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# EXISTENCE AND UNIQUENESS OF MILD SOLUTIONS FOR FRACTIONAL ORDER MIXED INTEGRODIFFERENTIAL EQUATIONS WITH NONLOCAL CONDITIONS

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Abstract. A fractional order mixed integrodifferential equation is studied in this article, and some sufficient conditions for existence and uniqueness of mild solutions for the equation is established by Banach fixed point theorem and Kransnoseslkii fixed point theorems, respectively.

## 1. Introduction and preliminaries

This article is concerned with the existence and uniqueness of mild solution of the following fractional order differential equation with nonlocal condition:

$$\begin{aligned} D^q x(t) + Ax(t) &= f\Big(t, x(t), \int_0^t g(t,s)x(s)ds, \int_0^T h(t,s)x(s)ds\Big), \quad t \in [0,T], \\ x(0) + k(x) &= x_0, \end{aligned} \tag{1.1}$$

where $0 < q < 1, T > 0$, and $-A$ generates analytic compact semigroup $\{S(t)\}_{t\geq 0}$ of uniformly bounded linear operators on a Banach space $X$ with norm $\|\cdot\|$, that is, there exist $M > 1$ such that $\|S(t)\| \leq M$, and without loss of generality, assume $0 \in \rho(A)$. $f$ is a continuous mapping defined on $[0,T] \times X_\alpha^3$ and $k$ is defined on $C([0,T], X_\alpha)$, where $X_\alpha = D(A^\alpha)$, for $0 < \alpha \leq 1$, the domain of the fractional power of $A$. $g \in C(D \times X_\alpha, X_\alpha), h \in C(D_0 \times X_\alpha, X_\alpha)$, where $D = \{(t,s) \in \mathbb{R}^2 : 0 \leq s \leq t \leq T\}, D_0 = [0,T] \times [0,T]$.

For the sake of the shortness let

$$Gx(t) = \int_0^t g(t,s)x(s)ds, \qquad Hx(t) = \int_0^T h(t,s)x(s)ds \tag{1.2}$$

and

$$G^* = \sup_{t\in[0,T]} \int_0^t g(t,s)ds < \infty, \qquad H^* = \sup_{t\in[0,T]} \int_0^T h(t,s)ds < \infty. \tag{1.3}$$

Recently, fractional differential equations have been of great interest. For example, Li[6] discussed the existence and uniqueness of mild solution for

$$\begin{aligned} \frac{d^q x(t)}{dt^q} &= -Ax(t) + f\big(t, x(t), Gx(t)\big), \quad t \in [0,T], \\ x(0) + g(x) &= x_0. \end{aligned} \tag{1.4}$$

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Li and Guérékata[7] studied mild solutions of the fractional integrodifferential equations as follows

$$\frac{d^q x(t)}{dt^q} + Ax(t) = f(t, x(t)) + \int_0^t a(t-s)g(s, x(s))ds, \quad t \in [0, T], \quad x(0) = x_0. \tag{1.5}$$

For detailed discussion on this topic, refer to the monographs of Kilbas et al.[4], Miller and Ross [8], Pazy [9], Podlubny [10], Smart [11], and the papers by Anguraj et al.[1], Benchohra et al.[2], Guo and Liu [3], Lakshmikantham et al.[5] and the references therein.

Applying Banach fixed point theorem and Krasnoselskii fixed point theorem, we obtain a result of existence and uniqueness of mild solutions for equation (1.1).

The following notations, definitions, and preliminary facts will be used throughout this paper.

Let $C_\alpha$ denote the Banach space $C([0, T], X_\alpha)$ endowed with the sup norm given by

$$\|x\|_\infty := \sup_{t\in[0,T]} \|x\|_\alpha, \quad x \in C_\alpha. \tag{1.6}$$

**Lemma 1.1**[9] (1) $X_\alpha = D(A^\alpha)$ is a Banach space with the norm $\|x\|_\alpha := \|A^\alpha x\|$ for $x \in D(A^\alpha)$.

(2) $S(t) : X \to X_\alpha$ for each $t > 0$ and $\alpha > 0$.

(3) For each $u \in D(A^\alpha)$ and $t \geq 0, S(t)A^\alpha u = A^\alpha S(t)u$.

(4) For each $t > 0, A^\alpha S(t)$ are bounded on $X$ and there exist $M_\alpha > 0$ such that

$$\|A^\alpha S(t)\| \leq M_\alpha t^{-\alpha}. \tag{1.7}$$

**Definition 1.2** A continuous function $x : [0, T] \to X$ is called a mild solution of (1.1) if

$$x(t) = S(t)\big(x_0 - k(x)\big) + \frac{1}{\Gamma(q)} \int_0^t (t-s)^{q-1} S(t-s) f\big(s, x(s), Gx(s), Hx(s)\big)ds \tag{1.8}$$

for $t \in [0, T]$.

**Theorem 1.3**(Krasnoselskii fixed point theorem,[11]) Let $D$ be a closed convex and nonempty subset of a Banach space $X$, and $A, B$ be two operators such that

(i) $Ax + By \in D$ whenever $x, y \in D$,

(ii) $A$ is compact and continuous,

(iii) $B$ is a contraction mapping.

Then there exists $z \in D$ such that $z = Az + Bz$.

Now list the following hypotheses for convenience.

(H1) $f : [0, T] \times X_\alpha^3 \to X$ is continuous and there exists a function $m(\cdot) : [0, T] \to \mathbb{R}^+$ such that

$$\|f(t, x, y, z)\| \leq m(t), \ \forall x, y, z \in C_\alpha, \tag{1.9}$$

and

$$\int_0^t (t-s)^{q-1-\alpha} m(s)ds \leq M_m < \infty, \quad t \in [0, T]. \tag{1.10}$$

(H2) there exists a function $l(\cdot) : [0, T] \to \mathbb{R}^+$ such that

$$\|f(t, x_1, y_1, z_1) - f(t, x_2, y_2, z_2)\| \leq l(t) \max\{\|x_1 - x_2\|_\alpha, \|y_1 - y_2\|_\alpha, \|z_1 - z_2\|_\alpha\},$$
$$\forall x_1, y_1, z_1, x_2, y_2, z_2 \in C_\alpha, \tag{1.11}$$

and

$$\int_0^t (t-s)^{q-1-\alpha} l(s)ds \le M_l < \infty, \quad t \in [0,T]. \tag{1.12}$$

(H3) function $k : C_\alpha \to X_\alpha$ is continuous and there exists $b > 0$ such that

$$\|k(x) - k(y)\|_\alpha \le b\|x-y\|_\infty, \ \forall x, y \in C_\alpha. \tag{1.13}$$

## 2. Existence and Uniqueness of a Mild Solution

In this section, a few sufficient conditions of existence and uniqueness of a mild solution for equation (1.1) will be given.

**Theorem 2.1** Assume $-A$ is the infinitesimal generator of an analytic compact semigroup $\{S(t)\}_{t\ge 0}$ with $\|S(t)\| \le M, t \ge 0$, and $0 \in \rho(A)$. If $x_0 \in X_\alpha$, (H1)-(H3) hold, and $Mb\Gamma(q) + M_\alpha M_l \max\{1, G^*, H^*\} < \Gamma(q)$, then equation (1.1) has a unique mild solution $x \in C_\alpha$.

**Proof.** Set $K = \sup_{x\in C_\alpha} \|k(x)\|_\alpha$ and choose $r$ such that

$$r \ge M\big(\|x_0\|_\alpha + K\big) + \frac{M_\alpha M_m}{\Gamma(q)}. \tag{2.1}$$

Let $B_r = \big\{x \in C_\alpha : \|x\|_\infty \le r\big\}$.

Define a mapping $F : C_\alpha \to C_\alpha$ by

$$(Fx)(t) = S(t)\big(x_0 - k(x)\big) + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1} S(t-s) f\big(s, x(s), Gx(s), Hx(s)\big)ds. \tag{2.2}$$

For each $x \in B_r$ and $t \in [0,T]$, by Lemma 1.1, we have

$$\begin{aligned}\big\|(Fx)(t)\big\|_\alpha \le& \|S(t)\|\big(\|x_0\|_\alpha + K\big) \\ &+ \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1}\big\|A^\alpha S(t-s) f\big(s, x(s), Gx(s), Hx(s)\big)\big\|ds \\ \le& M\big(\|x_0\|_\alpha + K\big) + \frac{M_\alpha}{\Gamma(q)}\int_0^t (t-s)^{q-1}(t-s)^{-\alpha} m(s)ds \\ \le& M\big(\|x_0\|_\alpha + K\big) + \frac{M_\alpha M_m}{\Gamma(q)} \le r, \end{aligned} \tag{2.3}$$

which means $Fx \in B_r$. For each $x, y \in C_\alpha, t \in [0,T]$, we deduce that

$$\begin{aligned}&\big\|(Fx)(t) - (Fy)(t)\big\|_\alpha \\ &\le \big\|S(t)\big(k(x) - k(y)\big)\big\|_\alpha + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1} \\ &\quad \big\|S(t-s)\big[f\big(s, x(s), Gx(s), Hx(s)\big) - f\big(s, y(s), Gy(s), Hy(s)\big)\big]\big\|_\alpha ds \\ &\le M\big\|k(x) - k(y)\big\|_\alpha + \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1} \\ &\quad \big\|A^\alpha S(t-s)\big[f\big(s, x(s), Gx(s), Hx(s)\big) - f\big(s, y(s), Gy(s), Hy(s)\big)\big]\big\|ds \\ &\le Mb\|x-y\|_\infty + \frac{M_\alpha}{\Gamma(q)}\int_0^t (t-s)^{q-1} \\ &\quad (t-s)^{-\alpha} l(s) \max\big\{\|x-y\|_\alpha, \|Gx - Gy\|_\alpha, \|Hx - Hy\|_\alpha\big\}ds\end{aligned}$$

$$\begin{aligned}&\le Mb\|x-y\|_\infty+\frac{M_\alpha}{\Gamma(q)}\int_0^t(t-s)^{q-1-\alpha}l(s)ds\max\{1,G^*,H^*\}\|x-y\|_\alpha\\&\le\left(Mb+\frac{M_\alpha M_l}{\Gamma(q)}\max\{1,G^*,H^*\}\right)\|x-y\|_\infty,\end{aligned}\tag{2.4}$$

which ensures

$$\big\|(Fx)(t)-(Fy)(t)\big\|_\infty\le\left(Mb+\frac{M_\alpha M_l}{\Gamma(q)}\max\{1,G^*,H^*\}\right)\|x-y\|_\infty<\|x-y\|_\infty.\tag{2.5}$$

Then the conclusion follows from the Banach fixed point theorem.

**Theorem 2.2** Assume $-A$ is the infinitesimal generator of an analytic compact semigroup $\{S(t)\}_{t\ge0}$ with $\|S(t)\|\le M, t\ge0$, and $0\in\rho(A)$. If (H1), (H3) hold, $Mb<1$, and the function $s\to m(s)(t-s)^{-\alpha}$ is integrable on $[0,t]$, then equation (1.1) has a mild solution for each $x_0\in X_\alpha$.

**Proof.** Let $K$ and $B_r$ be the same as in Theorem 2.1.

Define two mappings $A,B:X_\alpha\to X_\alpha$ by

$$\begin{aligned}&(Ax)(t)=\frac{1}{\Gamma(q)}\int_0^t(t-s)^{q-1}S(t-s)f\big(s,x(s),Gx(s),Hx(s)\big)ds,\\&(Bx)(t)=S(t)\big(x_0-k(x)\big).\end{aligned}\tag{2.6}$$

(i) Obviously, $Ax+By\in B_r,\ \forall x,y\in B_r$.

(ii) It is declared that $A$ is continuous. Let $\{x_n\}$ be a sequence of $B_r$ such that $x_n\to x$ in $B_r$. Then the continuity of $f$ ensures that

$$f\big(s,x_n(s),Gx_n(s),Hx_n(s)\big)\to f\big(s,x(s),Gx(s),Hx(s)\big).\tag{2.7}$$

For $t\in[0,T]$, we obtain

$$\begin{aligned}&\|(Ax_n)(t)-(Ax)(t)\|_\alpha\\&\quad=\frac{1}{\Gamma(q)}\Big\|\int_0^t(t-s)^{q-1}S(t-s)\big[f\big(s,x_n(s),Gx_n(s),Hx_n(s)\big)\\&\qquad\qquad-f\big(s,x(s),Gx(s),Hx(s)\big)\big]ds\Big\|_\alpha\\&\quad\le\frac{1}{\Gamma(q)}\int_0^t(t-s)^{q-1}\big\|A^\alpha S(t-s)\big[f\big(s,x_n(s),Gx_n(s),Hx_n(s)\big)\\&\qquad\qquad-f\big(s,x(s),Gx(s),Hx(s)\big)\big]\big\|ds\\&\quad\le\frac{M_\alpha}{\Gamma(q)}\int_0^t(t-s)^{q-1-\alpha}\big\|f\big(s,x_n(s),Gx_n(s),Hx_n(s)\big)\\&\qquad\qquad-f\big(s,x(s),Gx(s),Hx(s)\big)\big\|ds.\end{aligned}\tag{2.8}$$

According to the fact that

$$\big\|f\big(s,x_n(s),Gx_n(s),Hx_n(s)\big)-f\big(s,x(s),Gx(s),Hx(s)\big)\big\|\le2m(s),\ \forall s\in[0,T],\tag{2.9}$$

and the function $s \to 2m(s)(t-s)^{-\alpha}$ is integrable on $[0,t]$, the Lebesgue Dominated Convergence Theorem guarantees that

$$\int_0^t (t-s)^{q-1-\alpha}\big\|f\big(s,x_n(s),Gx_n(s),Hx_n(s)\big)-f\big(s,x(s),Gx(s),Hx(s)\big)\big\|ds \to 0$$

as $n \to \infty$. (2.10)

Therefore,

$$\lim_{n\to\infty}\|(Ax_n)(t)-(Ax)(t)\|_\infty = 0. \tag{2.11}$$

(iii) It is claimed that $A$ is compact.

First to show that $A$ is uniformly bounded on $B_r$.

$$\begin{aligned}\|(Ax)(t)\|_\alpha =&\frac{1}{\Gamma(q)}\Big\|\int_0^t (t-s)^{q-1}S(t-s)f\big(s,x(s),Gx(s),Hx(s)\big)ds\Big\|_\alpha\\ \le&\frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1}\|A^\alpha S(t-s)f\big(s,x(s),Gx(s),Hx(s)\big)\|ds\\ \le&\frac{M_\alpha}{\Gamma(q)}\int_0^t (t-s)^{q-1-\alpha}m(s)ds\\ \le&\frac{M_\alpha M_m}{\Gamma(q)}.\end{aligned} \tag{2.12}$$

Next to prove that $(Ax)(t)$ is equicontinuous. Let $0<t_1<t_2<T$ and $\epsilon>0$ be small enough, then we have

$$\begin{aligned}&\|(Ax)(t_2)-(Ax)(t_1)\|_\alpha\\ \le&\frac{1}{\Gamma(q)}\Big\|\int_0^{t_1}\big[(t_2-s)^{q-1}-(t_1-s)^{q-1}\big]S(t_1-s)f\big(s,x(s),Gx(s),Hx(s)\big)ds\Big\|_\alpha\\ &+\frac{1}{\Gamma(q)}\Big\|\int_{t_1}^{t_2}(t_2-s)^{q-1}S(t_2-s)f\big(s,x(s),Gx(s),Hx(s)\big)ds\Big\|_\alpha\\ &+\frac{1}{\Gamma(q)}\Big\|\int_0^{t_1}(t_2-s)^{q-1}\big[S(t_2-s)-S(t_1-s)\big]f\big(s,x(s),Gx(s),Hx(s)\big)ds\Big\|_\alpha\\ =&I_1+I_2+I_3.\end{aligned} \tag{2.13}$$

By (1.7) and (H1), we get

$$\begin{aligned}I_1 =& \frac{1}{\Gamma(q)}\Big\|\int_0^{t_1}\big[(t_2-s)^{q-1}-(t_1-s)^{q-1}\big]S(t_1-s)f\big(s,x(s),Gx(s),Hx(s)\big)ds\Big\|_\alpha\\ \le& \frac{1}{\Gamma(q)}\int_0^{t_1}\big[(t_2-s)^{q-1}-(t_1-s)^{q-1}\big]\|A^\alpha S(t_1-s)f\big(s,x(s),Gx(s),Hx(s)\big)\|ds\\ \le& \frac{M_\alpha}{\Gamma(q)}\int_0^{t_1}\big[(t_2-s)^{q-1}-(t_1-s)^{q-1}\big]\frac{m(s)}{(t_1-s)^\alpha}ds\\ \le& \frac{M_\alpha}{\Gamma(q)}\int_0^{t_1-\epsilon}\big[(t_2-s)^{q-1}-(t_1-s)^{q-1}\big]\frac{m(s)}{(t_1-s)^\alpha}ds\\ &\qquad\qquad+\frac{M_\alpha}{\Gamma(q)}\int_{t_1-\epsilon}^{t_1}(t_2-s)^{q-1}\frac{m(s)}{(t_1-s)^\alpha}ds\\ =& I_1'+I_1''.\end{aligned} \tag{2.14}$$

It follows from the assumption of $m(s)$ that $I_1'$ tends to 0 as $t_1 \to t_2$. For $I_1''$, we can see that $I_1''$ tends to 0 as $t_1 \to t_2$ and $\epsilon \to 0$.

It can be seen from (1.7) and (H1) that

$$\begin{aligned} I_2 &= \frac{1}{\Gamma(q)}\Big\| \int_{t_1}^{t_2} (t_2 - s)^{q-1} S(t_2 - s) f\big(s, x(s), Gx(s), Hx(s)\big) ds \Big\|_\alpha \\ &\le \frac{1}{\Gamma(q)} \int_{t_1}^{t_2} (t_2 - s)^{q-1} \big\| A^\alpha S(t_2 - s) f\big(s, x(s), Gx(s), Hx(s)\big)\big\| ds \\ &\le \frac{M_\alpha}{\Gamma(q)} \int_{t_1}^{t_2} (t_2 - s)^{q-1} \frac{m(s)}{(t_2 - s)^\alpha} ds \to 0 \quad \text{as} \quad t_1 \to t_2. \end{aligned} \tag{2.15}$$

Furthermore,

$$\begin{aligned} I_3 \le& \frac{1}{\Gamma(q)}\Big\| \int_0^{t_1-\epsilon} (t_2 - s)^{q-1} \big[S(t_2 - s) - S(t_1 - s)\big] f\big(s, x(s), Gx(s), Hx(s)\big) ds \Big\|_\alpha \\ &+ \frac{1}{\Gamma(q)}\Big\| \int_{t_1-\epsilon}^{t_1} (t_2 - s)^{q-1} \big[S(t_2 - s) - S(t_1 - s)\big] f\big(s, x(s), Gx(s), Hx(s)\big) ds \Big\|_\alpha \\ \le& \frac{1}{\Gamma(q)} \int_0^{t_1-\epsilon} (t_2 - s)^{q-1} \Big\| S\big(\frac{t_2 - t_1}{2} - \frac{t_2 - s}{2}\big) - S\big(\frac{t_1 - s}{2}\big)\Big\| \\ &\qquad \cdot \Big\| A^\alpha S\big(\frac{t_1 - s}{2}\big) f\big(s, x(s), Gx(s), Hx(s)\big)\Big\| ds \\ &+ \frac{M_\alpha}{\Gamma(q)}\Big\| \int_{t_1-\epsilon}^{t_1} (t_2 - s)^{q-1} \big[\frac{m(s)}{(t_2 - s)^\alpha} + \frac{m(s)}{(t_1 - s)^\alpha}\big] ds \\ \le& \frac{2^\alpha M_\alpha}{\Gamma(q)} \int_0^{t_1-\epsilon} (t_2 - s)^{q-1} \Big\| S\big(\frac{t_2 - t_1}{2} - \frac{t_2 - s}{2}\big) - S\big(\frac{t_1 - s}{2}\big)\Big\| \cdot \frac{m(s)}{(t_1 - s)^\alpha} ds \\ &+ \frac{M_\alpha}{\Gamma(q)}\Big\| \int_{t_1-\epsilon}^{t_1} (t_2 - s)^{q-1} \big[\frac{m(s)}{(t_2 - s)^\alpha} + \frac{m(s)}{(t_1 - s)^\alpha}\big] ds \\ =& I_3' + I_3''. \end{aligned} \tag{2.16}$$

Applying the compactness of $S(t)$ in $X$ implies the continuity of $t \mapsto \|S(t)\|$ for $t \in [0, T]$; integrating with $s \mapsto m(s)(t_1 - s)^{-\alpha} \in L^1_{\text{loc}}([0, t_1], \mathbb{R}^+)$, we see that $I_3'$ tends to 0, as $t_1 \to t_2$. For $I_3''$, it follows from the assumption of $m(s)$ that $I_3''$ tends to 0 as $t_1 \to t_2$ and $\epsilon \to 0$.

Therefore, $\|(Ax)(t_2) - (Ax)(t_1)\|_\alpha \to 0$ as $t_1 \to t_2$, which do not depend on $x$. Thus, $A(B_r)$ is relatively compact. In virtue of the Arzela-Ascoli Theorem, $A$ are compact.

(iv) $B$ is a contraction mapping. In fact,

$$\|(Bx)(t) - (By)(t)\|_\alpha \le \|S(t)\| \|k(x) - k(y)\|_\alpha \le Mb\|x - y\|_\infty < \|x - y\|_\infty \tag{2.17}$$

ensures that

$$\|(Bx)(t) - (By)(t)\|_\infty < \|x - y\|_\infty. \tag{2.18}$$

Now the proof is completed by Krasnoselskii fixed point theorem.

**Remark 2.3** Theorems 2.1 and 2.2 extend and improve the Theorems 3.1 and 3.2 of Li[6], Theorems 3.1 and 3.2 of Li and Guérékata[7].

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