

# Estimating link travel time functions for heterogeneous traffic flows on freeways

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## SUMMARY

Oversized vehicles, such as trucks, significantly contribute to traffic delays on freeways. Heterogeneous traffic populations, that is, those consisting of multiple vehicle types, can exhibit more complicated travel behaviors in the operating speed and performance, depending on the traffic volume as well as the proportions of vehicle types. In order to estimate the component travel time functions for heterogeneous traffic flows on a freeway, this study develops a microscopic traffic-simulation based four-step method. A piecewise continuous function is proposed for each vehicle type and its parameters are estimated using the traffic data generated by a microscopic traffic simulation model. The illustrated experiments based on VISSIM model indicate that (i) in addition to traffic volume, traffic composition has significant influence on the travel time of vehicles and (ii) the respective estimations for travel time of heterogeneous flows could greatly improve their estimation accuracy. Copyright © 2016 John Wiley & Sons, Ltd.

**KEY WORDS:** travel time function; microscopic traffic simulation; heterogeneous traffic flow; BPR functions; cars and trucks; freeways

## 1. INTRODUCTION

Freeways are an important piece of the traffic network of a large nation. Vehicles using the freeways exhibit a wide variety of operational and driver characteristics, and thus constitute a heterogeneous user population. These vehicles can be classified according to their physical dimensions, weights, intended uses, and their dynamic characteristics into classes such as passenger cars, light trucks, heavy trucks, buses, and so on. Compared with urban traffic, freeway traffic is much more heterogeneous and contains a large percentage of trucks. The travel speeds, operational characteristics, sizes, and headways of trucks on freeways are quite different from that for cars. Mixing cars and trucks on the freeway results in larger delays because heterogeneous vehicle types share the same road space. The fast-moving cars may suffer from the field of vision interference and increased lane changing. Trucks also slow down the traffic stream because of their limited acceleration and deceleration capabilities. Hence, the average travel time functions of trucks and cars are not identical, and furthermore depend on the traffic volume as well as composition. An independent estimated link travel time function for different vehicle type is essential for traffic engineering and transportation planning, such as the build–operate–transfer (BOT) highway projects [1] and traffic network analysis [2].

One of the most commonly used link travel time estimators, the Bureau of Public Roads (BPR) function, is formulated as a polynomial function with respect to the ratio of traffic volume to capacity space [3]. However, the standard BPR function does not account for heterogeneity in traffic flows. Likewise, the technique of converting all vehicle types into a single class using a passenger car unit (PCU) factor does not account for the operational differences between these types. Thus, there is a

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need for travel time functions that considers both volume and the proportions of trucks in the traffic stream.

### 1.1. Relevant studies

With respect to travel time functions for heterogeneous traffic flows, there are three main approaches in the literatures. The first approach collects real traffic data to calibrate an empirical travel time function for a particular freeway segment with heterogeneous traffic. For example, based on the study and traffic data of the Southern California Association of Governments [4], Wu *et al.* [5] proposed a single link travel time function for all the vehicle types using the BPR-type function:

$$t_a = t_a^0 \left[ 1 + 0.15(Q/y)^4 \right] \quad (1)$$

Here,  $t_a$  denotes the average travel time of the study road segment  $a$ ,  $t_a^0$  denotes the free-flow travel time, and  $Q, y$  are the aggregated traffic flow and road capacity in PCUs. Here, the effects of traffic composition are included in the variable  $Q$ , as follows:

$$Q = \sum_{k \in \mathbf{K}} [PCU(\rho_k, adj_k, y, g, l) \times f_k] \quad (2)$$

where  $f_k$  is the measured flow for vehicle type  $k$ ,  $\mathbf{K}$  is the set of vehicle types and the PCU conversion factor is a function of the percentage of  $k$ th vehicle type ( $\rho_k$ ), the road grade ( $g$ ), the road length ( $l$ ), and an adjustment factor ( $adj_k$ ) for vehicle type  $k$ . However, to conduct such a field experiment, which cover a wide range of traffic volumes and compositions a typical freeway, will encounter with many practical limitation and difficulties.

The second approach develops travel time functions based on the theoretical analysis, for use in urban network modeling and analysis applications. Lam and Huang [6] employed a combined travel time function of multiple vehicle types for the combined trip distribution and traffic assignment problem by adopting the volume-adjusted approach. They assumed that all types of vehicles on a road segment  $a$  have the same average travel time  $t_a$ , computed by:

$$t_a = t_a^0 \left[ 1 + a_0(Q/y)^4 \right], \text{ where } Q = \sum_k Q_k + \theta Q_{\text{bus}} \quad (3)$$

where  $a_0$  is a parameter used for indicating the congestion level of links;  $Q$  is the hourly traffic volume of all vehicle types under effects of traffic composition;  $Q_k, Q_{\text{bus}}$  are the hourly traffic volume of type  $k$  and buses, respectively; and  $\theta$  is the equivalent PCU values of a bus.

Consistently, Noriega and Florian [7] used the following BPR-type function to differentiate travel times among vehicle types, by introducing a time factor  $\phi_k$  for each vehicle type:

$$t_{a,k} = t_a^0 \times \phi_k \left[ 1 + a_0(Q/y)^{b_0} \right] \quad (4)$$

where  $t_{a,k}$  is the travel time for vehicle type  $k$  on link  $a$ ;  $\phi_k$  is a vehicle type time factor for vehicle type  $k$ ;  $a_0, b_0$  are the BPR parameters. Si *et al.* [8] also proposed a single link travel time function for mixed traffic. In their analysis, the average travel time  $t_a$  for a two-way road with central separation areas is given by:

$$t_a = t_a^0 \left[ 1 + a_1(Q_{\text{car}}/y)^{b_1} \right] \left[ 1 + a_2(Q_{\text{bus}}/y)^{b_2} \right] \quad (5)$$

where  $Q_{\text{car}}$  and  $Q_{\text{bus}}$  are the standard PCU volumes of cars and buses, and  $a_1, b_1, a_2, b_2$  are coefficients to be calibrated. Moreover, Sun *et al.* [9] suggested the following capacity-adjusted method for the link

travel time function of  $k$ th vehicle type  $t_{a,k}$  when both general vehicles and public bus run on the same urban road:

$$t_{a,k} = t_a^0 \left[ 1 + a_k \left( \frac{Q + Q_{\text{bus}}}{y - \theta \rho_{\text{bus}}} \right)^{b_k} \right] \quad (6)$$

where  $Q, Q_{\text{bus}}$  are the hourly traffic volume of general vehicles and buses;  $\rho_{\text{bus}}$  is the proportion of buses;  $\theta$  is the equivalent PCU values of bus;  $a_k, b_k$  are the assumed coefficients of vehicle type  $k$ . Equation (6) indicates that the public buses have negative impacts on the road capacity. In short, researchers have made use of the above travel time functions to study urban network traffic. However, although these volume-adjusted, time-factor, and capacity-adjusted methods can capture the impacts of truck composition to some extent, they cannot reflect the differences of particular freeway segments. In other words, these link travel time functions may be reasonable theoretically, but lack of relevant data support.

The third approach makes use of microscopic traffic simulation models to generate traffic data and estimate travel time functions for vehicle types. As recognized in the Highway Capacity Manual (HCM), the simulation-based approach has become a viable method for investigating and assessing the performance of traffic system [10]. This method does not suffer from the problems of data availability and coverage inherent in the empirical method [11]. Yun *et al.* [12] developed a single average speed-flow function for the mixed traffic using a microscopic traffic simulation model (CORSIM). The mixed formula they proposed is

$$V = \frac{V_0}{1 + a_0 \times f(\rho)^\gamma (Q/y)^{b_0}} \quad (7)$$

where  $V$  and  $V_0$  denote the average and free-flow travel speed of all vehicles on the road;  $f(\rho)$  is a function of truck proportion to the average speed;  $a_0, b_0, \gamma$  are coefficients to be calibrated from data. In addition, Moridpour *et al.* [13] formulated a similar BPR-type function to reveal the impacts of heavy trucks on their surrounding traffic by using AIMSUN. According to their analysis, the travel time function is expressed as

$$t_a = t_a^0 \left[ 1 + a_0 \times f(\rho) (Q/y)^{b_0} \right] \quad (8)$$

where function  $f(\rho) = \lambda \rho^\gamma$  represents the influence of heavy trucks on the average travel time. However, these simulation studies usually focused on developing either an average link travel time function for all vehicle types, or a regression model for a specific vehicle type under a given traffic composition.

Based on the analysis of the three approaches, it can be seen that the standard methodology for estimating travel time functions is to estimate the parameters of the proposed model by collecting field data from selected locations. However, this traditional method requires the collection of data related to the various volume and composition of traffic, which is very difficult and expensive. Another challenge is that there will be many unexpected difficulties in field experiments, and some of them are really hard to control or overcome. Thus, we prefer to choose the alternative method by using microscope traffic simulation technology. Commercial traffic simulators, such as VISSIM, CORSIM, can be used to generate large quantities of ‘clean’ traffic flow and composition data [12, 14]. These simulation experiments can be set up to cover the traffic flow and compositions that are not often seen in practice.

It has long been recognized that microscopic traffic simulation models are useful tools for mimicking the movement of vehicles on freeways. In this study, we use VISSIM. Other examples of microscopic traffic models include CORSIM, TRANSIMS, MITSIMLab, PARAMICS, and AIMSUN. The model underlying VISSIM is based on the psychophysical driver behavior theory proposed by Wiedeman [15]. The basic concept of the psychophysical model is that drivers of faster-moving vehicles are more sensitive to the changes in speed and distance of traveling vehicles located ahead of them

[16]. To correctly reproduce traffic phenomena, it is necessary to calibrate the parameters involved in a microscopic traffic model before they are used for any traffic analysis. As shown in Table I, several researchers have developed calibration procedures for this purpose. From another aspect, they also confirmed that the calibrated microscopic simulation model could represent the field condition and mimic interactions among the vehicles.

### 1.2. Objectives and contributions

In the planning of freeway projects, the travel time functions play an important role on the estimation of traffic volume and the selection of toll charges for various vehicle types. From the earlier review, we can see that no research has been carried out so far to distinguish the different travel time functions of different vehicle types under various traffic compositions on a freeway. To precisely estimate travel time for each vehicle type, it is worthwhile to investigate their travel time functions separately by taking into account the effects of traffic volume, road capacity, and mixed traffic interactions. The objective of this study is to propose a tangible method for estimating the travel time function of each vehicle type on a freeway by making use of traffic data generated from a microscopic traffic simulation model. The proposed method will be applied to develop travel time functions for heterogeneous traffic flows based on the calibrated VISSIM model.

The specific contributions of this study are threefold. Firstly, this study develops a simulation-based four-step method to estimate the travel time functions for heterogeneous freeway traffic. Secondly, a piecewise BPR-type function is proposed to compute travel times for different vehicle types by means of the proposed four-step method, which not only considers the influence of congestion levels, but also the effects of traffic composition. Thirdly, two VISSIM-based experiments are conducted to illustrate the estimation process of travel time functions for the three vehicle types on freeways (passenger cars, light trucks, and heavy trucks). These experiments demonstrate the validity of the proposed four-step method, and their calibrated functions can be used on many relevant freeway traffic studies.

The remainder of this paper is organized as follows. Section 2 describes the simulation based four-step method as a viable research methodology. Section 3 elaborates the VISSIM based experiment design. Section 4 presents the regression results obtained from the VISSIM experiments, and summarizes the results and possible applications. A general conclusion is provided in Section 5.

## 2. RESEARCH METHODOLOGY

Let us consider a single direction of a freeway, which connects its entrance and exit stations directly, as shown in Figure 1. Assume the freeway to be uninterrupted, that is, devoid of traffic control signal and

Table I. Summary of calibration studies on microscopic traffic simulation models.

Authors	Simulation tool	Calibrated parameters	Optimization methodology	Link type
Gomes <i>et al.</i> [17]	VISSIM	Car-following, vehicle performance parameters	Trial and error	Freeway
Kim <i>et al.</i> [16]	VISSIM	Car-following, vehicle performance parameters	GA with NPST*	Freeway
Laufer [18]	VISSIM	Car-following, vehicle performance parameters	Sensitive analysis	Freeway
Kim and Rilett [19]	CORSIM, TRANSIMS	Car-following, driver aggressiveness factor	Simplex algorithm	Freeway
Schultz and Rilett [20]	CORSIM	Driver behavior/vehicle performance parameters	Automated GA	Freeway
Ma <i>et al.</i> [21]	PARAMICS	Global, local parameters	SPSA*	Freeway
Lee <i>et al.</i> [22]	PARAMICS	Global parameters	SPSA*	Freeway
Balakrishna <i>et al.</i> [23]	MITSIMLab	Driver behavior parameters	SPSA*	Freeway
Hourdakis <i>et al.</i> [24]	AIMSUN	Global, local parameters	Trial and error	Freeway

\*Note: GA, genetic algorithm; SPSA, simultaneous perturbation stochastic approximation algorithm; NPST, nonparametric statistical technique; global parameters, mean target head, mean reaction time, and others; local parameters, link headway factor, link reaction factor, and others.

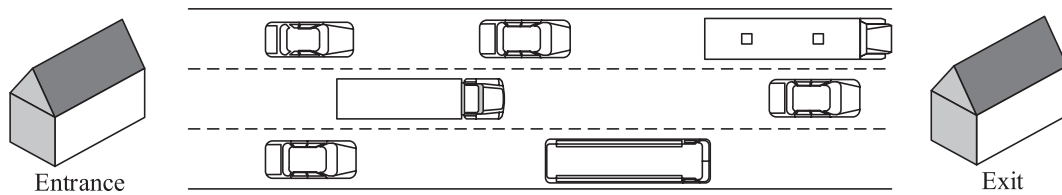


Figure 1. Heterogeneous traffic on one direction of a freeway.

interferences of traffic flow in opposite directions. There are many such freeways, such as the Beijing Capital Airport Expressway, and some segments of the Chengdu-Mianyang Expressway in China. We further assume that all vehicle types share the same road space, and their mutual interactions vary widely with the changes of vehicle traffic composition. In addition, there are no lane restrictions for vehicles on the freeway.

As we have pointed out, to deduce the travel time function for each vehicle type on the freeway, the standard methodology by collecting field data from selected locations has the practical difficulties and capital shortage problems. In contrast, the microscopic traffic simulation methodology offers a more efficient way to generate the heterogeneous traffic data and overcome the mentioned practical limitations. It could mimic travel time changes under effects of various factors by accounting for vehicle size, acceleration, deceleration, and lane-changing behaviors in macro simulation models. Then, the average travel time can be calculated from the generated travel time set of all vehicles on the hypothetical freeway. Therefore, this research uses the four-step methodology, shown in Figure 2, to construct the travel time functions for each vehicle type on freeways.

We now elaborate each of the four steps. Step 1 aims to calibrate the microscopic traffic simulated model selected for the generation of traffic data. When working with a simulation model, an essential concern is its ability to reproduce traffic phenomenon correctly. To achieve adequate reliability and credibility for the microscopic traffic simulation model, it is necessary to conduct a rigorous calibration using real data. Note that several methods have been proposed in the literature for calibrating

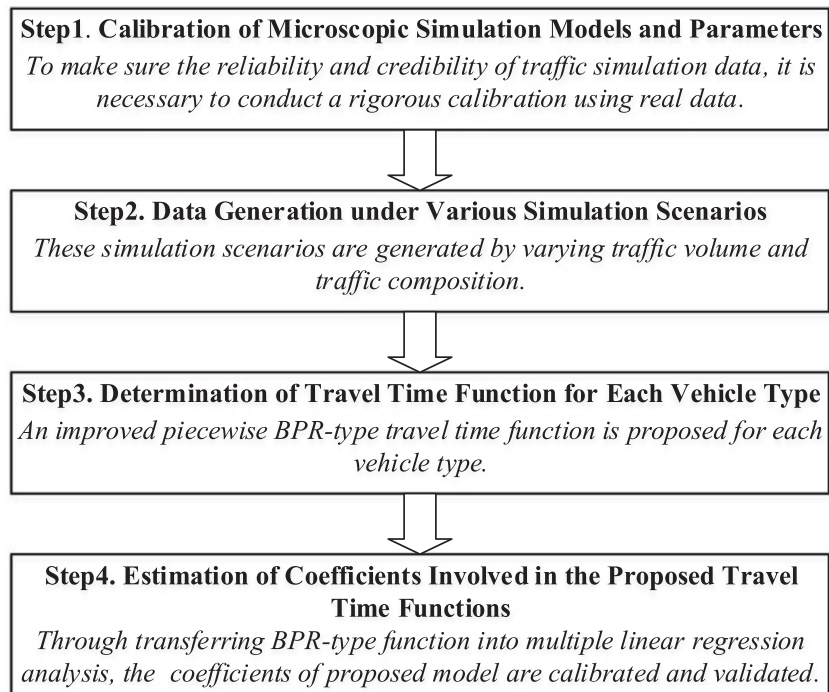


Figure 2. Flowchart of the microscopic traffic simulation-based four-step method. BPR, Bureau of Public Roads.

microscopic traffic simulation models. As shown in Table I, the parameters to be calibrated mainly include car-following coefficients in vehicle performance parameters and the local link parameters.

Step 2 attempts to generate heterogeneous traffic data over a variety of simulation scenarios. To create these scenarios, we identify that traffic volume and traffic composition are two key determinants affecting the travel times. Traffic volume is the primary determinant affecting the travel time because of capacity limitations and the impact of merging flows. Traffic composition is the secondary determinant based on different vehicle performance characters and drive behaviors of various vehicle types. For example, Figure 3 depicts the VISSIM data for travel times of cars and trucks under several traffic compositions. It can be easily seen that the travel times of both cars and trucks increase as traffic volume of the freeway increases. Also, the travel times of vehicles, especially cars, increase with the increasing percentage of trucks within a certain range.

The process of generating synthetic data involves the simulation of hundreds of scenarios with different volumes, compositions, and random seeds. For instance, in the case of cars and trucks, we simulate 40 different levels of traffic flow, ranging from free-flow to full congestion, and 15 different

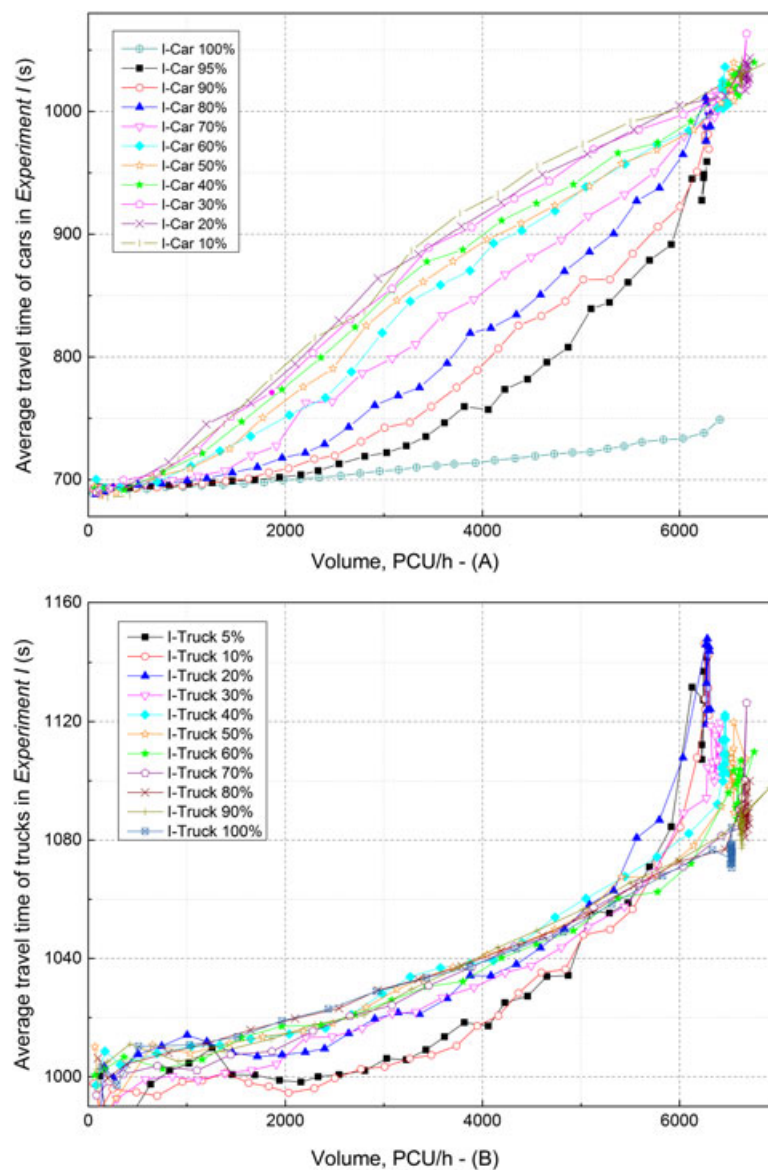


Figure 3. An illustration of collected data from VISSIM simulations. PCU, passenger car unit.



compositions, for a total of 600 scenarios. Moreover, each scenario needs to be run with several random seeds to obtain an average value. The average value of each scenario is collected as one data point for the estimation of travel time function for each vehicle type in next step.

Step 3 intends to determine the travel time function for each vehicle type. As illustrated in Figure 3, the travel time curves for car and trucks are all similar to the standard BPR curves expressed by Equation (1). This result is consistent with the cited literatures, which shows that the relationship between average travel time and volume/capacity ( $V/C$ ) ratio resembles the BPR function under a given traffic composition. To address the travel times of heterogeneous traffic, another term related to traffic composition needs to be included into the equation. We denote this additional term with  $f_k$  ( $\rho_0, \rho_1, \rho_2, \dots, \rho_{K-1}$ ) for the  $k$ th vehicle type, where  $\rho_k$  is the percentage of vehicle type  $k$ ,  $k \in \mathbf{K}$  and  $\mathbf{K} := \{k, k=0, 1, \dots, K-1\}$  is the set of vehicle types. Hence, the travel time function for the  $k$ th vehicle type on the freeway can be formulated by

$$t_{a,k} = t_{a,k}^0 \left[ 1 + a_k f_k(\rho_1, \rho_2, \dots, \rho_{K-1}) (Q/y)^{b_k} \right], \forall k \in \mathbf{K} \quad (9)$$

According to our preliminary study using the VISSIM experiments, the effects of traffic composition on the travel time of vehicles are fairly weak whenever the percentage of cars is below some certain level. In other words, the standard BPR function form is suitable when the percentage of cars is lower than a given percentage  $\phi$  (i.e., 50%–60% in our preliminary study). Therefore, the following piecewise continuous function is proposed as

$$t_{a,k} = \begin{cases} t_{a,k}^0 \left[ 1 + a_k f_k(\rho_1, \rho_2, \dots, \rho_{K-1}) (Q/y)^{b_k} \right], & \rho_0 \geq \phi \\ t_{a,k}^0 \left[ 1 + a_k (Q/y)^{b_k} \right], & \rho_0 < \phi \end{cases}, \forall k \in \mathbf{K} \quad (10)$$

where,  $t_{a,k}$  = link travel time of  $k$ th vehicle type on the freeway  $a$ ,  $k \in \mathbf{K}$ ;  $t_{a,k}^0$  = free-flow travel time of  $k$ th vehicle type;  $Q$  = total volume (PCU per hour per lane, PCU/h/l);  $y$  = freeway capacity (PCU/h/l);  $a_k$ ,  $b_k$  = coefficients to be calibrated for  $k$ th vehicle;  $\rho_0$  = percentage of passenger cars; and  $\phi$  = the threshold of the piecewise continuous function  $\phi \in [0, 1]$ . Here, the threshold  $\phi$  is an estimated value from the simulation scenarios, which specifies the two regression models whether the traffic composition is selected as a variable. To identify this threshold, we need to compare the different levels of traffic composition and find those scenarios that traffic composition has no effects on the average travel time. The Fisher's least significant differences (LSD) method is suggested to find the threshold  $\phi$  from these simulated levels of traffic composition in this randomized analysis of variance problem. It will compare the average travel time of a specific vehicle type under all traffic composition scenarios with the null hypotheses  $H_0: t_{a,0}(\rho_k) = t_{a,0}(\rho_j)$  (for all  $k \neq j$ ) using  $t$ -statistic under a given traffic volume [25]. After repeated analyses under various traffic volumes, the  $\phi$  will be identified.

For the function defined by Equation (10), it is important to find a suitable form for  $f_k(\rho_1, \rho_2, \dots, \rho_{K-1})$ . Based on the heterogeneous traffic data generated by VISSIM, a number of different function forms for  $f_k(\rho_1, \rho_2, \dots, \rho_{K-1})$  were tested and analyzed by using a statistical analysis software tool. Then it was found that the following polynomial function provided the best approximation:

$$f_k(\rho_1, \rho_2, \dots, \rho_{K-1}) = (1 + \rho_1)^{\gamma_{1,k}} (1 + \rho_2)^{\gamma_{2,k}} \dots (1 + \rho_{K-1})^{\gamma_{K-1,k}}, \forall k \in \mathbf{K} \quad (11)$$

where,  $\gamma_{1,k}, \gamma_{2,k}, \dots, \gamma_{K-1,k}$  are the coefficients to be calibrated for the  $k$ th vehicle type. In the aforementioned formula, the traffic composition has the relation  $\rho_0 + \rho_1 + \rho_2 \dots + \rho_{K-1} = 1$ , and the percentages of vehicles  $\rho_1, \rho_2, \dots, \rho_{K-1}$  are independent by excluding the percentage of cars  $\rho_0$ . This polynomial function form, based on the standard BPR function, is easy to calibrate and modify in the regression analyses. When  $\rho_n = 0$ ,  $(1 + \rho_n)^{\gamma_{n,k}} = 1, \forall n, k \in \mathbf{K}$ , it also ensures that even if the  $n$ th vehicle type is not present on the freeway, the proposed function form is still consistent and valid.

Step 4 aims to estimate the coefficients  $a_k, b_k$  and  $\gamma_{1,k}, \gamma_{2,k}, \dots, \gamma_{K-1,k}$  involved the improved travel time function by using the data generated in step 2. We first transform function on  $\rho_0 \geq \phi$  by taking the

logarithm on both sides of the equation after some proper arrangements. We thus have:

$$\ln\left(\frac{t_{a,k}}{t_{a,k}^0} - 1\right) = \ln(a_k) + \gamma_{1,k}\ln(1 + \rho_1) + \dots + \gamma_{n,k}\ln(1 + \rho_n) + b_k\ln(Q/y) \quad (12)$$

Equation (12) can be rewritten as the multiple linear equations:

$$Y_k = A_k + \gamma_{1,k}X_1 + \gamma_{2,k}X_2 + \dots + \gamma_{K-1,k}X_{K-1} + b_kX_N, \quad k \in \mathbf{K} \quad (13)$$

where  $Y_k = \ln\left(t_{a,k}/t_{a,k}^0 - 1\right)$ ,  $A_k = \ln(a_k)$ ,  $X_n = \ln(1 + \rho_n)$ ,  $n = 1, 2, \dots, K-1$ , and  $X_N = \ln(Q/y)$ .

The coefficients in Equation (13),  $A_k$ ,  $\gamma_{1,k}$ ,  $\dots$ ,  $\gamma_{K-1,k}$ , and  $b_k$ , can be estimated by linear regression method in which,  $Y_k$ ,  $k \in \mathbf{K}$  is the dependent variable, and  $X_1, X_2, \dots, X_{K-1}, X_N$  are the independent variables. To measure how well the generated data fits the estimated travel time function, a goodness-of-fit test need to be conducted. The coefficient of determination,  $R^2$ , is a commonly used statistical measure of how well the regression line approximates the simulated data points. A higher value of  $R^2$  indicates a stronger relationship between the dependent variable and the independent variables. Another test method is using the  $F$ -test to check whether there is a linear relationship between the dependent variable and independent variables. The standard error of the estimate (SEE) also quantifies that the level of accuracy for the estimation values from regression models. Finally, the regression model can be validated using another new set of simulated data that have different traffic composition and/or volumes.

### 3. VISSIM SIMULATION EXPERIMENT DESIGN

A hypothetical 20-km freeway is used to carry out the simulation based four-step method. This freeway has three lanes in one direction and connects its entrance and exit directly. It provides entirely uninterrupted traffic flow, meaning that traffic traveling on it devoid of interruptions by signal, intersections, or periodic delay. Vehicles on this hypothetical freeway are classified into three types: passenger cars/low-occupancy vehicles (LOV), two-axle light trucks (HGV\_MED), and five-axle heavy trucks (HGV\_LARGE).

In our experiment, we choose VISSIM as the microscopic traffic simulation model to generate traffic data for the hypothetical freeway. The main reason is that VISSIM is one of the most widely used microscopic multi-modal traffic flow simulation software, which contains a psychophysical car following model for longitudinal vehicle movement and a rule-based algorithm for lateral movements [15]. Another reason is that VISSIM is a mature software package with many user-controlled variables, and it can be started and accessed via APIs for various programming languages. Using the Application of Programming Interface (API), we were able to generate large number of scenarios runs in an automated fashion.

#### 3.1. Calibration of the VISSIM model

VISSIM includes a variety of user-controlled parameters that should be calibrated so that the simulated traffic pattern matches the observed field data. These parameters can be classified into two groups: vehicle performance parameters and driver behavior parameters. Gomes *et al.* [26] have successfully calibrated these parameters using 2001 traffic data of the Foothill Freeway (I-210 West) in California. The calibrated model includes demands for three separate vehicle types, defined by their operational characters, as shown in Table II.

The free-flow travel times on the I-210 West were recorded from vehicles traveling from the entrance to the egress point with their free-flow speed, excluding the waiting time outside of the freeway entrance. The average desired speeds were calibrated to be 104 km/h for LOV with a range of (96–112 km/h), 72 km/h for HGV\_MED, and HGV\_LARGE with a range of (64–80 km/h). These calibrated speeds reveal a general feature of the model that the desired travel speeds of cars and trucks are different. Hence, the speed limits are imposed for cars and HGV are different. For example, the speed limits on freeways are 115–130 km/h for car and 80–96 km/h for HGVs in the USA; the speed



Table II. Vehicle performance parameters used in VISSIM model.

Vehicle types	Cars (LOV)	Light trucks (HGV_MED)	Heavy trucks (HGV_LARGE)
Length (m)*	3.75–4.76	10.22–12.06	16.08–16.88
Width (m)*	1.86–2.07	2.5–3.17	2.5–2.93
Power (kw)*	50–120	80–1000	150–400
Weight (kg)*	700–1500	7000–10,000	25,000–40,000
Free-flow travel time (s)	690	960	990
Maximum acceleration*	(3.5 m/s <sup>2</sup> , 160 km/h)	(2.3/s <sup>2</sup> , 130 km/h)	(1.2 m/s <sup>2</sup> , 120 km/h)
Desired acceleration*	(3.5 m/s <sup>2</sup> , 160 km/h)	(2.3 m/s <sup>2</sup> , 130 km/h)	(1.2 m/s <sup>2</sup> , 120 km/h)
Maximum deceleration*	(−7.5 m/s <sup>2</sup> , −6 m/s <sup>2</sup> )	(−6.5 m/s <sup>2</sup> , −3 m/s <sup>2</sup> )	(−6.0 m/s <sup>2</sup> , 0 m/s <sup>2</sup> )
Desired deceleration*	(−2.8 m/s <sup>2</sup> , −2.8 m/s <sup>2</sup> )	(−1.9 m/s <sup>2</sup> , −1.9 m/s <sup>2</sup> )	(−1.3 m/s <sup>2</sup> , −1.3 m/s <sup>2</sup> )

\*Note: Resource data, vehicle type functions, and distributions comes from Gomes [27].

limits on freeways are 90 km/h for cars and 60 km/h for HGVs in Singapore [28]; in China, the speed limits are 120 km/h for cars and 90 km/h for HGVs [29].

Another group of calibration parameters is the driver behavior parameters, which involves a classification of driving reactions in response to the proceeding vehicles' speed and distance. Four driving modes are used by VISSIM: free driving, approaching, following, and braking. In each mode, the drivers behave differently, either trying to match a prescribed target speed or tracking to their following distance. The drivers switch from one mode to another as soon as they reach a certain threshold that can be expressed as a function of speed difference and distance. In VISSIM, these driving modes are specified by the Wiedemann psychophysical car-following model, and the basic calibration process for the car-following parameters is important. Table III gives the most influential driver parameters along with the calibrated values found by Gomes *et al.* [17] for Foothill Freeway test site. The car-following model of the freeway drivers involves 10 tunable parameters, and the remainder was left at their default values [15]. These parameters make the simulation of VISSIM match many qualitative aspects of Foothill Freeway operation, including the location of identified bottlenecks, initial time, and final times of mainline queues, extent of queues, utilization of the high-occupancy vehicle lanes, and onramp performance [17, 26].

### 3.2. Scenario design of VISSIM-based experiments

To validate the proposed travel time functions, we design the following two experiments. In the first experiment (Experiment I), two vehicle types, LOV and HGV\_LARGE, are considered with various combinations of traffic volume and traffic composition. In the second experiment (Experiment II), all of the three vehicle types, LOV, HGV\_MED, and HGV\_LARGE, are included under various combinations of traffic volume and composition. Each of the two experiments was run over the volume and composition combinations described below. In Experiment I, the passenger car percentage  $\rho_0$  was assigned with 15 different values ranging from 100% to 40% with a 5% or 10% decrement, and from 40% to 0% with a 10% decrement. The traffic volume was varied between 10 and 8000 veh/h with a 200 veh/h increment per scenario. In Experiment II, a larger number of simulation runs were performed. The percentages of the three vehicles types were altered by 5% or 10% when  $\rho_0$  decreases from 100% to 60%, and altered by 10% or 15% when  $\rho_0$  ranged from 60% to 0%. And traffic volumes increased from 10 to 8000 veh/h with a 200-veh/h interval at every of the designed 50 composition scenarios.

Table III. Calibrated CC values from Foothill Freeway (I-210 West) in California.<sup>#</sup>

Link type	CC0	CC1	CC4/CC5
Freeway	1.7	0.9	−2.0/2.0
Soft curve	1.7	1.1	−2.0/2.0
Hard curve	1.7	1.4	−2.0/2.0
Defaults	1.5	0.9	−0.35/0.35

<sup>#</sup>Note: Resource data come from Gomes [27].

Note that under some low traffic volume conditions, the generated average travel time of vehicles from microsimulation models is more random. Because when the traffic volume is very low on a large capacity freeway, the traveling vehicles approach to free flow traveling condition, and the errors of the expected average travel time in simulations will be larger. To reduce this error, one method is to repeat the lower volume experiments more times. Besides, because the BPR-type function is a monotonically increasing with traffic volume, another method is to remove some lower traffic volume. In Experiments I and II, we choose to remove the simulations that traffic volume is lower than 400 veh/h and repeat the simulation scenarios with volume between 400 and 1500 veh/h a few times of others.

Following the generation of simulated data, the freeway capacity and PCU values of each vehicle type need to be determined prior to estimating the travel time functions by using the regression technique. In this study, the lane capacity on the freeways is assumed at 2200 vehicles per hour per lane (veh/h/lane). Hence, the three-lane freeway capacity is  $\gamma = 6600$  veh/h, which is in accord with the study of VISSIM freeway capacity by Laufer [18]. In addition, for the equivalent PCU values of trucks, the typical value are 1.5–2 for single-unit trucks, and 2–3 for combination vehicles according to HCM 2010 [30]. Connecting the size of the simulation vehicles and the HCM 2010, the PCU values are set as 1.36 for light trucks and 2.45 for heavy trucks in this study.

#### 4. REGRESSION ANALYSIS OF TRAVEL TIME FUNCTIONS

All the simulation scenarios designed for Experiments I and II were coded using Visual Basic for Applications, calling for VISSIM 6.0, and run on a personal computer with Intel Core (TM) i7-3.4 GHz CPU, 16 GB RAM, and Windows 7 Enterprise operating system. After recording the travel time of each simulated vehicle on the freeway with respect to different scenarios, we made use of the linear regression technique to estimate the parameters of the piecewise travel time function shown in Equation (10) based on the transformed equations expressed by Equation (13).

##### 4.1. Travel time regression functions of Experiment I

Experiment I considers two vehicle types (cars and heavy trucks) under various traffic compositions for the hypothetical freeway. The VISSIM-based simulation generated 600 sample points from 15 traffic composition scenarios with 40 traffic volume scenarios. Applying to the proposed function form (10) and (11), the regression results, including the parameters and goodness-of-fit statistics, are listed at Table IV. The parameters for the standard BPR-type function were also estimated for comparison. In this experiment, the threshold of traffic composition in model (10) is identified as  $\phi = 60\%$  for both cars and heavy trucks by applying Fisher LSD method. Besides, the estimated parameters in Table IV are all statistically significant, which are proved by the null hypotheses that these parameters are insignificant and the results that  $p$ -values of all these parameters are all less than 0.01.

The piecewise travel time functions for passenger cars and heavy trucks in Experiment I can be expressed as follows:

Table IV. Estimated parameters and good-of-fit statistics for Experiment I.

Vehicle types		Parameters			Goodness-of-fit statistics			
		$a_k$	$\gamma_{1,k}$	$b_k$	$R^2$	SEE	$F$	Sample size
Cars $k=0$	$\rho_0 \geq 60\%$	<b>0.29</b>	<b>2.62</b>	<b>1.97</b>	0.94	0.23	1094	367
	$\rho_0 < 60\%$	<b>0.62</b>	\	<b>1.26</b>	0.95	0.17	2571	214
	$\rho_0 \in (0, 1)$	0.50	\	1.82	0.79	0.38	897	581
Heavy trucks $k=1$	$\rho_0 \geq 60\%$	<b>0.12</b>	\	<b>1.87</b>	0.82	0.37	648	328
	$\rho_0 < 60\%$	<b>0.10</b>	\	<b>1.26</b>	0.96	0.09	853	199
	$\rho_0 \in (0, 1)$	0.11	\	1.67	0.78	0.31	1209	527

$$\text{For cars, } t_{a,0} = \begin{cases} t_{a,0}^0 \times \left[ 1 + 0.29(1 + \rho_1)^{2.62} (Q/y)^{1.97} \right], & \rho_0 \geq 60\% \\ t_{a,0}^0 \times \left[ 1 + 0.62(Q/y)^{1.26} \right], & \rho_0 < 60\% \end{cases} \quad (14)$$

$$\text{For heavy trucks, } t_{a,1} = \begin{cases} t_{a,1}^0 \times \left[ 1 + 0.12(Q/y)^{1.87} \right], & \rho_0 \geq 60\% \\ t_{a,1}^0 \times \left[ 1 + 0.10(Q/y)^{1.26} \right], & \rho_0 < 60\% \end{cases} \quad (15)$$

To measure how well the regression models fit the VISSIM simulation data, a number of goodness-of-fit statistics are commonly evaluated. Firstly, a high value of coefficient of determination  $R^2$  indicates the simulation data fit our regression model well. Through comparing the value of  $R^2$ , it can be seen that the regression results of the proposed function are much better than the standard BPR-type function, especially for cars. For example, with regard to regression result for cars, the value of  $R^2$  in our proposed function are 0.94 and 0.95, respectively, in contrast to  $R^2=0.79$  for the standard BPR-type function; the improved functions for heavy trucks ( $R^2=0.82, 0.96$ ) also perform much better than standard BPR function ( $R^2=0.78$ ). It is worth mentioning that if the values of  $R^2$  are almost the same by comparing the regression travel time functions by including and excluding the traffic composition, we suggest to choose the standard BPR-type function to simplify the function form. This is why traffic composition is not included in some of our regression models. Secondly, the low SEE values reveals that the piecewise continuous models used to estimate the travel time of various vehicles have a high level of accuracy, and the high values of  $F$  statistics also indicate the piecewise continuous models are statistically significant.

We further validate the calibrated travel time functions by predicting travel times of heterogeneous traffic and comparing them with the actual values. Hence, we use traffic compositions (e. g.,  $\rho_0=0.83, \rho_1=0.17$ ) that have not yet been used in the VISSIM simulation. According to Figure 4, it can be found that all of the new simulation values of  $\rho_0=0.83, \rho_1=0.17$  belong to the 95% prediction intervals of the regression travel time. After checking some other randomly generation scenarios, the results indicate that these calibrated travel time functions are able to predict new observations on this freeway. Because the VISSIM model was calibrated using the traffic data from Foothill Freeway (I-210 West), it is expected that our calibrated travel time function will reproduce the traffic data well.

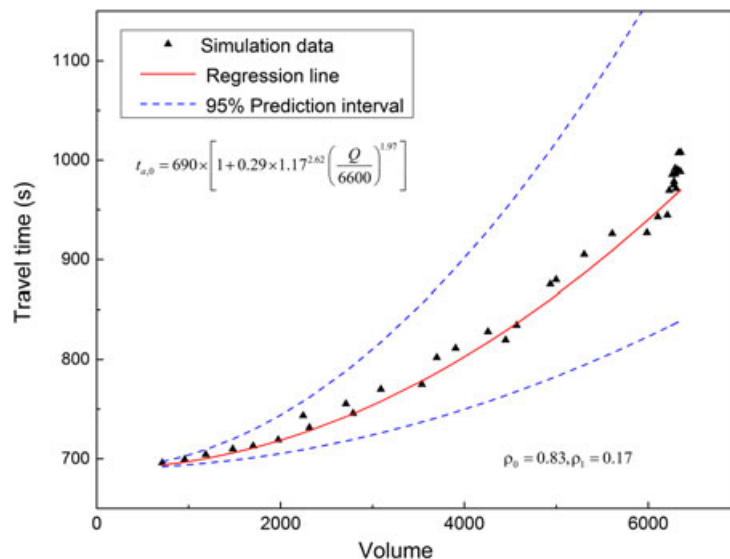


Figure 4. Ninety-five percent prediction intervals for simulation data at  $\rho_0=0.83, \rho_1=0.17$ .

Table V. Estimated parameters and goodness-of-fit statistics for Experiment II.

Vehicle types		Coefficients				Goodness-of-fit statistics			
		$a_k$	$\gamma_{1,k}$	$\gamma_{2,k}$	$b_k$	$R^2$	SEE	$F$	Sample size
Cars $k=0$	$\rho_0 \geq 55\%$	<b>0.29</b>	<b>1.52</b>	<b>1.98</b>	<b>1.55</b>	0.92	0.17	1763	1351
	$\rho_0 < 55\%$	<b>0.57</b>	\	\	<b>1.12</b>	0.98	0.14	7649	472
	$\rho_0 \in (0, 1)$	0.49	\	\	1.38	0.81	0.22	3580	1823
Heavy trucks $k=1$	$\rho_0 \geq 55\%$	<b>0.12</b>	\	\	<b>1.98</b>	0.88	0.22	3304	1372
	$\rho_0 < 55\%$	<b>0.106</b>	\	\	<b>1.78</b>	0.89	0.19	3011	495
	$\rho_0 \in (0, 1)$	0.112	\	\	1.89	0.86	0.25	6068	1867
Light trucks $k=2$	$\rho_0 \geq 55\%$	<b>0.08</b>	<b>1.17</b>	<b>0.57</b>	<b>2.07</b>	0.92	0.23	1639	985
	$\rho_0 < 55\%$	<b>0.10</b>	\	\	<b>1.84</b>	0.93	0.16	4833	502
	$\rho_0 \in (0, 1)$	0.11	\	\	2.02	0.89	0.32	6421	1487

#### 4.2. Travel time regression functions of Experiment II

Experiment II considers the three vehicle types: passenger cars, light trucks, and heavy trucks on the hypothetical freeway. The VISSIM-based simulation generated about 2000 sample points through 40 traffic volume scenarios from 10 to 8000 veh/h inputs at every of the 50 vehicle composition scenarios. According to these simulation data, the threshold of the piecewise continuous travel time function for the three vehicle types is obtained as  $\phi = 55\%$  by the Fisher LSD method. The corresponding regression results and goodness-of-fit statistics are summarized in Table V, and the regression results for standard BPR function are also included for comparison. It should be pointed out that all of the  $p$ -values of the estimated parameters in Table V were less than 0.01 meaning that these estimated parameters are statistically significant.

The piecewise travel time functions for the three vehicle types in Experiment II are summarized as follows:

$$\text{For cars, } t_{a,0} = \begin{cases} t_{a,0}^0 \left[ 1 + 0.29(1 + \rho_1)^{1.52} (1 + \rho_2)^{1.98} (Q/y)^{1.55} \right], & \rho_0 \geq 55\% \\ t_{a,0}^0 \left[ 1 + 0.57(Q/y)^{1.12} \right], & \rho_0 < 55\% \end{cases} \quad (16)$$

$$\text{For light trucks, } t_{a,2} = \begin{cases} t_{a,2}^0 \left[ 1 + 0.08(1 + \rho_1)^{1.17} (1 + \rho_2)^{0.57} (Q/y)^{2.07} \right], & \rho_0 \geq 55\% \\ t_{a,2}^0 \left[ 1 + 0.10(Q/y)^{1.84} \right], & \rho_0 < 55\% \end{cases} \quad (17)$$

$$\text{For heavy trucks, } t_{a,1} = \begin{cases} t_{a,1}^0 \left[ 1 + 0.12(Q/y)^{1.98} \right], & \rho_0 \geq 55\% \\ t_{a,1}^0 \left[ 1 + 0.106(Q/y)^{1.78} \right], & \rho_0 < 55\% \end{cases} \quad (18)$$

Similar to Experiment I, the high  $R^2$  values of the estimated piecewise continuous travel time function for passenger cars and light trucks indicate a strong relationships among the link travel time, V/C ratio, and traffic compositions (at  $\rho_0 \geq 55\%$ ). And the fitting results of our estimated travel time functions are more accurate than the standard BPR function, revealed by a much higher  $R^2$  values. Furthermore, both the low SEE and high values of  $F$  statistics presented in Table V indicate that these regression travel time functions are statistically significant. Note that the percentage of light trucks has an insignificant effect ( $p$ -value  $> 0.05$ ) on the travel time of heavy trucks from our regression results analyses, so the standard BPR-type function forms are chosen for the piecewise function of heavy trucks.

As with Experiment I, the validation of regression models was performed using the simulation results on new traffic compositions. Figure 5 shows that all the simulated data points fall in the 95%

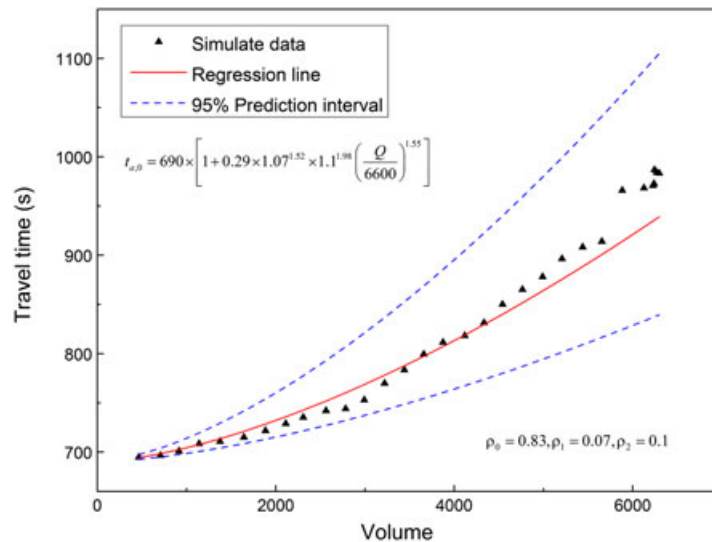


Figure 5. Ninety-five percent prediction intervals for simulation data at  $\rho_0 = 0.83, \rho_1 = 0.07, \rho_2 = 0.1$ .

prediction interval of the estimated travel time functions for the new traffic composition ( $\rho_0 = 0.83, \rho_1 = 0.07, \rho_2 = 0.1$ ).

#### 4.3. Properties of the piecewise continuous travel time functions

In the earlier experiments, through making use of our proposed four-step method, the piecewise continuous travel time functions are finally deduced, which provide a more accurate estimation of the travel time for various vehicle types. It is worth mentioning that to reveal the practical average travel times on freeways, vehicles' queue time and waiting time outside the freeway under over saturated traffic flow are not included in our simulation experiments. Because it is unrealistic for a freeway to have traffic volume larger than its capacity, and the stopped delay of vehicles under over saturated traffic flow always has high randomness. Figure 6 depicts the travel time functions of cars against traffic volume in Experiments I and II under different traffic compositions. Based on these two experiments, we note some interesting features of heterogeneous traffic on freeways are revealed by the piecewise travel time functions, and they are elaborated in the succeeding discussion.

Different characteristics of various vehicle types not only play a significance role on their own travel times, but also have distinctive influences on the travel times of other vehicle types. As we have discussed, one main reason for the interactions between vehicles on freeways come from differences of the average free-flow speed between vehicle types. When a fast-moving car travels behind a slow-moving truck, it needs to reduce its speed or change lane to overtake this truck. According to estimated parameters shown in Tables IV and V, it also can be seen that the fastest-moving cars have the widest range of travel times and the slowest-moving heavy trucks have the smallest travel time range. In other words, the vehicle type with the slowest free-flow speed (i.e., heavy trucks) shows a dominant impact on the average travel time of vehicles. Moreover, the vehicle operating and performance characteristics also affect the average travel times of vehicles. For example, in Experiment II, although the free-flow speeds of two-axle light trucks and five-axle heavy trucks are similar, they still have distinct travel time functions. These differences may come from their different operating and performance characters.

Traffic composition has the most significant impact on the travel time of cars, but the weakest impact on heavy trucks. The parameters  $\gamma_{1,k}, \gamma_{2,k}, \dots, \gamma_{K-1,k}, k \in \mathbf{K}$  indicate how sensitive the travel time of the  $k$ th vehicle type on freeways is to the traffic composition. From the regression models of travel time (14)–(18), it can be seen that both the heavy trucks and light trucks have significant impacts on the travel time of cars; in contract, the traffic composition is ineffective on the travel time of heavy trucks, which are only expressed by a piecewise continuous function with two standard BPR-type curves.



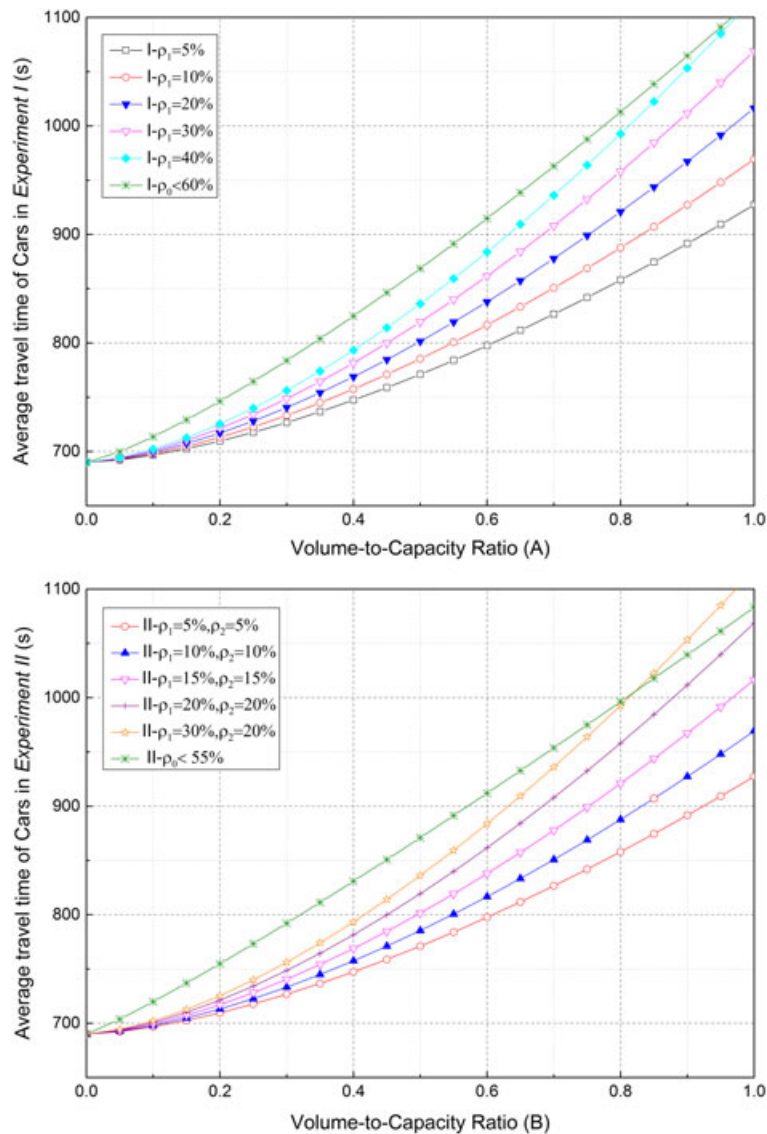


Figure 6. Travel time–volume relationships of cars in Experiments I and II under different traffic compositions.

Moridpour *et al.* [13] mentioned the possible reason is the operational limitations (acceleration/deceleration and maneuverability) of heavy trucks make them have difficulties to adjust their speed according to the surrounding traffic.

Moreover, the thresholds of the piecewise continuous function may be different when the vehicle types on the specified freeway are different. This boundary point indicates that the effects of traffic composition are fairly weak, while the percentage of cars is lower than some certain level. When the characteristics of vehicle types are changed, the thresholds will also change. For example, the threshold is 60% in Experiment I with two vehicle types and 55% in Experiment II with three vehicle types. Similar thresholds have been derived by previous studies [12].

#### 4.4. Possible applications of the piecewise continuous travel time functions for heterogeneous traffic flows

The possible applications of the estimated travel time functions for heterogeneous traffic are discussed below, which could make up the deficiencies of using the same travel time function for different vehicle types. Firstly, the travel time functions are important in the planning of freeway projects. To plan a

BOT freeway project, the road user costs, mainly including the travel time and toll charge, need to be estimated accurately for the interests of private firm and the government, because these costs determine whether the users choose this highway or not [31]. Because the willing-to-pay of different vehicle types is generally different in practice, their travel times and toll charges also should be estimated, respectively. The optimal tolls of different vehicle types may be misled by a homogenous travel time function. Secondly, in most studies of urban transportation networks, the traffic vehicles choose their traveling routes based on their average travel time costs [2]. However, if the average travel time of each vehicle type deviates from their actual value a lot, then the traffic assignment results will be unbelievable. Thus, it is clear that an average estimation of travel time function for all vehicle types in the earlier situations is insufficient, and our proposed respective link travel time function can be applied to provide an independent estimation of travel time costs for each vehicle type. In summary, the respective travel time function for each type of vehicles on freeways improves the estimation accuracy of road user cost.

## 5. CONCLUSIONS

This study has developed the microscopic traffic simulation based four-step method to estimate the travel time functions of heterogeneous traffic flows on a freeway. The piecewise continuous BPR-type functions were proposed to formulate travel time for each type of vehicles. Two experiments for two and three vehicle types were conducted to assess the validity of the method. A VISSIM model calibrated using real traffic data was adopted to generate the traffic data for the estimation of parameters. The two experiments demonstrated that traffic composition does play a significant role on the travel time. To be more specific, traffic composition has the most significant impacts on the travel time of cars, but the weakest impacts on the travel time of heavy trucks. In other words, the free-flow travel speed of slow vehicles has a dominant impact on the travel times in heterogeneous flows. In addition, the regression results also revealed that the travel time functions have distinct differences and their goodness-of-fit statistics were improved significantly by estimating their travel time functions separately. A more accurate estimation of the travel time functions could make an important contribution to both BOT freeway project studies and transportation network analysis when the heterogeneity of traffic is taken into account.

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