

# FIR Filter Sharpening by Frequency Masking and Pipelining-Interleaving Technique

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**Abstract**—This paper focuses on the improvements of digital filters with a highly sharp transition zone on the Xilinx FPGA chips by combining a sharpening method based on the amplitude change function and frequency masking and PI (Pipelining-Interleaving) techniques.

A linear phase requires digital filter realizations with Finite Impulse Response (FIR) filters. On the other hand, a drawback of FIR filters applications is a low computational efficiency, especially in applications such as filter sharpening techniques, because this technique uses processing the data by repeated passes through the same filter.

Computational efficiency of FIR filters can be significantly improved by using some of the multirate techniques, and such a degree of computation savings cannot be achieved in multirate implementations of IIR (Infinite Impulse Response) filters. This paper shows the realization of a filter sharpening method with FIR filters combined with frequency masking and PI (Pipelining-Interleaving) technique in order to effectively realize the filter with improved characteristic. This realization at the same time keeps the good features of FIR filters such as the linear phase characteristic.

**Index Terms**—Digital filters, Field programmable gate arrays, FIR filters, Filtering theory, Programmable logic devices.

## I. INTRODUCTION

The first approach to improving stop-band attenuation with a filter sharpening method was to process the data by repeated passes through the same filter. Each pass, while increasing the minimum stop-band attenuation  $k$  times, also increases the pass-band ripple also  $k$  times in decibels. It also increases the order of the equivalent filter.

The following method for filter sharpening was a method, based on the idea of the amplitude change function [1]. With this method, a signal is also processed several times with the same filter, but the output signal is formed from the input signal and other filtering stages output signals in a specific order. This method was restricted to symmetric, finite impulse response (FIR) filters with a constant group delay.

However, the fact that FIR filters order is considerably higher than that of an equivalent IIR filter, and that FIR filters computational efficiency is considerably poorer, has resulted in searching for new methods, with IIR filters in the sharpening structure [2, 3]. These solutions are based on the addition of new blocks to the sharpening structure, in order to eliminate the influence of initial filter non-linear phase characteristics. The advantage of this solution is increased computational efficiency, which is achieved with IIR filters

as an initial filter in the sharpening structure. On the other hand, in order to improve computational efficiency, nonlinear phase characteristics of sharpening filter are obtained, and this fact is the main drawback of these methods.

In contrast to such a solution, in this paper, a sharpening filter is implemented with FIR filters. Computational efficiency is achieved with frequency masking and PI technique instead of IIR filter usage. This technique uses the fact that computational requirements for FIR filters can be reduced by the sampling rate conversion, and such a degree of computation savings cannot be achieved with multirate IIR filters implementations [4]. This solution keeps the good properties of FIR filters (especially a linear phase characteristic). At the same time, significant savings of hardware resources were achieved. By combining both techniques, frequency masking for a sharp transition band and filter sharpening for high stop-band attenuation, filter characteristics can be considerably improved.

This paper presents a design method that consists of a low-order FIR filter realized with a frequency masking technique, and included in the sharpening method. The overall structure is realized in PI technique. The advantage of an FPGA implementation of the proposed filter sharpening method is illustrated by several examples.

Section II describes basic principles of sharpening method based on the idea of the amplitude change function. In this section is suggested frequency masking technique as an initial filter, in sharpening method realization.

Section III gives an efficient solution for polynomial realization of amplitude change function, with modified PI technique. It also shows how to include suggested initial filter in overall structure.

This structure is realized with Xilinx FPGA chips in section IV. This section shows way of realization and simulation results with amplitude characteristic of overall filter.

All these properties of sharpening method with frequency masking and PI technique are used in section V for narrowband filter realization. Filter narrowing is performed by increasing frequency masking factor  $M$  while PI technique is used for efficient realization.

## II. FIR FILTER SHARPENING METHOD AND FREQUENCY MASKING TECHNIQUE

### II.a PROPERTIES OF FIR FILTER SHARPENING METHOD

The basic principle of sharpening method based on the idea of the amplitude change function is shown in Figure 1. Input data sequence is running through a filter giving both the output sequence and the residual sequence.

The residual is then added to the original signal and this sum is again run through the filter, giving the overall filtering operation

$$\begin{aligned} H(z)[1 + 1 - H(z)] &= H(z)[2 - H(z)] \\ &= H(z)[1 + H_r(z)], \end{aligned} \quad (1)$$

where

$$H_r(z) = 1 - H(z), \quad (2)$$

is the filter residual.

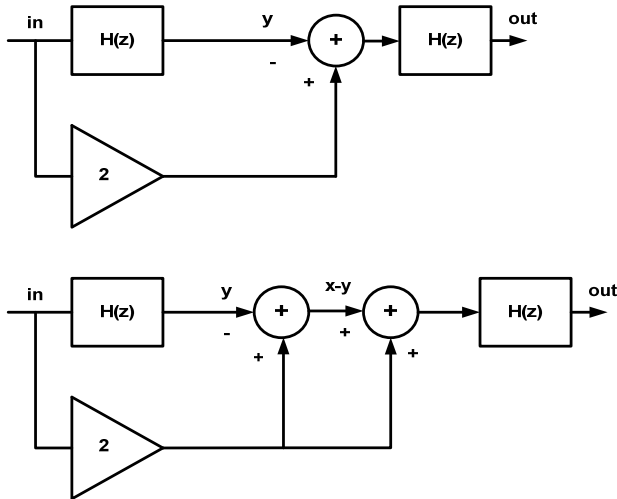


Figure 1. Principle of sharpening method based on the idea of the amplitude change function

This overall operation is called *twicing*. Figure 2 shows the second-order amplitude change function (solid line) and several other functions (dashed lines). With linear phase filters (filters with a constant group delay), by multiple filter using, it is possible to realize such amplitude change functions as to make improvements in stop-band, pass-band or in both.

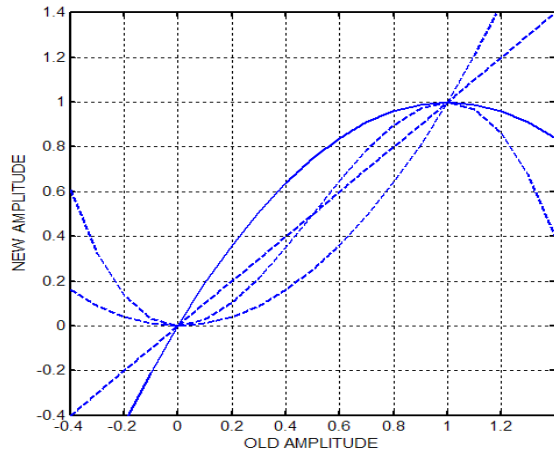


Figure 2. Amplitude change functions

### II.b MULTIRATE FIR FILTER SHARPENING METHOD REALIZATION

Using the fact that multirate techniques applied to FIR filters keep a linear phase characteristic, it seems logical to use a multirate FIR filter as an initial filter, in sharpening method realization. As the sharpening method uses a polynomial realization of filter transfer function, from different multirate methods, it is necessary to find an appropriate method, applicable in practice. The main problem with polynomial functions realization with PI technique, are critical loop limitations. These limitations are caused by feedback loops and sample rate conversions, when combining signals with different sample rates.

Frequency masking technique is a multirate technique which is appropriate for sharp linear phase filters realizations [5, 6, and 7]. This advantage derives from the frequency masking filter structures (Figure 3).

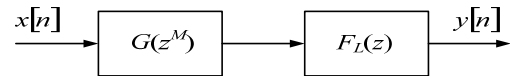


Figure 3. Cascade connections of periodic model filter and masking filter

Although the frequency masking technique is a multirate technique, actually there is no sampling rate change through the filter. Narrowing of the filter bandwidth is performed by replacing each model filter unit delay  $z^{-1}$  with the delay block  $z^{-M}$ , where  $M$  is an integer. In this way, the frequency response of the filter  $G(z^{j\omega})$  is made periodic. The FIR filter in the cascade  $F_L(z)$  is used to eliminate (mask) the images from the periodic model filter frequency response. Figure 4 shows the characteristics of model filter  $G(z)$ , periodic model filter  $G(z^M)$ , masking filter  $F_L(z)$  and overall narrow-band low-pass filter  $H_L(z)$ .

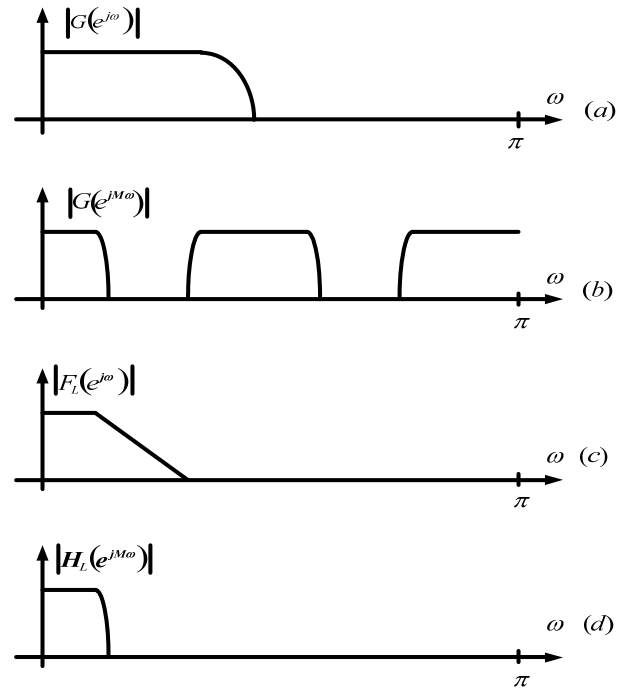


Figure 4. A cascade of two identical filters realized by using a single filter

The important outcome of the proposed approach is that the transition band of the overall filter  $B_{ov}$  is  $M$  times smaller

than the model filter transition band  $B_{\text{mod}}$ ,

$$B_{\text{ov}} = B_{\text{mod}} / M. \quad (3)$$

Consequently, the pass-band bandwidth is also reduced by the same factor, and filtering is performed with non-recursive filters, without sample rate conversions, and without feedback loops. In practice, this means realization without up-sampler and down-sampler blocks. Hence, this method is suitable for sharpening method design.

### III. POLYNOMIAL FUNCTIONS REALIZATIONS USING MODIFIED PIPELINING-INTERLEAVING (PI) TECHNIQUE

If two independent signal sequences  $\{x_1(n)\}$  and  $\{x_2(n)\}$  are required and if they have to be filtered by the same filter, an alternative to using two separate filters is a multirate implementation [8, 9], which is shown in Fig. 5.

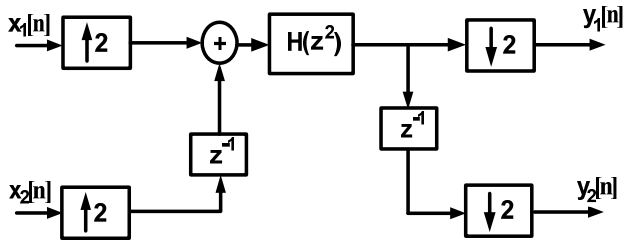


Figure 5. Pipelining-Interleaving (PI) techniques

This structure uses a single (pipelined/interleaved) filter for two identical filter implementations. Moreover, the clock rate for this implementation must be double the data rate.

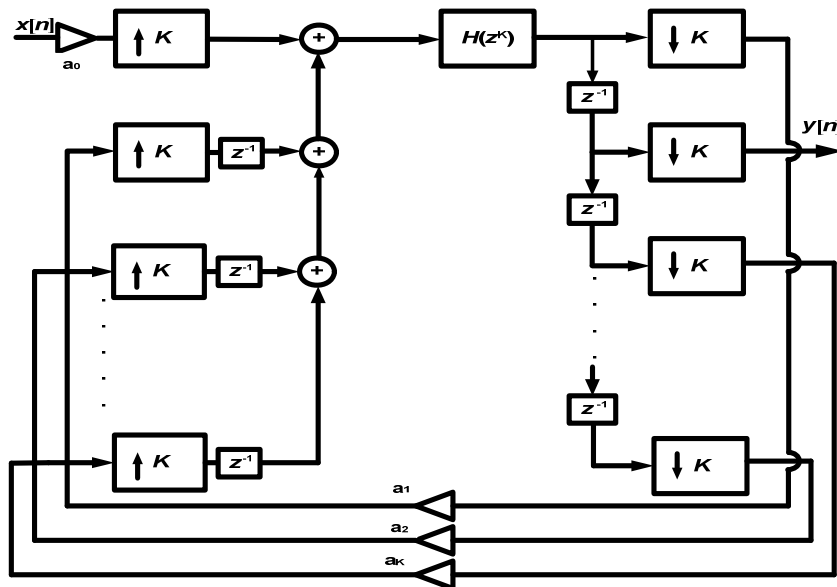


Figure 7. A cascade of two identical filters realized by using a single filter Realization of polynomial functions of  $H(z)$

The signal at the output  $y$  from Figure 7 would be:

$$Y(z) = X(z) \cdot H(z), \quad (4)$$

where

If only one input signal sequence is to be filtered, the first output sequence  $\{y_1(n)\}$  can be returned to the second input  $x_2(n)$ . In this way  $H(z^2)$  is used for implementing  $H^2(z)$  as shown in Fig. 6.

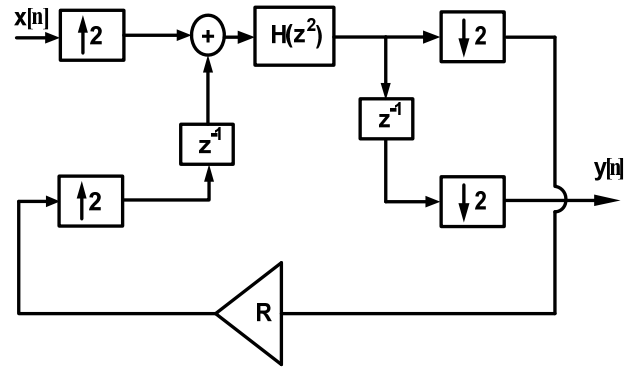


Figure 6. A cascade of two identical filters realized by using a single filter

This pipelined/interleaved implementation of a digital filter can easily be extended to  $H(z^K)$  ( $K$  is an arbitrary positive integer), and to the implementation of polynomial functions of  $H(z)$  [10]. Fig. 7 gives such an implementation. It should be noted that that today we got a different opportunities for sample rate conversion change [11, 12], even with rational factor of change.

$$H(z) = a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + \dots + a_N z^{-N}. \quad (5)$$

For such filter transfer function, a single filter can be used, but the clock rate must be  $N$  times the data rate. If the sampling rate is much lower than the clock frequency at which the system works, the use of parallel hardware

unnecessarily consumes resources available elements on the chip. The main reason for application of the PI technique is that signals are processed in parallel and thus avoid the realization when the signal processing is performed sequentially.

Amplitude change functions are actually polynomial transfer functions of initial filter. In our case, the amplitude

change function is realized with FIR filters and frequency masking technique, thus these filters are ideal candidates for the modified PI technique implementation. In addition, with a, for example, fifth order amplitude change function and PI implementation, by changing only polynomial coefficients (they can be reloadable), a bunch of fifth or smaller order functions can be realized.

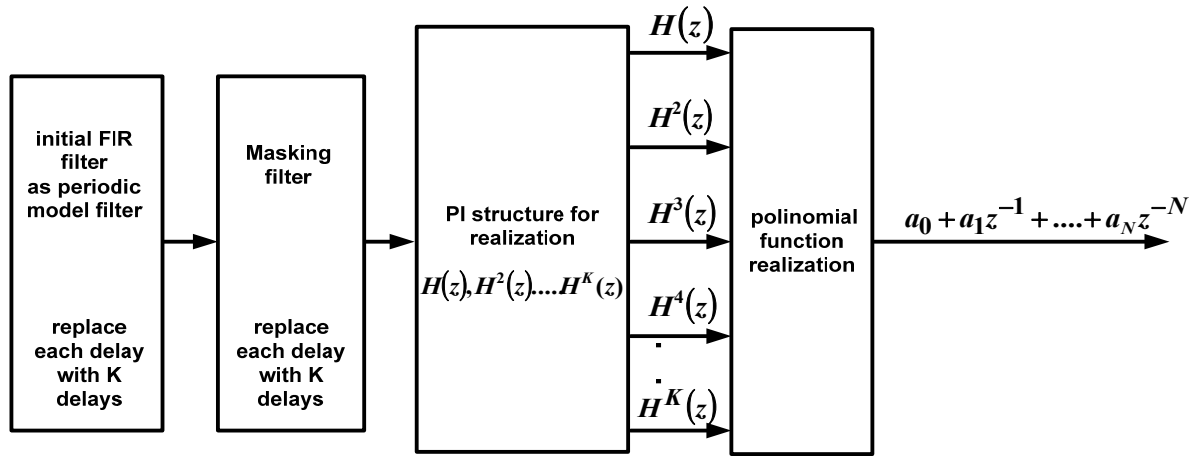


Figure 8. Block diagram of overall sharpening filter

#### IV. FPGA REALIZATION OF SHARPENING FILTER

This section presents the implementation structures of sharpening frequency masking FIR filters when they are implemented with the *Xilinx FPGA* chips.

Let's consider the realization of sharpening FIR narrowband low-pass filter with the *Xilinx FPGA Virtex5* series using PI realization and frequency masking technique, as shown in Figure 8. Filter is implemented using the *Xilinx's System Generator*. What is realized first is a block of frequency masking filter, whereby a model filter and masking filters are realized as FIR filters.

Then such a block, as a subsystem, is used for amplitude change function implementation. As this implementation means polynomial functions realizations, this structure is implemented using a modified PI technique. So, in the end, at the output of structure from Figure 8, we obtain polynomial functions realizations, i.e. our sharpening filter output.

##### IV.a LOW-PASS DIGITAL FILTER IMPLEMENTATION USING FREQUENCY MASKING TECHNIQUE

Software tools for designing structures that digital programmable hardware manufactures provide for users in recent years have become more complete and offer more and more freedom in design. The user has the option to choose whether to use one of the solutions from a wide range of forms for a specific digital structure, or design will start from beginning, using the standard programming languages [13, 14, 15], original manufacturer tool, or use some of the standard tools (including Matlab) and then again via software manufacturers make a compilation in VHDL, Verilog HDL, or the Verilog code [16, 17].

In our example, filter design starts from frequency masking filter realization, by using the *Xilinx's System Generator* tools *FDA tool* and *FIR compiler tool* [18]. Such

a FIR filter will be included in the further procedure (frequency masking and modified PI technique).

A model filter can be a lower order filter with poorer specifications:

$$\omega_{pass} = 0,1 \cdot \pi \text{ rad / sample}$$

$$\omega_{stop} = 0,4 \cdot \pi \text{ rad / sample}$$

$$A_{pass} = 0,5 \text{ dB}$$

$$A_{stop} = 40 \text{ dB}$$

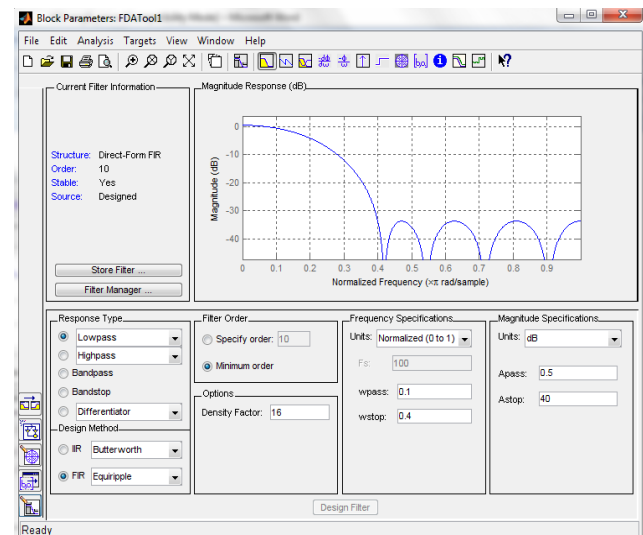


Figure 9. Model filter design using FDA tool

This model filter requires a masking filter in order to eliminate (mask) the images from the periodic model filter frequency response, with specifications:

$$\omega_{pass} = 0,1 \cdot \pi \text{ rad / sample}$$

$$\omega_{stop} = 0,7 \cdot \pi \quad \text{rad / sample}$$

$$A_{pass} = 0,5 \quad \text{dB}$$

$$A_{stop} = 40 \quad \text{dB}$$

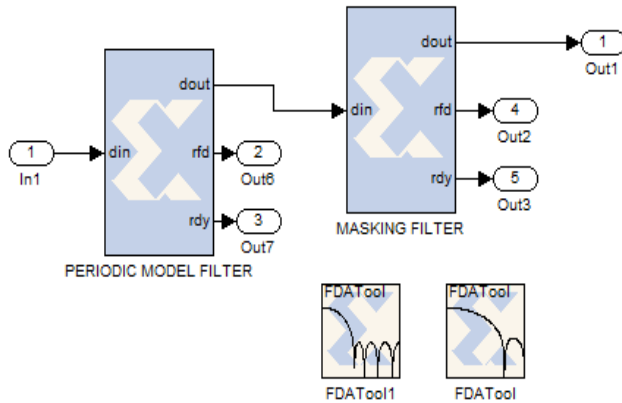


Figure 10. Frequency masking technique realizations

For such masking filter specifications, we will use a third order FIR equiripple filter and in this manner realize a frequency masking filter as illustrated in Figure 9.

Both filters are realized using *FIR compiler* and *FDA tool* (Figure 9 and Figure 10). A periodic model filter is obtained from the model filter by replacing each delay with two delays with choosing FIR compiler option “Filter type” to be “interpolated” and zero packing factor to be “2” (for  $M = 2$ ).

In this stage, with model filter definition and with factor  $M$  selection, overall filter pass-band and stop-band frequencies are determined. The following sharpening method is used only to improve filter attenuation in the stop-band and reduce ripple in the pass-band. So, with choosing  $M = 2$ , the characteristic of frequency masking filter, recorded in out 1 (Figure 10) will be:

$$\omega_{pass} = 0,05 \cdot \pi \quad \text{rad / sample}$$

$$\omega_{stop} = 0,2 \cdot \pi \quad \text{rad / sample}$$

$$A_{pass} = 0,5 \quad \text{dB}$$

$$A_{stop} = 40 \quad \text{dB}.$$

This characteristic is shown in Figure 11.

#### IV.b PI REALIZATION OF AMPLITUDE CHANGE FUNCTION

After frequency masking filter selection, our multirate filter should be included in amplitude change functions realization. As discussed, it will be an efficient PI realization, in order to decrease FIR filter complexity. Amplitude change function, in general, can be an arbitrary polynomial function  $H(z)$ ,

$$H(z) = a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + \dots + a_K z^{-K}. \quad (6)$$

With PI filter realizations, in a multirate filter each delay must be replaced with  $K$  delays. For a periodic model filter, this operation is already partially performed, so

instead of twice, delays replacements can be performed once, and in the following manner:

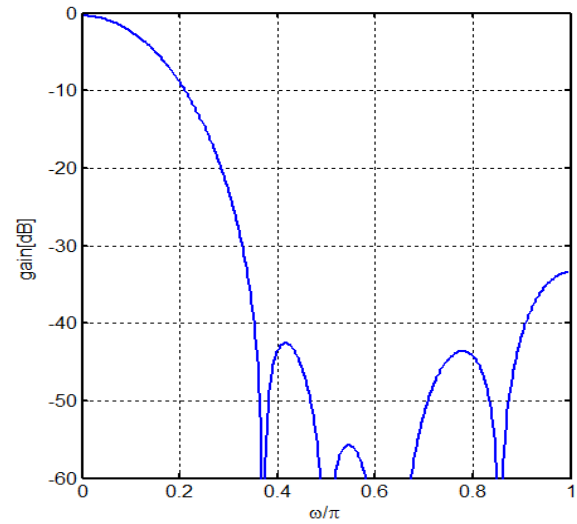


Figure 11. Amplitude characteristic of frequency masking filter

- Periodic model filter – replace each delay with  $M \cdot K$  delays,  $M$  is the frequency masking factor, and  $K$  is the  $H(z)$  order. This operation can be performed by using “FIR compiler” option “Filter type” to be “interpolated” and zero padding factor to be “ $M \cdot K$ ”,
- Masking filter - replace each delay with  $K$  delays. This operation can be performed by using “FIR compiler” option “Filter type” to be “interpolated” and zero padding factor to be “ $K$ ”.

The frequency masking filter defined in this manner can be included in the structure shown in Figure 12, which presents the FPGA implementation of the fourth order PI structure from Figure 6. On output ports (1-5), we will obtain  $1, H(z), H^2(z), H^3(z)$  and  $H^4(z)$  transfer function of input filter (filter between input port 2 and output port 6).

It should be noted that the additional delays in channels, as well as channels distribution, depend on the used multirate filter and selection of downsample/upsample method (the first value of frame is the last value of frame, with or without latency, etc.).

#### IV.c FPGA REALIZATION OF DIGITAL FILTER BY COMBINING SHARPENING, FREQUENCY MASKING AND PI TECHNIQUE

Now, after initial implementation of the FIR filter (with frequency masking technique), amplitude change function can be used in order to complete sharpening method realization. So, between input port 2 and output port 6 of the structure from Figure 12, we should connect our FIR digital filter realized with frequency masking technique. Signals on the output ports should be processed as a polynomial according to the amplitude change function.

Figure 13 shows a realization with the simplest amplitude change function  $F(H) = 3H^2 - 2H^3$ , and Table 1 gives an overview of a few amplitude change functions [1].

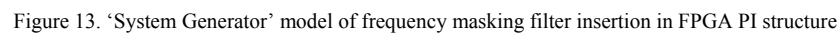
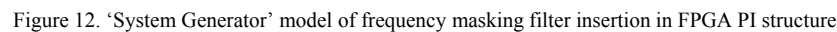


Figure 14 shows the amplitude characteristic of filter sharpening structure with amplitude change function  $F(H) = 3H^2 - 2H^3$ , and factor  $M = 2$  of frequency masking subsystem filter. Improvements in relation to initial filter characteristics in stop-band are significant (the new minimum stopband attenuation is about 30 dB better than initial). Figure 15 shows the pass-band of same filter, and we can see also improvements in the pass-band (pass-band ripple is reduced by 1 dB). At the same

Except for filter complexity, with a modified PI technique (Figure 12), the signal is processed several times consecutively across the same filter, and the influence of fixed point implementation is intensified more than in the



time, the sharpening filter retains the linear phase characteristic (Figure 16).

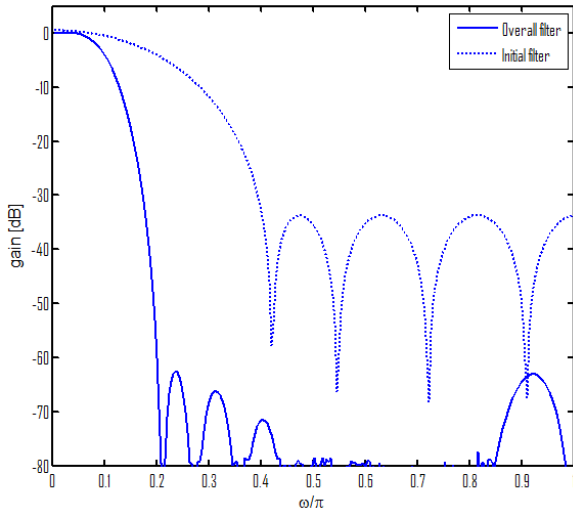


Figure 14. Amplitude characteristic of overall filter

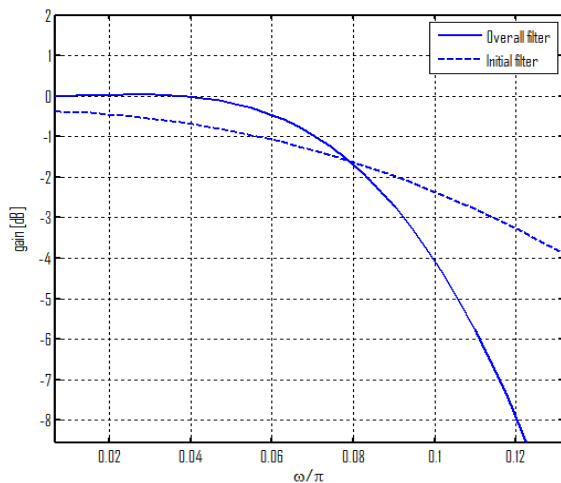


Figure 15. Amplitude characteristic of overall filter – Pass-band

## V. REALIZATION OF NARROWBAND DIGITAL FILTERS USING MODIFIED PI TECHNIQUE

FIR filters realized with a frequency masking and PI technique [19] can be used for sharpening method implementation. However, besides improving the filter performance in terms of pass-band and stop-band attenuation, with additional usage of frequency masking technique the filter performance can also be improved in terms of narrowband realization [20].

Filter pass-band and transition zone, with factor  $M$  increasing (Figure 1), can be significantly narrowed. On the other hand, with this narrowing, some new effects will occur.

The masking filter, whose role is to eliminate images from a periodic filter spectrum, now must be  $K$  times ( $K$  is the number of  $M_u$  factor reductions) narrower than the original masking filter. This causes increasing of masking filter order and overall filter complexity. At the end, with fixed point implementation, errors in the stop-band will be increased. This effect is illustrated in Figure 17, which shows the amplitude characteristic of our filter from Figure 11, for increased factors  $M = 4$  and  $M = 8$ ). Stop-band errors are increased with a growth of factor  $M$  and complexity of the filter.

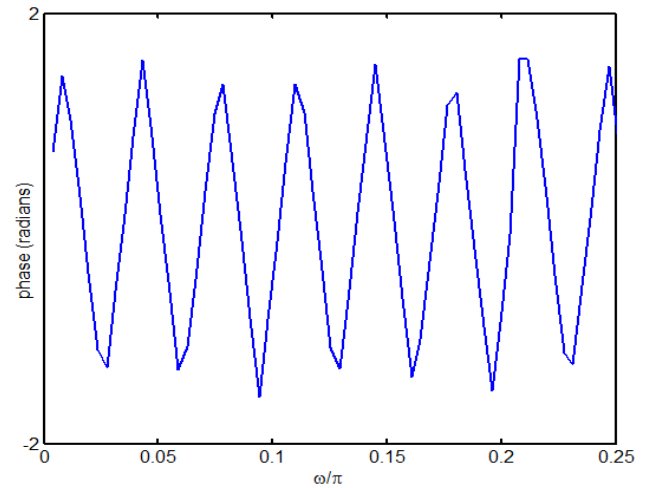


Figure 16. Phase characteristic of overall filter

In the case of very narrow-band bandpass filtering, as with other multirate realizations, in cases where the decimation/interpolation factor  $M$  is too large, say  $M \geq 10$ , it is preferable to use  $r$ -stages ( $r \geq 2$ ) multirate filters [3], according to the effect which occurs with a considerable increase of factor  $M$ . This effect is illustrated in Figure 18, which shows the pass-band of our filter from Figure 11, for increased factors  $M = 4$  and  $M = 8$ ). Unfortunately, with the sharpening method, a multistage approach cannot be used, because multistage filtering with two and more stages increases filter complexity. Since the sharpening method involves multiple uses of the same filter, (in the given example, third order amplitude change function is used), any complexity increase of the basic filter will significantly affect the overall filter characteristic.

So, for frequency masking FIR realization (and other multirate realizations) of initial filter with the sharpening method, for narrowband digital filters realization, we can use only low order factor  $M$  filters and multirate filters realized in one stage.

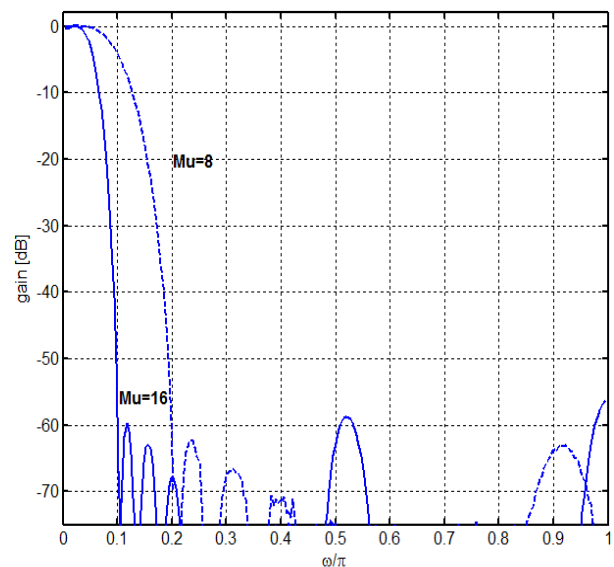
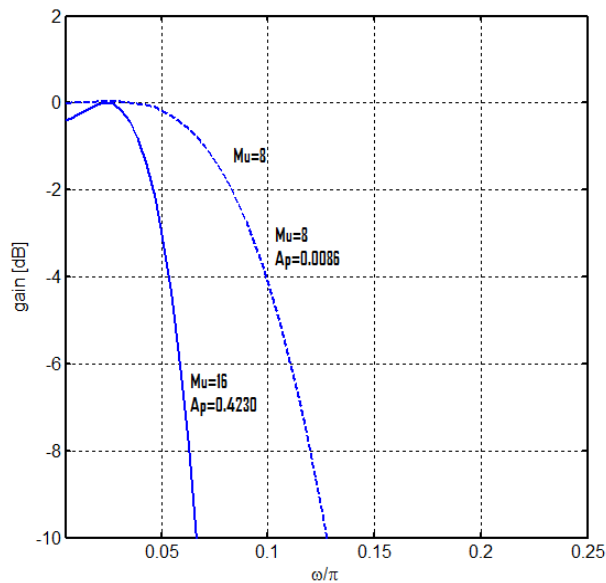


Figure 17. Stop-band errors with increased factor  $M$

Acceptable values of the  $M$  factor depend on initial filter specifications and amplitude change function order. The realized filter must be verified for each specific case. In our example, realization with  $M = 8$  is acceptable.

Figure 18. Pass-band attenuation Stop-band errors (increased factor  $M$ )

## VI. CONCLUSION

Low computational efficiency is the main drawback of FIR filter applications that involve cascade multiple filtering with the same filter. One such method is filter sharpening method that involves polynomial functions implementations.

The application of multirate multistage techniques combined with pipelining/interleaving technique provides a solution for the rationalization of hardware resources for applications that involve multiple use of the same filter several times or a hardware structure that can be rearranged so that the signal processing is performed in parallel. Combining the operations related to the implementation of PI procedure together with the operation related to the filter realization, it is possible to achieve additional improvements of filters structure. In this paper, for filter sharpening method, we introduce an original approach to algorithm development and digital filter design using pipelining/interleaving and frequency masking technique.

By combining these techniques with filter sharpening method, computational efficiency of overall sharpening FIR filter can be considerably improved, and, at the same time, the good features of FIR filters remain preserved, such as the linear phase characteristic.

With software capabilities offered by chip manufacturer, using the proposed technique, a general solution can be developed ("IP Core"), and used as a tool for narrow-band digital filter design.

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