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Adaptive Tracking and Obstacle Avoidance Control for Mobile Robots With Unknown Sliding

Regular Paper

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Abstract An adaptive control approach is proposed for trajectory tracking and obstacle avoidance for mobile robots with consideration given to unknown sliding. A kinematic model of mobile robots is established in this paper, in which both longitudinal and lateral sliding are considered and processed as three time-varying parameters. A sliding model observer is introduced to estimate the sliding parameters online. A stable tracking control law for this nonholonomic system is proposed to compensate the unknown sliding effect. From Lyapunov-stability analysis, it is proved, regardless of unknown sliding, that tracking errors of the controlled closed-loop system are asymptotically stable, the tracking errors converge to zero outside the obstacle detection region and obstacle avoidance is guaranteed inside the obstacle detection region. The efficiency and robustness of the proposed control system are verified by simulation results.

Keywords Wheeled mobile robot, Obstacle avoidance; Potential function, Adaptive control, Unknown sliding parameters, Sliding model observer

1. Introduction

The past last few decades have witnessed an ambitious research effort in the area of motion control of mobile robot (see [1–14] and the references therein). These works always assume that the mobile robots are subject to 'pure rolling without slipping', namely they keep nonholonomic constraints for controlling mobile robots. However, sliding effects have a critical influence on the performance of mobile robots that cannot be neglected. It means that we should deal with the mobile robot model with sliding induced from perturbed non-homonymic constraints for more a practical consideration. With this in mind, some researchers have studied approaches for controlling mobile robots considering skidding and slipping [15–19]. In [15, 16], only the skidding effect was considered. Wang and Low [17] proposed models of wheeled mobile robots (WMRs) considering the sliding of both wheels and analysed their controllability according to the manoeuvrability of mobile robots. They also designed controllers for path following and the tracking of mobile robots that took sliding into consideration

[18,19]. However, the information on skidding and slipping should be measured by a global positioning system (GPS), but only kinematics was used to design the controllers in [18,19]. Besides, the works [17–19] did not include any ideas for obstacle avoidance for mobile robots in the presence of wheel sliding.

Owing to the difficulty in handling both tracking and obstacle avoidance using one controller, there are few results available on the tracking control problem for nonholonomic mobile robots with obstacle avoidance, even though the problem is practical and important. Recently, some research work has investigated the problem on the kinematic level [20,21] and on the dynamic level [22-24]. The control approaches reported in [20,22] are commonly designed into tracking controllers with obstacle avoidance capability by using position tracking errors without coordinate transformation. However, some methods were developed without considering skidding and slipping effects [20-22] and obstacle avoidance is not considered in [23]. In [24], an adaptive controller is designed for trajectory tracking and obstacle avoidance in mobile robots and considers unknown sliding on the dynamic level by a backstepping approach. However, the design process of this controller is very complex and its implementation is not easy. This point motivates us to extend the study on tracking and obstacle avoidance in the presence of unknown sliding.

The main contributions of our work are the design of an adaptive control system, on the kinematics level, for tracking and obstacle avoidance for a class of mobile robots in the presence of unknown sliding. More specifically, in the theoretical part of this paper, we design a controller that guarantees tracking with bounded error and obstacle collision avoidance for mobile robot systems with unknown sliding. We assume that each robot knows its position and can detect the presence of any object within a given range. We apply this result to the control of the mobile robot system. Firstly, a kinematic model of mobile robots considering the influence of sliding is established where sliding is modelled as three time-varying parameters. Secondly, the time-varying sliding parameters are estimated by the sliding model observer online. The proposed adaptive controller is designed using Lyapunov design techniques where the angular velocities of the wheels are considered as the immediate controls to deal with unmatched sliding at the robot kinematics level. By using the Lyapunov stability approach with a potential function, we prove that the tracking errors of a controlled closed-loop system can converge asymptotically. The tracking errors converge to zero outside the obstacle detection region and no-collision between the robot and the obstacle is guaranteed inside the obstacle detection region, regardless of unknown sliding. Finally, simulation results are included to demonstrate the effectiveness of the proposed control approach.

This paper is organised as follows. In Section 2, we present the kinematic model of mobile robots considering the sliding influence, where sliding is modelled as three time-varying parameters. In Section 3, the sliding observer is employed to estimate sliding parameters and is introduced in detail. In Section 4, a controller is designed that guarantees tracking and obstacle avoidance for the mobile robot in the presence of sliding and the stability of the proposed control system is analysed. Simulation results are discussed in Section 5. Finally, Section 6 gives some conclusions.

2. Kinematic model of a wheeled mobile robot in the presence of sliding

A simple differentially steered WMR is shown as Fig.1. It has two differential driving wheels and two back caster wheels. The two driving wheels are powered independently by two DC servo motors respectively and have the same wheel radius.

To describe the motion features of a tracked mobile robot simply and rigorously in the general plane of motion, a fixed reference coordinate frame $F_1(X,Y)$ and a moving coordinate frame $F_2(x,y)$ which attaches to the robot body with the origin at the geometric centre O, are defined

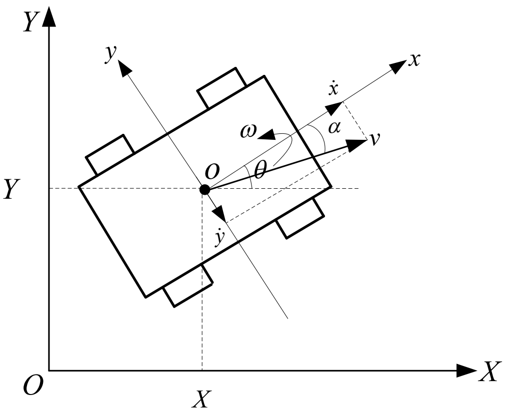


Figure 1. Tracked mobile robot with two independent driving wheels

The linear velocities of left and right driving wheels of mobile robot without sliding are represented as follows

$$
\begin{aligned}
v_L &= r\omega_L \\
v_R &= r\omega_R
\end{aligned}
\tag{1}
$$

where ω_L and ω_R are the angular velocities of the left and right wheels respectively, r is radius of the wheels. The longitudinal slip ratios of the left and right wheels of a tracked mobile robot are defined as follows [25]

$$
\begin{aligned}
i_L &= \frac{r\omega_L - v_L^s}{r\omega_L} \\
i_R &= \frac{r\omega_R - v_R^s}{r\omega_R}
\end{aligned}
\tag{2}
$$

where v_L^s and v_R^s are respectively, the linear velocities of the left and right wheels of the mobile robot in the presence of wheel slipping. The range of the longitudinal slip ratio i_R and i_L lies between $[0,1]$. The lateral sliding ratio of a tracked mobile robot is defined as [25]:

$$
\delta = \tan\alpha \tag{3}
$$

where α is the lateral sliding angle of a mobile robot (see Fig.1). From equation (2), the linear velocities of the left and right wheels of the mobile robot in the presence of wheel slipping are given as:

$$
\begin{aligned}
v_L^s &= r\omega_L(1-i_L) \\
v_R^s &= r\omega_R(1-i_R)
\end{aligned}
\tag{4}
$$

In the coordinate frame $F_1(X,Y)$ and in the absence of wheel slipping the kinematic model of the WMR is described by:

$$
\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{5}
$$

In the coordinate frame $F_2(x,y)$ a suitable model with sliding can be written as:

$$
\begin{aligned}
\dot{x} &= \frac{r\omega_L(1-i_L) + r\omega_R(1-i_R)}{2} \\
\dot{y} &= -\frac{r\omega_L(1-i_L) + r\omega_R(1-i_R)}{2}\delta \\
\dot{\theta} &= \frac{r\omega_R(1-i_R) - r\omega_L(1-i_L)}{b}
\end{aligned}
\tag{6}
$$

where b is the distance between the two driving wheels. As shown in Fig.1, the relationship of the coordinate transformation from $F_1(X,Y)$ to $F_2(x,y)$ is given by

$$
\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \tag{7}
$$

From equations (6) and (7), in the coordinate frame $F_1(X,Y)$, the kinematic model of the differential WMR with sliding is described as follows:

$$
\begin{aligned}
\dot{X} &= \frac{r\omega_L(1-i_L) + r\omega_R(1-i_R)}{2}(\cos\theta + \delta\sin\theta) \\
\dot{Y} &= \frac{r\omega_L(1-i_L) + r\omega_R(1-i_R)}{2}(\sin\theta - \delta\cos\theta) \\
\dot{\theta} &= \frac{r\omega_R(1-i_R) - r\omega_L(1-i_L)}{b}
\end{aligned}
\tag{8}
$$

where $[X,Y,\theta]^{\mathrm{T}}$ is the posture vector of mobile robot, θ is the heading angle of the WMR (the angle that is between the motion direction of robot and the positive direction of the X axis).

If $\dot{x} = \frac{r\omega_L(1-i_L) + r\omega_R(1-i_R)}{2}$ is defined as the longitudinal speed of the WMR in Fig.1, then, equation (8) can be rewritten as

$$
\begin{aligned}
\dot{X} &= \dot{x}(\cos\theta + \delta\sin\theta) \\
\dot{Y} &= \dot{x}(\sin\theta - \delta\cos\theta) \\
\dot{\theta} &= \frac{2\dot{x}}{b} - \frac{2r}{b}\omega_L(1-i_L) = \frac{-2\dot{x}}{b} + \frac{2r}{b}\omega_R(1-i_R)
\end{aligned}
\tag{9}
$$

If we define an auxiliary control input $\bar{u} = [v,\omega]^{\mathrm{T}}$, then the relationship between the auxiliary control input and effective control input $u = [\omega_L, \omega_R]^{\mathrm{T}}$ is regarded as:

$$
\begin{aligned}
\bar{u} = \begin{bmatrix} v \\ \omega \end{bmatrix} &= \begin{bmatrix} \dfrac{r(1-i_L)\omega_L + r(1-i_R)\omega_R}{2} \\ \dfrac{-r(1-i_L)\omega_L + r(1-i_R)\omega_R}{b} \end{bmatrix} \\
&= T\begin{bmatrix} \omega_L \\ \omega_R \end{bmatrix} = Tu
\end{aligned}
\tag{10}
$$

where the matrix $T = r\begin{bmatrix} \dfrac{1-i_L}{2} & \dfrac{1-i_R}{2} \\ \dfrac{-(1-i_L)}{b} & \dfrac{1-i_R}{b} \end{bmatrix}$,is a nonsingular matrix. From equation (10), effective control input $u = [\omega_L, \omega_R]^{\mathrm{T}}$ can be obtained as follows:

$$
\begin{bmatrix} \omega_L \\ \omega_R \end{bmatrix} = T^{-1}\begin{bmatrix} v \\ \omega \end{bmatrix} = \frac{1}{r}\begin{bmatrix} \dfrac{1}{1-i_L} & -\dfrac{b}{2(1-i_L)} \\ \dfrac{1}{1-i_R} & \dfrac{b}{2(1-i_R)} \end{bmatrix}\begin{bmatrix} v \\ \omega \end{bmatrix} \tag{11}
$$

Equation (8) can be written as:

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta + \delta\sin\theta & 0 \\ \sin\theta - \delta\cos\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{12}$$

As can be seen from equation (8), to solve the tracking control problem of a WMR with unknown sliding parameters i_R, i_L and δ, the top priority is to estimate the time-varying sliding parameters online and then to design the tracking controller on the basis of the sliding parameter estimations.

3. Robot sliding parameter estimation schemes

3.1 Design of nonlinear sliding model observer

The SMO (Sliding Model Observer) is a popular approach for state estimation, since it can deal with uncertainty in the system [26]. In this paper, an SMO [26-29] is designed to estimate sliding parameters, based on the kinematics model of the robot, sensor feedback of the robot's trajectory and the driving wheel speeds. Due to the inherent non-linear nature of the robot kinematics equations, it is a complex problem to obtain an estimation of the robot sliding parameters. The kinematics equations have to be linearized at a nominal trajectory when a linear estimator such as the Kalman Filter is applied to estimate the slipping parameters. Furthermore, measurements from inertia sensors include significant noise, it is very important for the observer to be robust against noise and model uncertainty. With an SMO, the control action switches from one value to another in finite time and this may cause chattering problems; to avoid this effect, the discontinuous terms go through a Low Pass Filter. An SMO is used to estimate the motor torque based on a single-input, single-output dynamics system as in [26]. In [28] two variables: disturbance and steering angle are estimated using an SMO, however the chattering is unavoidable due to the switching of sign function. The paper applies this approach to the multi-input, multi-output dynamics system and reduces the chattering by passing discontinuous terms from the observers through a low pass filter.

The observer takes the form as follows:

$$\begin{aligned} \dot{\hat{X}} &= \dot{x}\cos\theta + L_1 \operatorname{sgn}(X-\hat{X}) + L_2(X-\hat{X}) \\ \dot{\hat{\theta}} &= \frac{2\dot{x}}{b} + L_3 \operatorname{sgn}(\theta-\hat{\theta}) + L_2(\theta-\hat{\theta}) \\ \dot{\hat{\theta}}_1 &= -\frac{2\dot{x}}{b} + L_4 \operatorname{sgn}(\theta-\hat{\theta}_1) + L_2(\theta-\hat{\theta}_1) \end{aligned} \tag{13}$$

where $L_i > 0, 1 = 1, 2, \cdots 4$ are sliding model gains, $\dot{\hat{X}}$ is the estimated velocity X of the robot, $\dot{\hat{\theta}}$ is the estimated directional speed of the robot and yaw rate, $\dot{\hat{\theta}}_1$ is the estimated directional speed of the robot, sign function :

$\operatorname{sgn}(x) = \begin{cases} -1 & x \le 0 \\ 1 & x > 0 \end{cases}$, $\dot{\hat{\theta}}_1$ is used for designing an appropriate sliding surface which stabilizes all the posture variables.

Errors are defined as $\tilde{X} = X - \hat{X}$, $\tilde{\theta} = \theta - \hat{\theta}$ and $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$. By subtracting equation (9) from (13) the following error dynamics can be obtained as :

$$\begin{aligned} \dot{\tilde{X}} &= \delta\dot{x}\sin\theta - L_1 \operatorname{sgn}(X-\hat{X}) - L_2(X-\hat{X}) \\ \dot{\tilde{\theta}} &= -\frac{2r}{b}\omega_L(1-i_L) - L_3 \operatorname{sgn}(\theta-\hat{\theta}) - L_2(\theta-\hat{\theta}) \\ \dot{\tilde{\theta}}_1 &= \frac{2r}{b}\omega_R(1-i_R) - L_4 \operatorname{sgn}(\theta-\hat{\theta}_1) - L_2(\theta-\hat{\theta}_1) \end{aligned} \tag{14}$$

The error dynamics should converge to the sliding surface in a finite time by the appropriate choice of sliding gains, $X \to \hat{X}$, $\theta \to \hat{\theta}$, $\theta_1 \to \hat{\theta}_1$, then the equations (14) are reduced to:

$$\begin{aligned} &\delta\dot{x}\sin\theta - L_1 \operatorname{sgn}(X-\hat{X}) - L_2(X-\hat{X}) \approx 0 \\ &-\frac{2r}{b}\omega_L(1-i_L) - L_3 \operatorname{sgn}(\theta-\hat{\theta}) - L_2(\theta-\hat{\theta}) \approx 0 \\ &\frac{2r}{b}\omega_R(1-i_R) - L_4 \operatorname{sgn}(\theta-\hat{\theta}_1) - L_2(\theta-\hat{\theta}_1) \approx 0 \end{aligned} \tag{15}$$

If the driving wheels angular velocities ω_L, ω_R are measurable, the robot trajectory and sliding gains L_1, L_2, L_3, L_4 are given. Moreover, a low pass filter is applied to reduce the chattering of the SMO, then the sliding parameter estimations $\hat{\delta}$, $\hat{i}_L$, $\hat{i}_R$ can be calculated as follows:

$$\begin{aligned} \hat{\delta} &= \frac{L_1(\operatorname{sgn}(X-\hat{X}))_{LPF}}{\dot{x}\sin\theta} \\ \hat{i}_L &= 1 + \frac{b}{2r\omega_L} L_3(\operatorname{sgn}(\theta-\hat{\theta}))_{LPF} \\ \hat{i}_R &= 1 - \frac{b}{2r\omega_R} L_4(\operatorname{sgn}(\theta_1-\hat{\theta}_1))_{LPF} \end{aligned} \tag{16}$$

where $(\bullet)_{LPF}$ denotes an LPF (Low Pass Filter). The values of terms $L_1(\mathrm{sgn}(X-\hat{X}))_{LPF}$, $L_3(\mathrm{sgn}(\theta-\hat{\theta}))_{LPF}$, $L_4(\mathrm{sgn}(\theta_1-\hat{\theta}_1))_{LPF}$ can be determined. A first-order low-pass filter is described in Fig.2

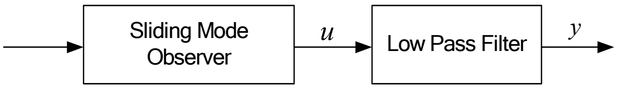


Figure2. Filtering processing of the estimation signal

The relationship between input u and output y of the LPF is given as:

$$\gamma \dot{y} + y = u,\ y(0) = u(0) \tag{17}$$

where $u = \left[\hat{i}_L, \hat{i}_R, \hat{\delta}\right]^{\mathrm{T}}$ and $\gamma = diag(\gamma_1, \gamma_2, \gamma_3),\ \gamma_1, \gamma_2, \gamma_3 > 0$ are the filter parameters, $L_1(\mathrm{sgn}(X-\hat{X}))_{LPF}$, $L_3(\mathrm{sgn}(\theta-\hat{\theta}))_{LPF}$ and $L_4(\mathrm{sgn}(\theta_1-\hat{\theta}_1))_{LPF}$ are the state vectors of the low-pass filter, y is the output and γ is the filter time constant and a very small value that lies within $0 \le \gamma_i \le 1, i = 1,2,3$.

3.2 Determination of switching gains

The switching gains $L_i, i = 1,3,4$ must be negative and large enough to satisfy the reaching condition of the sliding model. However, if it is too large, the chattering noise may lead to estimation errors. In this section, a proper selection of $L_i, i = 1,3,4$ is discussed.

In the following, the stability of the Sliding Model Observer (SMO) is discussed. The switching functions are defined as

$$s_1 = X - \hat{X}, s_2 = \theta - \hat{\theta}, s_3 = \theta_1 - \hat{\theta}_1 \tag{18}$$

To ensure stability, the SMO dynamics must exhibit the following characteristics: (1) The error state must reach the sliding surface from an arbitrary point in error space in a finite time and (2) the error dynamics must be stable in some neighbourhood of the sliding surface. To enforce stability, the sliding mode gains should be chosen such that $s_n \cdot \dot{s}_n < 0$, $n = 1,2,3$. It is shown in reference [26] that the error dynamics will converge in finite time if the conditions $s_n \cdot \dot{s}_n < 0$, $n = 1,2,3$ are satisfied. Applying this approach:

$$\begin{aligned} s_1\dot{s}_1 &= \dot{x}\delta s_1 \sin\theta - L_1 s_1 \,\mathrm{sgn}(s_1) - L_2 s_1^2 \\ &\le \dot{x}\delta s_1 \sin\theta - L_1 \left|s_1\right| \end{aligned} \tag{19}$$

From inequality (19), we know that if:

$$L_1 > \left|\dot{x}\delta \sin\theta\right| \tag{20}$$

then $s_1 \cdot \dot{s}_1 < 0$. Similarly:

$$\begin{aligned} s_2\dot{s}_2 &= \frac{2r}{b}\omega_L(1-i_L)s_2 - L_3 s_2 \,\mathrm{sgn}(s_2) - L_2 s_2^2 \\ &\le \frac{2r}{b}\omega_L(1-i_L)s_2 - L_3\left|s_2\right| \end{aligned} \tag{21}$$

From inequality (21), we know that if:

$$L_3 > \frac{2r}{b}\left|\omega_L(1-i_L)\right| \tag{22}$$

then $s_2\dot{s}_2 < 0$. Thus sliding mode can be enforced if:

$$L_4 > \frac{2r}{b}\left|\omega_R(1-i_R)\right| \tag{23}$$

then $s_3\dot{s}_3 < 0$.

In practice, $L_1 = \left|\dot{x}\sin\theta\right|$, L_2 is a small positive number, $L_3 = 4r / b\omega_L$ and $L_4 = 4r / b\omega_R$. If the lateral sliding angle α is expected to be large (as is the case for steep side slopes with loose soil) the trial-and-error method can be used to determine L_1.

4. Design of the tracking and obstacle avoidance controller

4.1 Potential function for obstacle avoidance

To deal with the obstacle avoidance of mobile robots in the presence of sliding, we consider the following potential function: [24]

$$V_{ob} = \left(\min\left\{0, \frac{d_{ro}^2 - L^2}{d_{ro}^2 - l^2}\right\}\right)^2 \tag{24}$$

where $l > 0$ and $L > 0$ with $L > l > b > 0$ are radii of the avoidance and detection regions (see Fig.3), respectively. The parameter l can be chosen by considering the radius of the mobile robot body. The distance function d_{ro} between the robot and the obstacle is:

$$d_{ro} = \sqrt{(X - X_o)^2 + (Y - Y_o)^2} \tag{25}$$

with the obstacle position (X_o, Y_o). The function (24) goes to infinity as the boundary of the avoidance region for the mobile robot approaches the obstacle and is zero outside the detection region.

The partial derivatives of the potential function V_{ob} with respect to the X and Y coordinates which would be required in the controller design procedure can be defined as:

$$\frac{\partial V_{ob}}{\partial X} = \begin{cases} \dfrac{4(L^2 - l^2)(d_{ro}^2 - L^2)}{(d_{ro}^2 - l^2)^3}(X - X_o), & \text{if } l < d < L \\ 0, & \text{otherwise} \end{cases} \tag{26}$$

$$\frac{\partial V_{ob}}{\partial Y} = \begin{cases} \dfrac{4(L^2 - l^2)(d_{ro}^2 - L^2)}{(d_{ro}^2 - l^2)^3}(Y - Y_o), & \text{if } l < d < L \\ 0, & \text{otherwise} \end{cases} \tag{27}$$

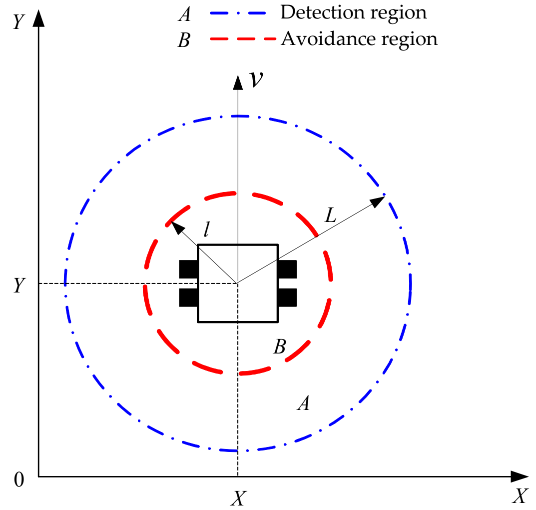


Figure 3. Wheeled mobile robot with avoidance and detection region

Assumption 1: The reference trajectory is smooth and satisfies:

$$|e_3| \neq \frac{\pi}{2}, \alpha - e_3 \neq \frac{\pi}{2} \tag{28}$$

where $e_3 = \theta - \theta_r$ is the orientation error and α is the lateral sliding angle of a mobile robot (in Fig.1). We define robot tracking errors: $e_1 = X - X_r$, $e_2 = Y - Y_r$.

Remark 1: Assumption 1 on the reference trajectory implies the following two conditions:

(1) Outside the detection region ($d_{ro} > L$) and for $(e_1, e_2) \neq (0,0)$ we have $\theta_r = \mathrm{A}\tan 2(-e_2, -e_1)$. The reference trajectory is such that it does not initiate sharp turns of 90° with respect to the current orientation of the robot. Note that this condition is not too restrictive since the robot can reorient itself on the spot if the condition is not satisfied and smoothness of the reference trajectory is a reasonable assumption in the case of robots subject to nonholonomic constraints.

(2) Inside the detection region ($l < d_{ro} < L$), we have $\theta_r = \mathrm{A}\tan 2(-e_2 - \frac{\partial V_{ob}}{\partial Y}, -e_1 - \frac{\partial V_{ob}}{\partial X})$.

The combination of obstacle position and reference trajectory might drive the robot into a singular configuration where Assumption 1 does not hold. One solution is to consider a perturbed desired orientation $\bar{\theta}_r$, instead of the desired orientation θ_r whenever (Assumption 1) is not satisfied. In particular, a robot can modify the desired orientation when Assumption 1 is not satisfied; θ_r is replaced with the following perturbed version: $\bar{\theta}_r = \theta_r + \bar{\varepsilon}$ where $\bar{\varepsilon} \neq 0$ is some small perturbation value. This condition guarantees that the system avoids singularities and deadlocks.

4.2 Design of controller

4.2.1 Control objective

The control objective is to design an adaptive control law for mobile robots considering the kinematics (8) with unknown sliding so that:

1. Outside the detection region ($d_{ro} \geq L$), the mobile robot tracks the reference trajectory generated by the following reference robot:

$$\begin{bmatrix} \dot{X}_r \\ \dot{Y}_r \\ \dot{\theta}_r \end{bmatrix} = \begin{bmatrix} \cos\theta_r & 0 \\ \sin\theta_r & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_r \\ \omega_r \end{bmatrix} \tag{29}$$

where X_r, Y_r and θ_r are the position and orientation of the reference robot and, v_r and ω_r are the linear and angular velocities of the reference robot.

2. Inside the detection region ($l < d_{ro} < L$), the mobile robot safely avoids obstacles under the influence of the reference trajectory $\dot{X}_r = \dot{Y}_r = 0$ and $\theta_r = \mathrm{A}\tan 2(-E_Y, -E_X)$ with:

$$\begin{cases} E_X = X - X_r + \dfrac{\partial V_{ob}}{\partial X} \\ E_Y = Y - Y_r + \dfrac{\partial V_{ob}}{\partial Y} \end{cases} \tag{30}$$

while all other signals remain semi-globally uniformly ultimately bounded.

Remark 2: In the control objective, the reference trajectory remains constant inside the detection region, that is, $\dot{X}_r = \dot{Y}_r = 0$ for $l < d_{ro} < L$. This means that when the mobile robot detects an obstacle in its trajectory, it keeps its reference to the last data received for a moment while trying to resolve the collision [24]. The reason for this choice is that collision avoidance has a higher priority than tracking, as a collision between the robot and obstacle could lead to system damage, which is more critical than temporary degeneration of tracking performance. Also, we define the desired orientation as $\theta_r = \mathrm{Atan2}(-E_Y, -E_X)$ inside the detection region. The reason is that when the mobile robot detects the obstacle, the direction of motion is related to the reference trajectory, the robot position and the change of potential function.

Assumption 2: The reference velocity $z_r = [v_r, \omega_r]^{\mathrm{T}}$ is bounded, where $v_r > 0$ is outside the detection region ($d_{ro} \ge L$) and $v_r = 0$ is inside the detection region ($l < d_{ro} < L$).

Remark 3: Assumption 2 is reasonable because this paper focuses on the trajectory tracking problem (i.e. $v_r > 0$) outside the detection region and the obstacle avoidance problem (i.e. $v_r = 0$ due to $\dot{X}_r = \dot{Y}_r = 0$) inside the detection region.

Assumption 3: From $\theta_r = \mathrm{A}\tan 2(-E_X, -E_Y)$, we can obtain:

$$\dot{\theta}_r = \frac{E_X \dot{E}_Y - \dot{E}_X E_Y}{E_X^2 + E_Y^2} \tag{31}$$

Hence:

$$\hat{\dot{\theta}}_r = \frac{E_X \hat{\dot{E}}_Y - \hat{\dot{E}}_X E_Y}{E_X^2 + E_Y^2} \tag{32}$$

where $\hat{\dot{\theta}}_r$ is a sufficiently smooth estimation of $\dot{\theta}_r$. In practice E_X and E_Y are noisy signals and in order to perform this task and avoid the effects of the noisy measurements we make use of a numerical differentiation based on algebraic nonlinear estimation to obtain $\hat{\dot{E}}_X$, $\hat{\dot{E}}_Y$. $\hat{\dot{E}}_X$, $\hat{\dot{E}}_Y$ are given as follows: [30]

$$\begin{cases} \hat{\dot{E}}_X = -\dfrac{3!}{T^3}\displaystyle\int_{t-T}^{t} [2T(t-\tau) - T] E_X(\tau) d\tau \\ \hat{\dot{E}}_Y = -\dfrac{3!}{T^3}\displaystyle\int_{t-T}^{t} [2T(t-\tau) - T] E_Y(\tau) d\tau \end{cases} \tag{33}$$

where $[t-T, t]$ is quite a short sliding time window. We assume that $\left|\hat{\dot{\theta}}_r - \dot{\theta}_r\right| \le \varepsilon$ for some small positive ε . Note that most of the variables in $\dot{\theta}_r$ can be measured, in fact we have:

$$\left|\hat{\dot{\theta}}_r - \dot{\theta}_r\right| = \frac{E_X(\dot{E}_Y - \hat{\dot{E}}_Y) - E_Y(\dot{E}_X - \hat{\dot{E}}_X)}{E_X^2 + E_Y^2} \tag{34}$$

where E_X , E_Y , $\sqrt{E_X^2 + E_Y^2}$ can be computed using the state measurements and desired values. Then E_X , E_Y are smooth almost everywhere, we have $(\dot{E}_X - \hat{\dot{E}}_X) \simeq (\dot{E}_Y - \hat{\dot{E}}_Y) \simeq o(T)$ and we can choose $\varepsilon \simeq o(T)$.

4.2.2 Design of controller

Define the tracking errors of a mobile robot as $e_1 = X - X_r$, $e_2 = Y - Y_r$, $e_3 = \theta - \theta_r$; the tracking error vectors of mobile robot are defined as $e = [e_1, e_2, e_3]^{\mathrm{T}}$ .From equation (12), the error dynamic equation of a mobile robot is obtained as:

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} v[\cos(e_3 + \theta_r) + \delta \sin(e_3 + \theta_r)] - \dot{X}_r \\ v[\sin(e_3 + \theta_r) - \delta \cos(e_3 + \theta_r)] - \dot{Y}_r \\ \omega - \dot{\theta}_r \end{bmatrix} \tag{35}$$

In the presence of sliding, we employ the Lyapunov direct method; the auxiliary control input is obtained as follows:

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} -k_1\sqrt{E_X^2 + E_Y^2}(\cos e_3 + \delta \sin e_3) \\ -k_2 e_3 + \hat{\dot{\theta}}_r \end{bmatrix} \tag{36}$$

where k_1 and k_2 are positive constants.

Now, if the sliding parameters i_R , i_L and δ that appear in (8) are unknown, we cannot choose directly the auxiliary control input as given by (36). Hence, we design a sliding model observer (13) to attain the control objective using estimations of i_R , i_L and δ . If $\hat{i}_L$, $\hat{i}_R$ and

$\hat{\delta}$ denote the estimations of i_R, i_L and δ respectively, then, from equation (36) we can obtain the auxiliary control input as:

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} -k_1\sqrt{E_X^2+E_Y^2}(\cos e_3 + \hat{\delta}\sin e_3) \\ -k_2 e_3 + \hat{\dot{\theta}}_r \end{bmatrix} \quad (37)$$

where $\hat{\dot{\theta}}_r$ is determined by equation(32), from equation (11). Actual control input can also be obtained as follows:

$$\begin{bmatrix} \omega_L \\ \omega_R \end{bmatrix} = \frac{1}{r}\begin{bmatrix} \frac{1}{1-\hat{i}_L} & -\frac{b}{2(1-\hat{i}_L)} \\ \frac{1}{1-\hat{i}_R} & \frac{b}{2(1-\hat{i}_R)} \end{bmatrix}\begin{bmatrix} v \\ \omega \end{bmatrix} \quad (38)$$

As can be seen from the above analysis, trajectory tracking and obstacle avoidance using a closed-loop control principle for the mobile robot can be described by the following scheme (See Fig.4).

4.2.3 Stability analysis of control system

Theorem 1: Consider system (12) and the reference trajectory described by (X_r, Y_r) that satisfies Assumptions 1–3. Consider also a static obstacle to be avoided that is located at (X_o, Y_o). Define the desired orientation as $\theta_r = \mathrm{A}\tan 2(-E_Y, -E_X)$. Then tracking with bounded error outside the detection region and obstacle avoidance inside the detection region are guaranteed if the following controller is applied:

$$\begin{cases} v = -k_1\sqrt{E_X^2+E_Y^2}(\cos e_3 + \delta\sin e_3) \\ \omega = -k_2 e_3 + \hat{\dot{\theta}}_r \end{cases}$$

for all gains $k_1 > 0$, $k_2 > 0$ and the singular case $\sqrt{E_X^2+E_Y^2} > 0$. Moreover, the tracking error can be reduced by increasing the value of the gains k_1 and k_2.

Proof: A Lyapunov function candidate is chosen as follows:

$$V = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{1}{2}e_3^2 + V_{ob} \quad (39)$$

The derivative of the Lyapunov function V is given by

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + \frac{\partial V_{ob}}{\partial X}\dot{X} + \frac{\partial V_{ob}}{\partial Y}\dot{Y} \quad (40)$$

From equations (35) and (36), the closed-loop error dynamic equation can be obtained as follows:

$$\begin{cases} \dot{e}_1 = -k_1\sqrt{E_X^2+E_Y^2}(\cos e_3 + \delta\sin e_3) \\ \qquad \times[\cos(e_3+\theta_r) + \delta\sin(e_3+\theta_r)] - \dot{X}_r \\ \dot{e}_2 = -k_1\sqrt{E_X^2+E_Y^2}(\cos e_3 + \delta\sin e_3) \\ \qquad \times[\sin(e_3+\theta_r) - \delta\cos(e_3+\theta_r)] - \dot{Y}_r \\ \dot{e}_3 = -k_2 e_3 + \hat{\dot{\theta}}_r - \dot{\theta}_r \end{cases} \quad (41)$$

From the equation $\theta_r = \mathrm{A}\tan 2(-E_Y, -E_X)$, we can obtain the following:

$$\sin\theta_r = \frac{E_Y}{\sqrt{E_X^2+E_Y^2}}, \cos\theta_r = \frac{E_X}{\sqrt{E_X^2+E_Y^2}} \quad (42)$$

Then equation (41) can be rewritten as:

$$\begin{cases} \dot{e}_1 = -k_1(\cos e_3 + \delta\sin e_3)[E_X(\cos e_3 + \delta\sin e_3) \\ \qquad - E_Y(\sin e_3 - \delta\cos e_3)] - \dot{X}_r \\ \dot{e}_2 = -k_1(\cos e_3 + \delta\sin e_3)[E_X(\sin e_3 - \delta\cos e_3) \\ \qquad + E_Y(\cos e_3 + \delta\sin e_3)] - \dot{Y}_r \\ \dot{e}_3 = -k_2 e_3 + \hat{\dot{\theta}}_r - \dot{\theta}_r \end{cases} \quad (43)$$

From equation (30) we know:

$$\begin{aligned} \frac{\partial V_{ob}}{\partial X} &= E_X - (X - X_r) = E_X - e_1 \\ \frac{\partial V_{ob}}{\partial Y} &= E_Y - (Y - Y_r) = E_Y - e_2 \end{aligned} \quad (44)$$

Moreover, notice that:

$$\dot{X} = \dot{e}_1 + \dot{X}_r, \dot{Y} = \dot{e}_2 + \dot{Y}_r \quad (45)$$

Substituting equations (43)~(45) into equation (40), we can obtain:

$$\begin{aligned} \dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + (E_X - e_1)(\dot{e}_1 + \dot{X}_r) \\ &\quad + (E_Y - e_2)(\dot{e}_2 + \dot{Y}_r) \\ &= e_3\dot{e}_3 + E_X\dot{e}_1 + E_X\dot{X}_r - e_1\dot{X}_r + E_Y\dot{e}_2 \\ &\quad + E_Y\dot{Y}_r - e_2\dot{Y}_r \\ &= -k_1(E_X^2+E_Y^2)(\cos e_3 + \delta\sin e_3)^2 \\ &\quad - e_1\dot{X}_r - e_2\dot{Y}_r + e_3(-k_2 e_3 + \hat{\dot{\theta}}_r - \dot{\theta}_r) \\ &\le -k_1(E_X^2+E_Y^2)(\cos e_3 + \delta\sin e_3)^2 \\ &\quad - e_1\dot{X}_r - e_2\dot{Y}_r - |e_3|(k_2|e_3| - \varepsilon) \end{aligned} \quad (46)$$

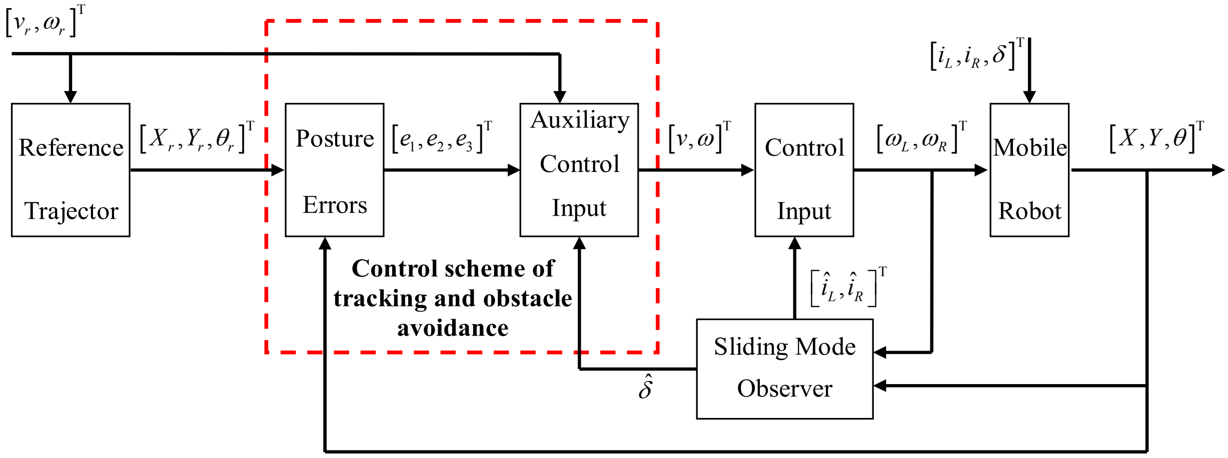


Figure4. Mobile robot trajectory tracking and obstacle avoidance control principle scheme

1. When the robot is outside the detection range ($d_{ro} > L$), we have

$$\frac{\partial V_{ob}}{\partial X} = \frac{\partial V_{ob}}{\partial Y} = 0 \tag{47}$$

Then, the inequality (46) becomes

$$\begin{aligned} \dot{V} &\le -k_1(e_1^2 + e_2^2)(\cos e_3 + \delta \sin e_3)^2 \\ &\quad - e_1\dot{X}_r - e_2\dot{Y}_r - |e_3|(k_2|e_3| - \varepsilon) \\ &= -\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}^{\mathrm{T}} A \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} - \begin{bmatrix} e_1 \\ e_2 \\ |e_3| \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \dot{X}_r \\ \dot{Y}_r \\ -\varepsilon \end{bmatrix} \\ &\le -\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}^{\mathrm{T}} A \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} + \left\| \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \right\| \left\| \begin{bmatrix} \dot{X}_r \\ \dot{Y}_r \\ -\varepsilon \end{bmatrix} \right\| \end{aligned} \tag{48}$$

where:

$$A = \begin{bmatrix} k_1(\cos e_3 + \delta \sin e_3)^2 & 0 & 0 \\ 0 & k_1(\cos e_3 + \delta \sin e_3)^2 & 0 \\ 0 & 0 & k_2 \end{bmatrix}$$

$\left\| \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \right\| = \sqrt{e_1^2 + e_2^2 + e_3^2}$. From Assumption 1, we know that $(\cos e_3 + \delta \sin e_3)^2 > 0$. Hence, $dV/dt < 0$ for:

$$\left\| \begin{bmatrix} e_1 & e_2 & e_3 \end{bmatrix}^{\mathrm{T}} \right\| > \frac{\left\| \begin{bmatrix} \dot{X}_r & \dot{Y}_r & \varepsilon \end{bmatrix}^{\mathrm{T}} \right\|}{\lambda_{\min}(A)} \tag{49}$$

where $\lambda_{\min}(A)$ denotes the minimum eigenvalue of matrix A . Therefore, the stability of the error dynamic equation and tracking with bounded error are guaranteed outside the detection region. Moreover, we can decrease the tracking error by increasing the gains k_1 and k_2 .

2. When the robot is inside the detection range($l < d_{ro} < L$), Assumption 2 implies $\dot{X}_r = \dot{Y}_r = 0$, the inequality(46) becomes:

$$\begin{aligned} \dot{V} &\le -k_1(\cos e_3 + \delta \sin e_3)^2 (E_X^2 + E_Y^2) \\ &\quad - |e_3|(k_2|e_3| - \varepsilon) \\ &\le -|e_3|(k_2|e_3| - \varepsilon) \end{aligned} \tag{50}$$

$\dot{V}$ is negatively defined for:

$$|e_3| > \frac{\varepsilon}{k_2} \tag{51}$$

Hence, as shown by Stipanovic et al. [20], since dV/dt is negatively defined V is non-increasing inside the detection region. Since:

$$\lim_{\|z - z_o\| \to l^+} V_{ob} = \infty \tag{52}$$

where $z = [X \quad Y]^{\mathrm{T}}$ and $z_o = [X_o \quad Y_o]^{\mathrm{T}}$ obstacle avoidance is guaranteed.

Remark 4. The singularity condition $E_X = E_Y = 0$ can occur in the following two cases: The first case occurs outside the detection region where:

$$\frac{\partial V_{ob}}{\partial X} = \frac{\partial V_{ob}}{\partial Y} = 0$$

which corresponds to $e_1 = e_2 = 0$ and this case can easily be handled using zero controllers $u = v = 0$. The second case occurs inside the detection region where the condition corresponds to a singularity in which the reference direction vector for tracking is of opposite but equal magnitude to the direction vector for avoiding collision; this results in a deadlock situation. This case can be handled by changing the reference trajectory to drive the robot out of the singularity. We do not investigate this case further in this paper.

5. Simulation results

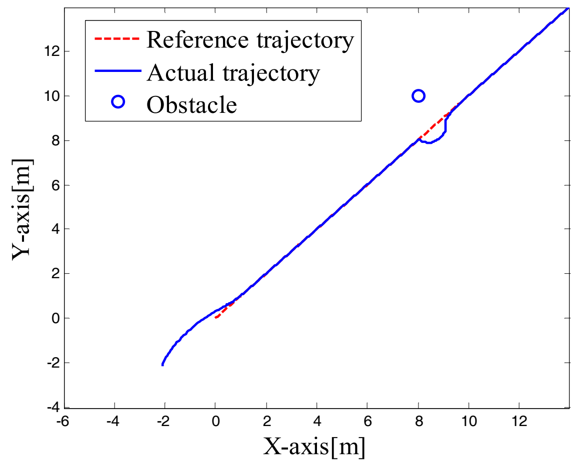
To validate the effectiveness of the proposed adaptive control scheme, we perform simulations for trajectory tracking and obstacle avoidance for mobile robots in the presence unknown sliding. In this simulation we choose the system parameters as $r = 0.125\text{m}$ and $b{=}0.5\text{m}$. The detection and avoidance radii are $L = 4\text{m}$ and $l = 2\text{m}$.

5.1 Straight line reference trajectory tracking

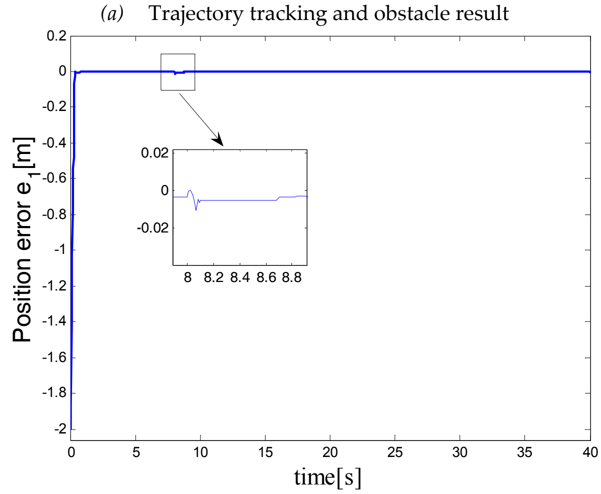
In this case, a straight line reference trajectory is considered. The equation of the straight line reference trajectory is given as $\begin{cases} X_r = t \\ Y_r = t \end{cases}$, where $0\text{s} \le t \le 40\text{s}$ is the simulation time. The reference velocities v_r and w_r are chosen as $v_r = 2\text{m/s}$ and $\omega_r = 0\text{rad/s}$. The obstacle is located at $(X_o, Y_o) = (8\text{m}, 10\text{m})$. The initial postures of the reference robot and the actual robot are set at $\left[X_r(0), Y_r(0), \theta_r(0)\right] = \left[0\text{m}, 0\text{m}, \frac{\pi}{4}\text{rad}\right]$ and $\left[X(0), Y(0), \theta(0)\right] = \left[-2\text{m}, -2\text{m}, \frac{\pi}{4}\text{rad}\right]$ respectively.

First, we assume that the wheels' sliding described by $\left[i_L, i_R, \delta\right] = \left[0.15, -0.15, 0.1\sin(0.2t)\right]$ only influences the mobile robot after $t = 8$. The controller and sliding model observer gains are chosen as $L_1 = \left|\dot{x}\sin\theta\right|$, $L_2 = 0.3$, $L_3 = 4r / b\omega_L$ and $L_4 = 4r / b\omega_R$. The parameters of the low pass filter are chosen as $\gamma_1 = 30$, $\gamma_2 = 20$ and $\gamma_3 = 50$. The tracking and obstacle avoidance results of the proposed control system are shown in Fig.5. Fig. 5*a* reveals that the proposed adaptive control system can overcome the effect of sliding while obstacle avoidance is guaranteed. Tracking errors of the proposed control system are shown in Figs. 5*b*~5*d*, we can observe in Figs. 5*b*~5*d* that the tracking errors converge asymptotically to zero except in the range that the mobile robot detects the obstacle because the sliding effect is considered. In addition, the distance between the robot and obstacle is shown in Fig. 5*e*. In this figure, notice that this distance is always larger than the avoidance radius $l = 2\text{m}$, namely there is no collision between the robot and the obstacle. Meanwhile, it can be seen from Fig.5 *f*~*h*, the siding parameters can be estimated precisely by the SMO.

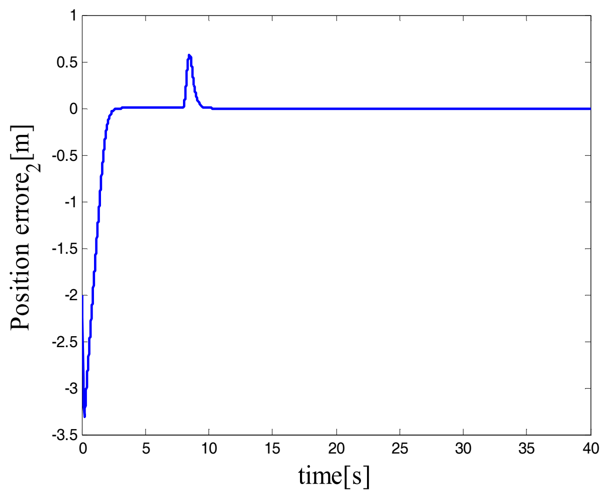
From Fig.5, we can see that the proposed control method can effectively overcome the wheels' sliding for the straight line trajectory tracking and obstacle avoidance of mobile robots.



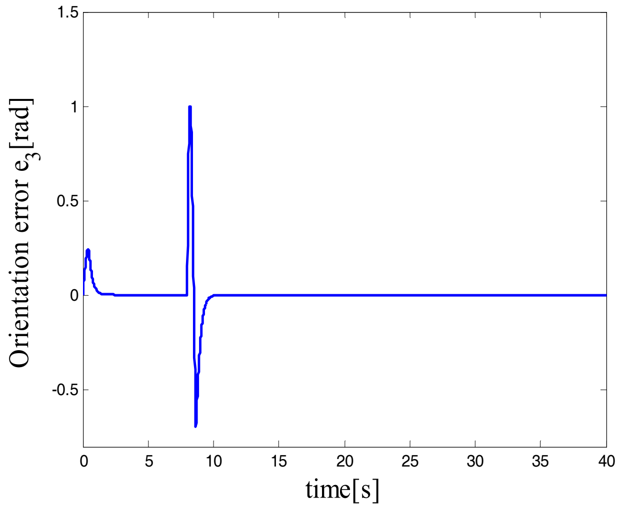
(*a*) Trajectory tracking and obstacle result



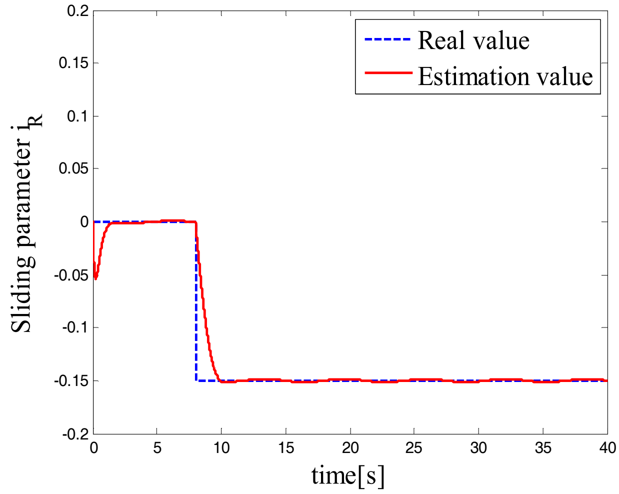
(*b*) X- axis tracking error e_1



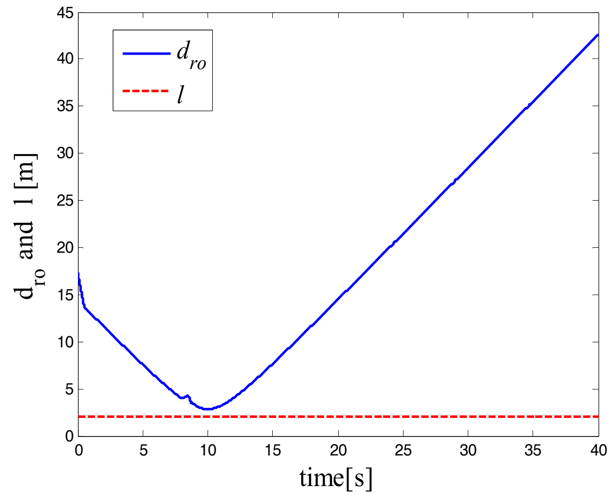
(*c*) Y- axis tracking error e_2



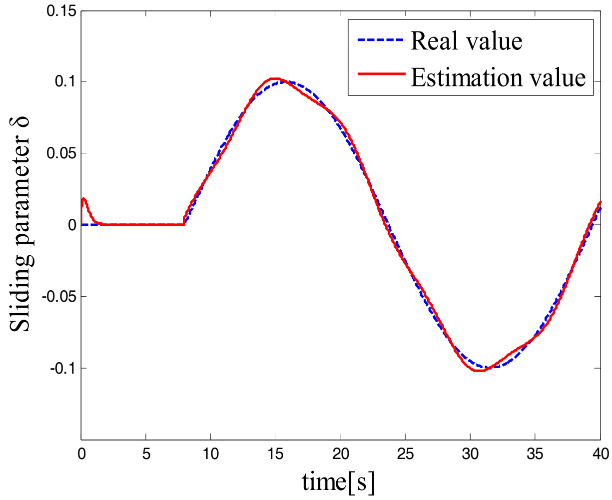
(d) Orientation tracking error e_3



(g) The sliding parameter i_R estimation

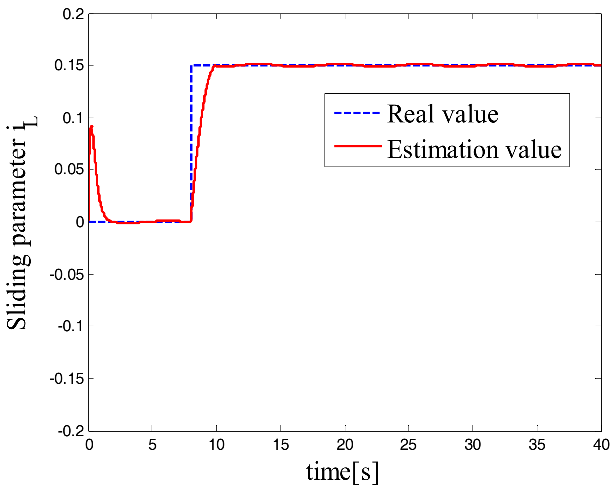


(e) The distance between the robot and the obstacle, and the avoidance region



(h) The sliding parameter δ estimation

Figure 5. simulation results for a straight line reference trajectory in the presence of wheel's sliding



(f) The sliding parameter i_L estimation

5.2 The curved line reference trajectory tracking

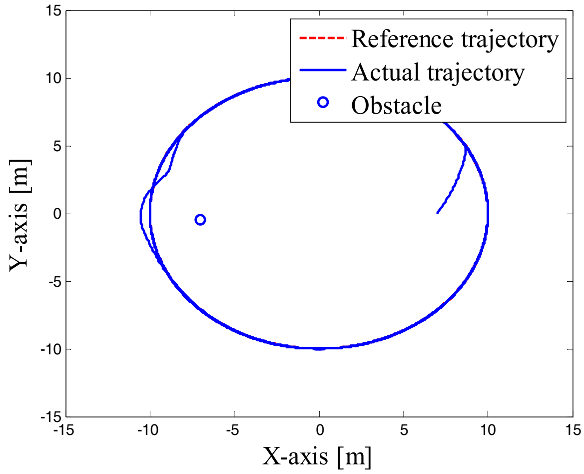
In this case, we consider a curved line reference trajectory generated by reference velocities $v_r = 2\text{m/s}$ and $\omega_r = 0.2\text{rad/s}$ for $0 \le t \le 40$. The equation of the straight line reference trajectory is given as: $\begin{cases} X_r = 10\cos(0.2t) \\ Y_r = 10\sin(0.2t) \end{cases}$

In addition, it is assumed that the obstacle is located at $(X_o, Y_o) = (-7, -0.5)$. The initial positions of the reference trajectory and the actual mobile robot are chosen as $[X_r(0), Y_r(0), \theta_r(0)]^{\mathrm{T}} = \left[10\text{m}, 0\text{m}, \frac{\pi}{4}\text{rad}\right]^{\mathrm{T}}$ and $[X(0), Y(0), \theta(0)]^{\mathrm{T}} = \left[7\text{m}, 0\text{m}, \frac{\pi}{4}\text{rad}\right]^{\mathrm{T}}$ respectively.

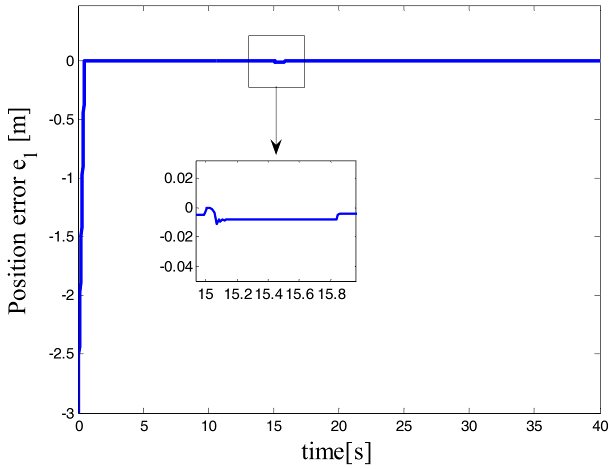
The wheels' sliding is described by:

$\left[i_L, i_R, \delta\right] = \left[0.15, -0.15, 0.6\cos(0.2t)\right]$ and influences the mobile robot after $t = 15$.

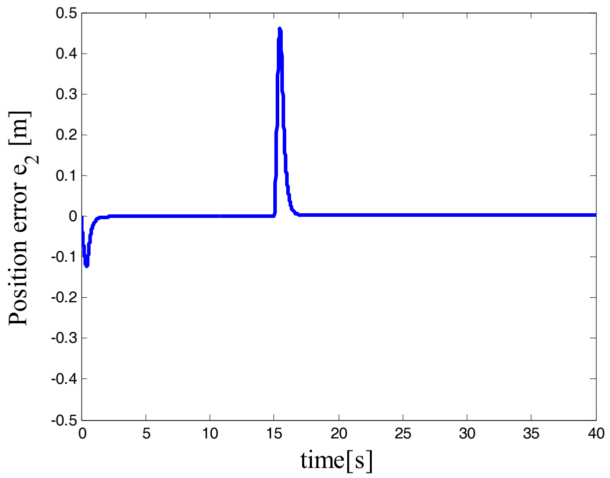
To simplify the complexity of the simulation, the control system parameters, control parameters and the parameters of the low pass filter are all the same as in the previous simulation.



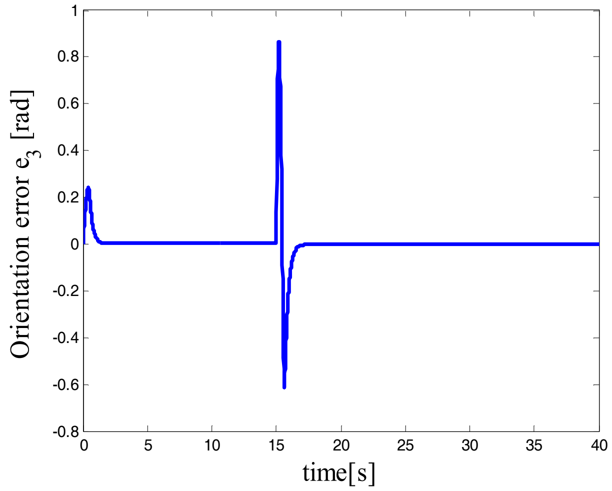
(a) Trajectory tracking and obstacle results



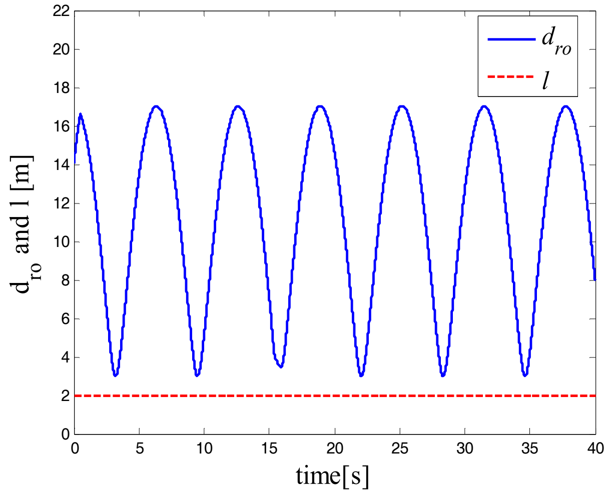
(*b*) X- axis tracking error e_1



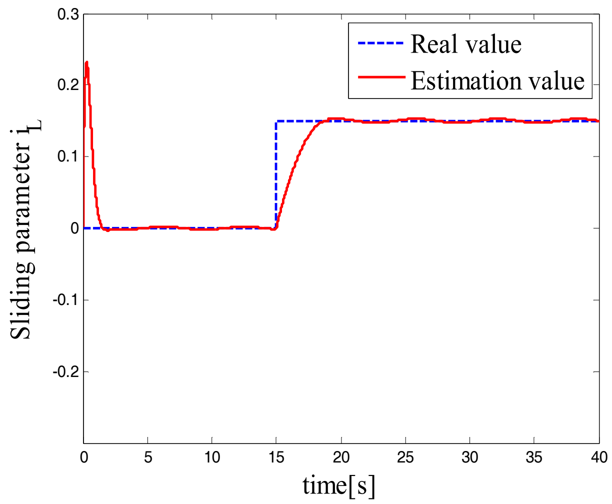
(*c*) Y- axis tracking error e_2



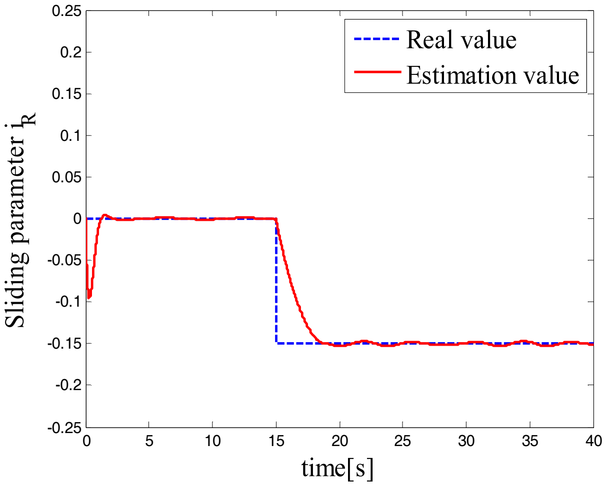
(*d*) Orientation tracking error e_3



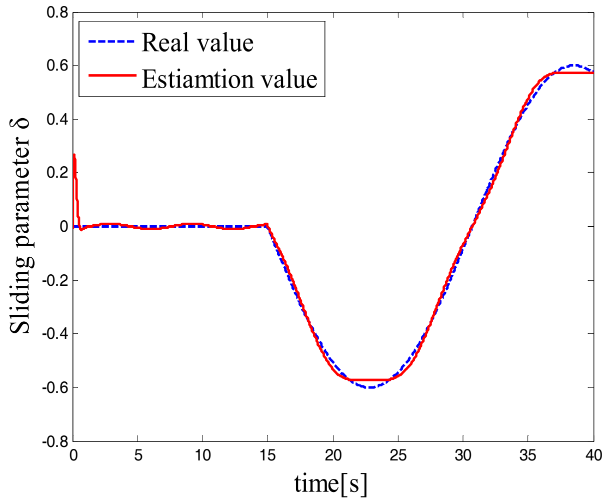
(*e*) The distance between the robot and the obstacle, and the avoidance region



(*f*) The sliding parameter i_L estimation



(g) The sliding parameter i_R estimation



(h) The sliding parameter δ estimation

Figure 6. Simulation results for a curved line reference trajectory in the presence of the wheels' sliding

The simulation results are shown in Fig. 6, where the proposed adaptive control system can compensate for the sliding effects and has good performance in obstacle avoidance (see Fig. 6*a*~*d*). In addition, Fig. 6*e* reveals that there is no collision between the robot and the obstacles. Fig.6 *f*~*h* show the three sliding parameters can be estimated accurately in real time by the SMO.

From Fig.6, we draw a conclusion that the proposed control approach has good tracking and obstacle avoidance performance for the curved path regardless of the effects of the unknown sliding.

Further, from Fig.5 and Fig.6, we discover that the proposed control method can avoid obstacles and overcome effectively the sliding influence for the given path tracking of mobile robots. This is mainly because the designed tracking control laws (equations (37) and (38)) have adaptive abilities and whose sliding parameters are adaptively modifying. What's more, when a robot's sliding parameters change, the tracking controller can automatically adjust these parameters to meet the demands of the mobile robot in the real environment by the sliding model observer. Even if the system sliding parameters i_L , i_R and δ change abruptly, the sliding model observer can still estimate sliding parameters rather accurately. Consequently, the adaptive tracking control algorithm has good robustness and the adaptive ability to face sliding parameter perturbations of the mobile robot.

6. Conclusions

We have presented an approach to design an adaptive controller for the tracking and obstacle avoidance of mobile robots in the presence of the wheels' unknown sliding at the kinematic level. The robot kinematic model has been induced from the model in the absence of sliding. A novel adaptive control system for mobile robots has been designed using the Lypunov design technique. Meanwhile, a sliding model observer is used to estimate sliding parameters online. We have proved its stability and have induced the control laws to compensate for unknown sliding from Lyapunov stability approach with the potential function. Finally, simulation results have shown the proposed controller has good tracking and obstacle avoidance performance and robustness against the unknown wheel sliding.

7. Acknowledgments

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