

Fixed-route taxi system: route network design and fleet size minimization problems

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SUMMARY

This paper introduces the taxi route network design problem (TXRNDP) for a fixed-route taxi service operating in Iran and, in similar form, in various other developing countries. The service operates fairly similar to regular transit services in that vehicles are only permitted to follow a certain predetermined route on the network. The service is provided with small size vehicles and main features are that vehicles only depart if full and that there are no intermediate boarding stops. In Iran the service attracts a high modal share but requires better coordination which is the main motivation for the present study. We develop a mathematical programming model to minimize the total travel time experienced by passengers while constraining the number of taxi lines, the trip transfer ratio and the length of taxi lines. A number of assumptions are introduced in order to allow finding an exact rather than heuristic solution. We further develop a linear programming solution to minimize the number of taxis required to serve the previously found fixed-route taxi network. Results of a case study with the city of Zanjan, Iran, illustrate the resulting taxi flows and suggest the capability of the proposed model to reduce the total travel time, the total waiting time and the number of taxi lines compared to the current taxi operation. Copyright © 2016 John Wiley & Sons, Ltd.

KEY WORDS: route network design problem; mixed integer programming; flexible transport services; taxi; developing countries

1. INTRODUCTION

When planning public transport networks, budget constraints mostly lead to trade-offs between network coverage and frequency. The larger the area over which the service is spread, the lower the frequencies. Lower frequency causes more expected waiting time, and, accordingly, the expected travel time increases. Therefore in many low density areas where demand is low flexible transport service (FTS) or semi-flexible transport services (SFTS) offer advantages over scheduled, conventional services [1].

FTS and SFTS are known as a range of services with greater flexibility in the size of fleet, lines, headways and obligatory stop locations than conventional public transport systems. Finn [2] has categorized FTSs into: (i) demand responsive transport (DRT) operated by buses, minibuses or micro-buses; (ii) shared taxis (sometimes known as taxi-buses); (iii) dynamic car-pooling; (iv) employee commuter programs; (v) car-sharing; and (iv) dedicated services for people with reduced mobility or other needs.

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FTSs are used increasingly in Europe and the US as part of the public transport mix to support conventional scheduled public transport [1]. As described in Velaga *et al.* [3], FTSs try to provide travelers with flexibility in choosing routes, times, modes of transport, service provider and payment systems. Well-implemented FTSs may have the ability to revive regular public transport services [4], especially when used as connections to areas concentrated on trunk corridors or where topographical constraints mean that regular public transport is hard to provide [5]. Interested readers are referred to Errico *et al.* [6] for more details on FTS.

In many developed countries therefore FTSs are used for feeding the conventional public transport network [7]. Further FTSs are introduced in order to provide suitable services for people with disabilities [8] or to improve social inclusion in low-density urban areas [9, 10]. In contrast, in developing countries, the underlying motivations to maintain FTSs are different. Outdated public transport systems with low quality fleets, poor safety and regularity pave the way for the FTSs to attain significant market shares. Furthermore, as discussed in Finn [2], FTS has some intrinsic advantages in developing countries such as the ability to provide services to places with low demand, to adapt services rapidly to fluctuating demand, to provide jobs for untrained workers.

Examples for large scale FTS in developing countries are numerous. To show the global scale we mention the tro-tro minibus in Accra of Ghana [11], the marshrutka minibuses in Tbilisi of Georgia [12], the minibus services in the principal cities of Kazakhstan [2], the Jeepneys in the Philippines [13, 14], the mini- and midi-buses in Rio de Janeiro [2], the shared “Servis” taxi-cars in Amman, Jordan [2], the Angkutan Kota in Indonesia [15, 16] and the Tuk-Tuk and Songtaew services in Thailand [15]. Finn [2] proposes that the organizational, operational and business relationship models of urban paratransit across the developing world contain valuable lessons that policy-makers and practitioners in the developed world can learn. In the following section we describe a particular type of paratransit that encounters market growth in Iran and other developing countries.

2. THE FIXED-ROUTE TAXI SERVICE

In this paper we focus on the “fixed-route taxi”. First, to avoid confusion, we stress that despite it being called “taxi” this is not a common taxi but a special type of FTS. The name taxi is because of the fact that the service is often operated with private cars that have a capacity of four passengers although sometimes also minivans with capacity of up to 14 passengers are used (see Figure 1). Also people in Iran commonly refer to this service as “taxi”.



Figure 1. Example of fixed-route taxis at terminals in Iran (source: own pictures).

Before discussing this service in more detail we note that there are also other taxi type services operating in Iranian cities with different service characteristics. There are particular taxis that can be shared between passengers and either booked (dial taxis) or hailed on-street (rounding taxis). Further, there are the usual exclusive taxis that are chosen by travelers willing to pay in order to travel alone.

In contrast to the service of interest here none of these services operates however on given routes. The *fixed-route taxi* instead starts and finishes at fixed and pre-determined taxi-terminals and uses fixed routes. Each car waits for a full load of passengers prior to departing. All passengers board the taxi together at one terminal and pay for a ride to a different fixed terminal. The service can hence be called a point-to-point service, with the exception that passenger can request the driver to stop in order to alight before the destination if they wish to do so. Boarding new passengers en-route is however not permitted by the service regulators. This ensures that the service is not delayed en-route. The routes and the number of taxis assigned to each “line” are adjusted several times per year and there is no real-time control and adjustment regarding the routes and the fleet size.

Figure 1 shows some of the vehicles at different types of terminals. Vehicles do not operate according to a schedule but wait until they are full. However, in most places they fill up quickly so that they provide frequent services and therefore the expected waiting time is in most cases less than for conventional public transport. On some high demand routes in Iranian cities we can observe taxi frequencies of several vehicles departing every minute. For example, in Zanzan city, the dispatch rate is around 200 taxis per hour on some routes. This indicates that the arrival of passengers, and hence departures of vehicles, can be assumed uniform which is a key assumption in our subsequent model.

We note further that long queues are rarely observed at terminals as the supply matches or exceeds demand (i.e. there is no capacity constraint issue). Given this and given that the service is not too dispersed the service operates hence almost continuously with few waiting times. Moreover, as there are no intermediate boarding points to allow transferring between two different lines and because there are no shared platforms for different lines at the terminals, the common-lines problem does not exist in this system. As a result, passengers of a specific OD mainly follow the shortest paths, that is they take those combination of taxi routes that bring them fastest to their destinations.

Besides the terminal restrictions, the Iranian transport authority imposes constraints on the fares. In general the taxi fares are fixed and non-negotiable. The fixed-route taxi fare is calculated mostly based on annual costs of operating and maintaining a taxi vehicle plus the annual income of a driver. Accordingly, in line with regional average income and living costs, also the taxi fare varies between cities. For example, the average fixed-route taxi fare in Tehran is 2000 IRR/km for each passenger but 1500 IRR/km in Zanzan. For comparison, the fare of fixed-route (shared) taxi is around 0.15 of the non-shared taxi fares.

In summary, the system has some obvious advantages and disadvantage. On the positive, for passengers on-board, the route taken to the terminal will be the shortest and the driver is not permitted to seek for other passengers, meaning that travel times do not increase because of detours and waiting times at stops. On the negative, it means that boarding points are limited and hence passengers must walk to the nearest taxi terminal (although there might be taxis closer to him on the road with vacant seats if some passengers alighted before the terminal). From an operator point of view, an advantage is the simplicity as there is no concern about schedule delays or bunching effects because of passenger queues at intermediate stops. The system is further currently mostly operated so that taxis travel forth and back between the same terminals. This clearly leads to problems if the demand is unbalanced. One possible solution to this problem discussed in this paper is that taxis operate tours between a number of terminals. To keep the system (with its advantages) close to the current operation we assume however that stops on the tour remain to be terminals. That is, all passengers have to alight and board (and pay) again at terminals.

In many cities this system is nowadays expanding in areas that are inadequately served by other public transit systems. Travel surveys in different cities of Iran show that on average, more than 10% of all passengers choose fixed-route taxis in their daily trips [17–20]. Further, a stated preference survey in Zanzan, the case study chosen later in this paper, shows that most passengers currently traveling by their own car to the CBD would change to fixed-route taxi when new guidelines restrict private vehicle access for entering the CBD.

Furthermore, because of its growth in market share and fleet size there is a need for the provision of route planning. The current approach to design the routes of this type of taxi in Iran is mainly by try and error procedure. The interest of decision makers is mainly toward selecting lines with more passengers and avoiding lines with less demand. This paper proposes a route network design model for the planning of the fixed-route taxi services. In line with the described service characteristics we retain the assumption that the vehicles only depart when all seats are occupied. Therefore an important trade-off is the aforementioned dilemma between providing too many, but infrequent, versus frequent, but few, lines. The model therefore minimizes the total passengers' travel time while controlling for maximum allowable transfers between ODs, maximum number of taxi lines and maximum allowable length of a taxi line.

The rest of the paper is organized as follows. Section 3 reviews the relevant taxi and public transport network and route design literature to clarify the main contribution of the paper. Section 4 introduces the problem of taxi service optimization through clarifying the main ideas behind the proposed taxi route design problem (TXRNDP) and describing the theoretical concepts for the model development. Section 5 proposes a mixed-integer linear programming model for the TXRNDP. Section 6 presents a simple method for determination of taxi fleet size for two-way taxi lines which is the conventional method in Iran. This section also presents a linear integer programming model (called MTFSP) to obviate the major shortcoming of the current conventional method. Section 7 presents the application results for the case study of Zanzan city. Finally, Section 8 concludes the paper and provides suggestions for further research in the field of fixed-route taxi service planning and operation.

3. TAXI ROUTE AND TRANSIT NETWORK OPTIMIZATION LITERATURE

Optimization of conventional taxi systems has been a point of interest for many researchers. The conventional framework for most studies involves limiting the passengers' waiting time as well as the number of taxis deployed, and increasing the revenue in a regulated market. Because the taxi literature however is based on different service characteristic than those outlined in previous section more relevant for our problem is literature on optimization of public transport services where vehicles also operate between fixed terminals.

There are a number of studies attempting to formulate the semi-flexible route design problem. Most of these studies have tried to derive a relationship between a (usually rectangular) service area and the amount of slack time by minimizing the total costs from different perspectives or by maximizing the overall benefit. For example, Fu [21] proposes a mathematical programming model which minimizes the cost elements associated with the operator and users minus the service benefit formulated by a single objective function. In Fu's study the service design problem is to determine the optimal amount of slack time, i.e. the difference between the expected route time and the direct running time that should be allocated to a service route. The service route should serve a number of fixed (and scheduled) stops and, if called, deviates to other stops, the number of which are constrained. There are a number of studies methodologically following the lines of Fu's work, e.g. Quadrifoglio et al. [22], Zhao and Dessouky [23] and Alshalalfah and Shalaby [24]. There is further another set of studies which try to optimally determine the dimension of the service area covered by the flex-route service and the maximum allowable diversions from the basic route (e.g. [25, 26]). Good literature reviews on the route design problem of semi-flexible services can be found in Koffman [27], Potts *et al.* [28] and Errico *et al.* [6]. We note, however, that, to the best of our knowledge, none of this literature is directly transferable to the problem discussed in this paper because of the aforementioned service characteristics.

Transit network design usually refers to setting routes and frequencies for transit services. In this sense, the TXRNDP presented here can be considered a special case of such a design problem. Because the taxis do not have intermediate stops the identification of routes collapses to the determination of a set of feasible origin-destination pairs (terminals). Assuming that taxis travel on shortest paths the terminals plus the routes taken by the taxis then span the "taxi route network". Clearly our problem at hand is simpler than the classic transit network design reviewed in the following. As this review section and the following methodology will show we aim to utilize this simplification for our solution method.

One of the main challenges in designing public transport networks has been the problem complexity because of multiple objectives, operational constraints and various factors affecting passengers' behavior. Given this complexity, in general, there are two main methodological approaches to the public transport network design problem: (i) heuristics or meta-heuristics, and (ii) analytical or mathematical programming formulations [29]. The main idea behind studies using the first approach has been to generate a number of reasonable candidate routes and then to select the best possible (or optimal) configuration set of the routes (see e.g. [30–47]; and [48]).

The latter approach has been built on adopting deterministic algorithms to achieve exact solutions, yet with simplifying assumptions regarding the operational constraints and passengers' behavior (see e.g. [49–54]; and [55]). More generally, Baaj and Mahmassani [56] discuss the problems associated with complex, non linear programming models for network design such as computation times and the possibility of solutions being trapped in local minima.

Although we refer to “taxi routes” we emphasize, in line with our discussion in Section 2, that the taxi related literature, is based on different service characteristics than those for the fixed-route taxi system dealt with in this paper. Because conventional taxis do not operate on fixed routes and a single passenger is sufficient for a vehicle to depart, most studies focus on the demand and supply interaction and resulting service characteristics such as waiting time for passengers and a. For this the basic assumptions that the demand for taxi services is a decreasing function of the expected fare and the expected traveler waiting time is employed. In such systems, the expected traveler waiting time decreases with the total vacant taxi-hours and the cost of operating a taxi per unit time is a constant (see e.g. [57–59]; [60]; [44, 61, 62]; and [63–66]). In contrast, fare setting, passenger behavior in markets with insufficient supply or vacant-taxi hours are not of concern for this study.

4. PROBLEM ILLUSTRATION AND CONCEPTUAL FRAMEWORK

To ensure reasonable computational times and practical applicability the present study seeks to use a linear, mixed integer programming formulation to solve the TXRNDP. We emphasize that the non-existence of intermediate stops simplifies the problem at hand. Our model can hence be considered a (far) variant of the Minimum Cost Multi-commodity Flow Problem (see Tomlin [67], Kennington [68], and Ouorou et al. [69]). To allow such a formulation we further have to make another simplification: We avoid an explicit incorporation of waiting times in the model by reasoning that by bounding the number of lines we can do so because of the extremely high service frequency as discussed before and illustrated later with the case study. In line with our observations, we assume further that supply and demand are matched so that there would be no queues at terminals. This is a more discussable assumption, although we believe is acceptable as demand is fairly regular between days and taxi operators usually adjust the service frequency quickly to demand changes.

In order to achieve a high frequency network with acceptable network coverage we propose the four-step framework illustrated in Figure 2. The main idea is to limit the number of taxi lines while satisfying the taxi demand between all origin–destination (OD) pairs with acceptable levels of comfort and travel times. In Step 1 the traffic analysis zones (TAZs) are aggregated into main zones in order to avoid ODs with low demand as well as the number of terminals. The second step is the key part of the model that is described in the next section and limits the number of route sections. If this step is not performed a city with n zones would require $n(n - 1)$ fixed route taxi lines because there are no intermediate stops. Step 3 converts the results into flows. Finally, after the determination of taxi lines between the main zones, a number of intra-zonal lines should be designed to provide suitable coverage over each main zone area and to connect different parts of the main zone to the taxi terminal located inside it. We note that some characteristics of the fixed-route taxis operating in intra-zonal lines are different from those operating between main zones; for example, on intra-zonal lines taxis can pick up passengers en-route unlimitedly. They mostly act as a feeder of main transit lines (see e.g. [70], and [71]). Although the “pre-” and “post-analysis” Steps 1 and 4 are important, in this paper we focus on the methodology that can be used for Steps 2 and 3.

To illustrate the purpose of Step 2, consider a city with four zones under two scenarios. In scenario (A) all zones are directly connected, i.e. there are $n = 6$ taxi lines as shown in Figure 3a. In (B) the total number of taxi lines is reduced to $n' = 3$ (see Figure 3c). Solid lines in 3a and c show the operated

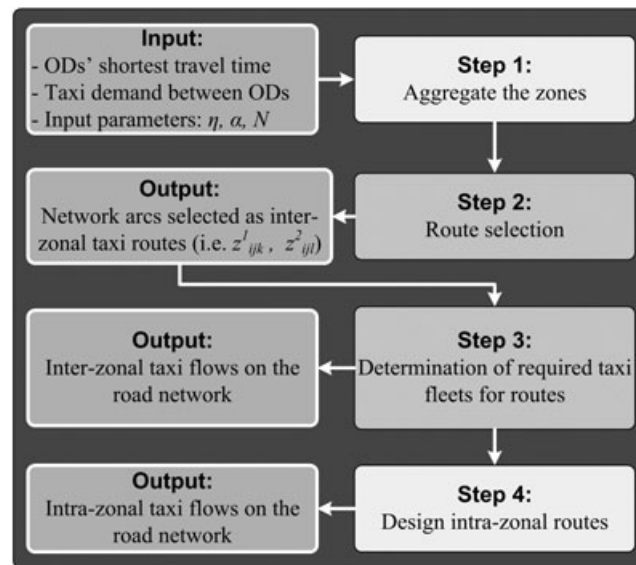


Figure 2. Conceptual framework for fixed taxi route network design problem. Focus of this paper is on Steps 2 and 3.

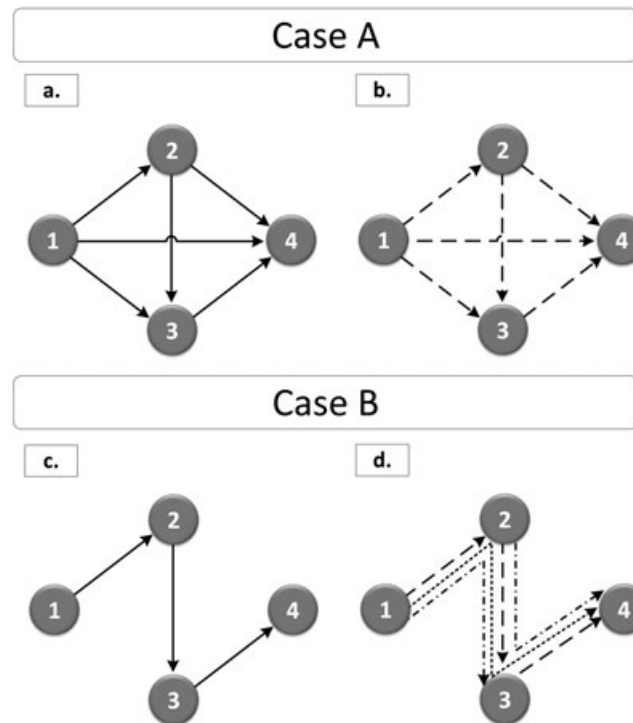


Figure 3. Illustration of taxi lines and resulting passenger trips.

routes. Figures 3b and d show the resulting passenger trips. In Case A there are only direct trips; instead, in Case B, passengers have to transfer. Assume further that there is a demand of 4 passengers/min from for OD pairs (1,2), (1,3), (2,3), (2,4) and (3,4). Table I then shows the resulting passenger movements and required service frequencies f_{ij} for line (i,j) which is obtained by

Table I. Example for the taxi lines problem.

	Case A, 6 taxi lines		Case B, 3 taxi lines	
	Passenger flows q_{ij} (pas/min)	f_{ij} (veh/min)	Passenger flows q_{ij} (pas/min)	f_{ij} (veh/min)
Line (1→2)	4	1	12	3
Line (1→3)	4	1		
Line (1→4)	4	1		
Line (2→3)	4	1	16	4
Line (2→4)	4	1		
Line (3→4)	4	1	12	3
Number of movements	24		40	
Evaluation criteria				
Transfer ratio α	$\alpha = 24/24 = 1$		$\alpha = 40/24 = 1.67$	
W	12 min		6 min	
T	24 t		40 t	
N	6 t		10 t	

$$f_{ij} = q_{ij}/k \quad (1)$$

where q_{ij} denotes the flow on a taxi line between nodes (zones or terminals), i and j , and k the taxi capacity which is assumed to be four passengers per vehicle for all lines. The table also includes the *transfer ratio* α , defined as the total passenger movements divided by total passenger trips. The higher the ratio, the larger the total demands for taxi lines. We are further evaluating the effect of reducing the number of taxi lines on the total passenger waiting time W , the on-board travel time T as well as the total fleet size FS . Following our assumptions of uniform passenger arrivals and regular demand-matched service departures, we obtain:

$$W = \sum_{ij} \left(q_{ij} \frac{0.5}{f_{ij}} \right) \quad (2)$$

$$T = \sum_{ij} q_{ij} t_{ij} \quad (3)$$

$$FS = \sum_{ij} f_{ij} t_{ij} \quad (4)$$

where t_{ij} denotes the travel time on a taxi line between nodes i and j . The last three rows in Table I show these indices for the two scenarios. The travel time of all lines are assumed to be identical and equal to t . This example shows that reducing the number of taxi lines has decreased the passengers' waiting time by half but increased the fleet size, the passengers total travel time as well as the transfer ratio. This "paradoxical" result regarding the effect of lines on waiting times is a direct consequence of the service characteristic that taxis wait at the terminals until the vehicles are full. Therefore more bundled demand means more vehicle departures and less waiting times.

In line with this small example that illustrates the conflicting evaluation criteria, we seek to propose a model for designing a fixed-route taxi network, aimed at minimizing the total travel time within a network while limiting the number of lines (denoted by N) as well as the transfer ratio. The following section proposes a solution for this problem. In the formulation we aim to minimize T given constraints on α and N . We do not include waiting time W in the problem as this is determined only subsequently by the line flows obtained in Step 3 of the framework shown in Figure 2. Instead we only obtain the zones that should be connected. This formulation without waiting time allows us to formulate the problem as a mixed integer problem.

5. FORMULATION OF THE ROUTE SELECTION PROBLEM

Two further detailed specifications of the proposed model are as follows: First, it is ensured that each OD pair is connected with at most 2 transfers to limit the inconvenience for passengers. Second, passengers have to decide for a specific line at each terminal and are assumed to take the shortest path. In other words, the common (attractive) lines problem is not an issue here. Given the layout of typical terminals this is a reasonable assumption. In addition to previous notation we introduce the following variables and parameters:

Sets/indices:

$I = \{1, 2, 3, \dots\}$ Set of zones (terminals), indexed by i, j, k, l .

Input variables and parameters:

\tilde{d}_{ij} Modeled demand between i and j
 τ_{ij} Shortest travel time from i to j in the (congested) network in minutes
 θ Average transfer time between two taxi lines.
 $\bar{\eta}$ Allowable ratio of travel time over shortest travel time for the same OD pair
 $\bar{\alpha}$ Allowable (maximum) transfer ratio
 \bar{N} Maximum number of taxi lines
 M Large number.
 δ_{ij} 1 if $i=j$, else 0

Each taxi line is characterized by terminals located in different zones. In line with current operation in Iran, we presume that taxis travel forth and back on the same line so that for our problem only the demand for the direction with larger demand is of interest. We therefore transform the demand matrix \mathbf{D} into an upper triangle matrix $\tilde{\mathbf{D}}$ with:

$$\tilde{d}_{ij} = \begin{cases} \max\{d_{ij}, d_{ji}\} & \text{if } i > j \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

Route travel times as well as θ , $\bar{\eta}$, $\bar{\alpha}$ are used as further input parameters to our problem. θ is used to account for the inconvenience when having to make a transfer. As a default value we suggest a penalty equivalent to 5-min travel time. In addition there are three sets of decision variables:

Decision variables:

x_{ij} binary variables to define whether there is a taxi line or not between zones i and j
 t_{ij} resulting travel time in designed taxi network from terminals i to j .
 z_{ijk}^p indicates whether k is the p^{th} transfer node when traveling from i to j

Because we presume that there can be no more than two transfers we have z_{ijk}^1 and z_{ijl}^2 which are defined as:

$$z_{ijk}^p = \begin{cases} 1 & \text{if } k \text{ is the } p^{\text{th}} \text{ transfer point on the shortest path from } i \text{ to } j \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

For computational convenience, we define that there will be one k for which the term takes on the value 1 for each pair $\{i, j\}$. In case terminals are connected with less than 2 transfers k will take on values of the origin and/or terminal. For example, as shown in Figure 4a, if there is no transfer in the taxi network connecting nodes 5 and 9, then $z_{5,9,5}^1 = 1$ and $z_{5,9,9}^1 = 1$. The dashed lines indicate that there is no transfer between two successive nodes. If the model solution is one transfer at say node 3, $z_{5,9,3}^1 = 1$ and $z_{5,9,9}^1 = 1$ (Figure 4b). If the model solution includes an additional transfer then also z^2 takes a value of 1 for a k different to j , i.e. in below with a second transfer at node 6, $z_{5,9,3}^1 = 1$ and $z_{5,9,6}^2 = 1$ (Figure 4c). Therefore, after solving the route selection problem, one can obtain the number

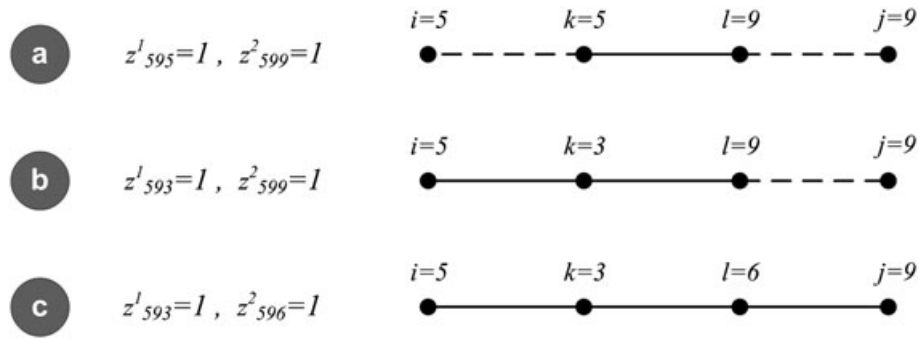


Figure 4. Example for possible states of z_{ijk}^p . In a) terminals {5,9} are connected directly, in b) with one transfer at node 3 and in c) with two transfers at nodes 3 and then 6.

of directly connected terminals with $\sum_{ij|i < j} z_{iji}^1$, the number of terminals connected via exactly one transfer with $\sum_{ij|i < j} z_{ijj}^2 - \sum_{ij|i < j} z_{iji}^1$ and the number of terminals connected by two transfers with $\frac{(|I|-1) \times |I|}{2} - \sum_{ij|i < j} z_{ijj}^2$.

With these assumptions and definitions the route selection problem can then be formulated with the following mathematical programming model:

$$\text{Min} \sum_i \sum_j \left\{ \tilde{d}_{ij} \left(t_{ij} + \sum_k z_{ijk}^1 (1 - \delta_{ik}) \theta + \sum_l z_{ijl}^2 (1 - \delta_{lj}) \theta \right) \right\}. \quad (7)$$

Connectivity constraints

$$z_{ijk}^1 \leq x_{ik} \quad \forall i, j | i < j, k \in I \quad (8)$$

$$z_{ijl}^2 \leq x_{lj} \quad \forall i, j | i < j, l \in I \quad (9)$$

$$(2 - z_{ijk}^1 - z_{ijl}^2) M + x_{kl} \geq 1 \quad \forall i, j | i < j, k, l \in I \quad (10)$$

$$\sum_k z_{ijk}^1 = 1 \quad \forall i, j | i < j \in I \quad (11)$$

$$\sum_l z_{ijl}^2 = 1 \quad \forall i, j | i < j \in I \quad (12)$$

$$1 - z_{iji}^1 + z_{ijj}^2 > 0 \quad \forall i, j | i < j \in I \quad (13)$$

$$x_{kl} = x_{lk} \quad \forall l, k \in I \quad (14)$$

Travel time constraints

$$\sum_i \sum_{j \neq i} x_{ij} \leq 2\bar{N} \quad (15)$$

$$(2 - z_{ijk}^1 - z_{ijl}^2)M + t_{ij} \geq \tau_{ik} + \tau_{kl} + \tau_{lj} \quad \forall i, j | i < j, k, l \in I \quad (16)$$

$$t_{ij} \leq (1 + \bar{\eta})\tau_{ij} \quad \forall i, j | i < j \in I \quad (17)$$

$$\sum_i \sum_j \left\{ \tilde{d}_{ij} \left(\sum_k z_{ijk}^1 (1 - \delta_{ik}) + \sum_l z_{ijl}^2 (1 - \delta_{lj}) \right) \right\} \leq (\bar{\alpha} - 1) \sum_i \sum_j \tilde{d}_{ij} \quad (18)$$

Solution space constraints

$$t_{ij} \geq 0 \quad \forall i, j \in I \quad (19)$$

$$z_{ijk}^1, z_{ijl}^2, x_{ij} \in \{0, 1\} \quad \forall i, j, k, l \in I \quad (20)$$

The objective function (7) minimizes total travel time plus transfer penalty of taxi users. The terms $(1 - \delta_{ik})$ and $(1 - \delta_{lj})$ in the objective function are used to obtain the number of transfers made by passengers traveling from i to j . If there is a transfer the predefined transfer penalty θ is added.

The first six constraints, (8) to (13), ensure that all terminals are connected with at most two transfers. Constraint (8) guarantees that each node k cannot be the first transfer point on a path from i to j unless line (i, k) is selected. Constraint (9) does the same for the second transfer points. Constraint (10) ensures that if k and l are the first and second transfer points of the selected path from i to j then x_{kl} equals 1. In case both z_{ijk}^1 and z_{ijl}^2 become the constraint will reduce to $x_{kl} \geq 1$ and hence x_{kl} must be 1. Otherwise, if z_{ijk}^1 or z_{ijl}^2 become 0 the constraint will be reduced to $2M + x_{kl} \geq 1$ or $M + x_{kl} \geq 1$ and x_{kl} can optionally equal 0 or 1, because M or $2M$ is always greater than one. (If other terminal OD pairs force x_{kl} to be 1 then the above inequalities will not impose any conflict.)

Constraints (11) and (12) have been discussed above. Constraint ensures that if there is exactly one actual transfer point, i , the virtual transfer will be assigned to the destination point, j ; in this case, we have $z_{iji}^1 = 0$ and $z_{ijj}^2 = 1$ and hence $z_{ijk}^1 = 1$ must be true for one $k \neq i$, i.e. the actual transfer node. It is to be noted that omitting this constraint yields the same solution in terms of x_{ij} and t_{ij} ; it only supports the unique identification of the first transfer nodes from z_{ijk}^1 (and not z_{ijk}^2).

Constraint (14) ensures that all selected lines will be two-way lines. As discussed earlier, the model assumes an upper triangular demand matrix. Therefore, the starting path is established for $i < j$ and the returning path $j < i$ is assumed to be the same.

Constraint (15) limits the total number of taxi lines in order to ensure sufficient service frequency and low waiting times. Therefore, although we do not include waiting time into the objective function we indirectly control for it through this constraint. Constraint (16) obtains the travel time of selected paths between ODs. Similar to (10), in cases in which both z_{ijk}^1 and z_{ijl}^2 take the value of 1 the constraint becomes active as it reduces then to $t_{ij} \geq \tau_{ik} + \tau_{kl} + \tau_{lj}$. The minimization of the objective function will then ensure equality for these cases. Constraint (17) controls for maximum allowable increase in travel times of selected routes relative to the shortest (fastest) path. Passenger travel time is a function of on-board travel time between zones plus transfer times. Maximum allowable increase in travel time is achieved by applying $\bar{\eta}$ to the shortest possible travel time between each OD pair. Constraint (18) enforces the total number of transfers not to be more than $100(\bar{\alpha} - 1)$ percent of all trips. Finally, constraints (19) and (20) ensure that the decision variables remain within their respective solution space.

Clearly not every combination of $\bar{\alpha}$, $\bar{\eta}$ and \bar{N} will provide a feasible solution. To achieve a feasible solution for specific quantities of two parameters the third parameter can be first set to a desired small value, then if there exists no feasible solution this parameter should be increased (relaxed) until a reasonable solution is found. As a limiting case for the maximum number of taxi lines, clearly relaxing $\bar{N} = n(n - 1)$ will certainly result in a feasible solution. The model outputs are taxi paths between each OD pair which can be determined after the determination of the first and second transfer points (i.e. those k and l for which the values of z_{ijk}^1 and z_{ijl}^2 become 1) and selected lines (i.e. $x_{ij} = 1$).

The problem forms a mixed integer linear programming model that can be solved by existing commercial software such as GAMS/CPLEX. This facilitates practitioners to solve this route design problem easily by finding the global optimal solutions of the problem. We note however that the number of constraints in the problem is large, especially the set of constraints (10) and (16). If this forms a problem we suggest to introduce new decision variables for travel times via transfer points and to replace (16) with below two constraints:

$$M(1 - z_{ijk}^1) + \hat{t}_{ijk} \geq \tau_{ik} + \sum_l (\tau_{kl} + \tau_{lj}) z_{ijl}^2 \quad \forall i < j, k \in I \quad (21)$$

$$t_{ij} \geq \hat{t}_{ijk} \quad \forall i < j, k \in I \quad (22)$$

where \hat{t}_{ijk} denotes the travel time from i to j if k is selected as a first transfer point; otherwise zero.

This new formulation, although requiring an additional set of decision variables, reduces the $\frac{(|I|-1) \times |I|}{2} \times |I| \times |I|$ constraints of (16) with instead $\frac{(|I|-1) \times |I|}{2} \times |I| \times 2$ constraints.

It should be noted that, the above taxi route design problem uses a predetermined and fixed OD demand matrix. Clearly, in reality, a user can choose between different modes of transport serving a complex transportation system, based on the utilities offered to him/her by different modes. In general, the utility of each mode is a function of travel time. Therefore, the actual proportion of trips attracted to the taxi lines cannot be determined unless the travel time by the taxi routes between each OD pair is determined. To deal with this problem, we suggest solving the proposed model and the mode choice model iteratively until reaching a convergence.

6. DETERMINING THE FLEET SIZE FOR OD PAIRS

In this section, we first explain the common practice of determining the fixed-route taxi fleet size in Iran, and then propose an alternative method to obviate the major shortcoming of the current conventional method that can lead to overestimation of fleet size and un-needed vacant taxi-hours.

After determining the optimal taxi lines between main zones, the number of required taxis should be computed in order to meet the fixed demand while minimizing total vacant taxi-hours. To accomplish this, the minimum required two-way frequency is calculated with respect to the line demand determined in previous section. This is achieved with following steps:

- 1 Given passenger demand matrix $\tilde{D}[\tilde{d}_{ij}]$, obtain passenger flow matrix $Q[q_{ij}]$:
 - 1.1 Set $q_{ik} = q_{kl} = q_{lj} = 0$.
 - 1.2 For i, j, k, l from 1 to $|I|$,
 If $z_{ijk}^1 = z_{ijl}^2 = 1$ then: $q_{ik} \leftarrow q_{ik} + \tilde{d}_{ij}$, $q_{kl} \leftarrow q_{kl} + \tilde{d}_{ij}$ and $q_{lj} \leftarrow q_{lj} + \tilde{d}_{ij}$.
- 2 Determine the one-way frequency for each line (i, j) : $\tilde{f}_{ji} = q_{ij}/k$, where k is the capacity of each taxi.
- 3 Determine the two-way frequency for each line (i, j) and (j, i) : $\tilde{f}_{ij} = \tilde{f}_{ji} = \max\{\tilde{f}_{ji}, \tilde{f}_{ji}\}$.
- 4 Obtain the number of required taxis per line: $\tilde{n}_{ji} = \tilde{f}_{ji} \frac{t_{ij} + t_{ji}}{60}$.
- 5 Obtain the total number of required taxis: $\tilde{FS} = \sum_{i < j} \tilde{n}_{ij}$.

For illustration, consider the three node network in Figure 5. Suppose Figure 5c has been identified with the methodology described above and example travel times and required service frequencies obtained with Step 1 in this section are given. Steps 3 to 5 then lead to the number of required taxis for each taxi line and a total required taxi fleet of 135 vehicles:

$$\begin{aligned} \tilde{f}_{12} = \tilde{f}_{21} &= \max\{\tilde{f}_{12}, \tilde{f}_{21}\} = 161 \text{ (veh/hr)} \\ \tilde{n}_{12} &= 161 \times \frac{6 + 19}{60} = 67; \tilde{n}_{23} = 79 \times \frac{18 + 8}{60} = 34; \tilde{n}_{34} = 136 \times \frac{7 + 8}{60} = 34 \\ \tilde{FS} &= \sum_{i < j} \tilde{n}_{ij} = 135. \end{aligned}$$

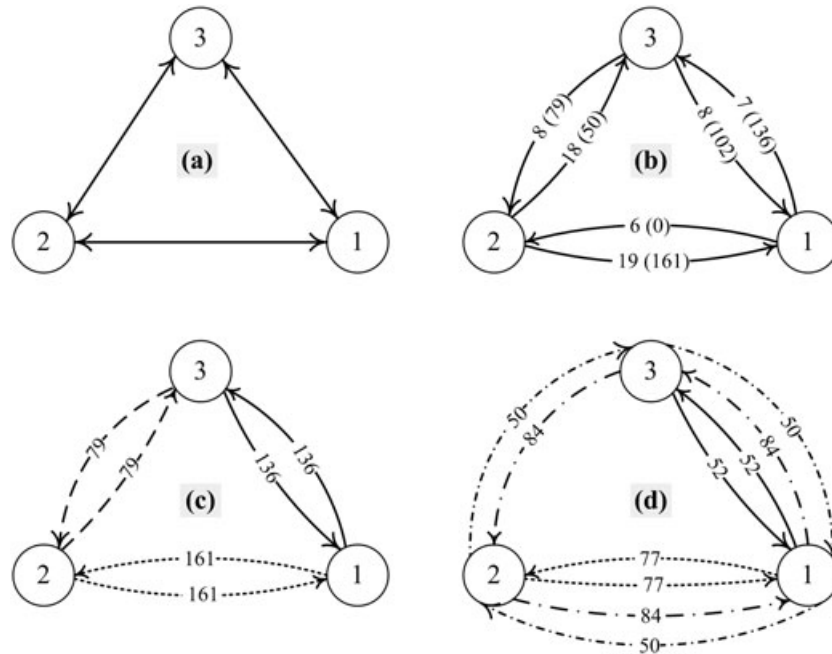


Figure 5. A simple fixed-route taxi network; (a): a graph representation of taxi lines; (b) travel times and required one-way frequencies (presented in parentheses); (c) taxi trips and two-way frequencies calculated from the conventional method; (d) taxi tour approach for reducing fleet size.

We remind that the above method does not utilize possible taxi tours, i.e. that taxis could travel from 1 to 2 and some of them return to 1 whereas others continue to 3 in line with operational characteristics of this type of service in Iran. Obviously, the more unbalanced the demand between the two terminals of a line or the more asymmetric the travel times, the more efficiency could be gained by introducing chains. We propose that by the use of a set of taxi tours that provide the closest possible \bar{f}_{ij} to the above one-way \bar{f}_{ij} quantities, excessive taxis produced by the above $\max\{\bar{f}_{ij}, \bar{f}_{ji}\}$ function can be avoided. This is illustrated in Figure 5d. The number of taxis for each taxi tour can be calculated as follows:

- Taxi Tour #1 (1) \rightarrow (3) \rightarrow (1): $52 \times \frac{7+8}{60} = 13$
 Taxi Tour #2 (1) \rightarrow (2) \rightarrow (1): $77 \times \frac{6+19}{60} = 32$
 Taxi Tour #3 (1) \rightarrow (3) \rightarrow (2) \rightarrow (1): $84 \times \frac{7+8+19}{60} = 48$
 Taxi Tour #4 (1) \rightarrow (2) \rightarrow (3) \rightarrow (1): $50 \times \frac{6+18+8}{60} = 27$

In total we obtain thence that $\hat{FS} = 120$ are needed which reduces the taxi fleet by 15 taxis compared to above conventional praxis. Based on this idea, the minimum taxi fleet size problem (MTFSP) can now be proposed as the following mix-integer linear programming model.

Sets/indices:

H Set of taxi tours, indexed by h .

Input variables and parameters:

- \bar{f}_{ij} One-way frequency \bar{f}_{ij} required online (i, j)
 τ_{ij} Shortest travel time from i to j in the (congested) network in minutes
 ξ_{ij} 1 if line (i, j) $i \neq j$ belongs to the set of lines determined through the TXRNDP.
 t_0 Maximum allowable time duration for each taxi tour
 m_0 Maximum number of taxi tours that can serve a specific line
 M Large number
 Decision variables:

\hat{f}_{ij}^h The one-way frequency of taxi tours h on line (i, j)
 y_{ij}^h 1 if taxi tour h traverses line (i, j) , otherwise 0.

$$\text{Min } \hat{F}S = \sum_i \sum_j \sum_h \hat{f}_{ij}^h \times \frac{\tau_{ij}}{60} \quad (23)$$

$$\sum_h \hat{f}_{ij}^h \geq \bar{f}_{ij} \quad \forall i, j \in I \quad (24)$$

$$\sum_i \hat{f}_{ik}^h - \sum_j \hat{f}_{kj}^h = 0 \quad \forall h \in H, k \in I \quad (25)$$

$$\sum_j y_{ij}^h \leq 1 \quad \forall i \in I, h \in H \quad (26)$$

$$y_{ij}^h \cdot M \geq \hat{f}_{ij}^h \quad \forall i, j \in I, h \in H \quad (27)$$

$$\sum_i \sum_j y_{ij}^h \tau_{ij} \leq t_0 \quad \forall h \in H \quad (28)$$

$$y_{ij}^h \leq \zeta_{ij} \quad \forall i, j \in I \quad (29)$$

$$\sum_h y_{ij}^h \leq m_0 \quad \forall i, j \in I \quad (30)$$

$$\hat{f}_{ij}^h \geq 0 \quad \forall i, j \in I, h \in H \quad (31)$$

$$y_{ij}^h \in \{0, 1\} \quad \forall i, j \in I, h \in H \quad (32)$$

As default values we set $t_0 = 60$ min and $m_0 = 5$. Similar to the TXRNDP model, the MTFSP model is a linear model that can globally be solved by available optimization software. Objective function (23) minimizes the fleet size, which is the sum of the optimal number of taxis working on different taxi tours. Constraint (24) is designated to satisfy the previously obtained required one-way frequencies on lines. Constraints (25) and (26) together guarantee the continuity of taxi trip chains, while restricting multiple exits of taxis belonging to a particular taxi tour from each node. Constraint (27) relates continuity variables y_{ij}^h and the optimal one-way frequencies \hat{f}_{ij}^h , so that if a particular line is not selected as a route for a specific taxi tour, the one-way frequency of the tour on that line must be zero. Constraint (28) limits each taxi tour to be traveled within a pre-determined time interval. Constraint (29) ensures that the resulting tours accommodate the lines determined through the TXRNDP model. (30) restrains the number of taxi trip tours that can serve a special line as too many lines serving the same route might be undesirable from an operators' point of view. Constraints (31) and (32) ensure the correct sign and the binary state of decision variables.

In fact, the proposed MTFSP model can be accounted as a new variant of the Vehicle Routing Problem (VRP). The MTFSP does not include the loop controlling constraints and therefore can be solved in reasonable CPU times. The reason that the loop controlling constraints are eliminated is that here taxis are not labeled to belong to certain groups making the same continuous tours, i.e. the MTFSP does not include vehicle assignment to tours. Instead, we only require a set of feasible continuous trip chains (loops) so that the frequency of each one-way line is met.

7. CASE STUDY: CITY OF ZANJAN

The case study illustrates the methods introduced in this paper by adopting it to the city of Zanjan, in Northwestern Iran. The city has a residential population of 0.4 million and is spread over an area of 60 km².

Zanjan is disaggregated into 150 TAZs by a previously established transportation master plan. We have aggregated these into 17 main zones as shown in Figure 6 which reflects the location of the taxi terminals. Trip survey data further show a modal share of 36.7% for fixed line taxis. There are 41 000 taxi trips in the morning peak hour of the year 2011, of which 33 570 trips are inter-zonal and 7430 trips are intra-zonal (within main zones). Link travel time data have been taken from the result of user equilibrium assignment performed by VISUM software and the OD matrix is estimated for morning peak hour based on a household OD survey calibrated with traffic volumes collected in May 2011.

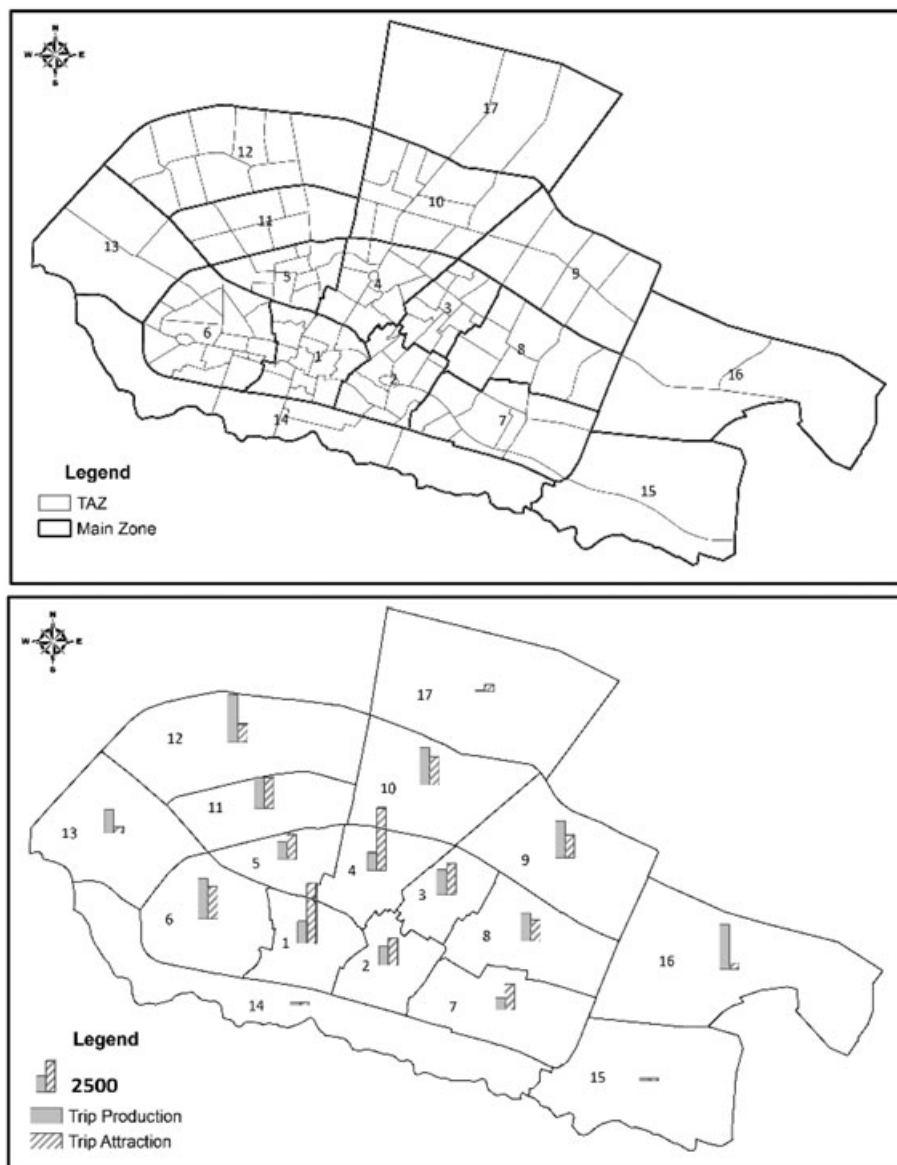


Figure 6. Traffic analysis zones (up) and taxi trips generated (down).

Table II illustrates the model results for different thresholds \bar{N} , $\bar{\eta}$ and $\bar{\alpha}$. Infeasible solutions resulting from too stringent parameter settings have been omitted. The transfer penalty parameter θ is assumed to be 5 min and it is assumed that all taxis are minibuses with a maximum capacity of $k=10$.

In Table II, the first row with 136 taxi lines reflects the current situation where there are taxi lines between all 17 terminals. The remaining rows show solutions for different feasible constraint settings. The resulting estimates of fleet size, waiting time and travel times have been obtained with the methods described in this paper. Table II also shows that the waiting time is an important part of the total travel time. For example, in the case of $\bar{\eta} = 0.2$ and $\bar{\alpha} = 1.2$, the total waiting time constitutes 38% of the total travel time. The advantage of limiting the number of taxi lines in terms of reducing total waiting times is apparent. In general, if the allowable transfer ratio is increased, the value of the objective function increases and the total waiting time and the number of taxi lines would decrease. If $\bar{\alpha}$ increases, the total travel time in the network decreases. Changes to the objective function because of variations in $\bar{\eta}$ are negligible. To further illustrate the effects of limiting the number of lines, Figure 7 depicts changes in total travel time in the network for more combinations of model parameters. We note the nonlinear relationship if the number of lines exceeds 80.

Judging which solution is optimal from the ones given in Table II will depend on the weight given to the different possible objectives. For illustration, we suggest that one good solution would be: $\bar{N} = 80$, $\bar{\eta} = 0.2$ and $\bar{\alpha} = 1.2$. In this case the waiting time decreases to 59% and the total travel time to 79% of their current values. Results of applying the MTFSP model to this solution show that the total number of taxis can be reduced from 1584 (i.e. the solution of the conventional method of two-way frequency) to 1486, which is approximately 6% smaller. In Figures 8 and 9 resulting network links and taxi flows

Table II. Model solutions for taxi lines using 2011 morning peak data.

\bar{N} (total number of lines)	$\bar{\alpha}$ (transfer ratio)	$\bar{\eta}$ (add. travel time ratio)	Total waiting time (min)	On-link travel time (min)	Total travel time (min)
136	—	—	180 531	173 173	353 704
90	1.2	0.2	119 469	173 695	293 164
90		0.3	119 469	173 730	293 199
80		0.2	106 195	174 124	280 319
80		0.3	106 195	174 133	280 327
70		0.2	92 920	174 836	267 757
70		0.3	92 920	175 058	267 978
60		0.2	79 646	176 173	255 819
60	1.3	0.3	79 646	176 150	255 796

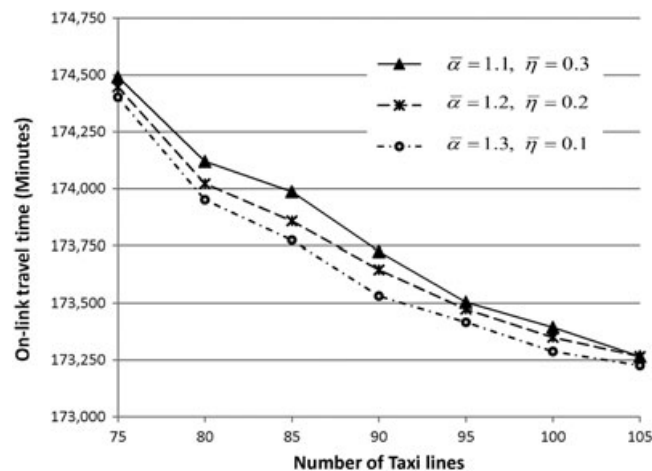


Figure 7. Trend of total travel time versus number of taxi lines.



Figure 8. Resulting network links with taxi lines.



Figure 9. Resulting number of taxis passing per hour.

are illustrated for this solution. The model selects 80 lines, meaning that 80 OD are served without transfers. We further find that 28 ODs are connected by one transfer and for no OD 2 transfers are needed.

To evaluate the efficiency of the model in terms of computation time we used a synthesized network with up to 50 terminals (zones) as a base network for experiments. We then executed the model for different actual numbers of OD pairs with GAMS version 23.5.1 on a system with cpu Intel core™ i5, 2.66 GHz, 4 G RAM. Results show that the computation time increases nonlinearly as the number of ODs increases. The problem is solvable (manageable) for at most 35×35 OD pairs Figure 10.

Finally, as illustrated in the framework (Figure 1), after determination of taxi lines between the main zones a number of intra-zonal lines should be designed to provide suitable coverage over the main zone area and to connect different parts of the main zone to the taxi terminal located within it. In contrast to the inter-zonal lines, these lines are usually shorter and without considerable overlap with other lines of this type. The number of taxis serving each line would be calculated based on the two-way frequency setting method, and is not presented here for brevity.

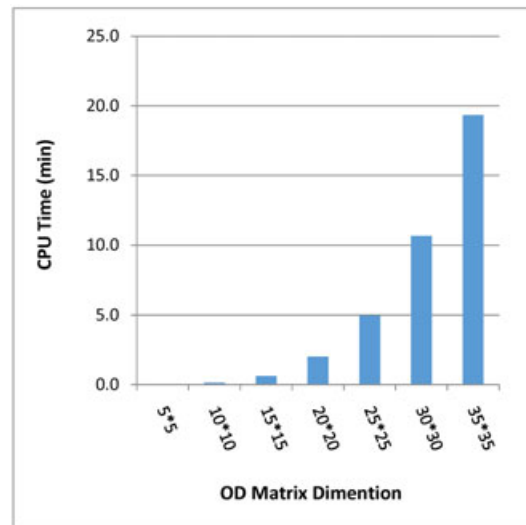


Figure 10. Effort required versus the number of terminals or O–D pairs.

8. CONCLUDING REMARKS AND FUTURE RESEARCH

The paper has proposed a general framework for planning fixed-route taxi services, a problem encountered in some variations in many developing countries. The “taxi route network design problem” (TXRNDP) has been developed to minimize the total travel time experienced within the fixed-route taxi network. The main idea is to reduce the number of taxi lines in order to make use of a main characteristic of the service which is its high service frequency and that all vehicles leave the terminal full. By means of a schematic example and a larger case study, it has been shown that reducing the number of lines can result in significant reductions in total travel and waiting times. The user cost of this service alteration is that it would increase the number of transfers in the network. Therefore, the transfer ratio has been constrained to an upper limit in the proposed model and the trade-off between these constraints illustrated. Specific results of applying the proposed TXRNDP model to Zanjan, Iran, show that if the number of taxi lines is reduced from currently 136 to 80 then the waiting time also decreases to 0.59 of its current value.

The main methodological novelty of this paper is the formulation of the route design problem as a mixed integer problem, whereas previous literature on similar network design problems is based on heuristics. For this to be feasible, we discuss the problem size restrictions and a number of assumptions had to be made especially on waiting and transfer time. Aiming to understand the significance of these assumptions should be topic for further research: In particular, in the proposed model, the transfer time between different lines has been assumed to be a constant quantity (i.e. average transfer time, termed θ) to prevent introducing nonlinearity to the model. If one wants to refine this assumption but remain linearity the analyst could set different θ for routes that are found to operate with high/low frequency and iterate the model until convergence is reached. Further research directions include the explicit inclusion of the operator’s profit in the optimization problem. This would allow considering the operator, passenger and possibly environmental perspective together in a social welfare maximization problem. Another topic for further work would be to deal with demand variation where demand is not stable or predictable.

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