^{-1} values [1] estimated from the OMI, YM, and H-twin methods is at least two orders of magnitude lower as compared to the results obtained according to conventional Hilbert transform method [1]. It is worthwhile to emphasize that the YM method outperforms the classic Yoshida (Y) method [2,3,8] to estimate the internal friction, Q^{-1} (Table 1).

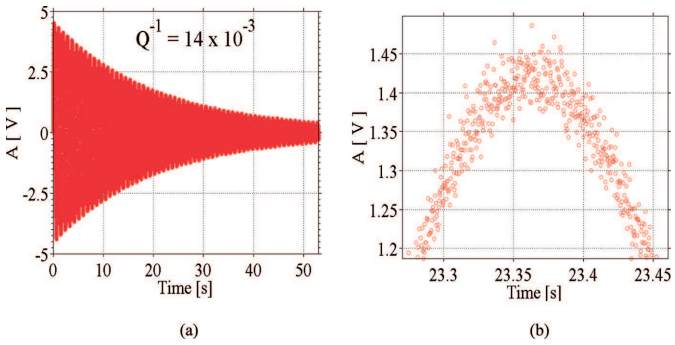

Fig. 2. (a) Illustration of exponentially damped harmonic oscillations with additive Gaussian noise for Q^{-1} = 0.014. (b) A part of the free decaying signal around a peak embedded in the experimental noise, S/N = 32 dB. The resonant frequency f_0 = 1.12345 Hz, the sampling frequency f_s = 3 kHz

Figure 2 illustrates the time series corresponding to the Q_1^{-1} = 0.014. The effect of noise is illustrated with zoom for the sampling frequencyf_s = 3 kHz (Fig. 2b.)

The dispersions in raw experimental internal friction data are frequently reported in the literature, which are brought about due to wrong choice of computing methods of the logarithmic decrement. Unsatisfactory results obtained from conventional Hilbert transform are shown in Figs. 1, 3, and Table 1. Figure 3 illustrates high-temperature raw experimental data biased by an undesirable scatter. The dispersion in experimental points is inevitable for free-elastic decays, i.e. free decays embedded in an experimental noise in a wide range of the signal-to-noise ratio, S/N (from 20 dB to 90 dB). This effect is observed for all damping levels (the case of extreme low damping level, i.e. Q^{-1} = 10^{-8}– 10^{-9}, is not discussed in this work.) This conclusion also has the virtue that it can be readily validated by various algorithms used to compute conventional Hilbert transform, available in commercial software packages. In contrast, the novel Hilbert-twin method yields the 'true envelope' [2] which, by definition, results in precise estimation of the logarithmic decrement, δ (Table 1, Fig. 1b).

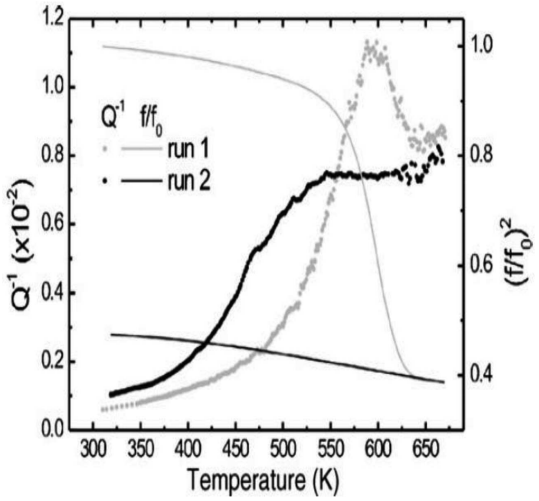

Fig. 3. Variation of the internal friction, Q^{-1}, and the normalized modulus of nanocrystalline MgH_2 during successive thermal runs [9] with visible dispersion in internal friction points

TABLE 1

The performance of different computing methods to estimate the mean value and the standard deviation of the internal friction for: Q^{-1} = 0.014, Q^{-1} = 0.005, and Q^{-1} = 0.0028 [1] (for 100 free-elastic decays). The results obtained from the classic Yoshida (Y) method [2,3,8] are provided for comparison. The resonant frequency f_0 = 1.12345 Hz, the sampling frequency f_s = 3 kHz, the number of oscillations N_{osc} = 30, the signal-to-noise ratio S/N = 32 dB

Computing method	Internal friction, Q^{-1}		
	Q_1^{-1} = 0.014	Q_2^{-1} = 0.005	Q_3^{-1} = 0.0028
HT – *Hilbert transform* [1]	~0.014±0.002	~0.005±0.001	~0.0028±0.0008
H-twin – *Hilbert-twin* [2]	0.013999±0.000011	0.004998±0.000013	0.002798±0.000012
OMI – *Optimization in Multiple Intervals* [2-6]	0.013999±0.000002	0.005000±0.000001	0.002799±0.000001
YM – *Yoshida-Magalas* [2,3]	0.013999±0.000004	0.004999±0.000003	0.002799±0.000003
Y – *Yoshida* [7]	0.014195±0.001758	0.004772±0.001549	0.002638±0.001349

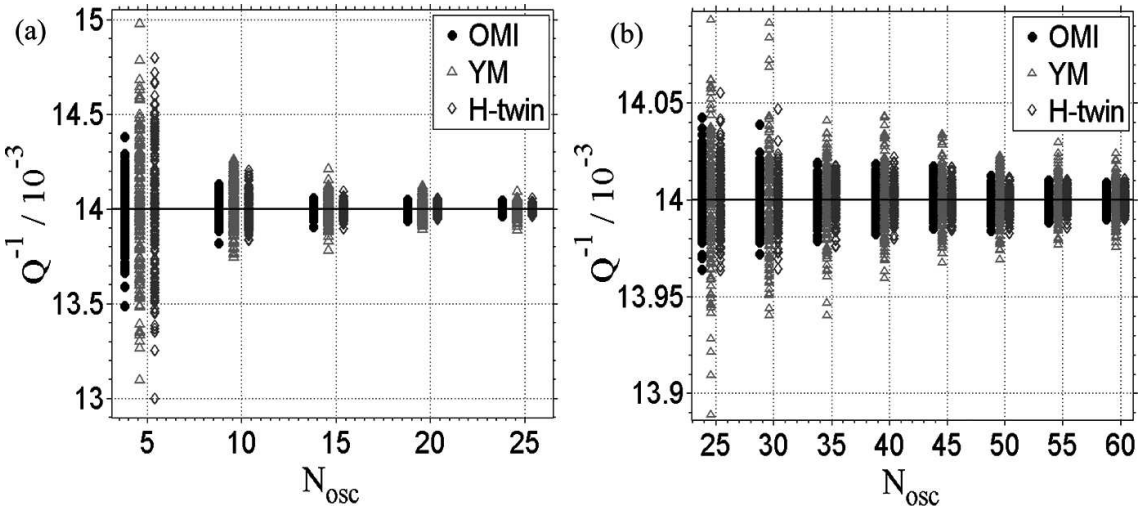

Fig. 4. Dispersion of the internal friction computed according to the OMI (•, black), YM (Δ, red), and Hilbert-twin (◊, blue) methods (vertical rows from left to right) for 100 free-elastic decays characterized by Q^{-1} = 0.014 and S/N = 32 dB. Each vertical row comprises 100 points of the estimated internal friction. Dispersion of the internal friction is a function of the number of free decaying oscillations, N_{osc}: (a) dispersion observed for the first 5, 10, 15, 20, and 25 oscillations; (b) dispersion observed for the first 25, 30, 35, 40, 45, 50, 55, and 60 oscillations. The resonant frequency f_0 = 1.12345 Hz, the sampling frequency f_s = 3 kHz. (For interpretation of the references to color in this figure legend, readers are referred to the web version of this article.)

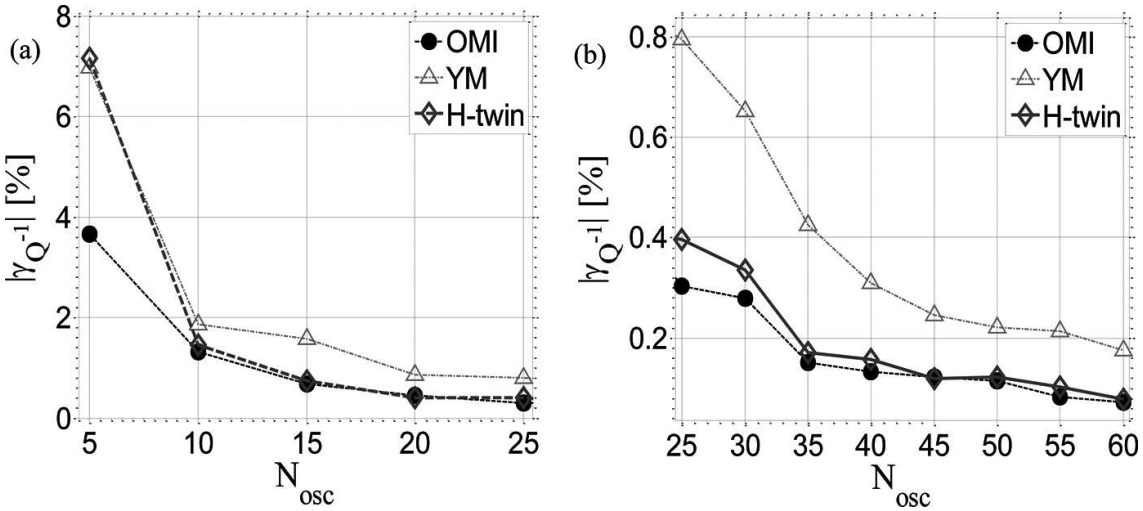

Fig. 5. The maximum relative error, $|\gamma_{Q^{-1}}|$, of the internal friction, Q^{-1}, computed according to the OMI (•, black), YM (Δ, red), and H-twin (◊, blue) methods as a function of the number of oscillations, N_{osc} (the number of periods). (a) The first 5, 10, 15, 20, and 25 oscillations; (b) the first 25, 30, 35, 40, 45, 50, 55, and 60 oscillations. Q^{-1} = 0.014. The resonant frequency f_0 = 1.12345 Hz, the sampling frequency f_s = 3 kHz, S/N = 32 dB. The maximum relative error estimated from conventional Hilbert transform, $|\gamma_{Q^{-1}}|$ =16.7% [1] is outside the figure

Figure 4 indicates the smallest dispersion in internal friction values obtained from the advocated methods: the OMI, YM, and H-twin as a function of the number of oscillations, N_{osc}. In general, dispersion in internal friction values varies as a function of theN_{osc}, the signal-to-noise ratio, S/N, and the sampling frequency.

Variation of the maximal relative error $\left|\gamma_{Q^{-1}}\right|$ as a function of the N_{osc} is shown in Fig. 5. The dispersion decreases with increasing the number of oscillations N_{osc} and thereby the results of computations are displayed in two separate figures for the increasing number of oscillations (Figs. 4a, 4b and 5a, 5b). The results shown in Figs. 4 and 5 confirm excellent performance of the OMI [2-7], YM [2,3], and H-twin methods [2]. Of special importance is the observation that the Hilbert-twin method yields slightly better estimation of the internal friction as compared to the YM method (Fig. 5) [2]. In both cases, however, the maximal relative errors $\left|\gamma_{Q^{-1}}\right|$ are extremely small. For 20 oscillations, the maximal relative error $\left|\gamma_{Q^{-1}}\right|$ is below 0.4% and 0.8% for the H-twin and YM method, respectively. For 45 oscillations the $\left|\gamma_{Q^{-1}}\right|$ is around 0.1% and 0.25% for the H-twin and YM method, respectively (Figs. 5a and 5b.)

Figures 1, 4, 5 and Table 1 illustrate a typical problem encountered in mechanical loss measurements of solids in the resonant domain. One of the main sources of high dispersion in experimental points is the wrong choice of the computation method to estimate the logarithmic decrement, δ. The computation of the logarithmic decrement from envelope of free decaying oscillations according to the Hilbert transform is biased by the presence of undesirable 'ripples' on envelope which results from an intrinsic property of oscillating envelope [2]. The 'ripples' are less detrimental in the case of very low-damping levels, only. Strong dispersion in estimated internal friction values occurs in low-, medium- and high-damping levels as an intrinsic property of the Hilbert transform.

It should be pointed out that computation of the logarithmic decrement according to conventional Hilbert transform proved inadequate in the case of exponentially damped free decaying oscillations embedded in noise and in noiseless free decays. This conclusion is valid for low- ($f_0 \approx 1$ Hz, Fig. 4) and high-frequency ($f_0 \approx$ kHz, Fig. 1) mechanical loss measurements.

3. Conclusions

1. It is not recommended to use the Hilbert transform available in commercial software packages to compute envelope of exponentially damped harmonic oscillations embedded in noise and estimate the logarithmic decrement from envelope.
2. The Hilbert-twin (H-twin) method is a novel method and algorithm, which eliminates the effect of detrimental 'ripples' on envelope and yields high precision in estimation of the logarithmic decrement.
3. The OMI, H-twin, and YM methods outperform classical methods [4-7], which are still widely used to compute the logarithmic decrement δ in the field of mechanical spectroscopy of solids.
4. The predominant role of careful selection of a computing method to estimate the logarithmic decrement is clearly demonstrated.
5. For exponentially damped time-invariant harmonic oscillations embedded in experimental noise the OMI, H-twin and YM methods yield excellent precision in estimation of the logarithmic decrement. Dispersion in estimated experimental internal friction values must be jointly analyzed as a function of noise, the number of oscillations (the number of periods), the sampling frequency, the resonant frequency, and the damping level.

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