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Risk-based stochastic equilibrium assignment model in augmented urban railway network

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SUMMARY

This study developed a methodology to model the passenger flow stochastic assignment in urban railway network (URN) with the considerations of risk attitude. Through the network augmentation technique, the urban railway system is represented by an augmented network in which the common traffic assignment method can be used directly similar to a generalized network form. Using the analysis of different cases including deterministic travel state, emergent event, peak travel, and completely stochastic state, we developed a stochastic equilibrium formulation to capture these stochastic considerations and give effects of risk aversion level on the URN performance, the passenger flow at transfer stations through numerical studies. Copyright © 2012 John Wiley & Sons, Ltd.

KEY WORDS: stochastic equilibrium assignment; urban railway; augmented network; risk aversion

1. INTRODUCTION

Urban railway traffic has been regarded as the most efficient method to alleviate the congestion because of its larger capacity, stable travel time, lower pollution, and rapid speed. Recently, a number of studies have been devoted to the passenger flow assignment problem in the transit network. However, most of methods are based on the well-developed traffic flow assignment model in the road network.

Compared with road traffic assignment, public transport assignment is much more complicated. Road networks only consist of the physical network of links, nodes, and may be turns; public networks consist, in addition, of an organizational network of routes, terminals, and transfers. Especially to the urban railway system, the running time between two stations is always stable; therefore, the main problem should be the analysis of discomfort in the train, the waiting time, and the transfer time in the stations. In the past years, many models are proposed and extended with the consideration of the effects of in-train and waiting time on the passenger's route choice decisions for transit networks. But in the urban railway network (URN), the in-train congestion and the comfort degree of the trip, which would affect the distribution of passenger flow in the whole network directly, should be considered in the route choice analysis [1,2].

It looks like that URN is simpler than the transit network. But unfortunately, URNs do not always work as expected because of uncertain input parameter or unforeseen events that are subject to stochastic variations in reality. Even minor events, such as vehicle breakdown, accident, *etc*., the system

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capacity would be degraded to different degrees. In order to provide more effective and reliable transport problem solutions, transport system analysts and engineers have to face the different sources of uncertainty when modeling transportation systems and its main components, such as travel demand, transport supply, interaction between demand and supply, and relevant analysis and assessments [3]. Therefore, flow assignments based on stochastic characteristics are more realistic and important that it becomes a hotspot of researches. In fact, stochastic user equilibrium (SUE) assignment has been widely used in the road traffic and is well-known as a general model that consistently unifies the concept of the stochastic assignment and Wardropian equilibrium. It overcomes the not only the shortcoming of the homogeneous user assumption in the Wardropian equilibrium but also includes the random effect of the stochastic assignment problem on a congested road network [4]. Sumalee *et al.* [5] considered the effects of stochastic demand on network reliability, which was also studied recently using multiple network demon approach [6] and cooperative game approaches [7]. In recent years, the effects of route choice decision on stochastic transportation network have attracted much attention ([8–12]), which is also an important factor in the modeling of route choice behavior in URN. However, the methods mentioned earlier cannot reveal the characteristics of passengers on the urban railway system with different risk attitude.

As a public transport mode, there exist transfer behaviors in the urban railway system, which make the flow assignment in URN different from that of in road network. In order to describe the route choice behaviors with the traditional SUE model, an augmented network and its link travel cost are constructed to solve the problem. The terms of travel cost, including in-train time, waiting time, transfer time, and delay time caused by the train, are stochastic variables that will influence the passengers' choice behaviors. Lo *et al.* [13] presented a framework to modeling multimodal network by transforming it to a state-augmented multi-modal network that can be treated as a simple network and can be combined with either deterministic or stochastic assignment procedures for the applications at hand. In our previous studies of transit assignment, a new method to solve the common-line problem in the transit network assignment is proposed by developing an augmented network [14]. However, these models almost are used to analyze the deterministic case that assumes that the network supply and the route cost are perfectly known.

In addition, faced with uncertainty about the travel cost in their route choice, passengers are required to make a trade-off between the travel cost and its uncertainty. This behavior is known as risk-taking behavior [15]. Many studies [16–19,5,20–22] have been performed to model this risk-taking behavior of route choices. Lo *et al.* [23] develops an approach to relate the travel time variability due to stochastic network link capacity variations with travelers' risk aversive route choice behaviors. Zhou and Chen [24] assumed the travel demand to follow log-normal distribution and proposed different risk-based assignment frameworks. Lam *et al.* [25] considered the rain effects on road network, where the link free-flow travel time is represented by a nondecreasing function of rainfall intensity, the link capacity is represented by a nonincreasing function of rainfall intensity, and travel demand is stochastic. Shao *et al.* [26] extended the model by Lam *et al.* [25] to consider multiple user classes. Recently, Ordóñez and Stier-Moses [27] extended the traffic assignment problem by adding random deviations and analyzed three specific equilibrium models with risk-averse users. Lam *et al.* [28] proposed a new risk-averse user equilibrium model to estimate the distribution of traffic flows over road networks with taking account the effects of accident risks because of the conflicting traffic flows at signalized intersections. Szeto *et al.* [29] proposed a nonlinear complementarity problem formulation for the risk-aversive stochastic transit assignment problem to account for different effects of on-board passengers and passengers waiting at stops. Within the current expected utility approach, Hensher *et al.* [30] incorporated attribute risk with a linear probability weighting function and then introduced attribute risk together with a nonlinear probability weighting function. Although many related works about risk-averse can be found recently, only a few research on the urban railway system cannot capture the route choice decision in different passenger situation, such as the peak passenger flow and the emergency passenger flow.

Because of the existence of transfer behaviors in URN, traditional traffic assignment model cannot be used directly; hence, an augmented network is developed in our studies. According to different modeling assumptions of passengers' responses to traffic states, this paper aims to study the route

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choice behaviors of passengers in URN by considering four different cases: deterministic travel state, emergent event, peak travel, and completely stochastic state, which corresponds to different randomness of travel cost.

The next section presents the augmented method to reconstruct URN. Section 3 discusses the notation, assumptions, and link cost function of the proposed model. The SUE of augmented URN is formulated and the algorithm is described in Section 4. A numerical example is given to illustrate the characteristics of the proposed model in Section 5. Finally, conclusions are given with some suggestions for future studies in Section 6.

2. AUGMENTED URN

Urban railway network constitutes a set of stations where the passengers can board, alight from, or change the route line; a set of the railway train running back and forth between different stations; and a set of lines. The urban railway system can be described using an augmented network according to the following principles:

- Each transfer station is described using augmented nodes, and each augmented node corresponds to one operation line at that station.
- Connect the augmented nodes respectively to their two neighboring stations using the in-train links corresponding to the transit line.
- If different lines can be transferred in the same station, the corresponding augmented nodes should be connected by transfer links.

When constructing an augmented URN, carefully consider the following. First, the number of augmented nodes of each station should be identical to the number of the operating lines through this station, i.e., there is just one in-train link in or out at each augmented node in the augmented URN. Second, there are two types of links in the augmented URN, i.e., the transfer links and the in-train links. In this paper, the former types of links are called virtual links and the later types of links (i.e., the in-train links) are called real links. Third, the number of in-train links connecting the neighboring stations is equal to the number of operating lines between the corresponding stations in the augmented URN.

There are some considerations in establishing the connectivity between the augmented stations using the transfer links when constructing the augmented URN.

(a) If two virtual stations belong to the same operation line, the cost (time) is set zero. Otherwise, the cost (time) is equal to the transfer time. It means that if passengers will not transfer, then their travel time between two virtual stations in the same line is zero. Otherwise, their travel time should be added a transfer cost.
(b) The self-loop and multilink are forbidden in the augmented network.
(c) The transfer should not happen between different lines at the stations that are connected with the origins or destinations.

Figure 1 provides an example of a simple URN to illustrate the aforementioned augmented method. In this example, the simple URN consists of five stations and two operating lines. Arbitrary two nodes are origin–destination (OD) pair.

Passengers from the origin S1 to destination S2 and S4 will pass through two lines. Therefore, node S3 is the transfer node, and the passenger must transfer from this node to reach their destination.

Based on the augmented rules above, the associated augmented urban railway transit (AURT) network is shown in Figure 2.

Figure 2 shows five stations *S*1, *S*2, *S*3, *S*4, and *S*5, where two lines (line 1 and line 2) go through respectively. Transfer station is extended into four augmented stations (yellow nodes). In all augmented stations, the transfer between different lines could be realized through the transfer links (virtual link) of relevant augmented stations. The augmented stations are connected by the corresponding in-train links (real link) with other real stations.

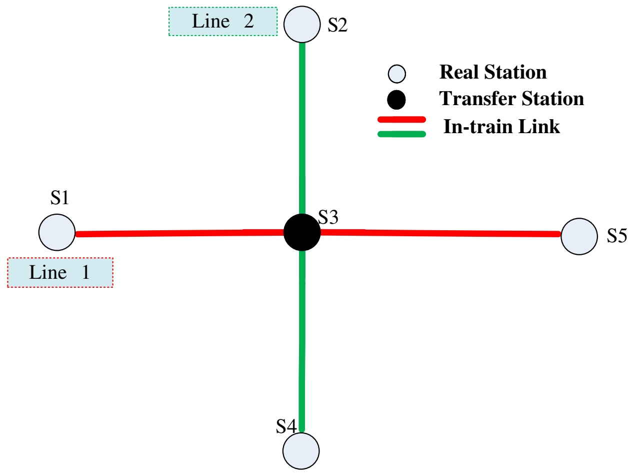

Figure 1. A simple URN.

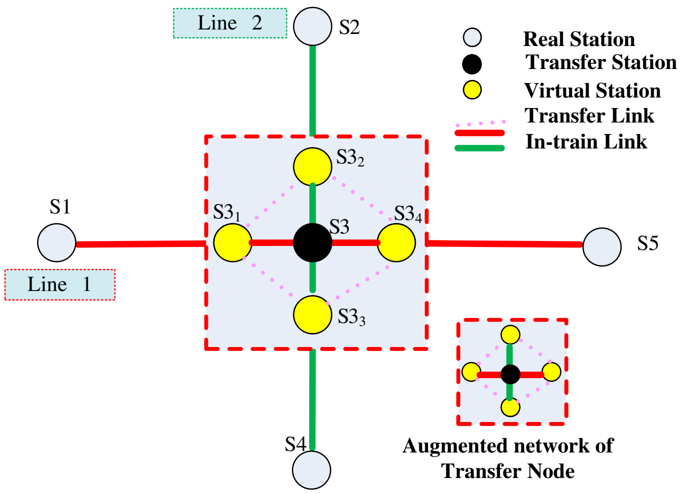

Figure 2. The AURT network for the example.

3. BASIC ASSUMPTIONS AND LINK TRAVEL COST FUNCTION

In this section, the mathematical formulations of the urban railway traffic flow assignment models for a stochastic network with cost uncertainty are presented.

3.1. Notations

$\hat{N}$: virtual and transfer station set in augmented network
A_1: in-train link set in original network
A_2^i: transfer link set in augmented network at transfer station i, $i \in \hat{N}$
RS: OD set
TN: transfer node set
rs: OD pair index, $rs \in RS$
p_{rs}: the set of all noncyclic paths connecting the OD pair $rs \in RS$
a: link index, $a \in A_1$

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λ: degree of risk aversion

d_{rs}: OD demand between pair rs

t_a: the in-train time of passenger on link a, $a \in A_1$

Y: the extra cost caused by congestion in the train

t'_a: the in-train cost of passenger on link a, $a \in A_1$

w: the waiting time

t_a^t: the transfer time between different lines, $a \in A_2^i$, $i \in \hat{N}$

t_a^d: delay time on link a, $a \in A_1$

$t_a(\bar{x}_a)$: the time of passenger on link a with the consideration of random cases

T_a: the link travel time (including the time of in-train, waiting, and transfer) with the consideration of risk aversion

C_k^{rs}: travel time (including the time of in-train, waiting, and transfer) with the consideration of risk aversion on the path k between OD pair rs

l_a: the length of link a, $a \in A_1$

L: the distance between two platforms in the same transfer node

v_t: the running speed of train

v: the walking speed of passenger

x_a: the flow of link a, $a \in A_1 \cup A_2^1 \cup A_2^2 \cdots A_2^i$, $i \in \hat{N}$

h_k^{rs}: traffic flow on path k between OD pair $rs \in RS$

X_i: the flow of transfer node i, $i \in \hat{N}$

f: the frequency of train

n_{seat}: the total seat number of train

c: the maximum capacity of train

β, γ, δ: parameters converting time to cost

3.2. Basic assumptions

To facilitate the model formulation, some assumptions are made throughout the paper and are explained in this section.

Assumption A1: The OD demand d_{rs} between arbitral two stations is not equal and nonuniform, which means that we consider the heterogeneous demand among stations, especially for the transfer stations.

Assumption A2: The link (route) cost is a random variable that follows the same distribution form as its components and the link travel time is defined additive.

Assumption A3: Passengers do not have full knowledge of the URN, which means that they only choose rationally according to their perceived utilities.

Assumption A4: Passengers are assumed to be cost-minimizing decision makers. They will choose their departure times and transit routes so as to minimize their total perceived disutilities of travel.

Assumption A5: There is only one train line between two neighboring stations, which means that there are no competitive lines in the same segment.

Assumption A6: Other travel modes are not considered, which means that passengers only choose the urban railway line to reach their destinations.

3.3. Link travel cost of passengers in URN

(1) Determined link travel cost in URN

Generally, the link travel cost of passenger in URN is composed of four types of cost: in-train time, waiting time, transfer time, and delay time caused by the train.

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3.3.1. In-train cost. In-train cost refers to the cost of passenger in the train on certain segment, which includes two parts: the pure time in the train and the cost caused by congestion. The former term is independent on the number of passengers in the train and can be represented by $t_a = l_a/v_t$. However, the latter term is related with many factors, such as the number of passengers and seats, etc. The function of discomfort mainly comes from the congestion that makes the passengers pay additional cost. This may be considered using different coefficients A (general congestion) and B (overcrowding) as follows and over a certain value when calculating the function of discomfort:

$$Y(x_a) = \begin{cases} A\dfrac{(x_a - fn_{\text{Seat}})}{fn_{\text{Seat}}} & fn_{\text{Seat}} < x_a \leq fc \\ A\dfrac{(fc - fn_{\text{Seat}})}{fn_{\text{Seat}}} + B\dfrac{(x_a - fc)}{fn_{\text{Seat}}} & x_a > fc \end{cases}. \tag{1}$$

Accordingly, the in-train cost can be written as:

$$\mathrm{t}'_a(x_a) = t_a + t_a Y(x_a). \tag{2}$$

3.3.2. Waiting time. Passengers always cannot go-board directly when they arrive at the station. So they have to wait the train on the platform. This time can be calculated using $w = \frac{\alpha}{f}$, where α is a parameter related to the passengers' arrival pattern, the frequency of train, and so on. $\alpha = 1$ corresponds to a uniform random passenger arrival distribution and a stable frequency of train. It is clear that the average waiting time is longer on lines with smaller frequencies. The waiting time will be a stochastic variable in some cases, such as in the emergent conditions, the special event, and so on.

3.3.3. Transfer time. The passengers may transfer to other lines before reaching their destinations. It is the time that passenger changes from one line to another in the same station that is related to the length of transfer channel, the density of passenger, the walking speed, and so on. In general, it can be represented by $t_a^t = \frac{L}{v}$ and $a \in A_2^i$. In reality, the transfer time will be a stochastic variable because of the stochasticity of walking speed and passenger's density.

3.3.4. Delay time. Because of the congestion, passengers have to determine in-board or waiting the next train with the consideration of the congestion degree. This will typically be the case on lines with heavier passenger flow. Obviously, the delay time is related to the passenger flow and given as the similar form to the BPR, $t_a^d(x_a) = \left(\frac{x_a}{c}\right)^\rho$, where ρ is the parameter.

3.3.5. Link travel cost in URN. The link travel cost in URN is the sum of the four types of cost mentioned earlier and can be represented as follows:

$$t_a(x_a) = t'_a + \beta w + \gamma t_a^t + w' t_a^d(x_a), a \in A_1 \tag{3}$$

(2) Stochastic link travel cost in URN

The URN's performance is always modeled as a deterministic approach because of its undisturbed railway line, relatively fixed timetable, and relatively stable OD demand. In reality, many of factors in the urban railway system are subject to stochastic variations. For example, link capacity degradations can be caused by many events, such as earthquake, large gathering, weather, passenger flow peak, vehicle breakdown, accident, and so on. Because of stochastic variations in the urban railway system, travel time becomes uncertain. We assumed that the waiting time, transfer time, and delay time may be random variables affected by stochastic events.

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$$\bar{w} = w + \eta_1, \bar{t}_a^t = t_a^t + \eta_2, \bar{t}_a^d = t_a^d + \eta_3$$

where $\bar{w}, \bar{t}_a^t, \bar{t}_a^d$ are random variables with measured travel time w, t_a^t, t_a^d and random error terms η_1, η_2, η_3.

Then, the travel time t_a becomes a random variable:

$$\overline{t_a(x_a)} = t_a^0 + t_a^0 Y(x_a) + \beta\bar{w} + \gamma\bar{t}_a^t + \delta\bar{t}_a^d \tag{4}$$

with its mean and variance expressed as:

$$E\left[\overline{t_a(x_a)}\right] = E\left[t_a^0 + t_a^0 Y(x_a) + \beta w + \gamma t_a^t + \delta\left(\frac{x_a}{c}\right)^\rho\right] \tag{5}$$

$$\mathrm{Var}\left[\overline{t_a(x_a)}\right] = E\left[\overline{t_a(x_a)}^2\right] - E^2\left[\overline{t_a(x_a)}\right] \tag{6}$$

Knowing that t_a^0 is deterministic, therefore, $E\left(t_a^0\right) = t_a^0$ and $\mathrm{Var}\left(t_a^0\right) = 0$. It is assumed that η_1, η_2, and η_3 are independent; the $E\left[\overline{t_a(x_a)}\right]$ and $E\left[\overline{t_a(x_a)}^2\right]$ can be simplified as follows:

$$E\left[\overline{t_a(x_a)}\right] = t_a^0 + t_a^0 Y(x_a) + E(\eta_1)w + E(\eta_2)t_a^t + E(\eta_3)\left(\frac{x_a}{c}\right)^\rho \tag{7}$$

$$\begin{aligned}
\mathrm{Var}\left[\overline{t_a(x_a)}\right] &= E\left[\left(t_a^0 + t_a^0 Y(x_a) + E(\eta_1)w + E(\eta_2)t_a^t + E(\eta_3)\left(\frac{x_a}{c}\right)^\rho\right)^2\right] \\
&-E^2\left(t_a^0 + t_a^0 Y(x_a) + E(\eta_1)w + E(\eta_2)t_a^t + E(\eta_3)\left(\frac{x_a}{c}\right)^\rho\right) \\
&= w^2\,\mathrm{Var}(\eta_1) + \left(t_a^t\right)^2\mathrm{Var}(\eta_2) + \left[\left(\frac{x_a}{c}\right)^\rho\right]^2\mathrm{Var}(\eta_3) \\
&= w^2\,\mathrm{Var}(\eta_1) + \left(t_a^t\right)^2\mathrm{Var}(\eta_2) + \left(\frac{x_a}{c}\right)^{2\rho}\mathrm{Var}(\eta_3)
\end{aligned} \tag{8}$$

The expression of Equations (7) and (8) allow the calculation of the expectation and variance of link travel time, provided that these terms, the expectation, and variance of random variables η_1, η_2, and η_3 can be evaluated.To simplify the exposition, we assume that these random variables follow the normal distribution with $E(\eta_1) = \psi_1$, $E(\eta_2) = \psi_2$, and $E(\eta_3) = \psi_3$ and $\mathrm{Var}(\eta_1) = \sigma_1$, $\mathrm{Var}(\eta_2) = \sigma_2$, and $\mathrm{Var}(\eta_3) = \sigma_3$. Then, we can get the following exposition:

$$E\left[\overline{t_a(x_a)}\right] = t_a^0 + t_a^0 Y(x) + \psi_1 w + \psi_2 t_a^t + \psi_3\left(\frac{x_a}{c}\right)^\rho \tag{9}$$

$$\mathrm{Var}\left[\overline{t_a(x_a)}\right] = \sigma_1 w^2 + \sigma_2\left(t_a^t\right)^2 + \sigma_3\left(\frac{x_a}{c}\right)^{2\rho} \tag{10}$$

Generally, the in-train cost is a fixed variable because the uncertainty of route choice behavior mainly comes from other three terms in Equation (3). According to the different assumptions of the passenger's responses to the flow, the analysis can be separated into four scenarios, listed below in descending order of complexity. Case A corresponds to the common passenger flow, including the deterministic in-train cost, waiting time, transfer time, and delay time (DW-DT&DD). Case B is the "emergent event" on the URN line represented by deterministic transfer time and doubly stochastic waiting and delay time (DT-SW&SD). Case C is thought of as the peak passenger flow in which the doubly stochastic transfer time and determinate waiting time are considered (SD&ST-DW). Additionally, Case D is the most complete specification in which all of the terms in Equation (3) are considered as stochastic variables (SD&ST&SW).

- Deterministic travel state (DW-DT&DD)

In this case, all of four terms are considered as deterministic variables in the link travel cost. Therefore, the problem can be regarded as a deterministic function and is simplified to their deterministic forms:

$$t_a(x_a) = t_a^0 + t_a^0 Y(x_a) + w + t_a^t + \left(\frac{x_a}{c}\right)^\rho. \tag{11}$$

- Emergent event (DT-SW&SD)

In some time, the emergent event, caused by the breakdown of railway, the failure of train, and so on, will have a great influence on the passengers' travel behavior. So in this case, the delay and waiting time will become uncertain. Then, Equations (7) and (8) can be given as follows:

$$E\left[\overline{t_a(x_a)}\right] = t_a^0 + t_a^0 Y(x) + \psi_1 w + t_a^t + \psi_3 \left(\frac{x_a}{c}\right)^\rho \tag{12}$$

$$\mathrm{Var}\left[\overline{t_a(x_a)}\right] = \sigma_1 w^2 + \sigma_3 \left(\frac{x_a}{c}\right)^{2\rho}. \tag{13}$$

- Peak travel (SD&ST-DW)

Train frequency is fixed according to the timetable in which the buffer time has been considered. But during peak period, passengers' walking speed, affected by the crowed passenger, would become slower. Therefore, the transfer time and delay time should be stochastic variables. Consequently, Equations (7) and (8) can be simplified to:

$$E\left[\overline{t_a(x_a)}\right] = t_a^0 + t_a^0 Y(x) + w + \psi_2 t_a^t + \psi_3 \left(\frac{x_a}{c}\right)^\rho \tag{14}$$

$$\mathrm{Var}\left[\overline{t_a(x_a)}\right] = \sigma_2 \left(t_a^t\right)^2 + \sigma_3 \left(\frac{x_a}{c}\right)^{2\rho}. \tag{15}$$

- Completely stochastic state (SD&ST&SW)

In this case, all of the terms of link travel cost are stochastic variables that represent the "true" behavior. The function is the original form of Equations (7) and (8).

4. RISK-BASED STOCHASTIC EQUILIBRIUM ASSIGNMENT MODEL

4.1. Risk attitude

In the trip, passengers are always assumed to choose the path that will minimize their expected travel time. This assumption represents the risk–neutral case. Although under travel time uncertainty, travelers consider both travel time variability and mean travel time. Sumalee and Xu [31] divided two classes of passengers. One is the risk–neutral behavior who considers only the mean travel time in the route choice decision. Another case is called risk-averse behavior that considers both the mean and variance of travel time in route choice model. In order to express the risk-based (prone or averse) passenger behaviors in stochastic system, expected link travel time and its variance are used to represent the disutility of that link,

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which means that passengers will give their choice decision not only based on their expected travel time but also on the risk of travel time variations. Therefore, the perceived travel time on link a is dependent on both the mean and the variance of the travel time of that link, which is defined as

$$T_a = E\left[\overline{t_a(x_a)}\right] + \lambda \operatorname{Var}\left[\overline{t_a(x_a)}\right], \tag{16}$$

Therefore, the travel time on path k can be represented by

$$\bar{C}_k^{rs} = \sum_a T_a \delta_{a,k}^{rs}, \tag{17}$$

where link–path incidence parameter $\delta_{ak}^{rs} = \begin{cases} 0 & \text{if route } a \text{ is on path } k \\ 1 & \text{otherwise} \end{cases}.1$

In the route choice decisions, because of random effects, passengers cannot calculate the travel time exactly. Their preferences toward each route can be described by a disutility measure including measured cost and an additive random error term. Let

$$C_k^{rs} = \bar{C}_k^{rs} + \varepsilon_k^{rs}$$

where C_k^{rs} is referred to as the perceived disutility and ε_k^{rs} is the random error term.

4.2. *Risk-based SUE model*

Consider a population of passengers who are about to take a trip between a given origin and a given destination. Because of variations in perception and exogenous factors (such as weather, lighting, *etc*.), the path costs are perceived differently by each traveler. Given his or her perception of travel cost, each passenger is assumed to choose the smallest travel cost route from origin to destination. Each traveler, however, will perceive path costs differently, and therefore, each traveler may choose a different path. Because the perceived travel cost of each path is a random variable, it is associated with some probability density function. This function gives the probability that a traveler randomly drawn from the population will associate a given travel time with that path [32]. The choice probability of path k between OD rs is:

$$P_k^{rs} = \Pr\left(C_k^{rs} \le C_l^{rs}, \forall l \ne k\right)$$

Given the OD trip rates d_{rs}, the stochastic equilibrium conditions can be characterized by the following equation:

$$h_k^{rs} = d_{rs} P_k^{rs} \tag{17}$$

Link flows satisfy the following flow conservation relationship. For each OD pair rs, the travel demand conservation constraint can be given by:

$$\sum_{k \in RS} h_k^{rs} = d_{rs}, \forall rs \in RS, h_k^{rs} \ge 0 \forall k \in p_{rs}, rs \in RS. \tag{18}$$

where h_k^{rs} is the flow on route k between OD pair rs. Then, the following relationships between the link and path flows have to hold:

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$$x_a = \sum_{rs \in RS} \sum_{k \in R_k} h_k^{rs} \delta_{a,k}^{rs}, \forall a \in E. \tag{19}$$

According to the cost function represented in Equation (16), the risk-based SUE model can be given as follows:

$$\min_{\mathbf{x}} Z(\mathbf{x}) = -\sum_{rs} q_{rs} M\left[\min_{k \in \mathbf{K}_{rs}} \left\{C_k^{rs}\right\} \middle| \mathbf{c}^{rs}(\mathbf{x})\right] + \sum_a x_a T_a(x_a) - \sum_a \int_0^{x_a} T_a\left(x'\right) dx', \tag{20}$$

where $T_a(x_a)$ considers both the stochastic and the risk aversion situations and is only related to the link passenger flow x_a. $\mathbf{c}^{rs}$ is the route cost between OD pair rs. It can be proved that the first-order conditions for Equation (20) equals the SUE condition Equation (17) [32].

Let $S_{rs}[\mathbf{c}^{rs}(\mathbf{x})] = M\left[\min_{k \in \mathbf{K}_{rs}} \left\{C_k^{rs}\right\} \middle| \mathbf{c}^{rs}(\mathbf{x})\right]$, we have two important properties [32]:

Property 1: $S_{rs}[\mathbf{c}^{rs}]$ is concave for the route cost $\mathbf{c}^{rs}$;

Property 2: $\frac{\partial S_{rs}[\mathbf{c}^{rs}]}{\partial c_k^{rs}} = P_k^{rs}(\mathbf{c}^{rs})$, where $P_k^{rs}(\mathbf{c}^{rs}) = P_k^{rs} = \Pr\left(C_k^{rs} \leq C_l^{rs}, \forall l \in p_{rs} - k\right)$;

It can be proved that the first condition of Equation (20) equals to the SUE condition Equation (17). Therefore, the programming can be solved easily, and the passenger flow can be obtained.

4.3. Solution algorithm

By reconstructing augmented network, the method of successive averages [32] can be used to solve the passenger flow assignment problem under stochastic conditions. The stochastic loading mechanism based on probit model is used in this paper.

4.3.1. Method of successive averages algorithm.

Step 1: Initialization. Reconstruct URN with augmentation method. Initialize the network parameters. Based on the initial cost $\left\{T_a^0\right\}$, perform the probit loading algorithm. Get the link passenger flow x_a^n, the iteration number $n := 1, \forall a$.

Step 2: Update cost. Let $T_a^n = T_a\left(x_a^n\right), \forall a$.

Step 3: Direction searching. Perform the probit load algorithm in the current cost $\left\{T_a^n\right\}$ resulting in new passenger flows $\left\{y_a^n\right\}$.

Step 4: Updating. Let $x_a^{n+1} = x_a^n + (1/n)\left(y_a^n - x_a^n\right), \forall a$ in the direction $\left\{y_a^n - x_a^n\right\}$.

Step 5: Check convergence. If $\sqrt{\sum_a \left(x_a^{n+1} - x_a^n\right)^2} / \sum_a x_a^n \leq \varepsilon$, $\forall a$ (ε is tolerance criterion).Stop; otherwise, set $n := n + 1$ and go to Step 2.

Step 6: Calculation the passenger flow in real network. According to the relationship between the virtual and real stations, get the passenger flow information in the original URN.

4.3.2. Probit loading algorithm.

P_Step 1: Initialization. Set the iteration number $m := 1$.

P_Step 2: Sampling. Generate the sample $T_a^{'m}, \forall a$, and assume it follows the normal distribution $\left[T_a^m, \beta' T_a^m\right]$.

P_Step 3: All-or-nothing passenger flow assignment. Based on $T_a^{'m}$, the OD flow q_{rs} is assigned in the shortest path. Get the link passenger flow x_a^{m}.

P_Step 4: Average passenger flow. Let $y_a^m = \frac{1}{m}\left[(m-1) y_a^{m-1} + x_a^m\right], \forall a$.

P_Step 5: Stop criteria. Stop according to stop criteria [32]; otherwise, set $m := m + 1$ and go to Step 2.

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5. NUMERICAL STUDIES

We consider a URN in Figure 3, which consists of 17 general nodes, 5 transfer nodes, and 3 operation lines. Arbitrary two nodes can be seen as an OD pair. Here, we assume that basic demand

$$d_{rs} = \begin{cases} q & \text{both } r \text{ and } s \text{ are not the transfer station} \\ 3q & \text{either } r \text{ or } s \text{ is the transfer station} \end{cases}, \forall r, s; r \neq s.$$

The link parameters are listed in Table 1. Table 2 gives the networks parameters.

The initial values of ψ_1, ψ_2, and ψ_3 in four cases are 1.3, 1.2, and 1, respectively, and $\sigma_1 = \psi_1$, $\sigma_2 = 3\psi_2$, and $\sigma_3 = 2\psi_3$, respectively.

5.1. Augmented network

According to the augmented network method, we plot the test URN as Figure 4. It can be seen that every transfer node is divided into four virtual nodes and six virtual links. If two virtual nodes belong

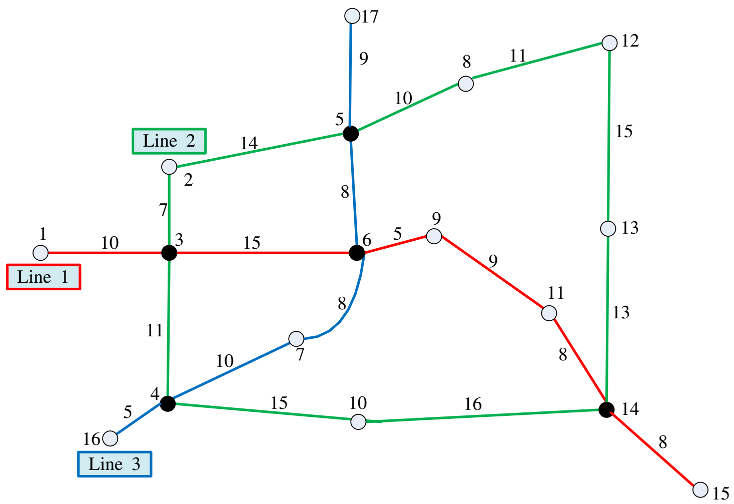

Figure 3. A test network.

Table I. Link parameters.

Link	Running time t_a	Capacity	Link	Running time t_a	Capacity
1–3	10	2400	5–17	9	2400
2–3	7	2400	6–7	8	2400
2–5	14	2400	6–9	5	2400
3–4	11	2400	8–12	11	2400
3–6	15	2400	9–11	9	2400
4–7	10	2400	10–14	16	2400
4–10	15	2400	11–14	8	2400
4–16	5	2400	12–13	15	2400
5–6	8	2400	13–14	13	2400
5–8	10	2400	14–15	8	2400

Table II. Network parameters.

A	B	p	n_{Seat}	β	γ	w'	$^{L}/_{v}$	ρ
1	2	2	360	1.6	1.6	1	8	2

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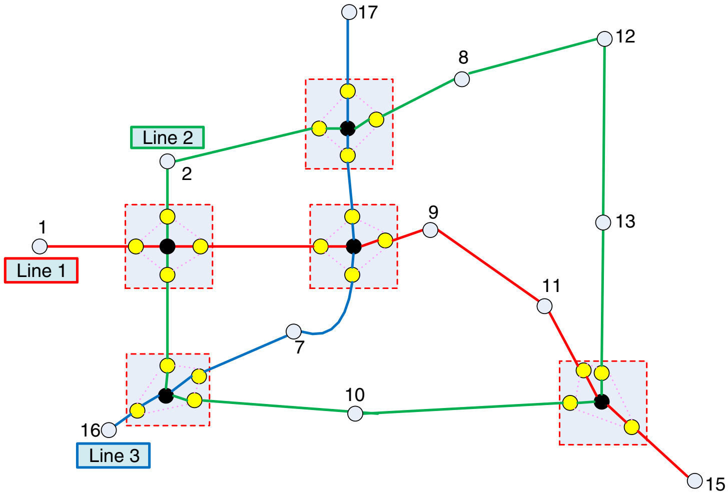

Figure 4. Augmented test network.

to the same operation line, then the cost (time) is set to zero. Otherwise, the cost (time) is equal to the transfer time. Therefore, the original figure can be extended to a graph with 37 nodes and 60 links.

For the augmented URN, the relationship between virtual link and original transfer station can be given by the following form:

$$X_i = \frac{1}{2} \sum_{a \in A_2^i} x_a, \forall i \in \hat{N}, \tag{21}$$

where $\hat{N}$ is the virtual station set of transfer station in URN and A_2^i is the virtual link set of transfer station i in URN.

5.2. *Effects of parameter τ on total system cost*

The purpose of this paper was to discover the effects of stochastic on URN performance. Therefore, we first examine the change of the system cost and flow against parameter τ that can be used to describe the fluctuation $\tau \frac{\sigma_i}{E_i}, i = 1, 2, 3$ of expected travel cost. Denote the total system cost $\text{TSC} = \sum_{a \in A} x_a T_a$. Figure 5 depicts TSC against varied values of parameter τ with demand $q = 1000$ for four cases. From Figure 5, we can find that with the increase of parameter τ, TSC will increase sharply that indicates that the larger τ will make the performance of traffic system worse for cases 2–4 and cause the passenger

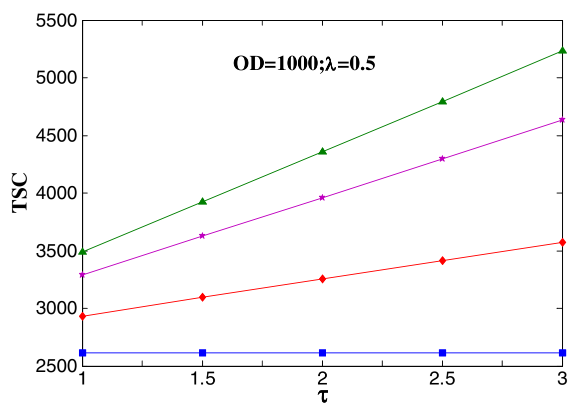

Figure 5. Effects of τ on TSC.

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crowding in the URN. Therefore, ignoring the stochastic effects can result in a sizeable underestimation of the actual TSC of URN.

5.3. *The flow distribution at the station*

Figure 6 shows the distribution of passenger flow at each station with different cases for $\lambda = 0.5$. We can see that the distribution of passenger flow is heterogeneous on URN for all cases, which means that the flow is homogeneous in most of stations and is higher in few stations (i.e., transfer stations). Therefore, these stations are prone to become bottlenecks and have a great influence on the performance of URN.

To further describe the differences among four cases, we give the flow ratio of other three cases to case 1 in Figure 7. It can be found that the case of DW-DT&DD has great distinction with other three cases, especially for DT-SW-SD (sub-graph of Figure 7). This can be explained easily. For the case DT-SW-SD, because of the stochastic delay time caused by emergent flows, many passengers are

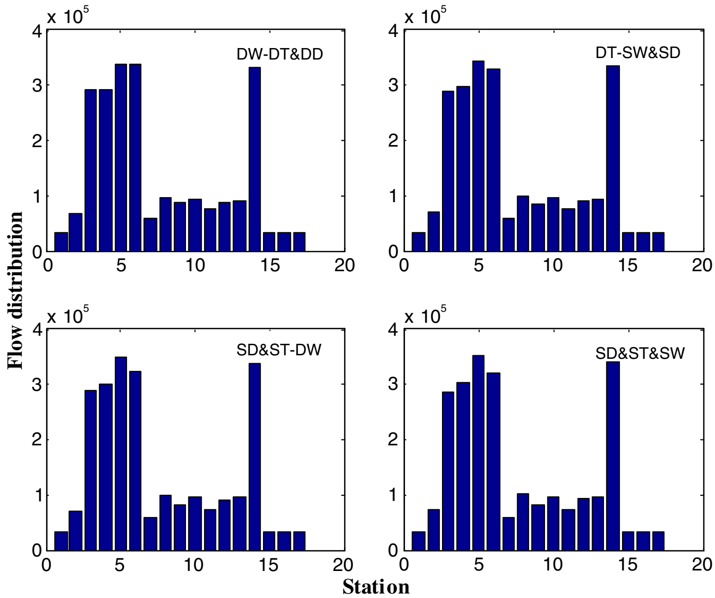

Figure 6. The distribution of passenger flow at each station for four cases with $\lambda = 0.5$ and $q = 1000$.

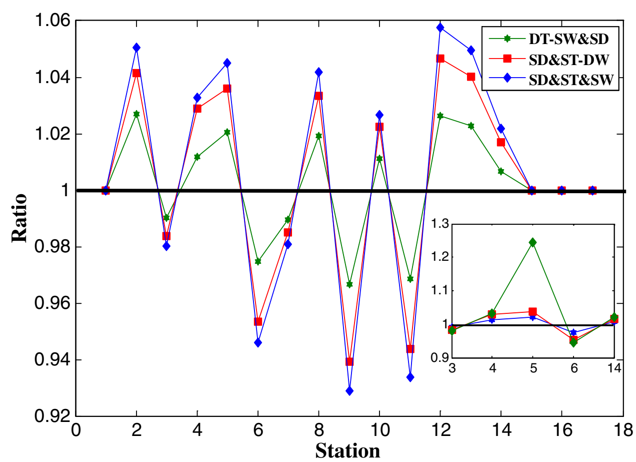

Figure 7. The ratio of other three cases with case 1 with $\lambda = 0.5$ and $q = 1000$.

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stranded in stations, especially in the transfer stations. That is, ignoring the stochastic effects cannot estimate the actual passenger flow exactly.

5.4. *The total transfer passenger flow*

Figure 8 gives the total transfer flow $\mathrm{TTF} = \sum_{i \in TN} X_i$ at the transfer stations against different risk aversion λ.

In URN, the transfer passenger flow is an important factor concerned by the managers because of occupying different operation lines in their travels. Figure 8 plots the TTF for four cases when $\lambda = 0.5$ and $q = 1000$. We can see that with the increase of risk-averse, the TTF would decrease sharply. This is because with the increase of λ, passengers will pay more attention on the risk-averse and they prefer the route without transfer stations and with smaller time fluctuation. Therefore, the total transfer passenger will decrease.

5.5. *Convergence test*

To further illustrate the efficiency of the solution algorithm, a convergence test of absolute deviations between iterations is given in the numerical example as shown in Figure 9. The results indicate that the algorithm can converge to a steady state within a short step.

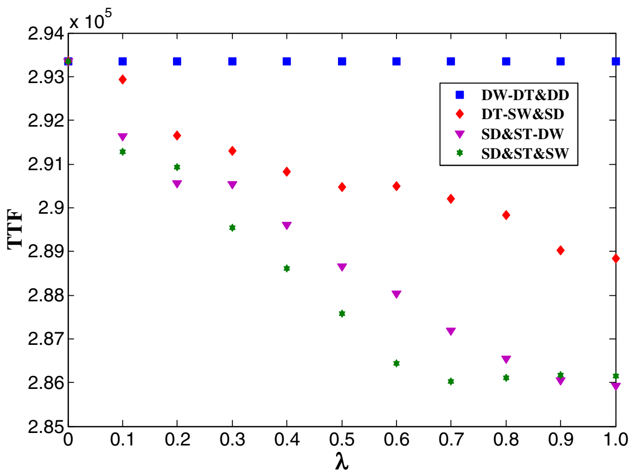

Figure 8. TTF for four cases when $\lambda = 0.5$ and $q = 1000$.

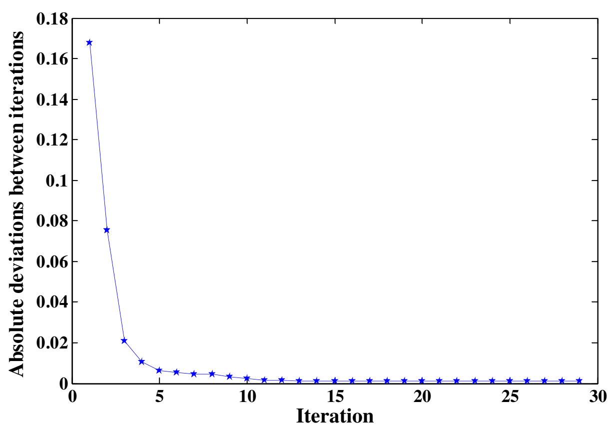

Figure 9. The convergence results for case 4 with $\lambda = 0.5$ and $q = 1000$.

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6. CONCLUSIONS

By developing an augmented URN, we analyze the passenger flow assignment under the stochastic conditions with the traditional traffic flow distribution model directly.

The total travel cost comprises four parts: in-train cost, waiting time, transfer time, and delay time related to the link passenger flow is proposed. By introducing the risk-averse behavior, we give the passenger disutility that depends on both the mean and the variance of the link travel time. Additionally, we investigate the TSC and the passenger flow distribution with the proposed stochastic passenger flow assignment model in four cases, respectively. The result shows that as consistent with our expectation, the stochastic factor plays important roles in the passenger travel behavior, and therefore should not be ignored in the reality, especially under highly congested URN.

However, both fixed OD matrix and capacity are assumed in this paper. In fact, the OD demand and capacity would fluctuate every day. Therefore, it would be interesting to examine the dynamic OD demand and capacity in network design problem in the further study. Moreover, the network design problem with multimode could be an important task of further investigations. Additionally, estimating link correlations is a difficult problem that needs a lot of investigating data; therefore, the assumption of independent link time distribution is adopted for simplification and mathematical clarity in the model development effort in this paper. In the future, it is essential to establish correlation between travel time components ignored in this paper.

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