3D OMP algorithm for 3D parameters estimation in bistatic MIMO radar

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Abstract: A three-dimensional (3D) signal sparse representation (SR) model that can be used for joint Doppler frequency, direction of departure and direction of arrival estimation in bistatic multiple-input–multiple-output radar is built. Then, a fast 3D orthogonal matching pursuit (OMP) algorithm is put forward to estimate the 3D parameters. First, the problem of targets 3D parameters joint estimation is decomposed into three estimation problems whose computation burden is decreased. Then, the high-resolution 3D parameters of all targets are got by solving a very small-scale SR problem. Numerical simulations verify that the proposed algorithm can get similar estimation accuracy and decrease the computational burden compared with the traditional 1D OMP algorithm.

1 Introduction

Multiple-input–multiple-output (MIMO) radar has received significant attention since its first appearance a decade ago and many literatures devote to researching the performance advantages of MIMO radar in significantly improved parameter identifiability [1]. More recently, sparse representation (SR)-based MIMO radar (SR-MIMO) becomes a hot field because of its advantage in fully exploiting the inherent sparseness of the scenario [2]. Nonetheless, most of the SR-based estimation approaches always rearrange the three-dimensional (3D) MIMO radar echo into a 1D signal. And it will lead to greatly increasing of the storage and computing complexity as the size of dictionary grows. This problem seriously restricts its application [3]. Besides, despite the above estimation problem could be solved by Bayesian method, orthogonal matching pursuit (OMP) algorithm basis pursuit algorithm etc., OMP is more suitable for application for its satisfactory estimation accuracy and low computational burden [4].

In this paper, a 3D signal SR model that is suitable for joint Doppler frequency, direction of departure (DOD) and direction of arrival (DOA) estimation in bistatic MIMO radar is built. Later, a novel 3D OMP algorithm which exploits the separable property of this estimation problem is proposed to solve the 3D parameters estimation problem.

2 Problem formulation

Assuming there is a bistatic MIMO radar with K transmitters and L receivers, where the transmit and receive arrays are collocated linear array. The interval between array elements is half of wavelength. Coherent processing interval contains N pulses. Targets residing in a certain range cell are characterised by Doppler frequency f_d, DOD φ and DOA θ. f_s, d_1 and d_2 are the pulse repetition frequency, the distance between transmitters and distance between receivers. Moreover, $f^{(D)} = f_d/f_s$, $f^{(T)} = d_1 \cos(\varphi)/\lambda$, $f^{(R)} = d_2 \cos(\theta)/\lambda$ are the normalised Doppler frequency, normalised DOD and normalised DOA. In the rest of this paper, we will directly refer to them as Doppler, DOD and DOA for simplicity.

Assume the transmitting waveforms are normalised orthogonal signals, i.e. $\boldsymbol{S}\boldsymbol{S}^{\mathrm{H}} = \boldsymbol{I}_K$, where $\boldsymbol{S} = [s_1, \quad s_2, \ \ldots, \ s_K]^{\mathrm{T}}$. Moreover, the received signal of the receive array after matched filtering can be expressed as

$$\begin{aligned}\boldsymbol{Y}_n &= \left[\sum_{m=1}^{M} a_m \mathrm{e}^{\mathrm{j}2\pi f_m^{(D)} n} \boldsymbol{a}_{\mathrm{R}}(f_m^{(R)}) \boldsymbol{a}_{\mathrm{T}}(f_m^{(\mathrm{T})})^{\mathrm{T}} \boldsymbol{S} + \boldsymbol{W}_n\right] \boldsymbol{S}^{\mathrm{H}} \\ &= \sum_{m=1}^{M} a_m \mathrm{e}^{\mathrm{j}2\pi f_m^{(D)} n} \boldsymbol{a}_{\mathrm{R}}(f_m^{(R)}) \boldsymbol{a}_{\mathrm{T}}(f_m^{(\mathrm{T})})^{\mathrm{T}} + \boldsymbol{Z}_n, \quad n = 1, 2, \ldots, N \end{aligned} \quad (1)$$

where a_m denotes the scattering coefficient of the mth target. $\boldsymbol{W}_n$ refers to the noise matrix. $\boldsymbol{Z}_n = \boldsymbol{W}_n \boldsymbol{S}^{\mathrm{H}}$. The term n is the launch time of the nth pulse. $\boldsymbol{a}_{\mathrm{R}}(f_m^{(\mathrm{R})}) = [1, \ \ldots, \ \exp(\mathrm{j}(L-1)2\pi f_m^{(\mathrm{R})})]$ and $\boldsymbol{a}_{\mathrm{T}}(f_m^{(\mathrm{T})}) = [1, \ \ldots, \ \exp(\mathrm{j}(K-1)2\pi f_m^{(\mathrm{T})})]$ are receive steering vectors and transmit steering vectors corresponding to the DOA and DOD *of* the mth target, respectively. Similarly, another vector $\boldsymbol{a}_{\mathrm{D}}(f_m^{(\mathrm{D})})$ is denoted by $\boldsymbol{a}_{\mathrm{D}}(f_m^{(\mathrm{D})}) = [1, \ \ldots, \ \exp(\mathrm{j}(N-1)2\pi f_m^{(\mathrm{D})})]$.

Applying the vectorisation operation to (1), the 3D received echo can be represented by the 1D signal model

$$\boldsymbol{y} = \sum_{m=1}^{M} a_m \boldsymbol{A}_m + \boldsymbol{z} \in \boldsymbol{C}^{NKL \times 1} \quad (2)$$

where $\boldsymbol{A}_m = \boldsymbol{a}_{\mathrm{D}}(f_m^{(\mathrm{D})}) \otimes \boldsymbol{a}_{\mathrm{T}}(f_m^{(\mathrm{T})}) \otimes \boldsymbol{a}_{\mathrm{R}}(f_m^{(\mathrm{R})})$ represents the steering vector of the mth target. Moreover, $y = [\mathrm{vec}(Y_1)^{\mathrm{T}}, \ \ldots, \ \mathrm{vec}(Y_N)^{\mathrm{T}}]^{\mathrm{T}} \in \boldsymbol{C}^{\mathrm{NKL}}$ and $z = [\mathrm{vec}(Z_1)^{\mathrm{T}}, \ \ldots, \ \mathrm{vec}(Z_N)^{\mathrm{T}}]^{\mathrm{T}} \in \boldsymbol{C}^{\mathrm{NKL}}$.

Then, discretising the range of Doppler, DOD and DOA to $K_{\mathrm{d}} > K$, $N_{\mathrm{s}} > N$ and $L_{\mathrm{a}} > L$ resolution grids, respectively, we can get the 3D parameters by solving the following 1D SR problem

$$\boldsymbol{y} = \boldsymbol{\Phi}\boldsymbol{\alpha} \quad (3)$$

where $\boldsymbol{\alpha} \in \boldsymbol{C}^{K_{\mathrm{d}}N_{\mathrm{s}}L_{\mathrm{a}} \times 1}$ is the sparse vector and the non-zero elements' position represent the estimation result. $\boldsymbol{\Phi} = [A_1, \quad A_2, \ \ldots, \ A_{K_{\mathrm{d}}N_{\mathrm{s}}L_{\mathrm{a}}}]$ is the Doppler–DOD–DOA dictionary and $\boldsymbol{A}_i$ represents its ith column.

The question is that solving (3) will bring about huge calculation burden because both the length of atoms of the dictionary KNL and atom's number $K_{\mathrm{d}}N_{\mathrm{s}}L_{\mathrm{a}}$ are often very huge.

Note that each column of $\boldsymbol{\Phi}$ can be decomposed into the Kronecker product of the steering vector of Doppler, DOD and DOA, that is, the dictionary $\boldsymbol{\Phi}$ are separable. So, the high-dimensional dictionary $\boldsymbol{\Phi}$ can be decomposed into three subdictionaries, i.e. the Doppler dictionary $\boldsymbol{\Phi}_{\mathrm{D}} = [\boldsymbol{a}_{\mathrm{D}}(f_1^{(\mathrm{D})}), \ \ldots, \ \boldsymbol{a}_{\mathrm{D}}(f_{K_{\mathrm{d}}}^{(\mathrm{D})})]$, the DOD dictionary $\boldsymbol{\Phi}_{\mathrm{T}} = [\boldsymbol{a}_{\mathrm{T}}(f_1^{(\mathrm{T})}), \ \ldots, \ \boldsymbol{a}_{\mathrm{T}}(f_{L_{\mathrm{a}}}^{(\mathrm{T})})]$ and the DOA dictionary $\boldsymbol{\Phi}_{\mathrm{R}} = [\boldsymbol{a}_{\mathrm{R}}(f_1^{(\mathrm{R})}), \ \ldots, \ \boldsymbol{a}_{\mathrm{R}}(f_{N_{\mathrm{S}}}^{(\mathrm{R})})]$. Moreover, $\boldsymbol{\alpha} \in \boldsymbol{C}^{K_{\mathrm{d}}N_{\mathrm{s}}L_{\mathrm{a}} \times 1}$ is a sparse steering vector which can be rearranged into a 3D sparse matrix, i.e. $\aleph = \mathrm{Ivec}(\boldsymbol{\alpha})_{K_{\mathrm{d}} \times N_{\mathrm{s}} \times L_{\mathrm{a}}} \in \boldsymbol{C}^{K_{\mathrm{d}} \times N_{\mathrm{s}} \times L_{\mathrm{a}}}$, where $\mathrm{Ivec}(\bullet)_{K_{\mathrm{d}} \times N_{\mathrm{s}} \times L_{\mathrm{a}}}$ represents rearranging the data into a $K_{\mathrm{d}} \times N_{\mathrm{s}} \times L_{\mathrm{a}}$-dimensional matrix.

So, (3) is possible solved efficiently by solving several equivalent problems which have less computation burden.

Input:
MIMO Radar received echo $\mathbf{y}$, targets' number M and discretising number of grids K_d, N_s, L_a.

Output:
Estimated result $f_m^{(D)}$, $f_m^{(T)}$ and $f_m^{(R)}$ (m=1,2,⋯,M) of all target.

Procedure:
- step1: Doppler estimation by solving Equation (5)
- step2: DOD estimation by solving Equation (6)
- step3: DOA estimation by solving Equation (7)
- step4: Joint 3D parameters estimation of all targets by performing 1D OMP to solve $\mathbf{y} = \mathbf{\Phi}_{new}\boldsymbol{\alpha}_{new}$

Fig. 1 *3D OMP algorithm*

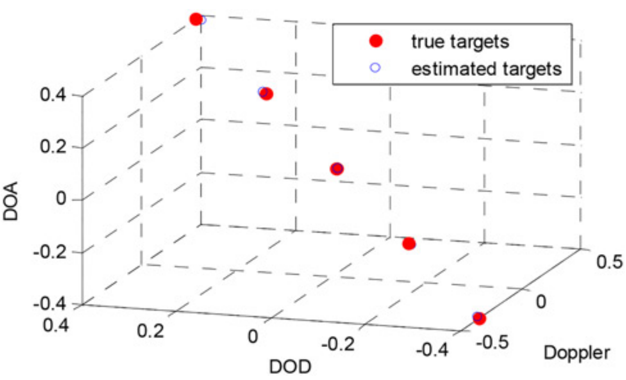

Fig. 2 *3D parameters estimation result of our method*

3 3D OMP algorithm

According to the property that the projection of $\aleph$ on its each dimension is sparse, we propose the 3D OMP algorithm to solve (3). Utilising the property of Kronecker product, i.e. $\mathrm{vec}(\mathbf{\Phi}_1\mathbf{\Lambda}\mathbf{\Phi}_2^{\mathrm{T}}) = ((\mathbf{\Phi}_2^{\mathrm{T}})^{\mathrm{T}} \otimes \mathbf{\Phi}_1)\mathrm{vec}(\mathbf{\Lambda})$, another form of (3) can be got

$$\boldsymbol{Y} = \mathbf{\Phi}_{\mathrm{R}}\mathbf{\Lambda}_1(\mathbf{\Phi}_{\mathrm{D}} \otimes \mathbf{\Phi}_{\mathrm{T}})^{\mathrm{T}} \tag{4}$$

where $\boldsymbol{Y} = \mathrm{Ivec}(y)_{L\times KN}$ and $\mathbf{\Lambda}_1 = \mathrm{vec}(\aleph)_{L_a\times K_d N_s}$.

Making $N_s = N$ and $L_a = L$, $\mathbf{\Phi}_{\mathrm{T}} = [\boldsymbol{a}_{\mathrm{T}}(f_1^{(\mathrm{T})}), \ldots, \boldsymbol{a}_{\mathrm{T}}(f_{L_a}^{(\mathrm{T})})]$ and $\mathbf{\Phi}_{\mathrm{R}} = [\boldsymbol{a}_{\mathrm{R}}(f_1^{(\mathrm{R})}), \ldots, \boldsymbol{a}_{\mathrm{R}}(f_{N_s}^{(\mathrm{R})})]$ will become invertible Vandermonde matrices. So, another form of (4), i.e. $\boldsymbol{Y}^{\mathrm{T}}[(\mathbf{\Phi}_{\mathrm{R}})^{-1}]^{\mathrm{T}} = (\mathbf{\Phi}_{\mathrm{D}} \otimes \mathbf{\Phi}_{\mathrm{T}})\mathbf{\Lambda}_1^{\mathrm{T}}$, can be got through multiplying $\mathbf{\Phi}_{\mathrm{R}}^{-1}$ and matrix transposing, where $(\mathbf{\Phi}_{\mathrm{R}})^{-1}$ could be calculated easily [5] by the property of Vandermonde matrix. Moreover, $\boldsymbol{\Lambda}_1^{\mathrm{T}}$ is still a sparse matrix because its non-zero elements exist in few rows. So, by multiplying the matrix $\boldsymbol{E}_{L_a\times 1}$ whose elements are all 1, we get $\boldsymbol{Y}^{\mathrm{T}}[(\mathbf{\Phi}_{\mathrm{R}})^{-1}]^{\mathrm{T}}\boldsymbol{E}_{L_a\times 1} = (\mathbf{\Phi}_{\mathrm{D}} \otimes \mathbf{\Phi}_{\mathrm{T}})\mathbf{\Lambda}_1^{\mathrm{T}}\boldsymbol{E}_{L_a\times 1}$, where $\mathbf{\Lambda}_1^{\mathrm{T}}\boldsymbol{E}_{L_a\times 1}$ is a $K_d N_s \times 1$-dimensional sparse steering vector. Utilising the property of Kronecker product again, $\mathrm{Ivec}(\boldsymbol{Y}^{\mathrm{T}}[(\mathbf{\Phi}_{\mathrm{R}})^{-1}]^{\mathrm{T}}\boldsymbol{E}_{L_a\times 1})_{K\times N} = \mathbf{\Phi}_{\mathrm{T}}\boldsymbol{\Lambda}_2(\mathbf{\Phi}_{\mathrm{D}})^{\mathrm{T}}$ is obtained, where $\mathbf{\Lambda}_2 = \mathrm{Ivec}(\boldsymbol{\Lambda}_1^{\mathrm{T}}\boldsymbol{E}_{L_a\times 1})_{N_s\times K_d}$. Moreover, by multiplying $\mathbf{\Phi}_{\mathrm{T}}^{-1}$ and matrix transposing, we can get $(\mathrm{Ivec}(\boldsymbol{Y}^{\mathrm{T}}[(\mathbf{\Phi}_{\mathrm{R}})^{-1}]^{\mathrm{T}}\boldsymbol{E}_{L_a\times 1})_{K\times N})^{\mathrm{T}}[(\mathbf{\Phi}_{\mathrm{T}})^{-1}]^{\mathrm{T}} = \mathbf{\Phi}_{\mathrm{D}}(\mathbf{\Lambda}_2)^{\mathrm{T}}$. After that, by multiplying the matrix $\boldsymbol{E}_{N_s\times 1}$ whose elements are all 1, we obtain

$$(\mathrm{Ivec}(\boldsymbol{Y}^{\mathrm{T}}[(\mathbf{\Phi}_{\mathrm{R}})^{-1}]^{\mathrm{T}}\boldsymbol{E}_{L_a\times 1})_{K\times N})^{\mathrm{T}}[(\mathbf{\Phi}_{\mathrm{T}})^{-1}]^{\mathrm{T}}\boldsymbol{E}_{N_s\times 1} = \mathbf{\Phi}_{\mathrm{D}}\boldsymbol{\lambda} \tag{5}$$

where $\boldsymbol{\lambda} = (\mathbf{\Lambda}_2)^{\mathrm{T}}\boldsymbol{E}_{N_s\times 1}$ and it is a $K_d \times 1$ sparse steering vector. It should be pointed out that the non-zero rows of $\boldsymbol{\lambda}$ represent the Doppler parameter of targets, thus we will get the high-resolution estimation result of Doppler by solving (5) based on the traditional 1D OMP algorithm and the computation burden is decreased for the problem's size is becoming small.

Similarly, letting $K_d = K$, $L_a = L$ or $K_d = K$, $N_s = N$, we can get the two equations as follows:

$$\mathrm{Ivec}[\mathrm{Ivec}(y)_{NL\times K}(\mathbf{\Phi}_{\mathrm{D}}^{-1})^{\mathrm{T}}\boldsymbol{E}_{K_d\times 1}]_{N\times L}(\mathbf{\Phi}_{\mathrm{R}}^{-1})^{\mathrm{T}}\boldsymbol{E}_{L\times 1} = \mathbf{\Phi}_{\mathrm{T}}\gamma \tag{6}$$

$$\mathrm{Ivec}[\mathrm{Ivec}(y)_{NL\times K}(\mathbf{\Phi}_{\mathrm{D}}^{-1})^{\mathrm{T}}\boldsymbol{E}_{K_d\times 1}]_{L\times N}(\mathbf{\Phi}_{\mathrm{T}}^{-1})^{\mathrm{T}}\boldsymbol{E}_{N\times 1} = \mathbf{\Phi}_{\mathrm{R}}\xi \tag{7}$$

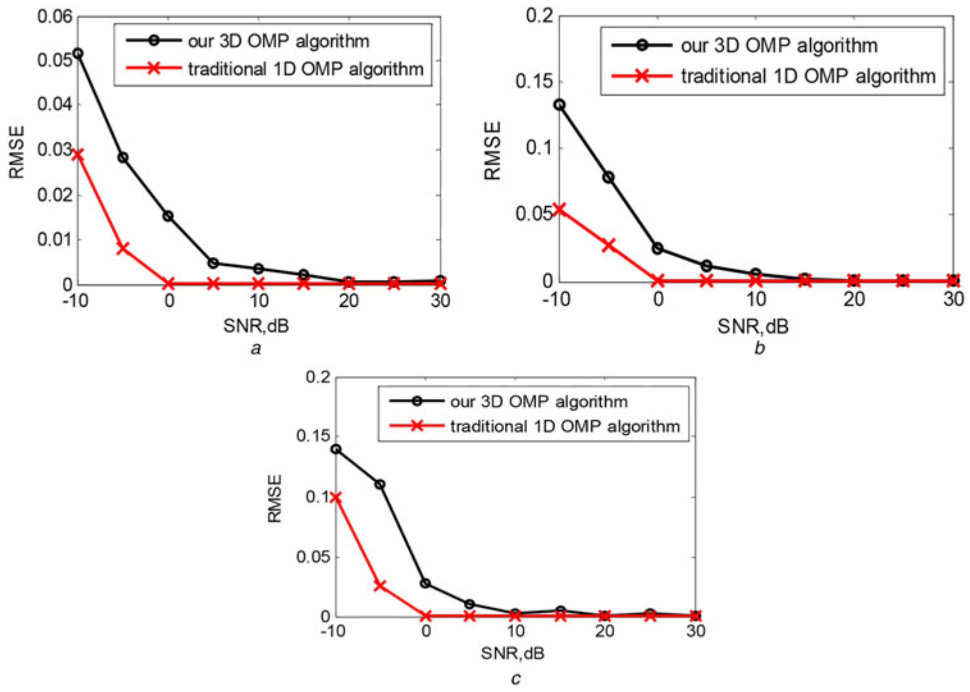

Fig. 3 *Average RMSE of estimation result for all targets*
a Doppler
b DOD
c DOA

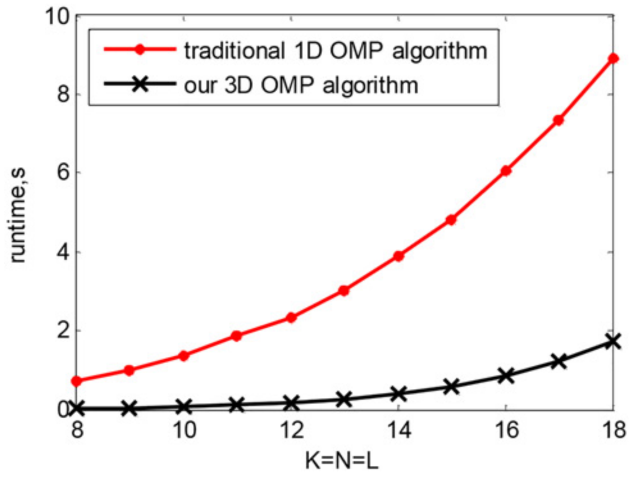

Fig. 4 *Runtime of 1D OMP and 3D OMP algorithms*

Moreover, the high-resolution DOD γ and DOA ξ also can be obtained through solving (6) and (7) by traditional 1D OMP algorithm.

Then, in order to estimate the 3D parameters of each target, a very small union dictionary denoted by $\boldsymbol{\Phi}_{\text{new}} = \boldsymbol{\Phi}_{\text{D}}^{M} \otimes \boldsymbol{\Phi}_{\text{T}}^{M} \otimes \boldsymbol{\Phi}_{\text{R}}^{M}$ is constructed, where the columns of $\boldsymbol{\Phi}_{\text{D}}^{M} \in \boldsymbol{C}^{K \times M}$, $\boldsymbol{\Phi}_{\text{T}}^{M} \in \boldsymbol{C}^{N \times M}$ and $\boldsymbol{\Phi}_{\text{R}}^{M} \in \boldsymbol{C}^{L \times M}$ are chosen from $\boldsymbol{\Phi}_{\text{D}}$, $\boldsymbol{\Phi}_{\text{T}}$ and $\boldsymbol{\Phi}_{\text{R}}$ according to the estimated result of Doppler, DOD and DOA, respectively. Then, solving the equation $\boldsymbol{y} = \boldsymbol{\Phi}_{\text{new}} \boldsymbol{\alpha}_{\text{new}}$ by 1D OMP, we can get the 3D parameters of all targets. In summary, the proposed 3D OMP algorithm consists of four main steps attained which is shown in Fig. 1.

4 Computational cost comparison

The most time-consuming procedure in 1D OMP is to find the most relevant atoms from the dictionary, for M targets, all of the computational amount is $O(\text{MKNLK}_{\text{d}} N_{\text{s}} L_{\text{a}})$ [4]. The 3D OMP algorithm could reduce the computational complexity to $O(\text{MKK}_{\text{d}}) + O(\text{MNN}_{\text{s}}) + O(\text{MLL}_{\text{a}}) + O(M^{4}\text{KNL})$ but get the same accuracy of original 3D parameters estimation method, which indicates that the proposed 3D OMP algorithm is more efficient.

5 Simulation results

Assuming that $M = 5$, i.e. the number of uncorrelated targets are five. The mth target's Doppler, DOD and DOA are $f_{m}^{(D)} = 0.2m - 0.55$, $f_{m}^{(T)} = 0.2m - 0.6$ and $f_{m}^{(\text{R})} = 0.2m - 0.6$, respectively. We provide three kinds of simulations. The discretising number of grids is 30, i.e. $K_{\text{d}} = N_{\text{s}} = L_{\text{a}} = 30$. The noise in all the simulation is additive white Gaussian noise.

For the first simulation, $\text{SNR} = 20\,\text{dB}$ and $K = N = L = 10$. The 3D parameters estimation performance of the proposed algorithm is shown in Fig. 2, which indicates that our method can get satisfactory estimation performance. For the second simulation, $K = N = L = 10$, we contrast the variation of average root-mean-square error (RMSE) of estimated result with different SNR for traditional 1D OMP and the proposed algorithms, and the result is presented in Fig. 3. About 500 trails Monte Carlo simulations are performed for each given SNR. Fig. 3 indicates that, when SNR < 5 dB, the traditional algorithm is better than the proposed algorithm. Nevertheless, when SNR > 10 dB, the accuracy of the proposed algorithm is comparable with traditional algorithm and the difference is small. In the third simulation, we contrast the computational performance of the two algorithms through the runtime of each algorithm. Fig. 4 shows the runtime of the two algorithms versus different $K = N = L$. We can observe that the 3D OMP is faster than the 1D OMP and the proposed method is more computationally efficient.

6 Conclusion

A 3D signal SR model that can be used for joint Doppler frequency, DOD and DOA estimation in bistatic MIMO radar is built. Then, a fast 3D OMP algorithm is put forward to estimate the 3D parameters. Numerical simulations verify that the proposed algorithm can get similar estimation accuracy and decrease the computational burden compared with the traditional 1D OMP algorithm.

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