

Fuzzy Saturated Output Feedback Tracking Control for Robot Manipulators: A Singular Perturbation Theory Based Approach

Regular Paper

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Received 19 May 2011; Accepted 19 Aug 2011

Abstract To deal with the problem of the output feedback tracking (OFT) control with bounded torque inputs of robot manipulators, we propose a generalized fuzzy saturated OFT controller based on singular perturbation theory. First, considering the fact that the output torque of joint actuators is limited, a general expression for a class of saturation functions is given to be applied in the control law. Second, to carry out the whole closed-loop control with only position measurements, linear and nonlinear filters are optionally involved to generate a pseudo signal to surrogate the actual velocity tracking error. As a third contribution, a fuzzy regulator is added to obtain a self-tuning performance in tackling the disturbances. Moreover, an explicit but strict stability proof of the system based on the stability theory of singularly perturbed systems is presented. Finally, numerical simulations on several sample controllers are implemented to verify the effectiveness of the proposed approach.

Keywords Robot, tracking control, singular perturbation, output feedback, bounded torque input

1. Introduction

For tracking problem of robot manipulators, it is crucial to overcome the situation of noisy velocity and acceleration measurements in actual working conditions. Output feedback tracking (OFT) controllers, which estimate those signals from position feedback signals and make the whole closed-loop control with only position measurements, have aroused increasing concerns in motion control related researches, especially the saturated OFT controllers for robot manipulators with bounded joint torque inputs (Loria and Nijmeijer, 1998, Moreno-Valenzuela et al., 2008a, Liu and Zhu, 2009).

With a review of previous studies, to solve the saturation problem caused by the limited power of the joint actuators, the most common method is to apply hyperbolic tangent function in the control law (Llama et al., 2001, Loria and Nijmeijer, 1998, Moreno-Valenzuela et al., 2008b, Moreno-Valenzuela et al., 2008a, Santibanez and Kelly, 2001, Moreno-Valenzuela et al., 2010). The first saturated OFT controller was proposed in (Loria and

Nijmeijer, 1998), where the hyperbolic tangent function was invoked in the control law to ensure the bound of the torque input, and a pseudo velocity error signal was obtained from a nonlinear filter containing only position tracking error, guaranteeing the whole closed-loop control without velocity or acceleration measurements. In (Santibanez and Kelly, 2001), a supplement considering the viscous friction at the robot joints was made to this scheme, to prove that the global asymptotic stability can be assured if large enough damping is presented. Afterwards, a general expression of a class of saturation function was defined by Moreno-Valenzuela et al. (2008a) in an attempt to generalize the results of Loria and Nijmeijer (1998), based on it, a uniform expression of a class of saturated OFT controllers was proposed, but it added an extra restriction on the permitted range of the velocity error by the nonlinear filter, which was used to achieve the OFT control. Motivated by it, a more generalized expression with a linear filter to generate the pseudo velocity error signal was given in (Liu and Zhu, 2009). Furthermore, as a particular case of the generalized controller proposed in (Liu and Zhu, 2009), a saturated OFT controller with error-gain contained arctangent function and fuzzy proportional and derivative (PD) regulator in the control law was designed in (Liu et al., 2010) to gain a more satisfactory dynamic tracking performance. More importantly, the stability theory of singularly perturbed systems was introduced to make an explicit stability proof of the saturated OFT controller (Moreno-Valenzuela et al., 2008a, Moreno-Valenzuela et al., 2008b, Liu et al., 2010, Liu and Zhu, 2009). It is remarkable that the singular perturbation theory was first used to tackle OFT control by Burkov (1998), but without mentioning the torque saturation problem.

With respect to the applications of fuzzy strategy, a saturated tracking controller equipped with a fuzzy PD regulator was contributed in (Llama et al., 2001), but it is a state feedback controller rather than an output feedback controller. Another fuzzy approach was proposed recently in (Santibanez et al., 2005), yet it aimed only to solve the set-point control problem. It seems that the design of fuzzy OFT controllers and the stability analysis of them become more complicated when encountering with robot manipulators subject to input saturation, let alone give a general formula of them (Andrieu and Praly, 2009, Hernandez-Guzman et al., 2008).

In this paper, we focus on finding a general expression of a class of fuzzy saturated OFT controllers, with achieving simultaneously both system stability and satisfactory tracking performance. In Section 2 the robot dynamics and some useful properties are presented. In Section 3 a class of saturation functions are given. Section 4 deals with the design of a generalized saturated OFT controller via fuzzy self-tuning PD. Section 5 presents a stability proof of the proposed controller from the perspective of singularly

perturbed systems. In Section 6 simulation comparisons are made among several fuzzy OFT controllers. Finally, some conclusions are given in Section 7.

Notations: throughout this paper, $\|x\|$ and $\|X\|$ stand for the Euclidean norm of vector x and induced L_2 norm of matrix X , respectively; x_m and x_M denote the minimum and maximum values of variable x , respectively; $\lambda_m\{X\}$ and $\lambda_M\{X\}$ stand for the smallest and largest eigenvalues of matrix X , respectively.

2. Dynamic Model and Properties

2.1 Dynamic Model

The dynamics of rigid serial n -link robot manipulator with revolute joints can be written as:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + F_v\dot{q} = \tau \quad (1)$$

where $q \in \mathbb{R}^n$ denotes the vector of joint angle, respectively, $M(q) \in \mathbb{R}^{n \times n}$ represents the inertia matrix, $C(q, \dot{q}) \in \mathbb{R}^{n \times n}$ is the centripetal-Coriolis matrix, $G(q) \in \mathbb{R}^n$ is the vector of gravity effect, $F_v = \text{diag}\{f_{v1}, f_{v1}, \dots, f_{vn}\} \in \mathbb{R}^{n \times n}$ ($f_{vi} > 0, i=1, 2, \dots, n$) is the viscous friction coefficient matrix, and $\tau \in \mathbb{R}^n$ is the vector of the input torque.

2.2 Useful Properties

Some useful properties on robot dynamic model (1) are listed as follows (Kelly et al., 2005).

Property 1 The inertia and centripetal-Coriolis matrices satisfy the following skew symmetric relationship:

$$x^T [\dot{M}(q) - 2C(q, \dot{q})]x = 0 \quad (2)$$

$$\dot{M}(q) = C(q, \dot{q}) + C(q, \dot{q})^T \quad (3)$$

Property 2 The inertia matrix $M(q)$ is symmetric, positive definite, and satisfies the following inequalities:

$$\lambda_m\{M(q)\}\|x\|^2 \leq x^T M(q)x \leq \lambda_M\{M(q)\}\|x\|^2 \quad (4)$$

Property 3 For all $x, y, z \in \mathbb{R}^n$, the centripetal-Coriolis matrix satisfies the following transformations

$$C(x, y)z = C(x, z)y \quad (5)$$

$$C(x, y+z) = C(x, y) + C(x, z) \quad (6)$$

Property 4 The centripetal-Coriolis, gravity terms can be bounded in the following manner

$$\|C(q, \dot{q})\| \leq k_C \|\dot{q}\|, \|G(q)\|_M \leq k_G \quad (7)$$

3. A Class of Saturation Functions

To make the torque inputs bounded, applying appropriate smooth saturation function in the control law is a preferred solution (Liu and Zhu, 2009, Liu et al., 2010, Loria and Nijmeijer, 1998, Moreno-Valenzuela et al., 2008b, Moreno-Valenzuela et al., 2008a, Santibanez and Kelly, 2001, Santibanez et al., 2005). It is noteworthy that, recently a continuous piecewise-differentiable increasing function was invoked in (Huang et al., 2008) to achieve the boundedness of the tracking control law, but it cannot ensure continuous differentiation at the section point, which may enable unsmooth torque outputs of the joint actuators, so as to exert flexible impact on the joint mechanism. Motivated by (Moreno-Valenzuela et al., 2008a), we have found a class of more general saturation functions (Liu and Zhu, 2009).

Define $\text{Sat}(\mathbf{x}, \Delta) = [\text{sat}(x_1, \sigma_1), \text{sat}(x_2, \sigma_2), \dots, \text{sat}(x_n, \sigma_n)]^T$, where $\mathbf{x} = [x_1, x_2, \dots, x_n]^T$, $\Delta = \text{diag}[\sigma_1, \sigma_2, \dots, \sigma_n]$, $\sigma_i \geq 1$, $i=1, 2, \dots, n$. In addition, we use σ_m and σ_M to represent the minimum and the maximum values of σ_i , respectively.

i) $\text{sat}(x_i, \sigma_i)$ is a monotone increasing function in a real domain, i.e., $\frac{\partial \text{sat}(x_i, \sigma_i)}{\partial x_i} > 0$, $\forall x_i \in \mathbb{R}$.

ii) $\text{sat}(x_i, \sigma_i)x_i \geq 0$, if and only if $x_i = 0$ and $\text{sat}(x_i, \sigma_i)=0$, that $\text{sat}(x_i, \sigma_i)x_i = 0$, $\forall x_i \in \mathbb{R}$.

iii) $|\text{sat}(x_i, \sigma_i)| \leq p$, $\|\text{Sat}(\mathbf{x}, \Delta)\| \leq \sqrt{n} p$, $\forall x_i \in \mathbb{R}$, $\mathbf{x} \in \mathbb{R}^n$.

iv) $\sigma_M \|\mathbf{x}\| \geq \alpha_1 \|\text{Sat}(\mathbf{x}, \Delta)\|$, $\forall \mathbf{x} \in \mathbb{R}^n$, where $\alpha_1 > 0$ is small enough.

v) $\text{Sat}(\mathbf{x}, \Delta)$ is continuously differentiable and satisfies $\lambda_M \left\{ \frac{\partial \text{Sat}(\mathbf{x}, \Delta)}{\Delta \partial \mathbf{x}} \right\} \leq \beta$, $\forall \mathbf{x} \in \mathbb{R}^n$, where $\beta > 0$ is large enough.

vi) There always exists a large enough constant $\alpha_2 > 0$, for all $\mathbf{x} \in \Omega_\eta$, $\sigma_m \|\mathbf{x}\| \leq \alpha_2 \|\text{Sat}(\mathbf{x}, \Delta)\|$ is satisfied, where $\Omega_\eta = \{\mathbf{x} \in \mathbb{R}^n: \|\mathbf{x}\| \leq \eta\}$, $\eta > 0$ is arbitrarily large.

vii) For all $\mathbf{x} \in \Omega_\eta$, there exists $\gamma_1 > 0$ small enough and $\gamma_2 > 0$ large enough to satisfy

$$\gamma_1 \|\text{Sat}(\mathbf{x}, \Delta)\|^2 \leq \sum_{i=1}^n \sigma_i \int_0^{x_i} \text{sat}(x_i, \sigma_i) dx_i \leq \gamma_2 \|\text{Sat}(\mathbf{x}, \Delta)\|^2.$$

As the properties above described, the improvements compared with (Moreno-Valenzuela et al., 2008a) can be summarized: we apply an extra positive gain “ σ_i ” to change the approaching behavior to saturation for function “ $\text{sat}(x_i, \sigma_i)$ ”, and define a generalized relationship more than multiplication and division between x_i and σ_i . Obviously, $\text{atan}(\sigma_i x_i)$ (Liu and Zhu, 2009, Liu et al., 2010) and $\tanh(\sigma_i x_i)$ (Moreno-Valenzuela et al., 2008b, Loria and Nijmeijer, 1998, Moreno-Valenzuela et al., 2008a) are particular cases of $\text{sat}(x_i, \sigma_i)$.

4. Design of a Generalized Fuzzy OFT Controller

4.1 Control Goal

The control objective is to design a controller with bounded inputs $|\tau_i| < \tau_{iM}$, $i=1, 2, \dots, n$, which guarantees the joint displacements $\mathbf{q}(t) \in \mathbb{R}^n$ converge asymptotically to the desired joint displacements $\mathbf{q}_d(t) \in \mathbb{R}^n$, where τ_i denotes the control torque input of the i -th joint, and correspondingly, τ_{iM} denotes the permitted maximum torque input of the i -th joint. In brief, the control goal can be expressed as

$$\forall \mathbf{e}(0) \in \mathbb{R}^n, \lim_{t \rightarrow \infty} \mathbf{e}(t) = \mathbf{0} \quad (8)$$

where $\mathbf{e}(t) = \mathbf{q}_d(t) - \mathbf{q}(t)$ is the position tracking error.

In addition, we assume $\mathbf{q}_d(t)$ is twice differentiable, $\mathbf{q}_d(t)$ and its first two derivatives are bounded for all $t \geq 0$:

$$\begin{aligned} \|\mathbf{q}_d(t)\| &\leq \|\mathbf{q}_d(t)\|_M \\ \|\dot{\mathbf{q}}_d(t)\| &\leq \|\dot{\mathbf{q}}_d(t)\|_M \\ \|\ddot{\mathbf{q}}_d(t)\| &\leq \|\ddot{\mathbf{q}}_d(t)\|_M \end{aligned} \quad (9)$$

4.2 Control Law

To carry out the trajectory tracking control with only position measurements, we use filter to generate pseudo signal ξ from the position tracking error \mathbf{e} , so as to surrogate the actual velocity tracking error $\dot{\mathbf{e}}$. The filter can be linear or nonlinear.

In linear form, the filter is designed as

$$\dot{\xi} = \mathbf{U}(\dot{\mathbf{e}} - \xi) \quad (10)$$

which is made up of two implementable parts:

$$\dot{\mathbf{r}} = \mathbf{U}\xi, \quad \xi = \mathbf{U}\mathbf{e} - \mathbf{r} \quad (11)$$

While in the nonlinear form, the filter can be given as

$$\dot{\xi} = \mathbf{U}[\dot{\mathbf{e}} - \text{Sat}(\xi, \Delta)] \quad (12)$$

which is made up of two implementable parts:

$$\dot{\mathbf{r}} = \mathbf{U}\text{Sat}(\xi, \Delta), \quad \xi = \mathbf{U}\mathbf{e} - \mathbf{r} \quad (13)$$

where $\mathbf{r} \in \mathbb{R}^n$ is an auxiliary variable; \mathbf{e} and ξ are the input and output of the filter, respectively; $\mathbf{U} = \text{diag}\{\mu_1, \dots, \mu_n\} \in \mathbb{R}^{n \times n}$, $\mu_i > 0$, $i = 1, 2, \dots, n$.

To facilitate the expressions, we use M, C, C_d, G, K_p and K_d to represent $M(q), C(q, \dot{q}), C(q, \dot{q}_d), G(q), K_p(e, \xi)$ and $K_d(e, \xi)$, respectively. Then a generalized control law is given as

$$\tau = K_p \text{Sat}(e, K_e) + K_d \text{Sat}(\xi, K_\xi) + M\ddot{q}_d + C_d\dot{q}_d + G + F_v\dot{q}_d \quad (14)$$

where $K_p = \text{diag}\{k_{p1}(e, \xi), k_{p2}(e, \xi), \dots, k_{pn}(e, \xi)\}$ is the diagonal matrix of proportional gain, $K_d = \text{diag}\{k_{d1}(e, \xi), k_{d2}(e, \xi), \dots, k_{dn}(e, \xi)\}$ is the diagonal matrix of derivative gain. $K_e \in \mathbb{R}^{n \times n}$ is the diagonal matrix of position error, $K_\xi \in \mathbb{R}^{n \times n}$ is the diagonal matrix of velocity error. $k_{pi}(e, \xi), k_{di}(e, \xi) > 0, k_{ei}, k_{\xi i} \geq 1, i=1, 2, \dots, n$.

By (4), (7), (9) and iii), the boundedness of the control input is assured with satisfying

$$\tau_{im} > \sup |\tau_i| = \sup (k_{pi} + k_{di}) p + \|M_i\|_M \|\ddot{q}_d\|_M + \|C_{di}\|_M \|\dot{q}_d\|_M + |G_i|_M + f_{vi} |\dot{q}_{di}|_M \quad (15)$$

where M_i, C_{di} and G_i denote the i -th rows of M, C_d and G , respectively.

To improve the dynamic performance of the controller, a fuzzy self-tuning regulator is added to make K_p and K_d adaptive to the change of e and ξ in the closed-loop control. The design details of the regulator will be described in the following subsection.

4.3 Fuzzy Self-tuning Regulator

The block diagram of the fuzzy self-tuning regulator is shown in Figure 1.

Fuzzification. The physical universe of discourse of e_i, ξ_i, k_{pi} and k_{di} can all be partitioned into the same fuzzy sets {NB, NM, NS, Z, PS, PM, PB}, matching along with the integer universe of discourse: $\{-3, -2, -1, 0, 1, 2, 3\}$. The membership function of e_i, ξ_i, k_{pi} and k_{di} are chosen as asymmetrical polynomial curve based membership functions for NB and PB, while triangular membership functions for NM, NS, Z, PS and PM.

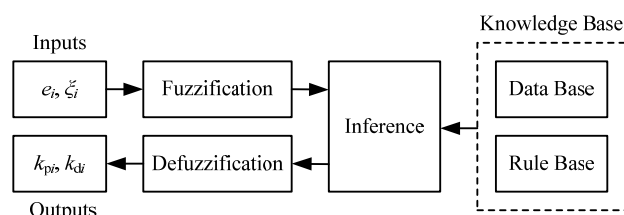


Figure 1. Structure of fuzzy self-tuning regulator

Knowledge Base and Inference. Knowledge base consists of two parts: data base and rule base. Data base provides membership functions related value of variables in fuzzy sets for inference, while the rule base offers the rules for inference engine, which is chosen as the Mamdani-type. To reduce the computational effort caused by massive inference rules of the fuzzy self-tuning, simplified rules are designed with neglecting the coupling relationship among the robot joints in the following form:

If e_i is A_i AND ξ_i is B_i , THEN k_{pi} is C_i AND k_{di} is D_i .

where A_i, B_i, C_i and D_i are certain elements of linguistic terms such as {NB, NM, NS, Z, PS, PM, PB}.

Defuzzification. The centroid method is selected in the defuzzification of outputs. It can be simply expressed as

$$k = \frac{\sum_{i=1}^n k_i u_i}{\sum_{i=1}^n u_i} \quad (16)$$

where k , as output of fuzzy regulator, denotes k_{pi} or k_{di} , k_i is the element in physical universe of discourse of k , u_i is the degree of membership of k_i .

Remarkably, when selecting PD gains, it is difficult to achieve simultaneously system stability and good performance (Hernandez-Guzman et al., 2008). In the flowing section, we will provide some constraints on the outputs of the fuzzy self-tuning regulator, to ensure both the exponential stability and satisfactory tracking performance of the system.

5. Stability Analysis

The theory of singularly perturbed systems (Khalil, 2007) is applied in this part to give a brief but strict stability proof of the proposed controller (Moreno-Valenzuela et al., 2008b, Moreno-Valenzuela et al., 2008a).

Theorem 1 Consider the nonlinear singularly perturbed system:

$$\dot{x} = f(t, x, z, \varepsilon) \quad (17)$$

$$\varepsilon \dot{z} = g(t, x, z, \varepsilon) \quad (18)$$

where $x \in \mathbb{R}^{n_1}, z \in \mathbb{R}^{n_2}, \varepsilon > 0$. Assume that the assumptions below are satisfied $\forall (t, x, \varepsilon) \in [0, +\infty) \times B_\eta \times [0, \varepsilon_0]$, $B_\eta = \{x \in \mathbb{R}^{n_1} : \|x\| \leq \eta\}$:

1. $f(t, 0, 0, \varepsilon) = 0$ and $g(t, 0, 0, \varepsilon) = 0$.
2. The equation $g(t, x, z, \varepsilon) = 0$ has an isolated root $z = h(t, x)$ such that $h(t, 0) = 0$.
3. The functions f, g, h and their partial derivatives up to the second order are bounded for $z - h(t, x) \in B_\rho$, where $B_\rho = \{x \in \mathbb{R}^{n_2} : \|x\| \leq \rho\}$.
4. The origin of the reduced system

$$\dot{\mathbf{x}} = f(t, \mathbf{x}, h(t, \mathbf{x}), 0) \quad (19)$$

is exponentially stable.

5. The origin of the boundary-layer system

$$\frac{d\mathbf{y}}{d\delta} = g(t, \mathbf{x}, \mathbf{y} + h(t, \mathbf{x}), 0) \quad (20)$$

is exponentially stable, uniformly in (t, \mathbf{x}) , where $\delta = t/\varepsilon$, and $\mathbf{y} = \mathbf{z} - h(t, \mathbf{x})$.

Then, there exists $\varepsilon^* > 0$ such that for all $\varepsilon^* > \varepsilon$, the origin of (17) and (18) is exponentially stable (Khalil, 2007).

Ordering $\mathbf{x} = [e^T \dot{e}^T]^T$, $\mathbf{z} = \xi$, $\mu_i = \mu$ ($i = 1, \dots, n$) and $\varepsilon = 1/\mu$, substituting (14) into (1), we obtain the similar form as (17) as described in Theorem 1:

$$\dot{\mathbf{x}}_1 = \dot{e} \quad (21)$$

$$\begin{aligned} \dot{\mathbf{x}}_2 = \ddot{e} = -\mathbf{M}^{-1} & \left[\mathbf{K}_p \text{Sat}(\mathbf{e}, \mathbf{K}_e) + \mathbf{K}_d \text{Sat}(\xi, \mathbf{K}_\xi) \right. \\ & \left. + (\mathbf{C} + \mathbf{C}_d + \mathbf{F}_v) \dot{e} \right] \end{aligned} \quad (22)$$

In the meantime, (10) and (12) can be written as follows, respectively.

$$\varepsilon \dot{\mathbf{z}} = \varepsilon \frac{d\xi}{dt} = \dot{e} - \xi \quad (23)$$

$$\varepsilon \dot{\mathbf{z}} = \varepsilon \frac{d\xi}{dt} = \dot{e} - \text{Sat}(\xi, \Delta) \quad (24)$$

When $\varepsilon = 0$, easily we can find (23) has an isolated root $\xi = \dot{e}$; and (24) has an isolated root $\text{Sat}(\xi, \Delta) = \dot{e}$, where we order $\Delta = \mathbf{K}_\xi$, the root can be written as $\text{Sat}(\xi, \mathbf{K}_\xi) = \dot{e}$.

Substitute the roots into (22), respectively, we can obtain

$$\begin{aligned} \dot{\mathbf{x}}_2 = \ddot{e} = -\mathbf{M}^{-1} & \left[\mathbf{K}_p \text{Sat}(\mathbf{e}, \mathbf{K}_e) + \mathbf{K}_d \text{Sat}(\dot{e}, \mathbf{K}_\xi) \right. \\ & \left. + (\mathbf{C} + \mathbf{C}_d + \mathbf{F}_v) \dot{e} \right]. \end{aligned} \quad (25)$$

$$\begin{aligned} \dot{\mathbf{x}}_2 = \ddot{e} = -\mathbf{M}^{-1} & \left[\mathbf{K}_p \text{Sat}(\mathbf{e}, \mathbf{K}_e) + \mathbf{K}_d \dot{e} \right. \\ & \left. + (\mathbf{C} + \mathbf{C}_d + \mathbf{F}_v) \dot{e} \right]. \end{aligned} \quad (26)$$

Then, (21) and (25) build up a reduced system (slow model) in the form of (19), while (21) and (26) build up another one.

For (23), let $\mathbf{y} = \mathbf{z} - \dot{e}$, $\delta = t/\varepsilon$, it can be written as a boundary-layer system (fast model) corresponding to the reduced system (21) and (25):

$$\frac{d\mathbf{y}}{d\delta} = \frac{d(\mathbf{z} - \dot{e})}{d\delta} = \frac{d\mathbf{z}}{d\delta} = \dot{e} - \xi \quad (27)$$

where \dot{e} is recognized as a constant for the scaled time variable δ .

Similarly, (24) can be written as a boundary-layer system (fast model) corresponding to the reduced system (21) and (26):

$$\frac{d\mathbf{y}}{d\delta} = \dot{e} - \text{Sat}(\xi, \Delta) \quad (28)$$

Remark: In contrast to the isolated root $\xi = \dot{e}$ of (23), from (24), we get the root $\text{Sat}(\xi, \mathbf{K}_\xi) = \dot{e}$ (where we order $\Delta = \mathbf{K}_\xi$). Note the definition of $\text{Sat}(\mathbf{x}, \Delta)$, property iii): $\|\text{Sat}(\mathbf{x}, \Delta)\| \leq \sqrt{n} p$, creates a bound to \dot{e} : $\dot{e} \leq \sqrt{n} p$, which may cause a too strict constraint condition $|\dot{e}| \leq p$. This results from applying a nonlinear filter described as (12) (Moreno-Valenzuela et al., 2008a), so we adopted the linear filter as (10) in the OFT design (Liu and Zhu, 2009). The following analysis also aims at the linear-filter case.

Proposition 1 For any initial state $[e^T(0) \dot{e}^T(0) \xi^T(0)]^T \in \Phi_\eta$, where $\Phi_\eta = \{\mathbf{x} \in \mathbb{R}^{3n}: \|\mathbf{x}\| \leq \eta\}$, $\eta > 0$ is arbitrarily large, if

$$\frac{\alpha_1 k_{dm}}{k_{\xi M}} + f_{vm} \left(\frac{\alpha_1}{k_{\xi M}} \right)^2 - k_c \|\dot{\mathbf{q}}_d\|_M \left(\frac{\alpha_2}{k_{\xi M}} \right)^2 > 0 \quad (29)$$

is satisfied, then there always exists $\varepsilon^* > 0$ such that for all $\varepsilon^* > \varepsilon$, the state space origin of the system (21)~(23) is exponentially stable.

Proof According to Theorem 1, the exponential stability proof of the singularly perturbed system (21)~(23) can be achieved by following the five steps below.

Step 1: From (21)~(23), we obtain $[e^T \dot{e}^T \xi^T]^T = [\mathbf{0} \ \mathbf{0} \ \mathbf{0}]^T$, when $[e^T \dot{e}^T \xi^T]^T = [\mathbf{0} \ \mathbf{0} \ \mathbf{0}]^T$.

Step 2: Ordering $\varepsilon = 0$, we can obtain the unique isolated root of (23): $\xi = \dot{e}$.

Step 3: Determine that the right sides of (21)~(23) and their partial derivatives up to the second order are bounded for $\xi - \dot{e} \in B_\rho$, where $B_\rho = \{\mathbf{x} \in \mathbb{R}^n: \|\mathbf{x}\| \leq \rho\}$.

Step 4: Stability of the reduced system (21) and (25). The state space origin $[e^T \dot{e}^T]^T = \mathbf{0}$ is the unique equilibrium point of the reduced system (21) and (25). Design a Lyapunov function $V(t, \mathbf{e}, \dot{e})$ (written as V for short):

$$\begin{aligned} V = \sum_{i=1}^n k_{pi} \int_0^{e_i} \text{sat}(e_i, k_{ei}) de_i + \frac{1}{2} \dot{e}^T \mathbf{M} \dot{e} \\ + v \dot{e}^T \mathbf{M} \text{Sat}(\mathbf{e}, \mathbf{K}_e) \end{aligned} \quad (30)$$

where the constant $v > 0$ is small enough to ensure the positive definiteness of Lyapunov function V .

Applying properties 2 and iiv), we obtain the inequality

$$V \geq \gamma_1 \left(\frac{k_{pi}}{k_{ei}} \right)_m \left\| \text{Sat}(\mathbf{e}, \mathbf{K}_e) \right\|^2 + \frac{1}{2} \lambda_m \{ \mathbf{M} \} \left\| \dot{\mathbf{e}} \right\|^2 - \nu \lambda_m \{ \mathbf{M} \} \left\| \dot{\mathbf{e}} \right\| \left\| \text{Sat}(\mathbf{e}, \mathbf{K}_e) \right\| \quad (31)$$

Therefore, it can be shown that, $\forall [\mathbf{e}^T \ \dot{\mathbf{e}}^T]^T \in B_\eta$, V is positive definite under the condition

$$\nu < \nu_1 = \sqrt{2\gamma_1 \lambda_m \{ \mathbf{M} \} \left(\frac{k_{pi}}{k_{ei}} \right)_m} / \lambda_m \{ \mathbf{M} \} \quad (32)$$

After taking the time derivative of (30), substituting (25) for $\ddot{\mathbf{e}}$ and applying property 1, we can obtain

$$\begin{aligned} \dot{V} = & -\dot{\mathbf{e}}^T \left[(\mathbf{C}_d + \mathbf{F}_v) \dot{\mathbf{e}} + \mathbf{K}_d \text{Sat}(\dot{\mathbf{e}}, \mathbf{K}_\xi) \right] \\ & + \nu \left[\ddot{\mathbf{e}}^T \mathbf{M} \text{Sat}(\mathbf{e}, \mathbf{K}_e) + \dot{\mathbf{e}}^T \dot{\mathbf{M}} \text{Sat}(\mathbf{e}, \mathbf{K}_e) \right. \\ & \left. + \dot{\mathbf{e}}^T \mathbf{M} \frac{\partial \text{Sat}(\mathbf{e}, \mathbf{K}_e)}{\partial \mathbf{e}} \dot{\mathbf{e}} \right] \end{aligned} \quad (33)$$

where the time derivative of the first part on the right of (30) can be written as $\dot{\mathbf{e}}^T \mathbf{K}_p \text{Sat}(\mathbf{e}, \mathbf{K}_\xi)$ by using the Newton-Leibniz formula.

According to properties 1~3, i)~vii) and the inequality $\|\dot{\mathbf{q}}\| \leq \|\dot{\mathbf{e}}\| + \|\dot{\mathbf{q}}_d\|$, we obtain

$$\begin{aligned} \dot{V} \leq & - \left[\begin{array}{c} \left\| \text{Sat}(\mathbf{e}, \mathbf{K}_e) \right\| \\ \left\| \text{Sat}(\dot{\mathbf{e}}, \mathbf{K}_\xi) \right\| \end{array} \right]^T \mathbf{S} \left[\begin{array}{c} \left\| \text{Sat}(\mathbf{e}, \mathbf{K}_e) \right\| \\ \left\| \text{Sat}(\dot{\mathbf{e}}, \mathbf{K}_\xi) \right\| \end{array} \right] \\ \leq & -\lambda_m \{ \mathbf{S} \} \left\| \left[\begin{array}{c} \left\| \text{Sat}(\mathbf{e}, \mathbf{K}_e) \right\| \\ \left\| \text{Sat}(\dot{\mathbf{e}}, \mathbf{K}_\xi) \right\| \end{array} \right]^T \right\|^2 \end{aligned} \quad (34)$$

where the entries of matrix $\mathbf{S} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}$ are

$$s_{11} = \nu k_{pm}, \quad s_{22} = T_1 - \nu T_2, \quad s_{12} = s_{21} = -\nu T_3,$$

$$\begin{aligned} T_1 &= k_{dm} \left(\frac{\alpha_1}{k_{\xi M}} \right) + f_{vm} \left(\frac{\alpha_1}{k_{\xi M}} \right)^2 - k_c \left\| \dot{\mathbf{q}}_d \right\|_M \left(\frac{\alpha_2}{k_{\xi m}} \right)^2, \\ T_2 &= \left(\sqrt{n} p k_c + \beta k_{em} \lambda_m \{ \mathbf{M} \} \right) \left(\frac{\alpha_2}{k_{\xi m}} \right)^2, \\ T_3 &= \left(k_c \left\| \dot{\mathbf{q}}_d \right\|_M + \frac{1}{2} f_{vm} \right) \left(\frac{\alpha_2}{k_{\xi m}} \right) + \frac{1}{2} k_{dm}. \end{aligned}$$

Specifically, if \mathbf{S} is positive definite, then \dot{V} is negative definite. Note that $s_{11} = \nu k_{pm} > 0$, by Sylvester's theorem, if the entries of \mathbf{S} satisfy $s_{11}s_{22} - s_{12}^2 > 0$, i.e.,

$$0 < \nu < \nu_2 = \frac{k_{pm} T_1}{k_{pm} T_2 + T_3^2} \quad (35)$$

then the positive definiteness of \mathbf{S} is guaranteed.

In addition, by properties 2, vi), and vii), with the same method of obtaining (34) we can obtain the inequality

$$\begin{aligned} V \leq & \left[\begin{array}{c} \left\| \text{Sat}(\mathbf{e}, \mathbf{K}_e) \right\| \\ \left\| \text{Sat}(\dot{\mathbf{e}}, \mathbf{K}_\xi) \right\| \end{array} \right]^T \mathbf{Q} \left[\begin{array}{c} \left\| \text{Sat}(\mathbf{e}, \mathbf{K}_e) \right\| \\ \left\| \text{Sat}(\dot{\mathbf{e}}, \mathbf{K}_\xi) \right\| \end{array} \right] \\ \leq & \lambda_m \{ \mathbf{Q} \} \left\| \left[\begin{array}{c} \left\| \text{Sat}(\mathbf{e}, \mathbf{K}_e) \right\| \\ \left\| \text{Sat}(\dot{\mathbf{e}}, \mathbf{K}_\xi) \right\| \end{array} \right]^T \right\|^2 \end{aligned} \quad (36)$$

where $\mathbf{Q} \in \mathbb{R}^{n \times n}$ is a constant symmetric matrix.

Then, from (34) and (36), we can obtain

$$\dot{V} \leq - \frac{\lambda_m \{ \mathbf{S} \}}{\lambda_m \{ \mathbf{Q} \}} V \quad (37)$$

i.e., $\forall [\mathbf{e}^T \ \dot{\mathbf{e}}^T]^T \in B_\eta$, the exponential stability of the reduced system (21) and (25) is assured, if $T_1 > 0$ (i.e., condition (29) is satisfied), and ν is small enough to meet the requirement of $0 < \nu < \min\{\nu_1, \nu_2\}$.

Step 5: Stability of the boundary-layer system (27). We define a Lyapunov function $W(\delta, \xi)$ (written as W for short):

$$W = \omega (\dot{\mathbf{e}} - \xi)^T (\dot{\mathbf{e}} - \xi) \quad (38)$$

where the constant ω is small enough.

The scaled time derivative of W is given by

$$\frac{dW}{d\delta} = -2\omega (\dot{\mathbf{e}} - \xi)^T (\dot{\mathbf{e}} - \xi) = -2W \quad (39)$$

So the exponential stability of the boundary-layer system (27) is guaranteed uniformly in (t, \mathbf{x}) , without any restrictions on the initial state conditions. Moreover, we find that the exponential convergence rate increases as the scaled time variable $\delta = t/\varepsilon$ increases, i.e., the smaller the ε the faster the exponential convergence of the boundary-layer system.

Till now, all five assumptions in Theorem 1 are satisfied. Consequently, there always exists $\varepsilon^* > 0$ such that $\forall \varepsilon > \varepsilon^*$ and $[\mathbf{e}^T(0) \ \dot{\mathbf{e}}^T(0) \ \xi^T(0)]^T \in \Phi_\eta$, the exponential stability of system (21)~(23) is guaranteed.

Importantly, to ensure the continuous stability of the overall system and the boundedness of the torque control

inputs. For the fuzzy regulator referred in Section 4.3, all the output values of k_{pi} and k_{di} should be restricted by (15) and (29).

6. Numerical Simulations

In this part, simulation comparisons between several fuzzy OFT controllers on a two-link direct-driven robot manipulator (see Figure 2) are presented to verify the effectiveness of the proposed approach.

The dynamics of the robot manipulator are given as

$$\begin{aligned} \mathbf{M} &= \begin{bmatrix} 3.3 + 0.24 \cos q_2 & 0.11 + 0.12 \cos q_2 \\ 0.11 + 0.12 \cos q_2 & 0.11 \end{bmatrix}, \\ \mathbf{C} &= \begin{bmatrix} -0.12 \dot{q}_2 \sin q_2 & -0.12 (\dot{q}_1 + \dot{q}_2) \sin q_2 \\ 0.12 \dot{q}_1 \sin q_2 & 0 \end{bmatrix}, \\ \mathbf{G} &= \begin{bmatrix} 48.02 \sin q_1 + 1.96 \sin(q_1 + q_2) \\ 1.96 \sin(q_1 + q_2) \end{bmatrix}, \\ \mathbf{F}_v &= \text{diag}\{2.5, 0.2\}, \\ \tau_{1M} &= 120 \text{ N} \cdot \text{m}, \\ \tau_{2M} &= 20 \text{ N} \cdot \text{m}. \end{aligned}$$

Without loss of generality, the desired position trajectories for each joint are given as

$$\begin{aligned} q_{d1}(t) &= \left[60(1 - e^{-3t^3}) + 20(1 - e^{-3t^3}) \sin(6t) + 10 \right]^\circ, \\ q_{d2}(t) &= \left[75(1 - e^{-2t^3}) + 105(1 - e^{-2t^3}) \sin(1.5t) + 15 \right]^\circ. \end{aligned}$$

Obviously, we can find that arctangent function $\text{atan}(\sigma_i x_i)$ (Liu et al., 2010, Liu and Zhu, 2009) and hyperbolic tangent function $\tanh(\sigma_i x_i)$ (Moreno-Valenzuela et al., 2008b) are two particular cases of the saturation function $\text{sat}(x_i, \sigma_i)$. As shown in Figure 3, in response to increasing values of σ_i , the zero-crossing slopes of function $\text{atan}(\sigma_i x_i)$ and $\tanh(\sigma_i x_i)$ steeply increase and rapidly approach saturation. Function $\text{atan}(\sigma_i x_i)$ has a wider range of $(-\pi/2, \pi/2)$ than $(-1, 1)$ of function $\tanh(\sigma_i x_i)$, and results in a more moderate approach to saturation for the same value of σ_i . These features have been proven to be helpful in improving the tracking performance (Liu and Zhu, 2009). Some other sample saturation functions such as $x_i(x_i^2 + 3)^{-1/2}$ and $x_i[\ln(\cosh x_i) + 3]^{-1}$ (Moreno-Valenzuela et al., 2008a) are also shown in Figure 3.

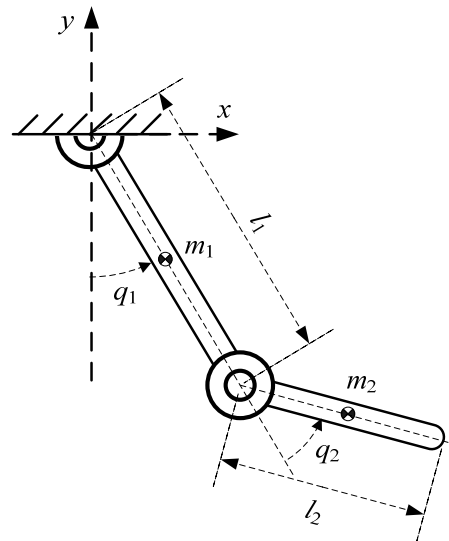


Figure 2. A two-link direct-driven robot manipulator

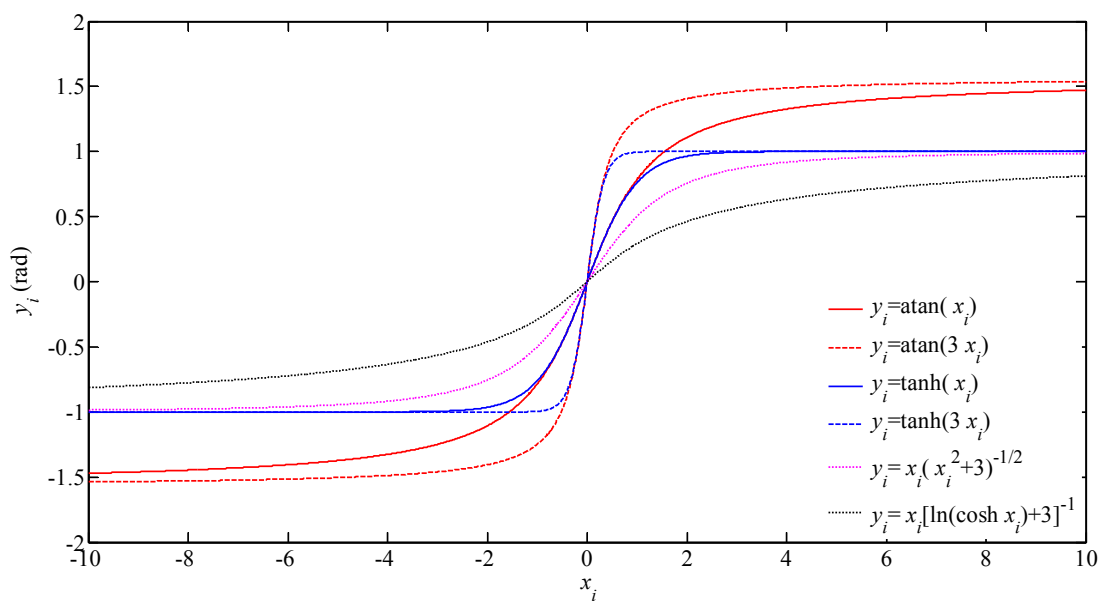


Figure 3. Curves of sample saturation functions

According to (14), (10) and (12), several sample controllers are given as

Controller “atan-linear”:

$$\begin{cases} \tau = K_p \text{Atan}(K_e e) + K_d \text{Atan}(K_\xi \xi) \\ \quad + M\ddot{q}_d + C_d \dot{q}_d + G + F_v \dot{q}_d \\ \dot{\xi} = U(\dot{e} - \xi) \end{cases} \quad (40)$$

which was proposed in (Liu et al., 2010), is a particular case of the generalized fuzzy OFT controller.

Controller “atan-nonlinear”:

$$\begin{cases} \tau = K_p \text{Atan}(K_e e) + K_d \text{Atan}(K_\xi \xi) \\ \quad + M\ddot{q}_d + C_d \dot{q}_d + G + F_v \dot{q}_d \\ \dot{\xi} = U[\dot{e} - \text{Atan}(K_\xi \xi)] \end{cases} \quad (41)$$

where $\text{Atan}(K_e e) = [\text{atan}(k_{e1} e_1), \text{atan}(k_{e2} e_2), \dots, \text{atan}(k_{en} e_n)]^T$, $\text{Atan}(K_\xi \xi) = [\text{atan}(\xi_1, k_{\xi 1}), \text{atan}(\xi_2, k_{\xi 2}), \dots, \text{atan}(\xi_n, k_{\xi n})]^T$.

Controller “tanh-linear”:

$$\begin{cases} \tau = K_p \text{Tanh}(K_e e) + K_d \text{Tanh}(K_\xi \xi) \\ \quad + M\ddot{q}_d + C_d \dot{q}_d + G + F_v \dot{q}_d \\ \dot{\xi} = U(\dot{e} - \xi) \end{cases} \quad (42)$$

which has the similar form of the controller proposed in (Moreno-Valenzuela et al., 2008b).

Controller “tanh-nonlinear”:

$$\begin{cases} \tau = K_p \text{Tanh}(K_e e) + K_d \text{Tanh}(K_\xi \xi) \\ \quad + M\ddot{q}_d + C_d \dot{q}_d + G + F_v \dot{q}_d \\ \dot{\xi} = U[\dot{e} - \text{Tanh}(K_\xi \xi)] \end{cases} \quad (43)$$

which has the similar form of the controller proposed in (Moreno-Valenzuela et al., 2008a).

In addition, the controller proposed in (Loria and Ortega, 1995) (called “none-linear”) can be written as

$$\begin{cases} \tau = K_p e + K_d \xi \\ \quad + M\ddot{q}_d + C_d \dot{q}_d + G + F_v \dot{q}_d \\ \dot{\xi} = U(\dot{e} - \xi) \end{cases} \quad (44)$$

To make fair comparisons, we apply the same gains: $K_e = \text{diag}\{3, 2\}$, $K_\xi = \text{diag}\{3, 2\}$, $U = \text{diag}\{500, 500\}$, and the same fuzzy self-tuning regulator for each controller. When selecting the physical universe of discourse for PD gains

(K_p and K_d), by conditions (15) and (29), we can set the following restrictions:

$$k_{p1} + k_{d1} < 56.2, \quad k_{p2} + k_{d2} < 7.8; \\ \min\{k_{d1}, k_{d2}\} = k_{dm} > 0.38.$$

The parameters and fuzzy rules for the inference of the fuzzy PD self-tuning regulator are set as shown in Tables 1 and 2, respectively.

Variables	e_1, e_2 (°)	ξ_1, ξ_2 (°)	$k_{p1},$ k_{p2}	$k_{d1},$ k_{d2}
Physical Universe of Course	[-12, 12]	[-15, 15]	[2, 41]	[0.8, 14]
Quantization Factor	1/4, 1/4	1/5, 1/5	2/13, 5/4	5/11, 5
Integer Universe of Course	{-3, -2, -1, 0, 1, 2, 3}			

Table 1. Parameters of the Fuzzy PD Regulator

$k_{pi},$ k_{di}	ξ_i						
	NB	NM	NS	Z	PS	PM	PB
e_i	NB	PM,	PS,	PS,	PS,	Z,	NS,
		Z	NS	NM	NS	Z	NS
	NM	PM,	Z,	PM,	PM,	Z,	PS,
		PS	Z	NS	NS	Z	Z
	NS	PM,	PS,	PB,	PM,	PM,	Z,
		PM	PM	NS	NB	NS	PS
	Z	PB,	PM,	PB,	PM,	PB,	NS,
		PB	PB	NM	NB	Z	PM
	PS	PB,	PS,	PM,	PM,	PM,	Z,
		PM	PM	NS	NB	NS	PM
	PM	PM,	Z,	PS,	PM,	PM,	Z,
		Z	PS	NS	NM	NS	PS
PB	PM,	NS,	PS,	Z,	Z,	NS,	Z,
	NS	Z	NM	NM	NM	Z	NS

Table 2. Fuzzy Rules for PD Gains

The architecture comparison of the five controllers is shown in Table 3.

Controller	Saturation Function	Filter
atan-linear	atan	linear
atan-nonlinear	atan	nonlinear
tanh-linear	tanh	linear
tanh-nonlinear	tanh	nonlinear
none-linear	NONE	linear

Table 3. Architecture Comparison

Furthermore, to test the anti-disturbance capability of the controllers, we applied man-made step torques as external disturbances during the 7th second of the total execution

time 16s, with amplitudes of 0.6 N·m to the first joint and 0.2 N·m to the second one.

To make overall appraisalment to the performance of each controller, we adopt five criteria as follows:

Adjusting time — a period from the start to the moment tracking error e_i falls into the area of $\pm 0.05^\circ$;

Overshoot — the maximum value of $|e_i|$ during the adjusting procedure;

Recovery time — a period from the end of disturbances to the moment tracking error e_i falls into the area of $\pm 0.05^\circ$;

Maximum deviation — the maximum value of $|e_i|$ from the start of disturbances to the end of the recovery procedure;

Maximum torque — the maximum value of $|\tau_i|$ during the whole procedure.

As shown in Figures 4, 5 and Table 4, controller “atan-linear” has the shortest adjusting time and recovery

time, the smallest maximum deviation, and a relative small overshoot, representing superior capacity of dynamic response and anti-disturbance to the others with continuous asymptotic stability. This is because we applied a fuzzy PD self-tuning regulator whose outputs, k_{pi} and k_{di} , are properly constrained by (15) and (29), and invoked a saturation function with a more moderate approach to saturation and wider range, arctangent, in the control law.

Moreover, as shown in Figures 6, 7 and Table 4, controller “atan-linear” has the smallest maximum torque control input, while controller “none-linear” fails to keep the initial torque inputs in the given limited range, with maximum torque of about 593 Nm to the first joint and 119 Nm to the second one, just for the reason that there is no saturation function in the control law to create bounded inputs as the other controllers.

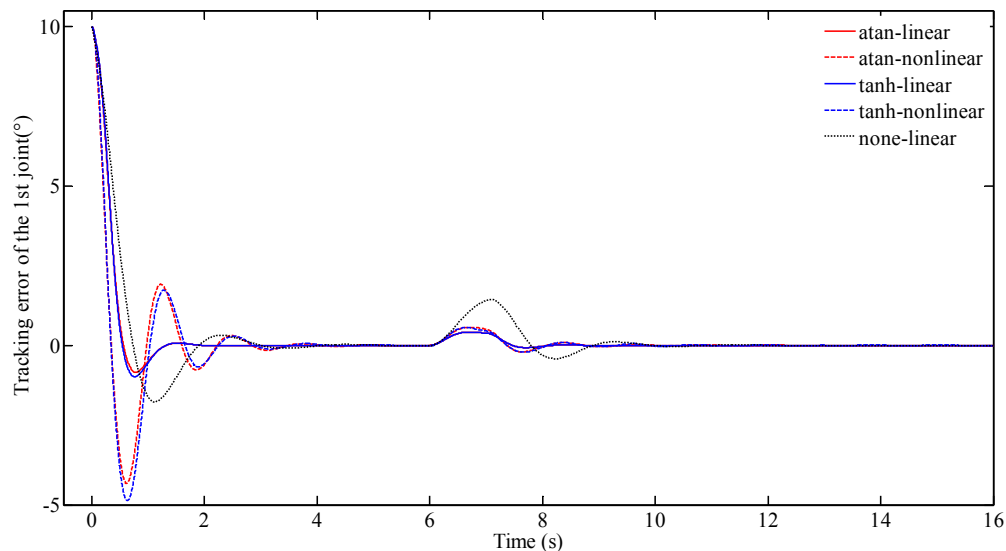


Figure 4. Tracking error of the first joint

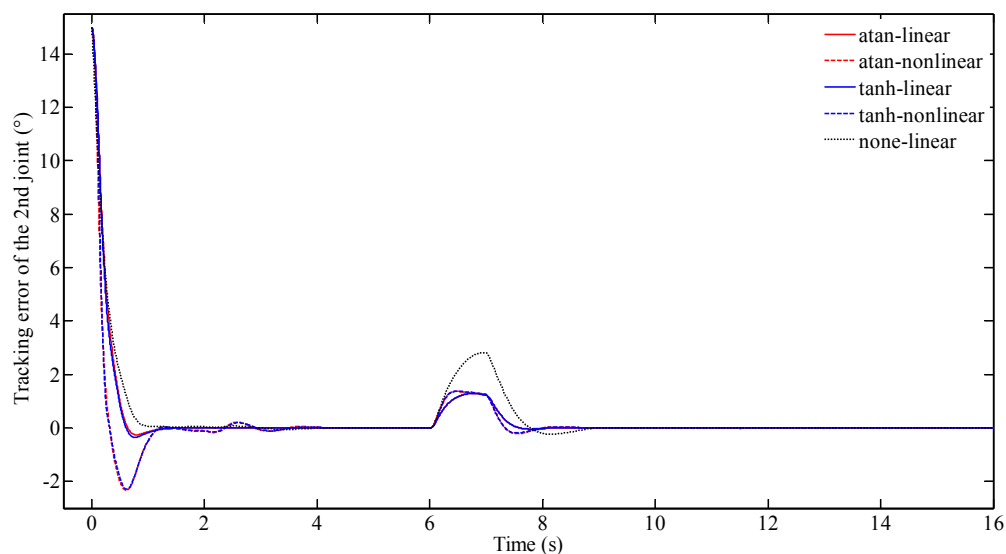


Figure 5. Tracking error of the second joint

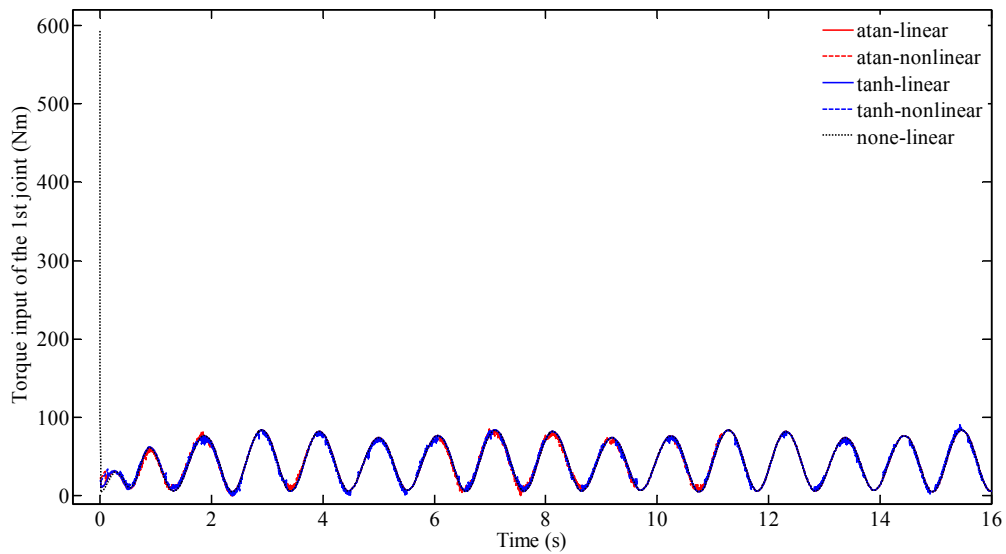


Figure 6. Torque input of the first joint

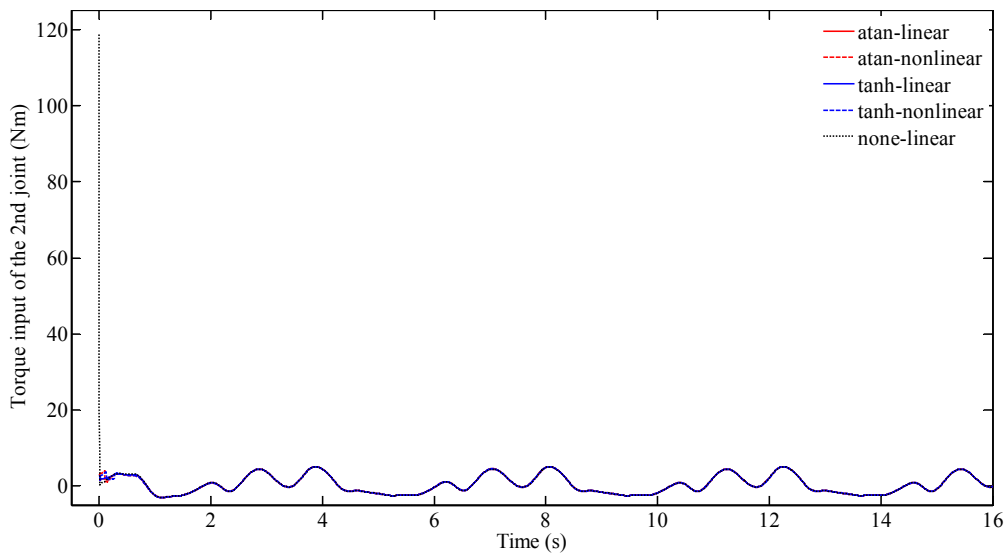


Figure 7. Torque input of the second joint

Parameter		Value				
		atan-linear	atan-nonlinear	tanh-linear	tanh-nonlinear	none-linear
Adjusting time (s)	1st joint	1.696	3.778	1.709	3.904	3.818
	2nd joint	1.204	3.772	1.239	3.404	3.528
Overshoot (°)	1st joint	0.844	4.332	0.983	4.862	1.770
	2nd joint	0.259	2.331	0.349	2.295	0.053
Recovery time (s)	1st joint	0.900	1.590	0.900	2.662	2.686
	2nd joint	0.526	0.908	0.526	0.938	1.774
Maximum deviation (°)	1st joint	0.419	0.567	0.419	0.561	1.444
	2nd joint	1.286	1.369	1.286	1.375	2.816
Maximum toque (Nm)	1st joint	84.039	85.530	84.039	90.770	592.692
	2nd joint	5.065	5.072	5.065	5.071	118.705

Table 4. Performance Comparison

7. Conclusions

In this paper, a generalized fuzzy saturated OFT controller has been developed for the trajectory tracking for robot manipulators with bounded torque inputs to the joint actuators. A general saturation function is invoked in the control law to ensure the boundedness of the torque inputs, and linear/nonlinear filters are optionally applied in the closed-loop control to eliminate the velocity measurements. A fuzzy regulator is used to make the PD gains vary as the position and pseudo velocity tracking errors change with time. More importantly, we made a brief but strict stability proof for the whole nonlinear systems by the stability theory of singularly perturbed systems. Benefiting from it, some proper constraints on certain gains have been obtained to ensure both the exponential stability and satisfactory tracking performance of the system. Based on the general formula of the proposed controller, several sample controllers were given to test the tracking performance. The comparison reveals that all the controllers work efficiently, and the “atan-linear” controller, which involves arctangent as the saturation function and linear filter in the control law, has better performance than the others.

8. References

- [1] Andrieu, V. & Praly, L. (2009). A unifying point of view on output feedback designs for global asymptotic stabilization. *Automatica*, Vol. 45, No. 8, pp. 2145-2158.
- [2] Burkov, I.V. (1998). Stabilization of a natural mechanical system without measuring its velocities with application to the control of a rigid body. *Journal of Applied Mathematics and Mechanics*, Vol. 62, No. 6, pp. 853-862.
- [3] Hernandez-Guzman, V.M., Santibanez, V. & Silva-Ortigoza, R. (2008). A new tuning procedure for PID control of rigid robots. *Advanced Robotics*, Vol. 22, No. 9, pp. 1007-1023.
- [4] Huang, C.Q., Peng, X.F., Jia, C.Z. & Huang, J.D. (2008). Guaranteed robustness/performance adaptive control with limited torque for robot manipulators. *Mechatronics*, Vol. 18, No. 10, pp. 641-652.
- [5] Kelly, R., Santibanez, V. & Loria, A. (2005). *Control of Robot Manipulators in Joint Space*. Berlin: Springer-Verlag.
- [6] Khalil, H.K. (2007). *Nonlinear Systems*. Beijing: Publishing House of Electronics Industry.
- [7] Liu, H.S. & Zhu, S.Q. (2009). A generalized trajectory tracking controller for robot manipulators with bounded inputs. *Journal of Zhejiang University - Science A*, Vol. 10, No. 10, pp. 1500-1508.
- [8] Liu, H.S., Zhu, S.Q. & Chen, Z.W. (2010). Saturated output feedback tracking control for robot manipulators via fuzzy self-tuning. *Journal of Zhejiang University - Science C*, Vol. 11, No. 12, pp. 956-966.
- [9] Llama, M.A., Kelly, R. & Santibanez, V. (2001). A stable motion control system for manipulators via fuzzy self-tuning. *Fuzzy Sets and Systems*, Vol. 124, No. 2, pp. 133-154.
- [10] Loria, A. & Nijmeijer, H. (1998). Bounded output feedback control of full actuated Euler-Lagrange systems. *Systems and Control Letters*, Vol. 33, No. 3, pp. 151-161.
- [11] Loria, A. & Ortega, R. (1995). On tracking control of rigid and flexible joint robots. *International Journal of Applied Mathematics and Computer Science*, Vol. 5, No. 2, pp. 101-113.
- [12] Moreno-Valenzuela, J., Santibanez, V. & Campa, R. (2008a). A class of OFT controllers for torque-saturated robot manipulators: Lyapunov stability and experimental evaluation. *Journal of Intelligent and Robotic Systems*, Vol. 51, No. 1, pp. 65-88.
- [13] Moreno-Valenzuela, J., Santibanez, V. & Campa, R. (2008b). On output feedback tracking control of robot manipulators with bounded torque input. *International Journal of Control, Automation, and Systems*, Vol. 6, No. 1, pp. 76-85.
- [14] Moreno-Valenzuela, J., Santibanez, V., Orozco-Manriquez, E. & Gonzalez-Hernandez, L. (2010). Theory and experiments of global adaptive output feedback tracking control of manipulators. *IET Control Theory and Applications*, Vol. 4, No. 9, pp. 1639-1654.
- [15] Santibanez, V. & Kelly, R. (2001). Global asymptotic stability of bounded output feedback tracking control for robot manipulators. *Proceedings of the 40th IEEE Conference on Decision and Control*, pp. 1378-1379.
- [16] Santibanez, V., Kelly, R. & Llama, M.A. (2005). A novel global asymptotic stable set-point fuzzy controller with bounded torques for robot manipulators. *IEEE Transactions on Fuzzy Systems*, Vol. 13, No. 3, pp. 362-372.