Simple evaluation of squared weight

I.J. Fair, E.R. Carle

Department of Electrical and Computer Engineering, University of Alberta, Edmonton, Canada
E-mail: ivan.fair@ualberta.ca

Published in *The Journal of Engineering*; Received on 15th June 2015; Accepted on 17th June 2015

Abstract: Squared weight is an excellent metric on which to base codeword selection in DC-free multimode coding. It has, however, been considered too complex to implement in practice. In this Letter, the authors present a simple approach to evaluate this metric that enables it to be implemented in high-speed digital logic.

1 Introduction

Constrained sequence codes are widely used to ensure that the coded sequence has characteristics that enable it to be conveyed accurately over a constrained channel [1]. These codes include the class of DC-free codes that are used to encode source data when the channel does not pass frequencies at or near $f = 0$ Hz [2].

Multimode coding has been shown to be an efficient and effective approach to encode data sequences to satisfy channel constraints [3]. As shown in Fig. 1, this approach involves representing source data words with a number of alternatives and selecting the alternative that best satisfies the constraints. Practical techniques to construct alternative representations include the scrambling of binary sequences [4] and Reed–Solomon-coded sequences [5], and use of the Hadamard transform [6] among other techniques.

Selection of the alternative that best meets the channel constraints is an integral component of multimode encoding. In the encoding of DC-free sequences, selection based on minimum squared weight (MSW) [3] has been shown to result in excellent suppression of low frequencies. However, this metric has been considered too complex for practical implementation, resulting in the proposal for other simpler, but less effective selection techniques.

In this Letter, we briefly review DC-free coding and the MSW metric. We then present a method to evaluate squared weight that does not involve multiplication or squaring operations and therefore is straightforward to implement in high-speed digital logic.

2 DC-free sequences

A sequence is DC-free if the continuous component of its power spectral density (PSD) is zero at $f = 0$ Hz. Consider the binary sequence

$$\{\ldots, x_{-1}, x_0, x_1, \ldots, x_n, \ldots\} \tag{1}$$

where the binary symbols take on values $x_n \in \{-1, +1\}$. The *running digital sum* (RDS) following the nth symbol in this sequence is evaluated as

$$R_n = \sum_{i=-\infty}^{n} x_i = R_{n-1} + x_n \tag{2}$$

where if the RDS is R_0 when $n = 0$

$$R_n = R_0 + \sum_{i=1}^{n} x_i = R_{n-1} + x_n, \quad n = 1, 2, 3, \ldots \tag{3}$$

Sequence (1) is DC-free if and only if its RDS R_n is bounded for all n [7].

The width of the spectral notch of a DC-free sequence can be characterised by its cut-off frequency ω_0, defined as the radian frequency at which the PSD falls to 1/2 [8]. Justesen demonstrated that the cut-off frequency and the variance of the RDS values are related as [8]

$$2\sigma_R^2 \omega_0 \simeq 1 \tag{4}$$

where σ_R^2, known as the *sum variance*, is defined as

$$\sigma_R^2 = E\{R_n^2\} \tag{5}$$

It follows from (4) that sequences with lower sum variance exhibit a wider spectral notch at DC.

3 MSW selection

Consider again the multimode encoder depicted in Fig. 1 in which fixed-length source words are mapped to J length-M alternatives

$$\mathbf{x}_j = \{x_{j,1}, x_{j,2}, \ldots, x_{j,M}\}, \quad j = 1, 2, \ldots, J \tag{6}$$

From these J alternatives the ‘best’ is selected as the codeword. If the objective is to construct a DC-free-coded sequence with a wide notch at DC, it follows from (4) and (5) that the equivalent time-domain objective is to construct a sequence with low sum variance. It was based on this observation that Immink and Patrovics proposed the MSW selection criterion [3]: from the alternatives, select the word with the lowest value of *squared weight*, where the squared weight of the jth length-M alternative is defined as

$$S_{j,M} = \sum_{m=1}^{M} R_{j,m}^2, \quad j = 1, 2, \ldots, J \tag{7}$$

Note that an initial value S_0 could be included in (7), but it would be

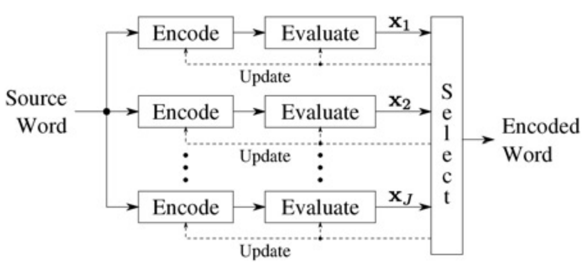

Fig. 1 *Multimode encoding*

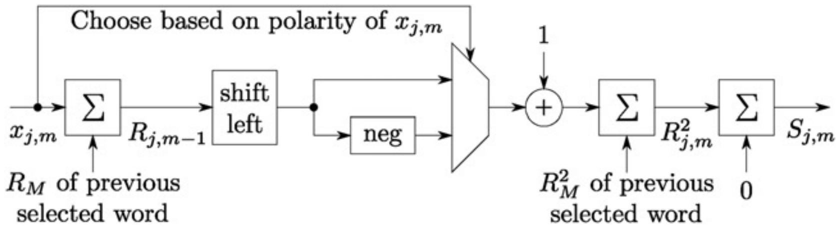

Fig. 2 *Simple evaluation of squared weight*

the same for all j because its value would depend only on the word selected in the previous encoding interval. Since this constant would not affect selection, it is ignored.

The modified MSW (MMSW) selection approach involves selecting, from those sequences whose $R_{j,M}$ falls within a predefined range, the sequence with the lowest $S_{j,M}$ [9]. Considerable analysis and simulation have verified the excellent performance that can be achieved with the MSW and MMSW selection criteria [3, 9].

MSW selection involves, for each alternative, (i) accumulation of symbol values to evaluate the RDS, (ii) evaluation of the squared RDS values and (iii) accumulation of the squared RDS values to evaluate the squared weight. Of these operations, it is the squaring of RDS values that has been considered too complex to allow for practical implementation in digital logic. Alternative selection methods which involve only accumulation, such as the minimum threshold overrun (MTO) criterion [3], have therefore been proposed.

4 Simple evaluation of squared weight

A simple approach to evaluate squared RDS values is based on the observation that in each symbol interval the RDS either increases or decreases by one. Let the initial value $R^2_{j,0}$, common to all alternatives $j, j = 1, 2, \ldots, J$, be the squared RDS value at the end of the previously selected word. If $x_{j,m} = +1$, then $R_{j,m} = R_{j,m-1} + 1$ and

$$\begin{aligned} R^2_{j,m} &= (R_{j,m-1} + 1)^2 \\ &= R^2_{j,m-1} + 2R_{j,m-1} + 1, \quad m = 1, 2, \ldots, M \end{aligned} \tag{8}$$

If $x_{j,m} = -1$ then $R_{j,m} = R_{j,m-1} - 1$ and

$$\begin{aligned} R^2_{j,m} &= (R_{j,m-1} - 1)^2 \\ &= R^2_{j,m-1} - 2R_{j,m-1} + 1, \quad m = 1, 2, \ldots, M \end{aligned} \tag{9}$$

It follows from (8) and (9) that squared weight can be evaluated in an iterative fashion without multiplication or squaring operations.

RDS values are evaluated by accumulating symbol values; these integer values are stored with an appropriate l-bit representation. Limiting l implicitly limits valid RDS values and results in an MMSW approach. Each RDS value is doubled with a single left-shift of the l-bit representation. This is either added to or subtracted from $R^2_{j,m-1}$ depending on the polarity of $x_{j,m}$, and the result is incremented by 1 to yield $R^2_{j,m}$. These squared RDS values are accumulated as indicated in (7) to determine the squared weight $S_{j,M}$.

Fig. 2 presents a block diagram of this simplified evaluation process. The accumulators in this figure are loaded immediately following selection of the previous word and prior to the start of the evaluation of the current alternative. Note that incrementing the value of $-2R_{j,m-1}$ by 1 prior to summing with $R^2_{j,m-1}$, as suggested in this figure, ensures that intermediate results within the $R^2_{j,m}$ accumulator are non-negative.

The incrementation by 1 in (8) and (9) can be avoided by noting that since this term is common to all alternatives $j, j = 1, 2, \ldots, J$, it has no effect on MSW selection. Let $\tilde{R}^2_{j,m}$ denote the value obtained by ignoring this term

$$\tilde{R}^2_{j,m} = R^2_{j,m} - m, \quad m = 1, 2, \ldots, M \tag{10}$$

Accumulating values of $\tilde{R}^2_{j,m}$ yields

$$\begin{aligned} \tilde{S}_{j,M} &= \sum_{m=1}^{M} \tilde{R}^2_{j,m} = \sum_{m=1}^{M} (R^2_{j,m} - m) \\ &= S_{j,M} - \sum_{m=1}^{M} m = S_{j,M} - \frac{M(M+1)}{2} \end{aligned} \tag{11}$$

Since the offset by $M(M+1)/2$ is independent of j, selecting the alternative with minimum $\tilde{S}_{j,M}$ results in the same decision as selecting the word with minimum $S_{j,M}$.

Note that $\tilde{R}^2_{j,M}$ and $\tilde{S}_{j,M}$ can be negative. Moreover, note that since the squared RDS of the previously selected word establishes $R^2_{j,0}$ for all j, an adjustment by M is required when updating accumulator values between encoding intervals. This adjustment is shown in Fig. 3.

We note that Figs. 2 and 3 depict metric evaluation requiring only summation, similar to the less effective MTO criterion. We also note that this approach can be extended to evaluation of higher-order metrics such as the sum of squared values of the kth-order RDS for DC^k-free multimode codes [10].

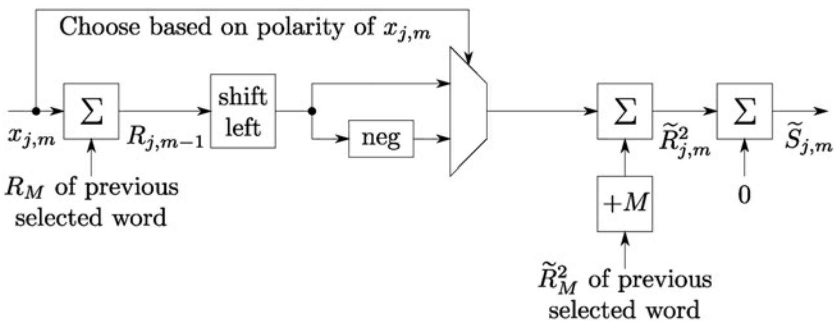

Fig. 3 *Alternate simple evaluation of squared weight*

5 Conclusion

We have presented a technique to evaluate squared weight in multimode coding that does not require multiplication or squaring. This approach lends itself well to implementation in high-speed digital logic.

6 Acknowledgment

This work was supported by the Natural Sciences and Engineering Research Council (NSERC) of Canada.

7 References

[1] Cattermole K.W.: 'Principles of digital line coding', *Int. J. Electron.*, 1983, **55**, (1), pp. 3–33

[2] Immink K.A.S.: 'Codes for mass data storage systems' (Shannon Foundation, 2004, 2nd edn.)

[3] Immink K.A.S., Patrovics L.: 'Performance assessment of DC-free multimode codes', *IEEE Trans. Commun.*, 1997, **45**, (3), pp. 293–299

[4] Fair I.J., Grover W.D., Krzymien W.A., *et al.*: 'Guided scrambling: a new line coding technique for high bit rate fiber optic transmission systems', *IEEE Trans. Commun.*, 1991, **39**, (2), pp. 289–297

[5] Kunisa A.: 'Comparison of two guided scrambling schemes for optical disks', *IEEE Trans. Consum. Electron.*, 2002, **48**, (3), pp. 484–488

[6] Copeland G., Tezcan B.: 'Disparity and transition density control system and method'. US Patent 6,304,196, 2001

[7] Pierobon G.: 'Codes for zero spectral density at zero frequency', *IEEE Trans. Inf. Theory*, 1984, **30**, (2), pp. 435–439

[8] Justesen J.: 'Information rates and power spectra of digital codes', *IEEE Trans. Inf. Theory*, 1982, **28**, (3), pp. 457–472

[9] Zhu Y., Fair I.J.: 'Modified minimum squared weight selection criterion for DC-Free multimode codes', *IEE Electron. Lett.*, 2005, **41**, (17), pp. 973–975

[10] Xin Y., Fair I.J.: 'High-order spectral-null multimode codes', *IEEE Trans. Commun.*, 2004, **52**, (8), pp. 1231–1237