NEW ZEALAND JOURNAL OF MATHEMATICS
Volume 42 (2012), 149-155

A NEW CHARACTERIZATION OF THE ALTERNATING GROUPS A_5 AND A_6

Alireza Khalili Asboei

(Received 12 September, 2012)

Abstract. Let G be a finite centerless group, let $\pi(G)$ be the set of primes p such that G contains an element of order p and let $n_p(G)$ be the number of Sylow $p-$subgroup of G, that is, $n_p(G)$=$|\mathrm{Syl}_p(G)|$. Set $\mathrm{NS}(G) := \{n_p(G)|\ p \in \pi(G)\}$. If NS($G$)=NS($M$), where M denotes one of the alternating simple groups A_5 or A_6, then $M \leq G \leq \mathrm{Aut}(M)$.

1. Introduction

If n is an integer, then we denote by $\pi(n)$ the set of all prime divisors of n. Let G be a finite group. Denote by $\pi(G)$ the set of primes p such that G contains an element of order p. A finite group G is a simple K_n-group if G is a simple group with $|\pi(G)| = n$. Denote by $(a,\ b)$ the greatest common divisor of positive integers a and b. If G is a finite group, then we denote by n_q the number of Sylow $q-$subgroup of G, that is, $n_q = n_q(G)$=$|\mathrm{Syl}_q(G)|$. All other notations are standard and we refer to [**10**], for example.

In 1992, Bi [**5**] showed that the group $L_2(p^k)$ can be characterized just by the orders of the normalizers of its Sylow subgroups. In other words, if G is a group and $|N_G(P)| = |N_{L_2(p^k)}(Q)|$, where $P \in \mathrm{Syl}_r(G)$ and $Q \in \mathrm{Syl}_r(L_2(p^k))$ for every prime r, then $G \cong L_2(p^k)$. This type of characterization is known for the following simple groups: $L_2(p^k)$ [**5**], $L_n(q)$ [**4**], $S_4(q)$ [**8**], the alternating simple groups [**7**], $U_n(q)$ [**9**], the sporadic simple groups [**2**] and ${}^2D_n(p^k)$ [**1**].

Set $\mathrm{NS}(G) := \{n_p(G)|\ p \in \pi(G)\}$. Let S be one of the above simple groups. It is clear that if $n_p(G) = n_p(S)$ for every prime p and $|G| = |S|$, then $|N_G(P)| = |N_S(Q)|$, where $P \in \mathrm{Syl}_p(G)$ and $Q \in \mathrm{Syl}_p(S)$. Thus by the above references, $G \cong S$. Now replace the condition NS(G) =NS(S) with the condition $n_p(G) = n_p(S)$ for every prime p and remove the condition $|G| = |S|$: in this case we can not conclude that G is isomorphic to S. For example, if $G = A_5 \times H$ where H is a finite nilpotent group such that $\pi(H) \subseteq \{2,\ 3,\ 5\}$, then NS($G$)=NS($A_5$)=$\{5,\ 6,\ 10\}$, but G is not isomorphic to A_5. So there are many finite groups G such that NS(G)=NS(A_5)=$\{5,\ 6,\ 10\}$.

In this paper, we show that the simple groups A_5 and A_6 are recognizable by NS(G), where G is a centerless group.

Main Theorem: Let G be a finite centerless group such that NS(G)=NS(M), where M denotes one of the simple groups A_5 or A_6. Then $M \leq G \leq$Aut(M).

2010 *Mathematics Subject Classification* 20D06, 20D20.
Key words and phrases: Finite group, Sylow subgroup.
The author would like to thank the referee for valuable comments.

2. Preliminary Results

In this section we collect some preliminary lemmas used in the proof of the main theorem.

Lemma 2.1. [**12**, Theorem 9.3.1] Let G be a finite solvable group and $|G| = m \cdot n$, where $m = p_1^{\alpha_1} \cdots p_r^{\alpha_r}$, $(m, n) = 1$. Let $\pi = \{p_1, ..., p_r\}$ and let h_m be the number of $\pi-$Hall subgroups of G. Then $h_m = q_1^{\beta_1} \cdots q_s^{\beta_s}$ satisfies the following conditions for all $i \in \{1, 2, ..., s\}$:
(1) $q_i^{\beta_i} \equiv 1 \pmod{p_j}$, for some p_j.
(2) The order of some chief factor of G is divisible by $q_i^{\beta_i}$.

Lemma 2.2. [**13**] If G is a simple K_3-group, then G is isomorphic to one of the following groups: A_5, A_6, $L_2(7)$, $L_2(8)$, $L_2(17)$, $L_3(3)$, $U_3(3)$ or $U_4(2)$.

Lemma 2.3. [**11**] Let G be a finite group and M be a normal subgroup of G. Both the Sylow $p-$number $n_p(M)$ and the Sylow $p-$number $n_p(G/M)$ of the quotient G/M divide the Sylow $p-$number $n_p(G)$ of G and $n_p(M)\ n_p(G/M) \mid n_p(G)$.

Lemma 2.4. [**14**] Let G be a simple group of order $2^a \cdot 3^b \cdot 5^c \cdot 7^d$, $abcd \neq 0$. Then G is isomorphic to one of the following groups: A_n, $n = 7, 8, 9, 10$; J_2; $L_2(49)$, $L_3(4)$, $O_5(7)$, $O_7(2)$, $O_8^+(2)$, $U_3(5)$ and $U_4(3)$.

Lemma 2.5. [**15**] Let G be a simple K_4-group. Then G is isomorphic to one of the following groups:
(1) A_7, A_8, A_9, A_{10}.
(2) M_{11}, M_{12}, J_2.
(3) (a) $L_2(r)$, where r is a prime and satisfies $r^2 - 1 = 2^a \cdot 3^b \cdot v^c$ with $a \geq 1$, $b \geq 1$, $c \geq 1$ and $v > 3$ is a prime;
(b) $L_2(2^m)$, where $2^m - 1 = u$, $2^m + 1 = 3t^b$, where $m \geq 2$, u, t are primes, $t > 3$, $b \geq 1$;
(c) $L_2(3^m)$, where $3^m + 1 = 4t$, $3^m - 1 = 2u^c$ or $3^m + 1 = 4t^b$, $3^m - 1 = 2u$, where $m \geq 2$, u, t are odd primes, $b \geq 1$, $c \geq 1$;
(d) $L_2(16)$, $L_2(25)$, $L_2(49)$, $L_2(81)$, $L_3(4)$, $L_3(5)$, $L_3(7)$, $L_3(8)$, $L_3(17)$, $L_4(3)$, $S_4(4)$, $S_4(5)$, $S_4(7)$, $S_4(9)$, $S_6(2)$, $O_8^+(2)$, $G_2(3)$, $U_3(4)$, $U_3(5)$, $U_3(7)$, $U_3(8)$, $U_3(9)$, $U_4(3)$, $U_5(2)$, $Sz(8)$, $Sz(32)$, ${}^3D_4(2)$, ${}^2F_4(2)'$.

Lemma 2.6. [**3**] Let α_i be a positive integer ($i = 1, ..., 5$), p a prime and $p \notin \{2, 3, 5, 7\}$. If G is a simple group and $|G| = 2^{\alpha_1} \cdot 3^{\alpha_2} \cdot 5^{\alpha_3} \cdot 7^{\alpha_4} \cdot p^{\alpha_5}$, then G is isomorphic to one of the following simple groups: A_{11}, A_{12}, M_{22}, HS, McL, He, $L_2(q)$ ($q = 26$, 53, 74, 29, 41, 71, 251, 449, 4801), $L_3(9)$, $L_4(4)$, $L_4(7)$, $L_5(2)$, $L_6(2)$, $O_5(49)$, $O_7(3)$, $O_9(2)$, $S_6(3)$, $O_8^+(3)$, $G_2(4)$, $G_2(5)$, $U_3(19)$, $U_4(5)$, $U_4(7)$, $U_5(3)$, $U_6(2)$, ${}^2D_4(2)$. If $p = 11$, then G is isomorphic to one of the following simple groups: A_{11}, A_{12}, M_{22}, HS, McL, $U_6(2)$.

Sylow's theorem implies that if p is prime, then $n_p = 1 + pk$. If $p = 2$, then n_2 is odd. If $p \in \pi(G)$, then

$$\begin{cases} p \mid (n_p - 1) \\ \\ (p,\, n_p) = 1 \end{cases} \qquad (*)$$

In the proof of the main theorem, we often apply $(*)$ and the above comments.

3. Proof of the Main Theorem

Case 1. Characterization of the group A_5

Let G be a finite centerless group such that NS(G)=NS(A_5)=$\{5,\, 6,\, 10\}$. First, we prove that $\pi(G) = \{2,\, 3,\, 5\}$. By Sylow's theorem, $n_p \mid |G|$ for every p, so by NS(G), we conclude that $\{2,\, 3,\, 5\} \subseteq \pi(G)$. On the other hand, by $(*)$ if $p \in \pi(G)$, then $p \mid (n_p - 1)$ and $(p,\, n_p) = 1$, which implies that $p \in \{2,\, 3,\, 5\}$. Therefore $\pi(G) = \{2,\, 3,\, 5\}$. Now $n_2(G) = 5$, $n_3(G) = 10$ and $n_5(G) = 6$.

We prove that G is a nonsolvable group. If G is a solvable group, since $n_5(G) = 6$ by Lemma 2.1, $3 \equiv 1 \pmod 5$, a contradiction. Hence G is a nonsolvable group.

Since G is a finite group, it has a chief series. Let $1 = N_0 \trianglelefteq N_1 \trianglelefteq \ldots \lhd N_{r-1} \trianglelefteq N_r = G$ be a chief series of G. Since G is a nonsolvable group, there exists a maximal non-negative integer i such that N_i/N_{i-1} is a simple group or a direct product of isomorphic simple groups and N_{i-1} is a maximal solvable normal subgroup of G. Now set $N_i := H$ and $N_{i-1} := N$. Hence G has the following normal series

$$1 \trianglelefteq N \lhd H \trianglelefteq G$$

such that H/N is non-abelian simple or H/N is a direct product of isomorphic non-abelian simple groups.

Since G is a K_3-group, H/N is a simple K_3-group or H/N is a direct product of simple K_3-groups. By Lemma 2.2, $H/N \cong A_5$, A_6 or $U_4(2)$. On the other hand, by Lemma 2.3, $n_p(H/N) \mid n_p(G)$ for every prime $p \in \pi(G)$, thus $H/N \cong A_5$.

Now set $\overline{H} := H/N \cong A_5$ and $\overline{G} := G/N$. Hence

$$A_5 \cong \overline{H} \cong \overline{H}C_{\overline{G}}(\overline{H})/C_{\overline{G}}(\overline{H}) \leq \overline{G}/C_{\overline{G}}(\overline{H}) = N_{\overline{G}}(\overline{H})/C_{\overline{G}}(\overline{H}) \leq \mathrm{Aut}(\overline{H}).$$

If $K = \{x \in G |\, xN \in C_{\overline{G}}(\overline{H})\}$, then $G/K \cong \overline{G}/C_{\overline{G}}(\overline{H})$. So $A_5 \leq G/K \leq \mathrm{Aut}(A_5) \cong S_5$. Therefore $G/K \cong A_5$ or $G/K \cong S_5$.

Suppose that $G/K \cong S_5$. We know that $n_2(S_5) = 15$ and $n_2(G) = n_2(A_5) = 5$. On the other hand, by Lemma 2.3, $n_2(S_5) = 15 \mid n_2(G) = 5$, a contradiction. Therefore $G/K \cong A_5$.

We show that $K = N$. Suppose that $K \neq N$. By Lemma 2.3, $n_p(K) = 1$ for every prime $p \in \pi(G)$, so K is a nilpotent subgroup of G. On the other hand, since $C_{\overline{G}}(\overline{H}) \cong K/N$ and N is a maximal solvable normal subgroup G, K is a nonsolvable normal subgroup of G, a contradiction. Thus $K = N$, so $G/N \cong A_5$.

We claim that $N = 1$. Suppose $N \neq 1$, and let Q be a non-trivial Sylow $q-$subgroup of N. Since N is nilpotent, Q is normal in G. Now if $P \in \mathrm{Syl}_p(G)$, then P normalizes Q and so if $p \neq q$, then $P \leq N_G(Q) = G$. Also NP/N is a Sylow p-subgroup of G/N. On the other hand, if $R/N = N_{G/N}(\ NP/N)$, then $R = N_G(P)N$. We know that $n_p(G) = n_p(G/N)$, so $|G : R| = |G : N_G(P)|$. Thus $R = N_G(P)$ and therefore $N \leq N_G(P)$. So $Q \leq N_G(P)$. Since $P \leq N_G(Q)$ and $Q \leq N_G(P)$, this implies that $[P, Q] \leq P$, $[P, Q] \leq Q$ and so $[P, Q] \leq P \cap Q = 1$. Thus $P \leq C_G(Q)$ and $Q \leq C_G(P)$, in other words P and Q centralize each other. Therefore $P \leq C_G(Z)$ where $Z = Z(Q)$. On the other hand, since

$C_G(Z) \trianglelefteq N_G(Z) = G$, $C_G(Z)$ is a normal subgroup of G. Also since N is a nilpotent group, $N \trianglelefteq C_G(Z)$. So $C_G(Z)/N \trianglelefteq G/N$. Since P is not contained in N and G/N is a simple group, $C_G(Z)/N = G/N$. Hence $C_G(Z) = G$. Therefore $Z \neq 1$ contained in the center of G, which is a contradiction. Hence $N = 1$ and $G \cong A_5$.

Case 2. Characterization of the group A_6
Let G be a finite centerless group such that NS(G)=NS(A_6)={10, 36, 45}. First we prove that $\pi(G) = \{2, 3, 5\}$. By Sylow's theorem, $n_p(G) \mid |G|$ for every p, so by NS(G), we conclude that $\{2, 3, 5\} \subseteq \pi(G)$. On the other hand, by $(*)$ if $p \in \pi(G)$, then $p \mid (n_p - 1)$ and $(p, n_p) = 1$, which implies that $p \in \{2, 3, 5, 7, 11\}$. Therefore $\pi(G) \subseteq \{2, 3, 5, 7, 11\}$. Thus $n_2(G) = 45$, $n_3(G) = 10$ and $n_5(G) = 36$. If $7 \in \pi(G)$, then $n_7(G) = 36$ and if $11 \in \pi(G)$, then $n_{11}(G) = 45$.

We prove that G is nonsolvable group. If G is a solvable group, then since $n_3(G) = 10$ by Lemma 2.1, $5 \equiv 1 \pmod 3$, a contradiction. Hence G is a nonsolvable group.

Since G is finite and nonsolvable, it has the following normal series

$$1 \trianglelefteq N \lhd H \trianglelefteq G$$

such that H/N is non-abelian simple or H/N is a direct product of isomorphic non-abelian simple groups.

Let $H/N = S_1 \times ... \times S_r$, where S_1 is a simple group and $S_1 \cong ... \cong S_r$. Since $\pi(G) \subseteq \{2, 3, 5, 7, 11\}$, S_1 is a simple K_3- or K_4- or K_5-group. We consider the following subcases:

Subcase a. Let $\pi(G) = \{2, 3, 5\}$. Hence H/N is a simple K_3-group or H/N is a direct product of simple K_3-groups. On the other hand, $n_p(H/N) \mid n_p(G)$ for every prime $p \in \pi(G)$, so by Lemma 2.2, $H/N \cong A_5$ or A_6.

If $H/N \cong A_5$ then, similar to the proof of Case 1, there exists a normal subgroup K such that $N \leq K$ and $A_5 \leq G/K \leq S_5$.

If $G/K \cong A_5$, then we prove that $K = N$. Suppose that $K \neq N$. Since $N < K$ and N is a maximal solvable normal subgroup G, K is a nonsolvable normal subgroup of G. On the other hand, $n_3(K) = 1$, $n_2(K) \mid 9$ and $n_5(K) \mid 6$, by Lemma 2.3. Because K is a nonsolvable, it has the following normal series

$$1 \trianglelefteq N_1 \lhd H_1 \trianglelefteq K$$

such that $H_1/N_1 \cong A_5$. Since $n_3(H_1/N_1) = n_3(A_5) \mid n_3(K) = 1$, we get a contradiction. Thus $N = K$. Therefore $G/N \cong A_5$.

Let P be a Sylow 5-subgroup of G. We know that PN/N is a Sylow 5-subgroup of $G/N = A_5$ and $N_G(P)N/N$ normalizes PN/N and hence has order 10 in G/N. So $|N_G(P)N| = 10 \times |N|$. Therefore the number of Sylow 5-subgroup of $N_G(P)N/N$ is 6. Since $N_G(P)N/N$ is solvable, by Lemma 2.1 we get a contradiction. Similarly if $G/K \cong S_5$, then $K = N$, a contradiction.

If $H/N \cong A_6$, then similar to the proof of Case 1, there exists a normal subgroup K such that $N \leq K$ and $A_6 \leq G/K \leq$Aut(A_6). So $G/K \cong A_6$, S_6, PGL(2, 9), M_{10} or PΓL(2, 9).

Let $G/K \cong A_6$. By Lemma 2.3, K is a nilpotent group and hence $K = N$.

We claim that $N = 1$. Let Q be a non-trivial Sylow $q-$subgroup of N. Since N is nilpotent, Q is normal in G. If $P \in$Syl$_p(G)$, then Q normalizes P; so if p is not q, then P and Q centralize each other. Let $C = C_G(Q)$, then C contains a full Sylow

$p-$subgroup of G for all primes p different from q, and thus $|G : C|$ is a power of q. Now let S be a Sylow $q-$subgroup of G. Then $G = CS$. Also if $Q > 1$, then $C_Q(S)$ is nontrivial, so $C_Q(S) \leq Z(G)$. Since by assumption $Z(G) = 1$, it follows that $Q = 1$. Since q is arbitrary, $N = 1$, as claimed. Therefore $G \cong A_6$.

If $G/K \cong S$, where S is one of the groups PGL(2, 9), M_{10} or PΓL(2, 9), then from $n_p(S) = n_p(G)$ for every prime p we prove $K = N$ and similar to the above discussion $N = 1$. Therefore $G \cong S_6$, PGL(2, 9), M_{10} or PΓL(2, 9).

Subcase b. Let $\pi(G) = \{2, 3, 5, 7\}$. Hence H/N is a simple K_3-group or H/N is a direct product of simple K_3-groups, or H/N is a simple K_4-group or H/N is a direct product of simple K_4-groups.

If H/N is a simple K_3-group or H/N is a direct product of simple K_3-groups, then by Lemma 2.2, $H/N \cong A_5$, A_6, $L_2(7)$, $L_2(8)$, $U_3(3)$ or $U_4(2)$.

If $H/N \cong L_2(7)$, $L_2(8)$, $U_3(3)$ or $U_4(2)$, then by Lemma 2.3, $n_2(L_2(7)) = 21 \mid n_2(G) = 45$, $n_3(L_2(8)) = 28 \mid n_3(G) = 10$, $n_2(U_3(3)) = 189 \mid n_2(G) = 45$ and $n_3(U_4(2)) = 160 \mid n_3(G) = 10$, a contradiction.

If $H/N \cong A_5$, then similar to the proof of Case 1, there exists a normal subgroup K such that $N \leq K$ and $A_5 \leq G/K \leq S_5$.

If $G/K \cong A_5$, then we prove that $K = N$. Suppose that $K \neq N$. By Lemma 2.3, $n_3(K) = 1$, $n_2(K) \mid 9$, $n_5(K) \mid 6$ and $n_7(K) \mid 36$. Since $N < K$ and N is a maximal solvable normal subgroup G, K is a nonsolvable normal subgroup of G. Therefore K has the following normal series

$$1 \trianglelefteq N_1 \lhd H_1 \trianglelefteq K$$

such that $H_1/N_1 \cong A_5$, A_6, $L_2(7)$, $L_2(8)$, $U_3(3)$, $U_4(2)$ or S, where S is one of the groups: A_n for $n = 7$, 8, 9, 10, J_2, $L_2(49)$, $L_3(4)$, $O_5(7)$, $O_7(2)$, $O_8^+(2)$, $U_3(5)$ and $U_4(3)$, by Lemma 2.4. Since $3 \in \pi(H_1/N_1)$ and $n_3(H_1/N_1) \mid n_3(K) = 1$, we get a contradiction. Thus $N = K$. Now $G/N \cong A_5$. This implies that $7 \in \pi(N)$ and the order of a Sylow 7-subgroup in G and N are equal. As N is normal in G, the number of Sylow 7-subgroups of G and N are equal. Therefore the number of Sylow 7-subgroups of N is 36. Since N is solvable by Lemma 2.1, $4 \equiv 1 \pmod 7$, a contradiction.

Similar to the above discussion if $G/K \cong S_5$, we get a contradiction. If $H/N \cong A_6$, then similar to the above discussion, we get a contradiction.

Now let H/N be a simple K_4-group or H/N be a direct product of simple K_4-groups. By Lemma 2.4, $H/N \cong S$, where S is one of the groups: A_n for $n = 7$, 8, 9, 10, J_2, $L_2(49)$, $L_3(4)$, $O_5(7)$, $O_7(2)$, $O_8^+(2)$, $U_3(5)$ and $U_4(3)$. Since $3 \in \pi(S)$ and $n_3(S) \mid n_3(G) = 10$, we get a contradiction.

Subcase c. Let $\pi(G) = \{2, 3, 5, 11\}$. Hence H/N is a simple K_3-group or H/N is a direct product of simple K_3-groups, or H/N is a simple K_4-group or H/N is a direct product of simple K_4-groups.

If H/N is a simple K_3-group or H/N is a direct product of simple K_3-groups, then by Lemma 2.2, $H/N \cong A_5$, A_6 or $U_4(2)$.

Let $H/N \cong U_4(2)$. By Lemma 2.3, $n_3(U_4(2)) = 160 \mid n_3(G) = 10$, a contradiction.

Let $H/N \cong A_5$. Similar to the proof of Subcase b, we get a contradiction.

Now let H/N be simple K_4-group. By Lemma 2.5, if $H/N \cong L_2(r)$, where r is a prime and satisfies $r^2 - 1 = 2^a \cdot 3^b \cdot v^c$ with $a \geq 1$, $b \geq 1$, $c \geq 1$ and $v > 3$

is a prime, then by $\pi(H/N) = \{2,\ 3,\ 5,\ 11\}$, $r = 11$. So $H/N \cong L_2(11)$; since $n_{11}(L_2(11)) = 12 \mid n_{11}(G) = 45$, this is a contradiction.

If $H/N \cong L_2(2^m)$, where $2^m - 1 = u$, $2^m + 1 = 3t^b$, where $m \geq 2$, u, t are primes, $t > 3$, $b \geq 1$, then $u,\ t \in \{3,\ 5,\ 11\}$, a contradiction.

If $H/N \cong L_2(3^m)$, where $3^m + 1 = 4t$, $3^m - 1 = 2u^c$ or $3^m + 1 = 4t^b$, $3^m - 1 = 2u$, where $m \geq 2$, u, t are odd primes, $b \geq 1$, $c \geq 1$, then $u,\ t \in \{3,\ 5,\ 11\}$, a contradiction.

For the other case by Lemma 2.5, we get a contradiction.

Subcase d. Let $\pi(G) = \{2,\ 3,\ 5,\ 7,\ 11\}$. Hence H/N is a simple K_n-group or H/N is a direct product of simple K_n-groups for $n = 3$, 4, or 5.

If H/N is a simple K_3-group or H/N is a direct product of simple K_3-groups or H/N is a simple K_4-group or H/N is a direct product of simple K_4-groups, then similar to the proof of Subcase a or b, we get a contradiction.

Let H/N be a simple K_5-group, or H/N is a direct product of simple K_5-groups. By Lemma 2.6, $H/N \cong A_{11}$, A_{12}, M_{22}, HS, McL or $U_6(2)$ and by $n_3(H/N) \mid n_3(G) = 10$, we get a contradiction. □

We conclude with a conjecture.

Conjecture: Let G be a finite centerless group such that NS(G)=NS(A_n), then $A_n \leq G \leq$Aut(A_n).

References

[1] A. Iranmanesh and N. Ahanjideh, *A characterization of $^2D_n(p^k)$ by order of normalizer of Sylow subgroups*, Int. J. Algebra, **2** (18) (2008) 853-865.

[2] A. Khosravi and B. Khosaravi, *Two new characterization of sporadic simple groups*, Pure Math. Appl, **16** (2005) 287-293.

[3] F. Z. Cao, *Diophantine Equation and its Application*, Shanghai Jiaotong University Press, 2000 (Chinese).

[4] J. Bi, *A characterization of $L_n(q)$ by the normalizers' orders of their Sylow subgroups*, Acta. Math. Sinica (New Ser), **11** (3) (1995) 300-306.

[5] J. Bi, *A characterization of $L_2(q)$*, J. Liaoning Univ (Natural Sciences Edition), **19** (2) (1992) 1-4. (Chinese)

[6] J. Bi, *A characterization of finite projective special unitary group $U_6(q)$*, J. Liaoning Univ (Natural Sciences Edition), **26** (4) (1990) 295-298. (Chinese)

[7] J. Bi, *Characterization of alternating groups by orders of normalizers of Sylow subgroups*, Algebra Colloq, **8** (3) (2001) 249-256.

[8] J. Bi, *On the group with the same orders of Sylow normalizers as the finite simple group $S_4(q)$*, Algebras Groups Geom, **18** (3) (2001) 349-355.

[9] J. Bi, *On the groups with the same orders of Sylow normalizers as the finite projective special unitary group*, Sci. China, Ser A, **47** (6) (2004) 801-811.

[10] J. H. Conway, R. T. Curtis, S. P. Norton and R. A. Wilson, *Atlas of Finite Groups*, Clarendon, Oxford, 1985.

[11] J. Zhang, *Sylow numbers of finite groups*, J. Algebra, **176** (10) (1995) 111-123.

[12] M. Hall, *The Theory of Groups*, Macmillan, New York, 1959.

[13] M. Herzog, *On finite simple groups of order divisible by three primes only*, J. Algebra, **120** (10) (1968) 383-388.

[14] W. Shi, *The simple groups of order $2^a3^b5^c7^d$ and Janko's simple groups*, J. Southwest Normal University, **4** (1987) 1-8. (Chinese)

[15] W. Shi, *On simple K_4-groups*, Chinese Sci. Bull, **36** (17) (1991) 1281-1283. (Chinese)

Alireza Khalili Asboei
Department of Mathematics,
Shariati Sari Mazandaran,
Iran
khaliliasbo@yahoo.com