



ANTI-PERIODIC SOLUTIONS FOR NEURAL NETWORKS WITH DELAYS AND IMPULSES

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Abstract- In this paper we investigate a class of artificial neural networks with delays subject to periodic impulses. By exploiting Lyapunov functions, we analyze the global exponential stability of an arbitrary solution with initial value being bounded by γ . Further, we discuss the existence of anti-periodic solutions by constructing fundamental function sequences based on a solution with initial value being bounded by γ . We also establish sufficient conditions to ensure the existence, uniqueness and exponential stability of anti-periodic solutions, which are new and easily verifiable. At last, we present a network with its time-series and phase graphics to demonstrate our results.

Key Words- Artificial neural networks (ANN), Anti-periodic solutions, Delays and Impulses, Exponential stability

1. INTRODUCTION

It is a significant fact that we spend a lot time in investigating artificial neural networks. We explore and study artificial neural networks, and we intend to reproduce products such as Robots, which possess functions similar to human's and can process signals and information. Mathematical models for artificial neural networks have been established. Neural networks, which are described as continuous differential dynamical systems, have been discussed, and there are extensive results about their dynamical behaviors including equilibriums, periodic solutions, anti-periodic solutions and their respective stability ([1],[2],[3],[4],[6],[10],[11]). But, in artificial networks for signal and image processing, the finites switching speed of amplifiers may cause delays in the transmission of signals, in addition, faulty elements may experience abrupt changes of state voltage and therefore the normal transient behaviors in processing signal and information are influenced. Consequently an artificial neural network with delays and impulses should be modeled. The dynamics of autonomous neural networks with delays and impulses have been considered recently, and the research are mainly focus on the existence and stability of an equilibrium state and periodic solution ([5],[7],[8],[19]). To the best of our knowledge, few authors have considered the problems of anti-periodic solutions for non-autonomous delayed networks with impulses. In the neural networks, the performance of the global stable anti-periodic solution may reveal the characteristic and stability of the mode of the signal. Sometimes, the existence and stability of a unique anti-periodic solution is a requirement in designing artificial neural networks, especially the existence of destabilizing factors such as delays and impulses. So we discuss anti-periodic problems on ANNs with delays and impulses. In this paper, we

first consider anti-periodic solutions for the following delayed differential system with impulses:

$$\left\{ \begin{array}{l} x_i'(t) = -c_i(t)x_i(t) + \sum_{j=1}^n a_{ij}(t)g_j(x_j(t-\tau_{ij}(t))) \\ \quad + \sum_{j=1}^n b_{ij}(t)f_j\left(\int_0^\infty K_{ij}(s)x_j(t-s)ds\right) + I_i(t), \quad t \neq t_k, \\ x_i(t_k^+) = (1+d_{ik})x_i(t_k), \quad i=1,2,\dots,n, \quad k=1,2,\dots. \end{array} \right. \quad (1)$$

Here n is the number of units in a neural network, $x_i(t)$ represent the state vector of the i th unit at time t . Functions $c_i(t) > 0$, $I_i(t)$, $a_{ij}(t)$, $b_{ij}(t)$, $\tau_{ij}(t)$, $g_j(t)$, $f_j(t)$, and $K_{ij}(t)$ have the similar meaning to paper [11]. $t_k \geq 0$ is the impulsive moments, and d_{ik} characterizes the impulsive jumps at time t_k for i th unit.

Throughout this paper, we assume the followings.

(H1) For $i, j=1,2,\dots,n$, $I_i, a_{ij}, b_{ij} : R \rightarrow R$ and $c_i, \tau_{ij} : R \rightarrow R^+$ are continuous functions, and there exists a constant $T > 0$ such that

$$\begin{aligned} c_i(t+T) &= c_i(t), \quad a_{ij}(t+T)g_j(u) = -a_{ij}(t)g_j(-u), \quad b_{ij}(t+T)f_j(u) = -b_{ij}(t)f_j(-u), \\ \tau_{ij}(t+T) &= \tau_{ij}(t), \quad I_i(t+T) = -I_i(t). \end{aligned}$$

In addition, we denote

$$I = \max_{1 \leq i \leq n} \{ \max_{0 \leq t \leq T} |I_i(t)| \}, \quad \tau = \max_{1 \leq i, j \leq n} \{ \max_{0 \leq t \leq T} |\tau_{ij}(t)| \}.$$

(H2) The sequence of times t_k ($k \in N$) satisfies $t_k < t_{k+1}$ and $\lim_{k \rightarrow +\infty} t_k = +\infty$, and d_{ik} satisfies $-2 \leq d_{ik} \leq 0$ for $i \in \{1,2,\dots,n\}$ and $k \in N$.

(H3) There exists a $q \in N$ such that

$$d_{i(k+q)} = d_{ik}, \quad t_{k+q} = t_k + T, \quad (k \in N).$$

(H4) For each $j \in \{1,2,\dots,n\}$, the activation functions $g_j, f_j : R \rightarrow R$ are continuous and there exist nonnegative constants L_j and $\bar{L}_j$ such that, for all $u, v \in R$,

$$g_j(0) = f_j(0) = 0, \quad |g_j(u) - g_j(v)| \leq \bar{L}_j |u - v|, \quad |f_j(u) - f_j(v)| \leq L_j |u - v|.$$

(H5) For $i, j \in \{1,2,\dots,n\}$, the delay kernels $K_{ij} : [0, +\infty) \rightarrow R$ are continuous and integrable.

(H6) There exist constants $\eta > 0$ and $\lambda > 0$ such that, for all $t > 0$,

$$(\lambda - c_i(t)) + \sum_{j=1}^n \bar{L}_j |a_{ij}(t)| e^{\lambda t} + \sum_{j=1}^n L_j |b_{ij}(t)| \int_0^\infty |K_{ij}(s)| e^{\lambda s} ds < -\eta < 0 \quad (i=1,2,\dots,n). \quad (2)$$

From inequality (2), we easily know

$$c_i(t) - \sum_{j=1}^n \bar{L}_j |a_{ij}(t)| e^{\lambda t} - \sum_{j=1}^n L_j |b_{ij}(t)| \int_0^\infty |K_{ij}(s)| e^{\lambda s} ds > 0 \quad (i=1,2,\dots,n).$$

For convenience, let $x = (x_1, x_2, \dots, x_n)^T \in R^n$, in which "T" denotes the transposition. We define $|x| = (|x_1|, |x_2|, \dots, |x_n|)^T$ and $\|x\| = \max_{1 \leq i \leq n} \{ |x_i| \}$. Obviously, the solution $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T$ of (1) has component $x_i(t)$, which is piece-wise continuous on $(t_0, +\infty)$. Moreover, $x_i(t)$ is differentiable on the open interval (t_{k-1}, t_k) , and $x_i(t_k^+)$ exists.

2. MAIN RESULTS

Lemma 2.1. Let (H1)-(H6) hold. Assume that $\bar{x}(t) = (\bar{x}_1(t), \bar{x}_2(t), \dots, \bar{x}_n(t))^T$ is a solution of system (1) with initial condition,

$$\bar{x}_i(s) = \bar{\varphi}_i(s), \quad |\bar{\varphi}_i(s)| < \gamma, \quad s \in (-\infty, 0],$$

where $\gamma > \frac{I}{\eta}$ and $i \in \{1, 2, \dots, n\}$. Then

$$|\bar{x}_i(t)| < \gamma \text{ and } |\bar{x}_i(t_k^+)| < \gamma \text{ for all } t > 0, \quad k \in N. \quad (3)$$

Proof. Suppose that (3) does not hold. Since $\bar{x}_i(t_k^+) = (1 + d_{ik})\bar{x}_i(t_k)$ and by (H2), then $|\bar{x}_i(t_k^+)| = |(1 + d_{ik})\bar{x}_i(t_k)| \leq |\bar{x}_i(t_k)|$. So, if $|\bar{x}_i(t_k^+)| > \gamma$, then $|\bar{x}_i(t_k)| > \gamma$. Thus, we may assume that there exist $i \in \{1, 2, \dots, n\}$ and $\sigma \in (t_k, t_{k+1})$ such that

$$|\bar{x}_i(\sigma)| = \gamma \text{ and } |\bar{x}_j(t)| < \gamma \text{ for all } t \in [0, \sigma), \quad j = 1, 2, \dots, n. \quad (4)$$

Calculating the upper left derivative of $|\bar{x}_i(t)|$, together with (H4), (2), (4), and $\gamma > \frac{I}{\eta}$, we have

$$\begin{aligned} 0 &\leq D^+ |x_i(\sigma)| \leq -c_i(\sigma) \bar{x}_i(\sigma) + \sum_{j=1}^n |a_{ij}(\sigma)| \cdot |g_j(\bar{x}_j(\sigma - \tau_{ij}(\sigma)))| \\ &\quad + \sum_{j=1}^n |b_{ij}(\sigma)| \left| f_j \left(\int_0^\infty K_{ij}(s) \bar{x}_j(\sigma - s) ds \right) \right| + |I_i(\sigma)| \\ &\leq -c_i(\sigma) \bar{x}_i(\sigma) + \sum_{j=1}^n \bar{L}_j \gamma |a_{ij}(\sigma)| + \sum_{j=1}^n L_j |b_{ij}(\sigma)| \int_0^\infty |K_{ij}(s)| \cdot |\bar{x}_j(\sigma - s)| ds + |I_i(\sigma)| \\ &\leq -c_i(\sigma) \bar{x}_i(\sigma) + \sum_{j=1}^n \bar{L}_j \gamma |a_{ij}(\sigma)| e^{\lambda \sigma} + \sum_{j=1}^n L_j \gamma |b_{ij}(\sigma)| \int_0^\infty |K_{ij}(s)| e^{\lambda s} ds + |I_i(\sigma)| < 0. \end{aligned}$$

It is a contradiction, which shows that (3) holds. The proof is now completed.

Lemma 2.2. Suppose that (H1)-(H6) are true. And assume that $x^*(t) = (x_1^*(t), x_2^*(t), \dots, x_n^*(t))^T$ is a solution of system (1) with initial value $\varphi^*(s) = (\varphi_1^*(s), \varphi_2^*(s), \dots, \varphi_n^*(s))^T$ ($s \in (-\infty, 0]$) being bounded by γ , and $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T$ is a solution of system (1) with initial value $\varphi(s) = (\varphi_1(s), \varphi_2(s), \dots, \varphi_n(s))^T$ ($s \in (-\infty, 0]$). Then there exist constants $\lambda > 0$ and $M > 1$ such that

$$|x_i(t) - x_i^*(t)| \leq M \|\varphi(s) - \varphi^*(s)\|_1 e^{-\lambda t}, \quad \forall t > 0, \text{ and } i = 1, 2, \dots, n.$$

$$\text{Here } \|\varphi(s) - \varphi^*(s)\|_1 = \max_{1 \leq i \leq n} \sup_{-\infty \leq s \leq 0} |\varphi(s) - \varphi^*(s)|.$$

Proof. Let $y(t) = x(t) - x^*(t)$. Then, for $i \in \{1, 2, \dots, n\}$, it is followed by

$$\begin{cases} y_i'(t) = -c_i(t)(x_i(t) - x_i^*(t)) + \sum_{j=1}^n a_{ij}(t)[g_j(x_j(t - \tau_{ij}(t))) - g_j(x_j^*(t - \tau_{ij}(t)))] \\ \quad + \sum_{j=1}^n b_{ij}(t)[f_j(\int_0^\infty K_{ij}(s)x_j(t-s)ds) - f_j(\int_0^\infty K_{ij}(s)x_j^*(t-s)ds)], \quad t \neq t_k, \\ y_i(t_k^+) = (1 + d_{ik})y_i(t_k), \quad k = 1, 2, \dots. \end{cases} \quad (5)$$

Considering the Lyapunov functional

$$V_i(t) = |y_i(t)|e^{\lambda t}, \quad i = 1, 2, \dots, n,$$

and calculating the upper left derivative of $V_i(t)$ along the solution $y(t) = (y_1(t), y_2(t), \dots, y_n(t))^T$ of system (5), we have, for $t \neq t_k$,

$$\begin{aligned} D^+V_i(t) &= \lambda e^{\lambda t} |y_i(t)| + e^{\lambda t} D^+ |y_i(t)| \leq (\lambda - c_i(t))V_i(t) \\ &\quad + \sum_{j=1}^n \bar{L}_j e^{\lambda t} |a_{ij}(t)| V_j(t - \tau_{ij}(t)) + \sum_{j=1}^n L_j |b_{ij}(t)| \int_0^\infty |K_{ij}(s)| V_j(t-s) e^{\lambda s} ds. \end{aligned} \quad (6)$$

And it is easily known that $V_i(t_k^+) = |1 + d_{ik}| V_i(t_k)$, where $i \in \{1, 2, \dots, n\}$.

Let $M > 1$ be an arbitrary real number and define $\|\varphi(s) - \varphi^*(s)\|_1 = \max_{1 \leq i \leq n} \sup_{-\infty \leq s \leq 0} |\varphi_i(s) - \varphi_i^*(s)|$. We claim that

$$V_i(t) = |y_i(t)|e^{\lambda t} < M \|\varphi(s) - \varphi^*(s)\|_1, \quad \text{for } \forall t \in [0, t_1], \quad i = 1, 2, \dots, n. \quad (7)$$

Obviously, (7) holds for $t = 0$. We first prove that (7) is true for $0 < t \leq t_1$. Otherwise, there exist $i \in \{1, 2, \dots, n\}$ and $\rho \in (0, t_1]$ such that

$$V_i(\rho) = M \|\varphi(s) - \varphi^*(s)\|_1 \text{ and } V_j(t) < M \|\varphi(s) - \varphi^*(s)\|_1 \quad \forall t \in [0, \rho], \quad j = 1, 2, \dots, n. \quad (8)$$

That is, $V_i(\rho) - M\|\varphi(s) - \varphi^*(s)\|_1 = 0$, and $V_j(t) - M\|\varphi(s) - \varphi^*(s)\|_1 < 0$ for $\forall t \in [0, \rho]$, $j = 1, 2, \dots, n$. Combining (8) with (6), we have

$$0 \leq D^+(V_i(t) - M\|\varphi(s) - \varphi^*(s)\|_1)_{t=\rho} = D^+V_i(\rho) \\ \leq [(\lambda - c_i(\rho)) + \sum_{j=1}^n \bar{L}_j |a_{ij}(\rho)| e^{\lambda \tau} + \sum_{j=1}^n L_j |b_{ij}(\rho)| \int_0^\infty |K_{ij}(s)| e^{\lambda s} ds] \cdot M\|\varphi(s) - \varphi^*(s)\|_1.$$

Thus

$$0 \leq (\lambda - c_i(\rho)) + \sum_{j=1}^n \bar{L}_j |a_{ij}(\rho)| e^{\lambda \tau} + \sum_{j=1}^n L_j |b_{ij}(\rho)| \int_0^\infty |K_{ij}(s)| e^{\lambda s} ds,$$

which contradicts to (H6). Therefore, (7) holds. From (7), we know that $V_i(t_1) = |y_i(t_1)| e^{\lambda t_1} < M\|\varphi(s) - \varphi^*(s)\|_1$, $i = 1, 2, \dots, n$. And since

$V_i(t_1^+) = |1 + d_{i1}| V_i(t_1) \leq V_i(t_1)$, it shows that $V_i(t_1^+) < M\|\varphi(s) - \varphi^*(s)\|_1$. Thus, for $t \in [t_1, t_2]$ we may repeat the above procedure, and obtain

$$V_i(t) = |y_i(t)| e^{\lambda t} < M\|\varphi(s) - \varphi^*(s)\|_1, \text{ for } t \in [t_1, t_2], i = 1, 2, \dots, n.$$

Further, we have

$$V_i(t) = |y_i(t)| e^{\lambda t} < M\|\varphi(s) - \varphi^*(s)\|_1, \text{ for } \forall t > 0, i = 1, 2, \dots, n.$$

That is

$$|x_i(t) - x_i^*(t)| \leq M\|\varphi(s) - \varphi^*(s)\|_1 e^{-\lambda t}, \quad \forall t > 0, \text{ and } i = 1, 2, \dots, n.$$

Theorem 2.1. If (H1)-(H6) are satisfied, then system (1) has exactly one T -anti-periodic solution $x^*(t) = (x_1^*(t), x_2^*(t), \dots, x_n^*(t))^T$, moreover, $x^*(t)$ is globally exponentially stable.

Proof. Let $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T$ be a solution of system (1) with initial conditions $\varphi(s) = (\varphi_1(s), \varphi_2(s), \dots, \varphi_n(s))^T$ ($s \in (-\infty, 0]$). Here $|\varphi_i(s)| < \gamma$ for $s \in (-\infty, 0]$ and $i = 1, 2, \dots, n$. By Lemma 2.1, the solution $x(t)$ is bounded and $|\bar{x}_i(t)| < \gamma$ for all $t \in [0, +\infty)$. By hypotheses (H1), we have, for $\forall h \in N$,

$$[(-1)^{h+1} x_i(t + (h+1)T)]' = (-1)^{h+1} x_i'(t + (h+1)T) = \\ -c_i(t)[(-1)^{h+1} x_i(t + (h+1)T)] + \sum_{j=1}^n a_{ij}(t) g_j((-1)^{h+1} x_j(t + (h+1)T - \tau_{ij}(t))) \\ + \sum_{j=1}^n b_{ij}(t) f_j(\int_0^\infty K_{ij}(s) (-1)^{h+1} x_j(t + (h+1)T - s) ds) + I_i(t), \quad t \neq t_k, i = 1, 2, \dots, n. \quad (9)$$

Further, by hypothesis of (H3), we have

$$(-1)^{h+1} x_i(t_k + (h+1)T)^+ = (-1)^{h+1} (1 + d_{i(k+(h+1)q)}) x_i(t_k + (h+1)T) \\ = (1 + d_{ik}) (-1)^{h+1} x_i(t_k + (h+1)T), \quad k = 1, 2, \dots. \quad (10)$$

Thus, for any natural number h , we obtain that $(-1)^{h+1}x(t+(h+1)T)$ is a solution of system (1). Hence, $-x(t+T)$ is also a solution of (1) with initial condition $\psi(s) = (\psi_1(s), \psi_2(s), \dots, \psi_n(s))^T$ ($s \in (-\infty, 0]$). Then, by Lemma 2.2, there exists a constant $M > 1$ such that

$$\begin{aligned} & \left| (-1)^{h+1}x_i(t+(h+1)T) - (-1)^h x_i(t+hT) \right| = \left| x_i(t+hT) - (-1)x_i(t+(h+1)T) \right| \\ & \leq M e^{-\lambda(t+hT)} \max_{1 \leq i \leq n} \left\{ \sup_{-\infty < s \leq 0} |\varphi_i(s) - \psi_i(s)| \right\} \leq 2M\gamma e^{-\lambda(t+hT)} \quad i=1,2,\dots,n. \end{aligned} \quad (11)$$

And

$$\begin{aligned} & \left| (-1)^{h+1}x_i(t_k+(h+1)T)^+ - (-1)^h x_i(t_k+hT)^+ \right| = \left| x_i(t_k+hT)^+ + x_i(t_k+(h+1)T)^+ \right| \\ & = (1+d_{ik}) \left| x_i(t_k+hT) + x_i(t_k+(h+1)T) \right| \\ & \leq 2M\gamma e^{-\lambda(t_k+hT)}, \quad \forall k \in N, \quad i=1,2,\dots,n. \end{aligned} \quad (12)$$

Further, for any natural number m , we have

$$(-1)^{m+1}x_i(t+(m+1)T) = x_i(t) + \sum_{h=0}^m [(-1)^{h+1}x_i(t+(h+1)T) - (-1)^h x_i(t+hT)], \quad t \neq t_k.$$

Then, for $i=1,2,\dots,n$, we have

$$\begin{cases} \left| (-1)^{m+1}x_i(t+(m+1)T) \right| \leq \left| x_i(t) \right| + \\ \quad \sum_{h=0}^m \left| (-1)^{h+1}x_i(t+(h+1)T) - (-1)^h x_i(t+hT) \right|, \quad t \neq t_k, \\ \left| (-1)^{m+1}x_i(t_k+(m+1)T)^+ \right| = \left| (1+d_{ik})(-1)^{m+1}x_i(t_k+(m+1)T) \right| \\ \leq \left| (-1)^{m+1}x_i(t_k+(m+1)T) \right| \end{cases} \quad (13)$$

In view of (11), (12) and (13), we know that $(-1)^{m+1}x_i(t+(m+1)T)$ is a fundamental sequence on any compact set of R . It follows that $(-1)^m x(t+mT)$ will converge uniformly to a piecewise continuous function $x^*(t) = (x_1^*(t), x_2^*(t), \dots, x_n^*(t))^T$ on any compact set of R .

Now we are in the position of proving that $x^*(t)$ is a T -anti-periodic solution of system (1). Firstly, it is easily known that $x^*(t)$ is anti-periodic, since

$$\begin{aligned} x_i^*(t+T) &= \lim_{m \rightarrow +\infty} (-1)^m x_i(t+T+mT) = - \lim_{m+1 \rightarrow +\infty} (-1)^{m+1} x_i(t+(m+1)T) = -x_i^*(t), \quad t \neq t_k, \\ \text{and} \quad x_i^*(t_k+T)^+ &= \lim_{m \rightarrow +\infty} (-1)^m x_i(t_k+T+mT)^+ = -x_i^*(t_k)^+. \end{aligned}$$

Further, from (9), and by hypothesis (H4) and (H5), we have, for $i=1,2,\dots,n$

$$\begin{aligned}
& | [(-1)^{h+1} x_i(t + (h+1)T)]' - [(-1)^{m+1} x_i(t + (m+1)T)]' | \\
& \leq | c_i(t) | \cdot | (-1)^{h+1} x_i(t + (h+1)T) - (-1)^{m+1} x_i(t + (m+1)T) | + \sum_{j=1}^n | a_{ij}(t) | \cdot \bar{L}_j \\
& \cdot | (-1)^{h+1} x_i(t + (h+1)T - \tau_{ij}(t)) - (-1)^{m+1} x_i(t + (m+1)T - \tau_{ij}(t)) | + \sum_{j=1}^n | b_{ij}(t) | \\
& \cdot L_j \cdot \int_0^\infty K_{ij}(s) | (-1)^{h+1} x_i(t + (h+1)T - s) - (-1)^{m+1} x_i(t + (m+1)T - s) | ds.
\end{aligned}$$

Since $\{(-1)^m x(t + mT)\}$ converges uniformly to a piecewise continuous function on any compact set of R , and a_{ij} , b_{ij} and c_i are continuous functions, therefore, $\{(-1)^{m+1} x_i(t + (m+1)T)\}'$ converges uniformly to a piece-wise continuous function on any compact set of R and $t \neq t_k$. Letting $m \rightarrow +\infty$ on both side of (9) and (10), we have

$$\begin{cases}
x_1^{*'}(t) = -c_i(t)x_i^*(t) + \sum_{j=1}^n a_{ij}(t)g_j(x_j^*(t - \tau_{ij}(t))) \\
\quad + \sum_{j=1}^n b_{ij}(t)f_j(\int_0^\infty K_{ij}(s)x_j^*(t-s)ds) + I_i(t), \quad t \neq t_k, \\
x_i^*(t_k^+) = (1 + d_{ik})x_i^*(t_k), \quad i = 1, 2, \dots, n, \quad k = 1, 2, \dots.
\end{cases}$$

Thus, $x^*(t) = (x_1^*(t), x_2^*(t), \dots, x_n^*(t))^T$ is a T -anti-periodic solution of system (1).

3. AN EXAMPLE

In this section, we illustrate our results by numerical simulation.

Example 3.1. Consider the following neural network consisting of two neurons $(x(t), y(t))$, which is described by

$$\begin{cases}
x'(t) = -2x(t) + \frac{1}{16} \sin(\pi t) g_1(x(t-1)) + \frac{1}{16} g_2(y(t-2)) \\
\quad + 0.1 \tanh \int_0^\infty K_{11}(s)x(t-s)ds + 0.4 \int_0^\infty K_{12}(s)y(t-s)ds + 3 \sin(\frac{\pi}{2}t) \\
y'(t) = -3y(t) + \frac{1}{16} g_1(x(t-2)) + \frac{1}{8} \sin(\pi t) g_2(y(t-3)) \\
\quad + 0.3 \tanh \int_0^\infty K_{21}(s)x(t-s)ds + 0.4 \int_0^\infty K_{22}(s)y(t-s)ds + 6 \sin(\frac{\pi}{2}t) \\
x(t_k^+) = (1 + d_k)x(t_k) \\
y(t_k^+) = (1 + p_k)y(t_k)
\end{cases} \quad t \neq 2k-1, \quad t_k = 2k-1, \quad k = 1, 2, \dots. \quad (14)$$

Here, it is assumed that the activation functions $g_1(x) = g_2(x) = \frac{1}{2}(|x+1| - |x-1|)$,

$f_1(x) = \tanh x$, $f_2(x) = x$, and the delay kernels $K_{11}(s) = e^{-0.5s}$, $K_{12}(s) = e^{-0.6s}$, $K_{21}(s) = e^{-0.7s}$, $K_{22}(s) = e^{-0.8s}$ for $0 \leq s \leq 20$ and $K_{11}(s) = K_{12}(s) = K_{21}(s) = K_{22}(s) = 0$ for $s > 20$. The connection weights functions are expressed by $a_{11}(t) = \frac{1}{16} \sin(\pi t)$, $a_{12}(t) = \frac{1}{16}$, $a_{21}(t) = \frac{1}{16}$, $a_{22}(t) = \frac{1}{8} \sin(\pi t)$, and $b_{11}(t) = 0.1$, $b_{12}(t) = 0.4$, $b_{21}(t) = 0.3$, $b_{22}(t) = 0.4$, respectively. $I_1(t) = 3 \sin(\frac{\pi}{2}t)$ and

$I_2(t) = 6 \sin(\frac{\pi}{2}t)$ are the external inputs. Obviously, network (14) is an anti-periodic system with anti-period T being 2 and impulsive jumps occur at time $t = 1, 3, \dots, 2k - 1$. Letting $d_k = p_k = 0$ for $\forall k \in N$, from the result of paper [11], we know that there exists unique 2-anti-periodic solution, which is globally exponentially stable. In Fig.1 and 2, we draw the time-series of “ $t - x(t)$ ” , “ $t - y(t)$ ” and the phase graph of “ $x(t) - y(t)$ ” for a solution with the initial value $(x(s), y(s)) = (4, -2)$ ($s \leq 0$) .

Further, we discuss the influence of impulsive effects on the existence of anti-periodic solutions. We choose the magnitude of impulsive jump $d_k = -0.5$ and $p_k = -1.5$ for all $k \in N$. It is easily verified that all conditions in Theorem 2.1 are satisfied. Thus, system (14) has exactly one 2-anti-periodic solution, and it is globally asymptotically stable. This result is demonstrated by the following numerical simulation in Fig.3 and 4 with the initial condition $(x(s), y(s)) = (4, -2)$ for $s \leq 0$. When t tends to the positive infinity, this solution will tends to the anti-periodic solution with anti-period being 2.

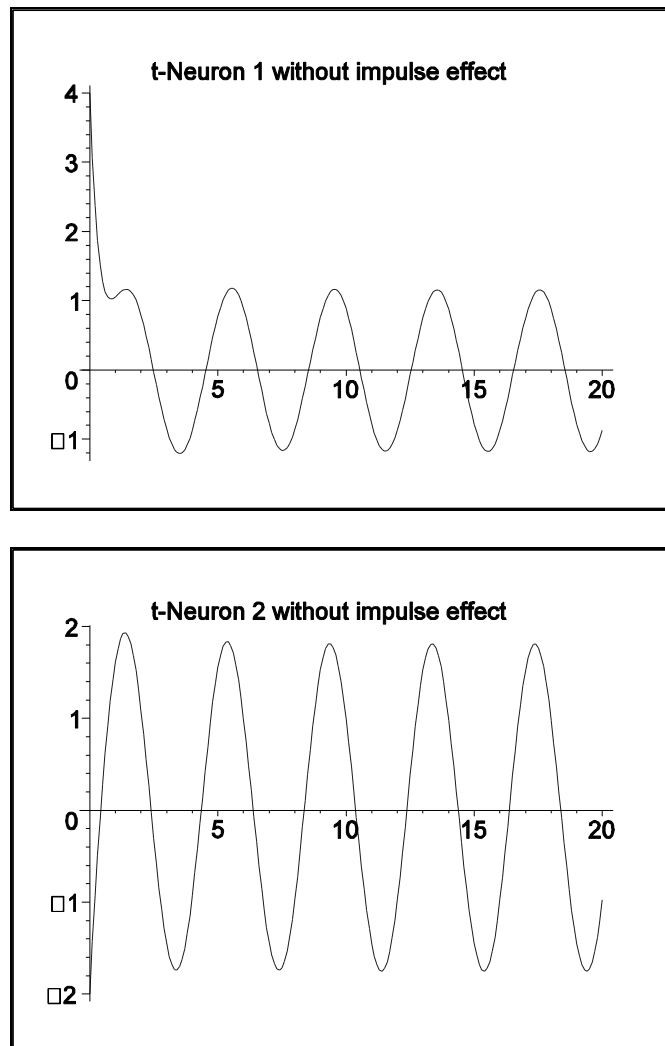


Figure 1. The time series graph of $x(t)$ and $y(t)$ for system (14) without impulses

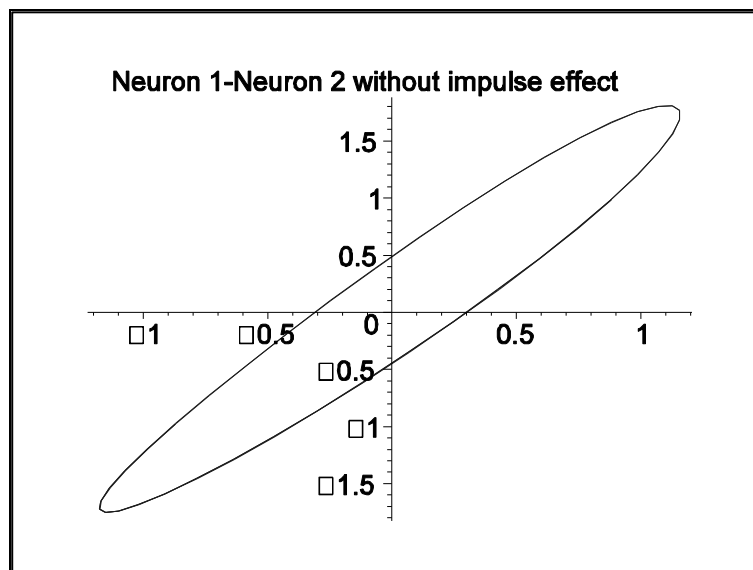


Figure 2. The graph of $x-y$ for (14) without impulses

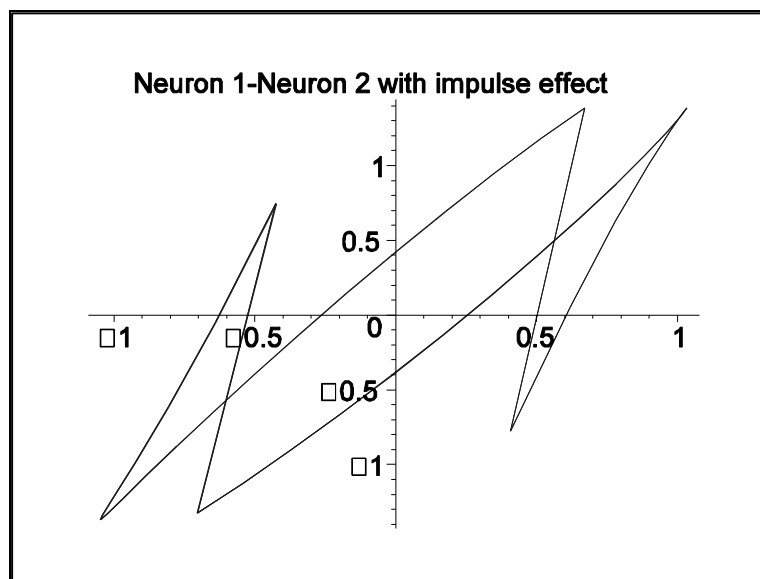


Figure 3. The graph of $x-y$ for (14) with impulses

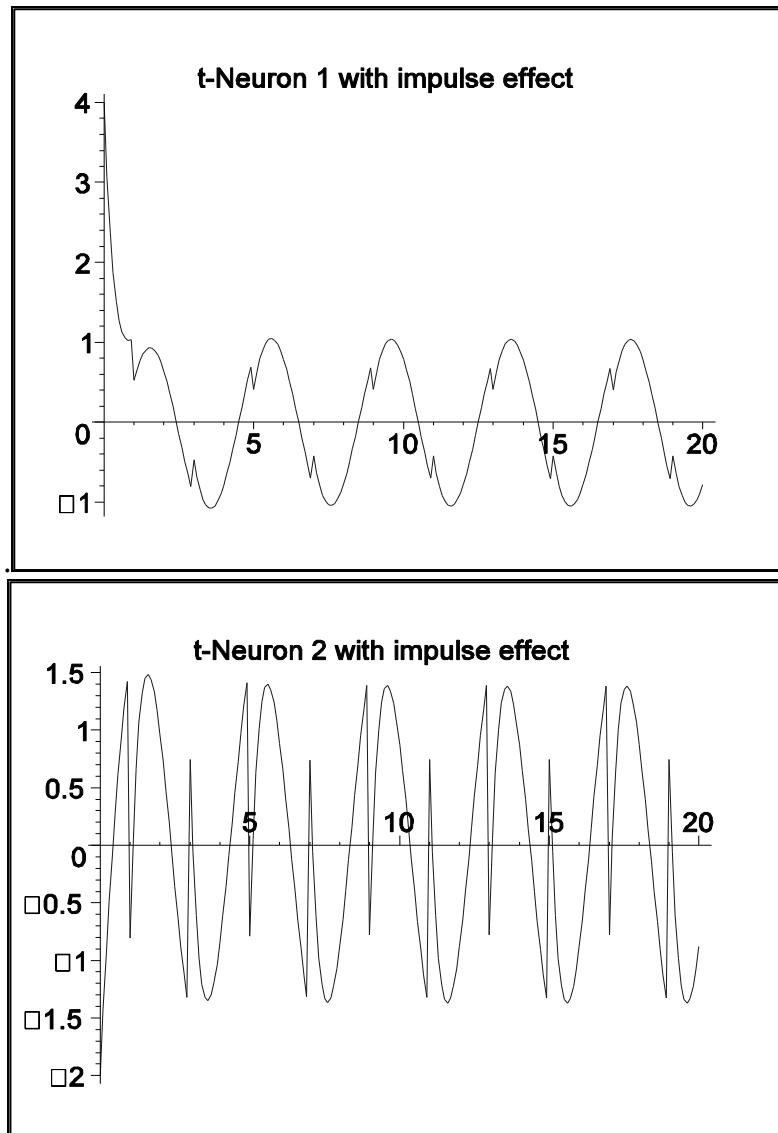


Figure 4. The time series graph of $x(t)$ and $y(t)$ for system (14) with impulses

4. CONCLUDING REMARKS

A set of sufficient conditions are derived in this work to guarantee the existence and global exponential stability of a unique anti-periodic solution for an artificial neural networks with delays and impulses. One of our conclusions is that every solution eventually converges exponentially to the anti-periodic solution under the impulsive jumps of the form $\Delta x_i(t_k) = d_{ik} x_i(t_k)$, if the frequency and the magnitude of the impulsive effects satisfy the hypotheses (H2) and (H3). Although continuous differential ANNs systems are studied extensively, but there are few results on anti-periodic solution for non-autonomous ANNs with delays and impulses.

Acknowledgement- This work is supported by Natural Science Foundation of Shanxi Province (No. 2008011002-2), and the Key Project of Chinese Ministry of Education (No. 210030), China.

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