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B. Madhu Rao, Jeroen Beliën



To cite this article:

B. Madhu Rao, Jeroen Beliën (2014) Case Article—Production Scheduling at Falcon Die Casting: A Comprehensive Example on the Application of Linear Programming and Its Extensions. INFORMS Transactions on Education 15(1):150-153. <https://doi.org/10.1287/ited.2014.0132ca>

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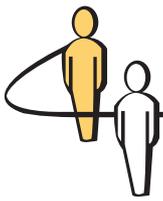
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## Case Article

# Production Scheduling at Falcon Die Casting: A Comprehensive Example on the Application of Linear Programming and Its Extensions

B. Madhu Rao

School of Business Administration, Stetson University, DeLand, Florida, 32726,  
bmr Rao@stetson.edu

Jeroen Beliën

Center for Information Management, Modeling and Simulation (CIMS), KU Leuven, B-1000 Brussels, Belgium,  
jeroen.belien@kuleuven.be

A comprehensive case study to introduce students to several concepts from linear, integer, and goal programming in an integrative manner is presented. Starting with a relatively simple production planning decision problem, the case progressively incorporates generalizations to introduce various modeling principles. Each variation to the case enables the teacher to engage in insightful discussion into the nature of the optimal solutions; the effect on the optimum solution of changes to the objective function or to key constraints. The case article also describes several ways in which the case can be used, ranging from a comprehensive introduction to a concluding capstone case in an undergraduate or graduate course in linear programming.

*Keywords:* linear optimization; production planning; case study; spreadsheet optimization

*History:* Received: July 2013; accepted: August 2014.

## 1. Introduction

In this paper we present a comprehensive case study, which, starting with a relatively simple production planning decision problem, progressively adds generalizations with the goal of introducing the students to several modeling principles from linear, integer and goal programming in an integrative manner. This case fills the need for a rich problem that incorporates multiple principles. The following are some of the pedagogical benefits of the case.

- Textbook examples typically introduce principles of linear, integer, and goal programming in separate examples and communicate them as discrete and isolated ideas. For example, textbook coverage of deviational variables (Ragsdale 2012, Chap. 7; Winston 2004, Chap. 4) is often included under goal programming and their uses in other settings such as in modeling unrestricted variables, dealing with infeasible models is not always discussed in detail.

- As a comprehensive and integrative example, this case study can serve as a capstone experience by reinforcing and connecting many apparently disparate ideas studied over a semester into a single

scenario. Each variation introduced in the case study also demonstrates the limitations of each modeling tool and shows how each of the limitations may be overcome with creative application of the general principles.

- By progressively observing the changes to the optimum solution as a result of each generalization, students can gain greater insight into the nature of the solutions generated by mathematical modeling and the effect of key constraints and the objective function on the optimum solution.

- A major challenge in teaching OR/MS courses, especially to students for whom OR/MS is not the primary field of study (e.g., MBA students), is to motivate them to recognize the need for OR/MS tools in decision making (Beliën et al. 2013). Examples based on specialized applications generally require the instructor and students to spend a considerable amount of time in understanding the background before the student can effectively model the decision problem under consideration. In such cases, “students lose track of the OR/MS aspects when drowning in

the details” (Trick 2004, p. 10). This case presents a reasonable compromise by presenting a realistic scenario in an efficient and compact manner.

- Extended discussion of decision making implications of nonstandard outcomes (e.g., unbounded optimum) are rarely carried out. For example, even when the result indicates no feasible solution, the manager still needs information to assist in decision making. “Coaxing” the model (e.g., use deviational variables) to generate practical recommendations along with information on how to correct the infeasibility will greatly assist the manager in making an informed decision. Similarly, implementation of the optimal solutions often need additional considerations and these issues are rarely discussed in the textbooks. This case considers these issues explicitly.

- This approach of progressively adding complexities to a basic decision problem is consistent with the commonly held wisdom that teaching new concepts and material is more effective when it is explicitly linked to material that students are already familiar with.

## 2. An Overview of the Case

The decision problem at the core of the case is the scheduling of weekly production of five products on five machines, where each product can be produced only on a subset of the machines. Each time the production on a machine changes from one product to another, a significant amount of machine time is spent on set-up. Regular time production capacity is insufficient to meet the demand and overtime production is generally necessary.

A key factor in the case is the time horizon for planning. The first part of the case guides the students to think in terms of weekly production by suggesting that due to technical limitations, each week’s production planning starts afresh, independent of the previous week’s production schedule. We will refer to this model and its variations as the *one-week model*. One means of addressing this decision problem is to formulate a linear programming model for production planning with the objective of meeting the customer demand with minimum overtime. A series of case questions requires the students to modify the one-week model to deal with increasingly complex decision making scenarios with each variation providing the motivation to use different modeling concepts. Students can gain insight into the nature of the solutions by observing the changes to the optimum solution resulting from the changes to the model. The second part of the case relaxes some of the technical limitations to permits the production schedules for two (or more) weeks to be developed simultaneously. We will refer to this model and its variations as the *two-week model*.

## 3. Possible Uses of the Case

This case can be used in courses in linear programming in undergraduate or graduate programs in operations research, industrial engineering or business administration. The case can be used in several ways depending on the academic background and interests and needs of the students.

1. In a first course in linear programming, the one-week model can be used for class discussion to provide the basic introduction to binary variables, creative linearization of potential nonlinear (e.g., maximum) functions into linear constraints and to introduce the value of deviational variables in mathematical modeling. Depending on the mathematical sophistication of the students, the two-week model may be used as a concluding capstone example in the course.

2. In a second course in linear programming, the one-week model can be used to refresh the concepts from the earlier course and the two-week model used to discuss advanced modeling issues as well as the decision making and implementation aspects.

3. The problem setting with multiperiod, multi-product production scheduling with setup times makes the case useful in graduate or advanced undergraduate courses in production planning. For this group of students, the class discussion may focus on the production planning issues with secondary discussion of the OR/MS aspects. Depending on the mathematical sophistication of the students, class discussion may or may not include the two-week model.

## 4. Teaching Suggestions

This case is designed to progressively add complexity to the scheduling problem and is most useful for classroom discussions rather than as an evaluative tool.

It is perhaps best to start the case discussion by introducing the case at the end of a class and ask the students to develop a feasible one-week schedule. This will highlight the inherent difficulty in solving the associated optimization problem. The next class can start with a discussion of the heuristics used by the students and the resulting solutions and then demonstrate the effectiveness and efficiency of a mathematical modeling approach.

For the one-week model, we recommend that sufficient class time be devoted to the discussion of the numerical results to highlight the changes to the optimum solution as more and more constraints and real-world considerations are added.

The modeling of setups is relatively straightforward for the one-week model, but more challenging for the two-week model. Incorporating setups in the two-week model provides an opportunity to develop

advanced skills in the use of binary variables, but this requires considerable class time in order to be effective.

Comparing and contrasting the two-week model with the Traveling Salesman Problem (TSP) is also a good learning opportunity for the students. In production scheduling with sequence-dependent setup times, explicit determination of the sequence of production is necessary and the TSP algorithm is often used to accomplish this. In the current example, with sequence-independent setup times, sequence of production is relevant only to the extent that the knowledge of the last item produced on a machine is necessary to schedule the following week's production. This could generate discussion on how partial information on the sequence of production (i.e., last item produced on each machine each week) can be obtained more easily than the complete sequence of production.

The two-week model has all the elements in place for extension to schedule three or more weeks of production at a time. This, while routine, is tedious and time consuming and is perhaps not a good use of class time. However, a discussion of the implementation of the multiweek schedules is well worth the class time. At the end of week 1 ( $t$  in general), when the final demand numbers for week 3 ( $t+2$  in general) are available, how should Falcon Die Casting (FDC) start planning for week 2 (or  $t+1$ ) production? One answer is to discard the week 2 (or  $t+1$ ) schedules determined by the linear programming (LP) model and generate an entirely new two-week production schedule for weeks  $t+1$  and  $t+2$  using the latest two-week demand data. In this approach, even though the week  $t+1$  production schedules are routinely discarded, scheduling week  $t$  production taking week  $t+1$  demands into consideration will improve the overall efficiency by yielding an advantageous initial setup condition on the machines for week  $t+1$ .

The final variation presented in the case deals with circumstances where the total production capacity, even with maximum overtime, is not sufficient to satisfy the weekly demand. Here, deviational variables can be used to deal with the infeasibility and generate a "practical" production schedule along with information on the extent of shortfall, so that the customer can be informed accordingly. This can be used to lead a discussion on the need to generate useful information for the manager, even when the model returns a nonstandard result such as unbounded optimum or no feasible solution.

Additional details and further teaching suggestions are provided in the accompanying case teaching note (available as restricted instructor material at <http://dx.doi.org/10.1287/ited.2014.0132ca>).

The sizes of the LP models are within the limits of the student versions for most standard LP software. The results presented in the teaching note were obtained using the student version of the Analytic Solver Platform (Analytic Solver Platform 2013) and confirmed by the corresponding AMPL (Fourer et al. 2003) CPLEX (IBM ILOG CPLEX Optimization Studio 2013) models. If formulated as a spreadsheet model, the most complex model in the case is within the size limits of a standard Excel Solver. We, however, recommend using the student version of the Analytic Solver Platform (Analytic Solver Platform 2013) to obtain the solutions in a reasonable time. The spreadsheet, AMPL and CPLEX models are available as supplemental material at <http://dx.doi.org/10.1287/ited.2014.0132ca> to interested faculty members.

## 5. Classroom Experience

The Falcon Die Casting case has been used in an advanced course on operations research and operations management at the KU Leuven (Brussels campus), Belgium, in the business engineering program (December 2013). There were 19 students in the class (typically 23 years old) who have already had an introductory course to linear and integer programming. The case teaching note lists the nine questions that we assigned.

The case was discussed in three consecutive classes spread over three weeks. In the first class, the case was introduced for about 30 minutes. The students were assigned the static one-week problem in which all machines have been reset during the weekend requiring new setups to start production as homework to be solved using the IBM ILOG CPLEX Optimization studio (Version 12.3). The second class (2 hours) started with a discussion of question 1 and the solution to question 2 followed by questions 3, 4, and 5. For each question, students were given 10–15 minutes to revise the model for the previous question after which the new model and the corresponding optimal solution were discussed. The problem settings, solutions and discussions were guided by a PowerPoint presentation (available as supplemental material). The remaining questions were tackled in the last class (3 hours) using the same approach.

Based on our experience, we believe that this case is very useful as a teaching case for the following reasons. First, starting with the basic one-week decision problem made it easy for students to be engaged with the problem (as described above) without having to deal with the more complicated issues such as the setup sequences. Second, the case encompasses many different modeling techniques (such as min-max objective, multiperiod problem setting, if-then

**Table 1** Summary of Student Responses

Level of agreement	S1	S2	S3	S4	S5	S6
Totally agree	1	5	3	7	8	2
Agree	8	9	9	7	6	9
Neither agree nor disagree	5	0	2	0	0	3
Disagree	0	0	0	0	0	0
Totally disagree	0	0	0	0	0	0
Total	14	14	14	14	14	14

constraints using binary variables, multiple objectives, Big-M constraints) which makes it highly suitable as a concluding case in a course in which these techniques have been addressed earlier. Third, the different solutions provide additional insight into the nature of the problems. By studying the solutions, the students can achieve better understanding of the problem setting. Fourth, the fact that this case is based on a real-life production scheduling problem makes it much more interesting to students than fictitious exercises from a textbook. Consequently, we experienced a motivated group of students which of course enriches the teaching experience.

An online survey with the following statements (S1, S2, . . . , S6) was sent to the students to obtain a formal evaluation of the case. 14 out of the 19 students responded and their responses are summarized in Table 1.

S1: The Falcon Die Casting case has increased my interest in optimization modeling.

S2: The Falcon Die Casting case has increased my insight in the use of optimization modeling for solving real-life production scheduling problems.

S3: The Falcon Die Casting case has improved my optimization modeling skills.

S4: I think that the Falcon Die Casting case has a surplus value compared to typical textbook exercises (see linear programming course) with respect to learning how to successfully apply optimization modeling in practice.

S5: In the Falcon Die Casting case, progressively adding complexities to the basic decision problem improved my understanding of the modeling process.

S6: In the Falcon Die Casting case, observing the changes to the optimal solution resulting from changes to the model provided me good insight on the nature of the solutions.

The high level of agreement with the first three statements shows that the Falcon Die Casting case has increased student interest in optimization modeling and, to an even larger extent, increased the modeling capabilities and insight into applying optimization modeling. All students agreed that the case has a surplus value compared to traditional textbook exercises (Statement 4) and confirmed that the approach of progressively adding complexities improved their

understanding of the modeling process (Statement 5). The added value of discussing the different solutions with respect to understanding the nature of the solutions was, though to a smaller extent, also confirmed by our students (Statement 6). The final open ended statement generated the following comments (translated from Dutch).

- Generally, a nice and interesting but “hard” case for students. The step-by-step approach is really required.
- In some cases, I would like to have some more explanation for certain constraints in the model, why exactly these formulations (and not other ones). Generally, a case from practice is always more interesting than theory.
- Very interesting and useful case.

These comments stress the importance of a step-by-step approach, indicating that certain extensions are far from straightforward and, consequently, sufficient attention should be paid to explaining the corresponding constraint formulations.

These results might be biased towards a positive experience because students tend to answer what teachers like to hear. It is also possible that the five students who did not complete the survey were not as impressed by the Falcon Die Casting case. Even if one assumes that all five nonresponding students disagreed with all the statements, it is clear that the majority of the 19 students still agrees with Statements 2, 4, and 5.

### Acknowledgments

The authors thank the two referees, the associate editor and the editor for the many excellent suggestions which greatly improved this paper. The authors are particularly indebted to the editor for the meticulous and painstaking reading of the final manuscript and for the many editorial improvements to the presentation of the paper. The authors gratefully acknowledge the academic licenses provided by the publishers of AMPL (Fourer et al. 2003), IBM ILOG CPLEX (IBM ILOG CPLEX Optimization Studio 2013), and Analytic Solver Platform (Analytic Solver Platform 2013).

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