Model Predictive Control for a Small Scale Unmanned Helicopter

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Abstract: *Kinematical and dynamical equations of a small scale unmanned helicoper are presented in the paper. Based on these equations a model predictive control (MPC) method is proposed for controlling the helicopter. This novel method allows the direct accounting for the existing time delays which are used to model the dynamics of actuators and aerodynamics of the main rotor. Also the limits of the actuators are taken into the considerations during the controller design. The proposed control algorithm was verified in real flight experiments where good perfomance was shown in postion control mode.*
Keywords: *MPC, unmmaned helicopter, flight control, helicopter modeling*

1. Introduction

Small-scale unmanned helicopters are widely used in many application fields because of their small size and superior flight characteristics, such as vertical take-off, landing, and hovering. They can be used in search and rescue after big natural disasters, patrol and surveillance, filming movies, suppression of smuggling, inspection of power lines, large bridges, dams and so on.

Helicopter models are strongly coupled, multi-variate, time delay nonlinear systems. Therefore the design of an elaborated control system for autonomous flight is a challenging task (Chen, W. & Wang, D. 2004). Many universities, such as Georgia Institute of Technology, Stanford, UC Berkley, Waterloo, etc. are researching on small scale unmanned helicopters.

Many controller design methods rely on dynamical equations which are difficult to obtain in a non-simplified form because of the aerodynamics of the main rotor. Even if the model is obtained, it may be hard to use it for controller design, because of its complexity. Often the model has to be simplified in order to be used in controller design. In (An, S. et al. 2005; Kadmiry, B. & Driankov, D. 2004; Kondak, K. et al. 2006; Kondak, K. et al. 2004; McLean, D. & Matsuda, H. 1998; Sanders, C. P. & DeBitettoc, P. A. 1998; Sugeno, M. et al. 1993) results on the dynamic modeling and control of helicopters are presented. In these papers, several controller design methods, such as PID, adaptive nonlinear control, neural network control, and fuzzy control are discussed. All these methods have one common drawback: They have first to measure the plant output and compare it with the desired output, and then they generate the control signal, so the controller output depends on the actual error and short part of the past state trajectory. Compared with these methods, model predictive control generates the control signal based not only on actual error and past state trajectory but also on the future behavior of the system. Even more, the control rules (gains) are changing in every control cycle in order to minimize the control error and satisfy the defined constraints (e.g. limits for controller outputs). For these reasons, a higher computational effort is required, but also superior performance is expected from a model predictive controller even if the model is rough(Xi, Y. 1993).

In (Balderud, J. & Wilon, D. I. 2002; Dutka, A. S. et al. 2003), Jonas Balderud and Arkadiusz S. Dutka used a model predictive controller to control a 2 DOFs toy helicopter. In the papers, the controller are used to control the elevation and azimuth angles. In our paper, the model predictive controller which control the rotation of the helicopteris is applied to a 6 DOFs helicopter.

The paper is organized as follows. Section 2 reviews multivariate state space model predictive control with constraints and section 3 explains the kinematic and dynamic model of the helicopter. Section 4 presents the proposed control algorithms. The experiments are shown in section 5. Finally, conclusions and future work are outlined in section 6.

2. MPC Algorithm

Model based predictive control makes an explicit use of a model of the plant to obtain the control signal by minimizing an objective function. MPC consists of tree steps: predicting the plant output at a future time

moment by use of the plant model, calculating a control sequence by minimizing an objective function, receding, which means that at each moment the horizon is shifted forward, and apply the first control signal of the sequence calculated at each step(Camacho, E. F. & Bordons, C. 1999).

MPC can use any kind of model description, e.g. impulse response, step response, transfer function, state space, to predict the plant output. In this paper we use a state space model:

$$x(k+1) = A x(k) + B u(k)$$
$$y(k) = C x(k) \tag{1}$$

where, x(k) is the n-dimension state vector of the plant, u(k) is the m-dimension vector of manipulated variables, y(k) is the p-dimension vector of the plant output.

The MPC algorithm minimizes an objective function (2) for obtaining the control law(Xi, Y. 1993):

$$\min J(k) = \left\| w(k) - y_{PM}(k) \right\|_Q^2 + \left\| \triangle u_M(k) \right\|_R^2 \tag{2}$$

where, w(k) is the reference signal,

$$w(k) = \begin{bmatrix} w_1(k) \\ \vdots \\ w_p(k) \end{bmatrix}, \quad w_i(k) = \begin{bmatrix} w_i(k+1) \\ \vdots \\ w_i(k+P) \end{bmatrix}, \quad i = 1, \cdots p$$

$y_{PM}(k)$ is the predictive output,

$$y_{PM}(k) = \begin{bmatrix} y_{1,PM}(k) \\ \vdots \\ y_{p,PM}(k) \end{bmatrix}, \quad y_{i,PM}(k) = \begin{bmatrix} y_{i,M}(k+1|k) \\ \vdots \\ y_{i,M}(k+P|k) \end{bmatrix}, \quad i = 1, \cdots p$$

$$y_{i,M}(k+j|k) = CA^j x(k) + \sum_{g=0}^{j-1} CA^{j-g-1} B \left[u(k-1) + \sum_{h=0}^{g} \triangle u(k+h) \right]$$

$\triangle u_M(k)$ is the differenced input,

$$\triangle u_M(k) = \begin{bmatrix} \triangle u_{1,M}(k) \\ \vdots \\ \triangle u_{m,M}(k) \end{bmatrix}, \quad \triangle u_{j,M}(k) = \begin{bmatrix} \triangle u_j(k) \\ \vdots \\ \triangle u_j(k+M-1) \end{bmatrix}, \quad j = 1, \cdots m$$

$$Q = block - diag(Q_1, \cdots Q_p), \quad Q_i = diag\left[q_i(1), \cdots q_i(P) \right], \ i = 1, \cdots, p$$
$$R = block - diag(R_1, \cdots R_m), \quad R_j = diag\left[r_j(1), \cdots r_j(M) \right], \ j = 1, \cdots, m$$

P is the cost horizon, and *M* is the control horizon. q_i and r_i are weights that penalize the changes of the error and manipulated variable.

For flight control it is important to limit the absolute and incremental values of plant inputs (actuators) generated by the controller:

$$u_{i,\min} \leq u_i(k+j-1|k) \leq u_{i,\max} \qquad i = 1, \cdots, m; j = 1, \cdots M$$
$$\triangle u_{i,\min} \leq \triangle u_i(k+j-1|k) \leq \triangle u_{i,\max} \qquad i = 1, \cdots, m; j = 1, \cdots M \tag{3}$$

Most controller design methods can not deal explicitly with constraints for plant inputs. Usually, the inputs are simply saturated according to the constraints. In the case of MPC the input and output constraints can be considered explicitly: The control signal is generated in an optimization procedure considering all defined constraints.

In order to get the optimal control variable $\triangle u_M(k)$, we must solve the equations (1-3). This is a standard quadratic programming (QP) problem of the form:

$$\min f(x) = \frac{1}{2} x^T H x + c^T x$$
$$s.t \quad Ax \leq b$$

which can be solved e.g. by Lemke algorithms or active set method.(Chen, B. 2004)

3. Helicopter Modeling

In (Kondak, K. et al. 2006) the main differences between small scale helicopters and full size helicopters are pointed out. These differences are: a much higher ratio of the main rotor mass to fuselage mass in case of the small scale helicopter, the rotation speed of the main rotor of small scale helicopters is higher than of most full size helicopters, and small scale helicopters have very stiff main rotors without flapping hinges. The conclusion was that the main rotor should be considered as a rigid body when model the rotational dynamics. It was shown that the inertial effects of the main rotor become the main component influencing the rotational dynamics of the whole mechanical system.

The model of the small scale helicopter is shown in Fig.1. Three PWM servo inputs, s_1, s_2, s_3, control the cyclic pitches of the Bell-Hiller bar (BHB), and the collective pitch of the main rotor P_{col}^{MR} through the lever system L^{MR}. The block A^{MR} describes the aerodynamics of the main rotor. The main rotor generates pitch and roll torques $T_{1,2}^{MR}$, drag torque T_3^{MR} and lifting force F_3^{MR}. The servo input s_5 controls the pitch on the tail rotor through block L^{TR} describing tail rotor servos and lever system. Block A^{TR} describes the aerodynamics of the tail rotor. F_2^{TR} and T_2^{TR} are the force and torque of the tail rotor respectively, generated in A^{TR}. The rotation speed of the engine is controlled by s_4. The outputs of the model are: $q_{1,2,3}$, the position of the helicopter reference point with respect to the inertial system, $q_{4,5,6}$, the Euler-angles 1-2-3 describing the orientation of the fuselage relative to the inertial frame, $u_{1,2,3}$, the translational velocity in the inertial frame, and $u_{4,5,6}$, the rotation speeds of the fuselage in the body frame, so that the angular velocity of the fuselage with respect to the inertial frame is defined as follows: $w = u_4 * f_1 + u_5 * f_2 + u_6 * f_3$.

The blocks A^{MR}, A^{TR} contain models for aerodynamic effects, but they can be approximated with linear functions in the area of operation (this was concluded from experimental results). F_3^{MR}, $T_{1,2}^{MR}$, F_2^{TR} are linear functions of P_{col}^{MR}, $P_{cyc1,2}^{MR}$, P_{col}^{TR} respectively. The blocks L^{MR}, L^{TR} can be derived using geometrical relationships.
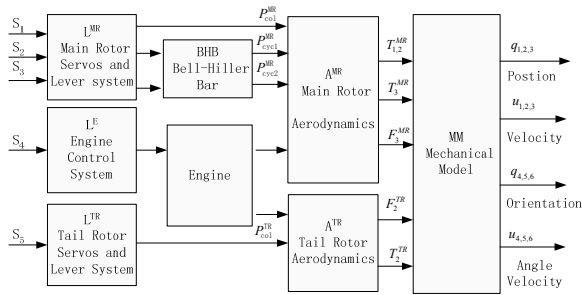
Fig. 1. Model of the small scale helicopter

Fig. 2. Attitude control

The experiments show that also the effects of the BHB can be approximated with a linear function.

After those significant simplifications, the relationship between servo inputs s_1, s_2, s_3, s_5 and abstract inputs F_3^{MR}, T_1^{MR}, T_2^{MR}, F_2^{TR} can be approximated with linear function, and the coefficients can be identified by experiments (Kondak, K. et al. 2007).

The kinematical equations for translation are:

$$\begin{aligned} \dot{q}_1 &= u_1 \\ \dot{q}_2 &= u_2 \\ \dot{q}_3 &= u_3 \end{aligned} \qquad (4)$$

and for rotation:

$$\begin{aligned} \dot{q}_4 &= (u_4 \cos (q_6) - u_5 \sin (q_6)) / \cos (q_5) \\ \dot{q}_5 &= u_4 \sin (q_6) + u_5 \cos (q_6) \\ \dot{q}_6 &= u_6 + \tan(q_5) (u_4 \cos (q_6) - u_5 \sin (q_6)) \end{aligned} \qquad (5)$$

The translation dynamical equations are derived considering the air resistance force $F_f = -sign(u)k_u u^2$ and have the following form:

$$\begin{aligned} M\dot{u}_1 &= F_3^{MR} \sin (q_5) - F_2^{TR} \cos (q_5) \sin(q_6) - sign(u_1)k_{u1}u_1^2 \\ M\dot{u}_2 &= -F_3^{MR} \sin (q_4) \cos (q_5) + F_2^{TR}(\cos (q_4) \cos (q_6) \\ &\quad - \sin (q_4) \sin(q_5) \sin(q_6)) - sign(u_2)k_{u2}u_2^2 \\ M\dot{u}_3 &= F_3^{MR} \cos (q_4) \cos (q_5) + F_2^{TR}(\sin (q_4) \cos (q_6) \\ &\quad + \cos (q_4) \sin(q_5) \sin(q_6)) - gM - sign(u_3)k_{u3}u_3^2 \end{aligned} \qquad (6)$$

The rotation dynamical equations are:

$$-dOT_1 F_2^{TR} + T_3^{MR} + (I_{11}^F - I_{22}^F)u_4 u_5 - (I_{33}^F + 2I_{11}^F)\dot{u}_6 = 0 \qquad (7)$$

$$T_1^{MR} + K_{1FTR2} F_2^{TR} + (K_{156}u_6 + K_{15}) u_5 + K_{1p4}\dot{u}_4 = 0 \qquad (8)$$

$$T_2^{MR} + T_2^{TR} + (K_{246}u_6 + K_{24}) u_4 + K_{2p5}\dot{u}_5 = 0 \qquad (9)$$

where, M is the mass of the helicopter and I_{11}^F, I_{22}^F, I_{33}^F are the moments of inertial in three axis of the fuselage. $K_{1p4}, K_{15}, K_{156}, K_{1FTR2}, K_{2p5}, K_{24}, K_{246}$ are parameters depending on the geometry of the mechanical construction of the helicopter, the mass and inertial numbers of the main rotor blades, and on the rotation speed of the main rotor (Kondak, K. et al. 2006).

4. Design of the controller

Similar to (Kadmiry, B. & Driankov, D. 2004; Kondak, K. et al. 2006) the presented helicopter controller is composed of two nested loops: The inner loop to control
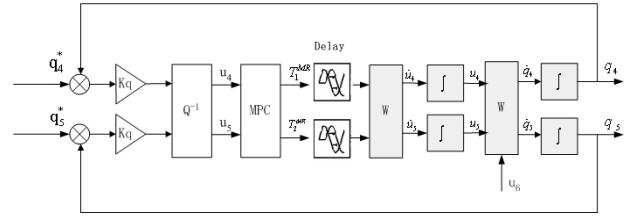
the attitude and the outer loop to control the position of the helicopter.

4.1 Attitude control

The scheme controlling the attitude angles q_4, q_5 is shown in Fig.2. In this scheme, the block Q is composed of the kinematic equations (5), and W is composed of the dynamical equations for rotation (8) and (9). Q^{-1} is the inversion of Q. Kq is the P-controller gain for angle control. The block MPC denotes the proposed model predictive controller. In (Kondak, K. et al. 2006), a decoupling block based on inversion of dynamical equations was used, so the accurate knowledge of the model parameters was required. Also the time delay between the controller outputs $T_{1,2}$ and corresponding generated torques on the main rotor was not explicitly considered in the controller design. The presented MPC can deal with the coupling and time delay and is robust against parameter variations of the system. In the presented control scheme the MPC is used for the attitude control, see Fig. 2, which increases the performance of this important part of the whole controller and makes the position controller more robust.

In order to design the MPC, a model presented in section 3 will be used. Here we consider $u_6=0$, F_2^{TR} and T_2^{TR} as disturbances. The equations (8)-(9) can then be rewritten in the following form:

$$\begin{bmatrix} \dot{u}_4 \\ \dot{u}_5 \end{bmatrix} = \begin{bmatrix} 0 & -\dfrac{K_{15}}{K_{1p4}} \\ -\dfrac{K_{24}}{K_{2p5}} & 0 \end{bmatrix} \begin{bmatrix} u_4 \\ u_5 \end{bmatrix} + \begin{bmatrix} -\dfrac{1}{K_{1p4}} & 0 \\ 0 & -\dfrac{1}{K_{2p5}} \end{bmatrix} \begin{bmatrix} T_1^{MR} \\ T_2^{MR} \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_4 \\ u_5 \end{bmatrix}$$

The equations above show a strong coupling between two corresponding axes of the helicopter frame.

The time delay of the plant is 0.12s (found in flight experiments), and both the sampling time and control time are 0.01s. The parameters for the presented controller were chosen as follows: predictive horizon $P=20$, control horizon $M=3$. The actuators constraints are defined using the following inequalities:

$-6 \, N.m \le T_1^{MR} \le 6 \, N.m$

$-6 \, N.m \le T_2^{MR} \le 6 \, N.m$

The error weight matrix *Q* and control weight matrix *R* were chosen as:

$$Q = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

$$R = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

For the heading control a simple P- controller was used. This simple controllers works well because the helicopter is equipped with a commercial gyroscope based controller GY401 which can be operated in AVCS mode to keep the helicopter in a fixed heading.

4.2 Position control

The position controller consists of a PID block and a F_{123}^{-1} block. The PID block generates the desired acceleration according to the position error. F_{123}^{-1} is the inversion of eqs.(6) and can be described by following eqs:

$$q_5 = \arcsin(\frac{M\dot{u}_1 + sign(u_1)k_1u_1^2}{F_3^{MR}})$$

$$q_4 = \arcsin(\frac{M\dot{u}_2 + sign(u_2)k_2u_2^2}{-F_3^{MR}\cos q_5})$$

where,

$$F_3^{MR} = \sqrt{\begin{array}{l}(M\dot{u}_1 + K_{u1}sign(u_1)u_1^2)^2 + \\ (M\dot{u}_2 + K_{u2}sign(u_2)u_2^2)^2 + (M\dot{u}_3 + K_{u3}sign(u_3)u_3^2 + Mg)^2\end{array}}$$

In Fig. 3, the *Inner Loop Controller* block is an attitude controller which takes the desired attitude angle $q_{4,5}^*$ as input, and generates the output torques T_1^{MR}, T_2^{MR}. The *Rotation Dynamics* block converts torques T_1^{MR}, T_2^{MR} into attitude angles $q_{4,5}$ which are described by eqs. (8) and (9). The block F_{123} represents the translational dynamics, described by eqs(6). This block converts attitude angle $q_{4,5,6}$ and lifting force F_3^{MR} into the acceleration of the translational movement $\dot{u}_{1,2,3}$. After two integrations the accelerations are integrated to the position of the helicopter $q_{1,2,3}$. In this scheme the grey blocks denote the helicopter model (plant) and the white blocks denote the controller parts.

5. Experiments

The experiments were performed in the laboratory for autonomously flying robots at Technische Universität Berlin. Fig. 4 shows the flight scene. The helicopter is fixed in a safety cage which is made of carbon tubes and has the mass of 1.2 kg. The four high brightness lamps (markers) are used to measure the actual position and orientation of the helicopter by a vision system.
The control algorithm runs on a control computer with a 800MHz CPU, a 128M RAM, 2 CAN-bus interfaces, 3 RS-232 interfaces, 1 RS-485 interface, and 1 Ethernet interface.
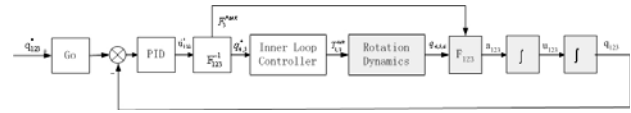

Fig. 3. Position control

Fig. 4. Real flight

This computer is powerful enough to handle the required mathematical computations of the control algorithms.
The onboard micro controller based on a Siemens SAB80C167 microcontroller deals with the angle velocity signals measured by gyroscopes ADXRS 300 and transferred to the control computer over the CAN-bus. The tree-cameras vision system (the three cameras are placed on the ceiling of the lab) measures the position and orientation of the helicopter which is sent to the ground computer through Ethernet. The translation velocities of the helicopter are determined by first order differentiation of the position coordinates
The parameters of the PID block and Kq of the attitude controller are calculated using the pole assignment method(Kondak, K. et al. 2006).
In the flight experiment, the helicopter started at position (0, -1.5, 0.4), then hovered in the position (0, -1.5, 0.9), then moved to the position (0, 0, 0.9). Fig. 5 shows the controller performing a hovering. From the figure, we can see that the position precision is 0.1m which is accurate enough for most practical applications. In Fig. 6, time t=30s, a step response is shown. We can see that the step response has no overshoot and oscillation when the position changes from (0, -1.5, 0.9) to (0, 0, 0.9). The performance of the controller is good.
It should be mentioned that the attitude angle q_4 and q_5 is not zero during in hovering mode. As can be seen from Fig .7, the mean value of angle q_4 is actually about 4.3 deg (0.075 rad) and q_5 is -2.9 deg (-0.05 rad). This can be analyzed by eqs. (4), as F_2^{TR} and q_6 (17 deg in real flight) are not zero. The maximal deviation from the mean value is about 2.9 deg..
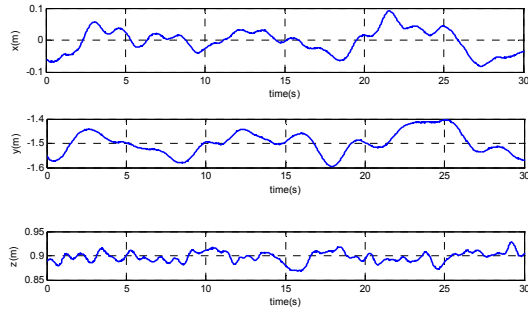
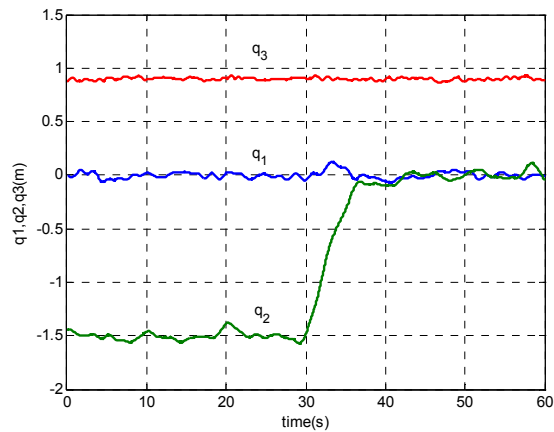Fig. 5. The position of the helicopter in hovering mode

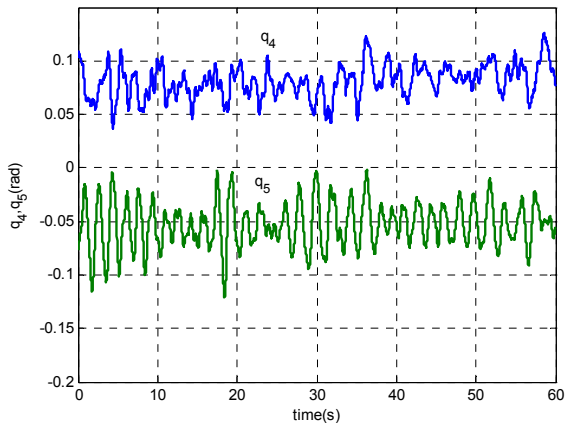Fig. 6. The step response of the helicopter in y axis

Fig. 7. The attitude angle q_4, q_5 of the helicopter

6. Conclusion

The controller can achieve very good performance by using the simplified linear aerodynamic model for the process of forces and torques generation on the main rotor. The nonlinear part of the controller is based on inversion of the kinematic equation for rotation and dynamical equations for translation. In order to achieve a robust and high performing attitude controller an MPC was used. The main advantages of the proposed control algorithm are the explicit consideration of time delay and actuator limits in the controller design. In addition, the

MPC increases the performance of the closed loop system due to the minimization of the control error using the prediction of the future system behavior.

The real flight experiments verify that the MPC controller used in our control system can work well. This is a new application of MPC on a 6-DOFs small scale helicopter. In future work we will investigate if the MPC controller for the translational part can increase the performance of the whole closed loop system.

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