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[Siberian Mathematical Journal](#)

January 2017, Volume 58, [Issue 1](#), pp 37–42 | [Cite as](#)

The height of faces of 3-polytopes

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Article

First Online: 04 March 2017

Received: 01 April 2015

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Abstract

The height of a face in a 3-polytope is the maximum degree of the incident vertices of the face, and the height of a 3-polytope, h , is the minimum height of its faces. A face is *pyramidal* if it is either a 4-face incident with three 3-vertices, or a 3-face incident with two vertices of degree at most 4. If pyramidal faces are allowed, then h can be arbitrarily large; so we assume the absence of pyramidal faces. In 1940, Lebesgue proved that every quadrangulated 3-polytope has $h \leq 11$. In 1995, this bound was lowered by Avgustinovich and Borodin to 10. Recently, we improved it to the sharp bound 8. For plane triangulation without 4-vertices, Borodin (1992), confirming the Kotzig conjecture of 1979, proved that $h \leq 20$ which bound is sharp. Later, Borodin (1998) proved that $h \leq 20$ for all triangulated 3-polytopes. Recently, we obtained the sharp bound 10 for triangle-free 3-polytopes. In 1996, Horňák and Jendrol' proved for arbitrarily 3-polytopes that $h \leq 23$. In this paper we improve this bound to the sharp bound 20.

Keywords

plane map, planar graph, 3-polytope, structure properties, height of face ,

The first author was supported by the Russian Foundation for Basic Research (Grants 15–01–05867 and 16–01–00499) and the State Maintenance Program for the Leading Scientific Schools of the Russian Federation (Grant NSh–1939.2014.1). The second author worked within the governmental task “Organization of Scientific Research.”

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References

1.
Steinitz E., “Polyeder und Raumeinteilungen,” *Enzykl. Math. Wiss. (Geometrie)*, vol. 3AB, no. 12, 1–139 (1922).
[Google Scholar](#)
2.
Lebesgue H., “Quelques conséquences simples de la formule d’Euler,” *J. Math. Pures Appl.*, vol. 19, 27–43 (1940).
[MathSciNet](#) [MATH](#) [Google Scholar](#)
3.
Borodin O. V., “Colorings of plane graphs: a survey,” *Discrete Math.*, vol. 313, no. 4, 517–539 (2013).
[MathSciNet](#) [CrossRef](#) [MATH](#) [Google Scholar](#)
4.
Ore O. and Plummer M. D., “Cyclic coloration of plane graphs,” in: *Recent Progress in Combinatorics* (W. T. Tutte, Ed.), Academic Press, New York, 1969, 287–293.
[Google Scholar](#)
5.
Borodin O. V., “Strengthening Lebesgue’s theorem on the structure of the minor faces in convex polyhedra,” *Diskretn. Anal. Issled. Oper. Ser. 1*, vol. 9, no. 3, 29–39 (2002).
[MathSciNet](#) [MATH](#) [Google Scholar](#)
6.
Borodin O. V., “Solving the Kotzig and Grünbaum problems on the separability of a cycle in planar graphs,” *Mat. Zametki*, vol. 46, no. 5, 9–12 (1989).
[MathSciNet](#) [MATH](#) [Google Scholar](#)
7.
Borodin O. V. and Ivanova A. O., “Describing 3-faces in normal plane maps with minimum degree 4,” *Discrete Math.*, vol. 313, no. 23, 2841–2847 (2013).
[MathSciNet](#) [CrossRef](#) [MATH](#) [Google Scholar](#)
8.
Borodin O. V., Ivanova A. O., and Kostochka A. V., “Describing faces in plane triangulations,” *Discrete Math.*, vol. 319, 47–61 (2014).
[MathSciNet](#) [CrossRef](#) [MATH](#) [Google Scholar](#)

9.

Kotzig A., “From the theory of Eulerian polyhedrons,” *Mat. Eas. SAV (Math. Slovaca)*, vol. 13, 20–34 (1963).

[MathSciNet](#) [MATH](#) [Google Scholar](#)

10.

Grünbaum B., “Polytopal graphs,” in: *Studies in Graph Theory. Part II* (Ed. D. R. Fulkerson), Washington, D. C., Math. Assoc. Amer., vol. 12, 1975, 201–224 (MAA Stud. Math.).

[Google Scholar](#)

11.

Plummer M. D., “On the cyclic connectivity of planar graphs,” in: *Graph Theory and Applications*, Springer-Verlag, Berlin, 1972, 235–242.

[CrossRef](#) [Google Scholar](#)

12.

Kotzig A., “Extremal polyhedral graphs,” *Ann. New York Acad. Sci.*, vol. 319, 569–570 (1979).

[Google Scholar](#)

13.

Borodin O. V., “Minimal weight of a face in planar triangulations without 4-vertices,” *Mat. Zametki*, vol. 51, no. 1, 16–19 (1992).

[MathSciNet](#) [MATH](#) [Google Scholar](#)

14.

Borodin O. V., “Triangulated 3-polytopes with restricted minimal weight of faces,” *Discrete Math.*, vol. 186, 281–285 (1998).

[MathSciNet](#) [CrossRef](#) [MATH](#) [Google Scholar](#)

15.

Avgustinovich S. V. and Borodin O. V., “Neighborhoods of edges in normal maps,” *Diskretn. Anal. Issled. Oper.*, vol. 2, no. 2–3, 3–9 (1995).

[MATH](#) [Google Scholar](#)

16.

Borodin O. V. and Ivanova A. O., “The vertex-face weight of edges in 3-polytopes,” *Sib. Math. J.*, vol. 56, no. 2, 275–284 (2015).

[MathSciNet](#) [CrossRef](#) [MATH](#) [Google Scholar](#)

17.

Borodin O. V. and Ivanova A. O., “Heights of minor faces in triangle-free 3-polytopes,” *Sib. Math. J.*, vol. 56, no. 5, 982–988 (2015).

[MathSciNet](#) [CrossRef](#) [MATH](#) [Google Scholar](#)

18.

Borodin O. V. and Loparev D. V., “The height of minor faces in normal plane maps,” *Diskretn. Anal. Issled. Oper.*, Ser. 1, vol. 5, no. 4, 6–17 (1998).

[MathSciNet](#) [MATH](#) [Google Scholar](#)

19.

Hornák M. and Jendrol' S., "Unavoidable sets of face types for planar maps," Discuss. Math. Graph Theory, vol. 16, no. 2, 123–142 (1996).

[MathSciNet](#) [CrossRef](#) [MATH](#) [Google Scholar](#)

20.

Jendrol' S. and Voss H.-J., "Light subgraphs of graphs embedded in the plane—a survey," Discrete Math., vol. 313, no. 4, 406–421 (2013).

[MathSciNet](#) [CrossRef](#) [MATH](#) [Google Scholar](#)

21.

Borodin O. V. and Woodall D. R., "The weight of faces in plane maps," Mat. Zametki, vol. 64, no. 5, 648–657 (1998).

[MathSciNet](#) [CrossRef](#) [MATH](#) [Google Scholar](#)

22.

Borodin O. V. and Woodall D. R., "Cyclic degrees of 3-polytopes," Graphs Comb., vol. 15, 267–277 (1999).

[MathSciNet](#) [CrossRef](#) [MATH](#) [Google Scholar](#)

23.

Borodin O. V., "Joint generalization of the theorems of Lebesgue and Kotzig on the combinatorics of planar maps," Diskret. Mat., vol. 3, no. 4, 24–27 (1991).

[MathSciNet](#) [MATH](#) [Google Scholar](#)

24.

Ferencová B. and Madaras T., "Light graphs in families of polyhedral graphs with prescribed minimum degree, face size, edge and dual edge weight," Discrete Math., vol. 310, 1661–1675 (2010).

[MathSciNet](#) [CrossRef](#) [MATH](#) [Google Scholar](#)

25.

Jendrol' S., "Triangles with restricted degrees of their boundary vertices in plane triangulations," Discrete Math., vol. 196, 177–196 (1999).

[MathSciNet](#) [CrossRef](#) [MATH](#) [Google Scholar](#)

26.

Kotzig A., "Contribution to the theory of Eulerian polyhedra," Mat. Eas. SAV (Math. Slovaca), vol. 5, 101–113 (1955).

[MathSciNet](#) [Google Scholar](#)

27.

Mohar B., Skrekovski R., and Voss H.-J., "Light subgraphs in planar graphs of minimum degree 4 and edge-degree 9," J. Graph Theory, vol. 44, 261–295 (2003).

[MathSciNet](#) [CrossRef](#) [MATH](#) [Google Scholar](#)

28.

Wernicke P., "Über den kartographischen Vierfarbensatz," Math. Ann., vol. 58, 413–426 (1904).

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About this article



Cite this article as:

Borodin, O.V. & Ivanova, A.O. Sib Math J (2017) 58: 37. <https://doi.org/10.1134/S0037446617010050>

- DOI <https://doi.org/10.1134/S0037446617010050>
- Publisher Name Pleiades Publishing
- Print ISSN 0037-4466
- Online ISSN 1573-9260
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