JOURNAL OF ADVANCED TRANSPORTATION
J. Adv. Transp. 2015; **49**:10–28
Published online 4 September 2013 in Wiley Online Library (wileyonlinelibrary.com). DOI: 10.1002/atr.1252

Analysis of factors that influence the sensor location problem for freeway corridors

Haijian Li[1,2], Honghui Dong[1], Limin Jia[1]*, Moyu Ren[2] and Shi Li[3]

[1]*State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University, Beijing 100044, China*
[2]*School of Traffic and Transportation, Beijing Jiaotong University, Beijing 10044, China*
[3]*Laboratoire Systèmes et Transports, Université de Technologie de Belfort, Montbéliard, 90010, France*

SUMMARY

The use of traffic sensors to acquire real-time traffic information for intelligent transportation systems is becoming increasingly common. It is a challenge to determine where these sensors should be located to maximize the benefit of their use. This paper aims to illuminate the interaction of the sensor location problem (SLP) and its influencing factors, and to reveal the influencing mechanisms between those factors and the optimal sensor numbers. Firstly, we sum up the factors that influence the SLP for freeway corridors in detail and present the mathematical formulation of each factor. Then, given the parameters, which are derived from those influencing factors, the maximum integration value model (MIVM) and simplified MIVM are proposed for addressing the SLP. Finally, a real world case study, in which the simplified MIVM is used, is presented to illustrate how these factors influence the optimal sensor numbers and the maximum integration value, and also leads to the typical influencing patterns of those factors for freeway corridors. The results of the case study also demonstrate the effectiveness of the model and problem solving scheme. What is more, the suggestions for using the findings hereof in practical applications are put forward. Copyright © 2013 John Wiley & Sons, Ltd.

KEY WORDS: traffic engineering; sensor location problem; influencing factors; freeway corridors; optimization method

1. INTRODUCTION

Congestion and oversaturated roads pose significant problems and create delays in every major city in the world. An important goal of intelligent transportation systems (ITS) is to build and extend traffic sensor networks to improve transportation systems' observability, productivity, and efficiency [1].

Intelligent transportation systems are considered as the best solution for improving the utility of transportation resources. An important function of ITS is real-time traffic information collection, which is performed using various types of traffic sensors, including traffic counting sensors and traffic speed sensors [2]. Sensor technologies (e.g., loop detectors, surveillance cameras, radio frequency identification, microwave sensors, and magnetic sensors) have been widely used on highway networks and urban arterial roads to estimate real-time traffic conditions and provide valuable information to both private individuals and public agencies. The effectiveness of ITS depends not only on the accuracy of the traffic information but also on the coverage of data collection over the transportation network [3].

*Correspondence to: Limin Jia, State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University, Beijing 100044, China. E-mail: jialm@vip.sina.com

Real-time traffic information enables road users to choose routes with less congestion, and enables traffic managers to respond promptly to congestion patterns and select control strategies efficiently [4]. Whether the traffic information collected by sensor technologies reflects real-time traffic conditions accurately depends mostly on the distribution and density of the traffic sensors. To obtain adequate traffic information, the density of traffic sensors should be high enough to cover the entire transportation network. If sensors are densely deployed over a transportation network and all of them collect real-time traffic data around their locations, the traffic status of the whole network can be obtained from the sensor data using some feasible algorithm. In addition, abnormal traffic states (e.g., traffic accidents and traffic jams) can be detected [4].

However, implementing these sensor technologies usually requires large investments. Therefore, the number and locations of sensors should be optimized to balance the required investments with the demand for traffic information. Considering the rapid development of ITS in modern cities, it is easy to see conflicts arising between the requirement for a certain density of fixed traffic sensors and budget limitations [2]. An important observation is that more traffic information can be obtained accurately, and the number of sensors can be significantly reduced if traffic sensors are installed rationally and their locations are selected effectively. It is important to find methods to work out a rational layout of traffic sensors that meets the requirements of ITS for reasonable investments [2]. Therefore, it is crucial to address the sensor location problem (SLP).

Just knowing the number of required sensors and the covered links of freeway network is not sufficient to solve the SLP. The patterns and mechanisms that some factors influence the optimal SLP solution need to be studied. These patterns and mechanisms include whether a link should have sensors installed, how many sensors a link requires, what are the best locations for these sensors, and how sensor layout strategies should be changed when changes occur in the traffic conditions (e.g., traffic volume, traffic control, or construction conditions). Many researchers have made contributions to the solution of the SLP and have proposed theoretical methods for optimizing sensor number and locations for freeway corridors [1,5–13]. These theoretical methods for optimizing SLP can be classified into two categories: count-based methods and speed-based methods. Count-based methods mainly address those applications, such as origin–destination (O–D) estimation [1,3,14,15], freeway bottleneck identification [13], and freeway incident detection [5]. Travel time estimates [6–12] always require data from speed-based methods.

Based on the traffic volume information estimated from sensor data, an O–D matrix of the transportation network can be calculated. A majority of researchers in this field use graph theory to obtain O–D matrix. They transform the SLP into a problem of graph theory and apply graph theories to obtain a solution [1,16]. The transportation network is modeled as a directed graph with vertices representing intersections and directed edges between the vertices representing roads. The traffic flow over the roads is described by a network flow function on the edges of the graph. Hence, the goal of the problem is to find the smallest subset of edges such that knowledge of the flow along them uniquely determines the flow everywhere on the graph. Speed-based methods are mostly applied to estimating travel time. In general, the traffic sensors (such as loop detectors and microwave detectors) that are used to measure vehicle speeds can also be used to estimate the traffic volume.

Graph theory based methods can solve problems for specific applications (O–D estimation and travel time estimation), but they have some intrinsic problems. First, there are many assumptions in graph theory. In addition, the solution process only identifies the smallest subset of edges. The edges on which sensors should be installed can be obtained, but the optimal sensor locations on edges (e.g., the middle, the front end, or somewhere else) cannot be determined. In a freeway network, the weights at different locations (e.g., on a ramp or a bridge) may be different; hence, it is necessary to consider the weights of different locations, which is an influencing factor in the SLP. Second, as these sensor technologies become more reliable and cost-effective, the demand for travel information is also growing. More than one sensor (note that if there are several sensors in a section of an edge, they are considered to be one sensor here) will be located on an edge. The optimal location of each sensor on an edge should therefore be determined.

However, there has been very little research on the factors that influence the SLP [2,17,18], especially for freeway corridors, and it can be difficult to see how these factors influence the optimal sensor number and locations. It can therefore be difficult to design a strategy to address the situation that

DOI: 10.1002/atr

arises when a factor changes. Leow *et al.* [17] proposed a novel method for the solution of the SLP. They identified some influencing factors (IFs), such as the cost of sensors, the level of service of the road, and the annual cost of congestion. They determined the optimal sensor spacing for a freeway, but they did not discuss how those factors influence their results. Hong and Fukuda [18] studied the influence of different sensor locations on the accuracy of traffic state estimation by using a velocity-based cell transmission model. They tested their method on a segment of the Tokyo Metropolitan Highway during rush hours. Li and Jia [2] discussed the principles of sensor layout, some of which can be treated as IFs of the SLP, but they did not develop any models to analyze those factors thoroughly.

1.1. Purpose and organization

The purpose of this paper is to fill these gaps. An analysis of IFs of the SLP for freeway and a transformation of those factors into model parameters with mathematical formulations will be presented. This paper also presents the use of a maximal integration value model (MIVM) [19] to analyze how those factors influence the optimal sensor locations and presents some strategies for SLP solution when those parameters are changed. Furthermore, the paper will illuminate the interaction of the SLP and its IFs, and analyze the influencing patterns of those factors intuitively. According to the distribution and trend of the optimal sensor numbers with different value of each factor, the paper also reveals the influencing mechanisms between those factors and the optimal sensor numbers.

This paper is organized as follows. In Section 2, the primary IFs are summed up and the model parameters are formulized. In Section 3, the MIVM and its simplified model are described, and the corresponding algorithms of those models are also proposed. Section 4 focuses on the sensitivity analysis of the model parameters and shows how these factors influence the optimal sensor numbers and the maximum integration value by using a numerical example. Conclusions are presented in Section 5.

2. INFLUENCING FACTORS AND THEIR MATHEMATICAL FORMULATIONS

2.1. Sensor location problem influencing factors

There are many factors (such as traffic information, sensor types, sensor locations, sensor cost, etc.) that may influence the optimal SLP solution. It can be divided into three categories of those IFs.

The first category of IFs is those related to the road. These include the sensor layout cost and sensor locations in the road. The layout cost mainly refers to the project cost of installing sensors, such as the expenses of setting up a pole or cutting pavement and the cost associated with closing traffic lanes. Perhaps sensors are placed in the same road, but the characteristics of the sensor locations may not be the same. Hence, different sensor locations should not be treated equally. The second category of IFs is those related to the sensors. Different types of sensors will result in different optimal layout schemes for the SLP. The third category of IFs are those related to the investment, which should be regarded as optional factors. If the budget is adequate, the SLP solution will not be influenced by the amount of the investment. Otherwise, it may be necessary to rethink the layout strategy and seek another scheme whose cost is within the limit of the allowable investment. The inner portion of Figure 1 shows the IFs and their relationships with the SLP.

2.1.1. Sensor layout cost

The sensor layout cost will influence the investment required for the project. When a sensor is installed, a pole or portal frame is set up to fix the sensor (such as a microwave sensor or camera) or the pavement is cut to embed the sensor (such as a loop sensor or magnetic sensor). Installing a traffic sensor usually requires a traffic lane closure. The time required for the lane closure may be varied. However, all sensor installations need funds (to set up poles or cut the pavement) and other costs (associated with lane closures). Table I shows estimates of the cost associated with sensor installation of different types for a typical intersection application [20]. The installation costs (layout costs) of different types of sensors are different (fifth column in Table I).

DOI: 10.1002/atr

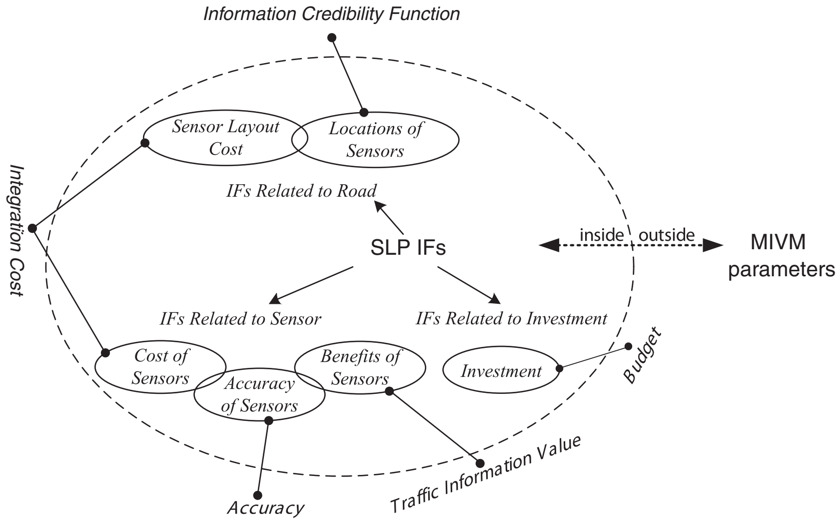

Figure 1. Influencing factors and their relationships with the sensor location problem.

2.1.2. Locations of sensors

The importance of sensor locations along a road may be different. Figure 2 shows a one-way road in a freeway corridor with point A near a ramp and point B in the middle. The traffic data derived from point A and point B at a fixed interval (e.g., 30 s, 1 min, 5 min, 10 min, etc.) can be considered a time series over a certain period (e.g., a week, a day, an hour, etc.). Defining $\mathbf{Vo}_{\mathrm{A}}(t)$ and $\mathbf{Vo}_{\mathrm{B}}(t)$ as the traffic volume time series at locations A and B, respectively, at time t, we have the following:

$$\mathbf{Vo}_{\mathrm{A}}(t) = \{Vol_{\mathrm{A}}(t - k\tau)\}, k \text{ is } \{0, 1, \ldots, N\} \tag{1}$$

$$\mathbf{Vo}_{\mathrm{B}}(t) = \{Vol_{\mathrm{B}}(t - k\tau)\}, k \text{ is } \{0, 1, \ldots, N\} \tag{2}$$

where τ is the sampling interval, N is the sampling number during a certain period, $Vol_{\mathrm{A}}(t)$ and $Vol_{\mathrm{B}}(t)$ are the traffic volumes at point A and point B at time t.

Correlation coefficient (CC) reflects the linear degree of correlation between two time series. The stronger the linear correlation is (or the larger the CC is), the more the two time series are a colinear representation. Here, we define the spatial correlation function (SCF) of two time series ($\mathbf{Vo}_{\mathrm{A}}(t)$ and $\mathbf{Vo}_{\mathrm{B}}(t)$) at locations A and B as follows:

$$\mathrm{SCF}\left(d_{\mathrm{A,B}}\right) = \mathrm{corr}(\mathbf{Vo}_{\mathrm{A}}(t), \mathbf{Vo}_{\mathrm{B}}(t)) \tag{3}$$

where $d_{\mathrm{A,B}}$ is the distance between locations A and B, corr is the CC of the two time series, that is

$$\mathrm{corr}(\mathbf{Vo}_{\mathrm{A}}, \mathbf{Vo}_{\mathrm{B}}) = \frac{\sum_{i=1}^{N}\left(\mathbf{Vo}_{\mathrm{A}i} - \overline{\mathbf{Vo}_{\mathrm{A}}}\right)\left(\mathbf{Vo}_{\mathrm{B}i} - \overline{\mathbf{Vo}_{\mathrm{B}}}\right)}{\sqrt{\sum_{i=1}^{N}\left(\mathbf{Vo}_{\mathrm{A}i} - \overline{\mathbf{Vo}_{\mathrm{A}}}\right)^2}\sqrt{\sum_{i=1}^{N}\left(\mathbf{Vo}_{\mathrm{B}i} - \overline{\mathbf{Vo}_{\mathrm{B}}}\right)^2}} \tag{4}$$

The difficulty of the installation at locations A and B may be different, which leads to different sensor layout costs. The location factor will certainly influence the SLP solution, so it is analyzed as one of the IFs.

DOI: 10.1002/atr

Table I. Estimated life-cycle costs for a typical freeway application (for six lanes).

	Device		Installation		Annual	
Sensor device	Unit quantity	Cost	Mounting	Cost	maintenance cost	Lifetime (year)
Inductive loop	12 loops	$9000 (including installation cost)	Under pavement	/	$700	5/15
Autosense II	6	$36 000	O	$3200	$600	7
ASIM IR 254	6	$4200	O/S	$3200/$1200	$600	7
Siemens PIR-1	6	$6600	O	$3200	$600	7
RTMS	One unit per direction	$6600	O/S	$2400/$400	$200	7
TC 26B	One unit per direction	$1470	O/S	$2400/$400	$200	7
SmarTek SAS-1	One unit per direction	$7000	S	$800	$400	7
Autoscope solo	One camera per direction	$9800	O/S	$3000/$1000	$400	10
VideoTrak 900	One camera per direction	$17 400	O/S	$3000/$1000	$400	10

O, overhead; S, sidefire.
Data from literature in [20], page 102.

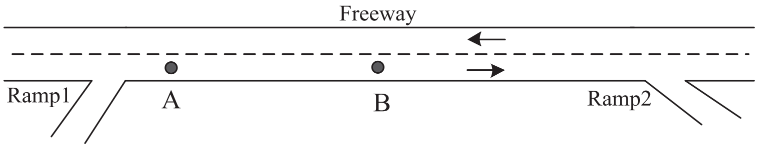

Figure 2. Sketch map of different location in a one-way road (point A is near the ramp, point B is in the middle).

2.1.3. Cost of sensors

Because sensor technologies continue to become more reliable and cost-effective, the demand for sensors is growing rapidly. Thousands of sensors are used in freeway networks for rapid development of ITS, and investments in traffic sensors and related facilities are growing accordingly. Furthermore, the sensor prices have not been reduced sufficiently. It is therefore necessary to consider the influencing factor of sensor cost. The cost of a sensor includes the sensor price (the device cost) and the maintenance cost over the sensor's lifetime. Table I also shows the device costs and the annual maintenance costs of different sensor types (third and sixth column in Table I), which are directly related to the investment required.

2.1.4. Benefits of sensors

The traffic information derived from sensors is significant for decision-making by managers and travelers, and it can also be used by traffic planning departments. When a sensor is placed in a road, it should provide some benefits. The benefits may be evident or implicit. Moreover, the importance of traffic information for ITS shows that traffic information must provide some value, just as traffic sensors must. The benefits provided by a sensor are referred to as the whole value realized in its lifetime. Some IFs may be related to the benefits provided by a sensor. These factors include the type of traffic information (e.g., traffic volume and traffic speed), the sensor locations (e.g., ramps, bridges, and road bends) and the lifetime of the sensor (e.g., 5, 7, and 10 years). The benefits of a sensor can be classified into two categories. The first category includes the direct benefits associated with the information type, sensor location, and lifetime. Direct benefits can be calibrated with respect to the reduction in congestion cost for the one-way road where the sensor is located. The other category includes the external benefits related to fuel savings, reductions in vehicle emissions and noise, and other external cost savings.

2.1.5. Accuracy of sensors

Like many other information technologies, most sensors are subject to performance disruptions because of technology flaws, system errors, adverse weather conditions, or intentional sabotage. Such failures may substantially impair traffic network coverage and surveillance effectiveness [4]. Table II shows the error rates associated with sensor devices in freeway field tests. In Table II, the error rates for both traffic counts and speed are presented. Table II also shows that the accuracy of sensor will change by sensor type (first column in Table II), mounting location (second column in Table II), and traffic

Table II. Error rates of sensor devices in freeway field tests.

Sensor	Mounting	Count (%)	Speed (%)	Evaluation organization
Inductive loop (Saw-cut)	Under pavement	2	5–10	TTI
Magnetic (3M microloop)	Pavement	2.5	1.4–4.8	MNDOT
Magnetic (3M microloop)	Bridge	1.2	1.8	MNDOT
Active Infrared (Autosense II)	Overhead	0.7	5.8	MNDOT
Microwave Radar (RTMS)	Overhead	2	7.9	MNDOT
Ultrasonic (TC 30)	Overhead	2	/	MNDOT
Passive Acoustic (SAS-I)	Sidefire	8–16	4.8–6.3	MNDOT
Video (Autoscope solo)	Overhead	5	2.5–7	MNDOT
Video (Autoscope solo)	Sidefire	2.1–3.5	0.8–3.1	TTI

TTI, Texas Transportation Institute; MNDOT, Minnesota Department of Transportation.
Data from literature in [20], page 74-75;.

DOI: 10.1002/atr

information type (third and fourth columns in Table II). Based on the data from literature in [20], it can also be concluded that for different locations (pavements, ramps, or bridges), the accuracy of a sensor for the same traffic parameter will be different. The accuracy of sensors will influence the optimal sensor locations and should be treated as an SLP influencing factor.

2.1.6. *Investment*

In the optimization problem, even though the investment changes, the optimal sensor number, and locations are fixed only if the IFs of roads and sensors are fixed. However, in an actual field project, the sensor number is constrained by the investment. In general, the investment consists of two main components. The first component is the fixed investment, which refers to the sensor cost and the layout cost (Table I, third and fifth columns). After all the sensor devices are installed, the second component of the investment is the annual maintenance cost (Table I, sixth column).

2.2. *Mathematical formulations of sensor location problem influencing factors*

Some IFs are qualitative. To build a mathematical model, some qualitative factors need to be transformed into quantitative factors. As shown in the outer part of Figure 1, the quantitative factors used as model parameters are given based on the SLP IFs. In Figure 1, the information credibility function (ICF) represents the location factors. The integration cost includes the layout cost and the sensor cost, and it represents the whole cost of a sensor. The traffic information value is derived from the sensor benefit factor and can be calibrated by estimating the direct benefits and external benefits. The last two model parameters, the accuracy and the budget, are derived from the sensor accuracy factor and investment factor.

2.2.1. *Information credibility function*

Traffic information is derived from those fixed sensors and it will be not balanced in the whole road network. In this paper, the ICF is proposed to represent the spatial distribution of traffic information derived from fixed sensors. In Figure 3, the value of a road's ICF at point x represents the credibility or believability of traffic information at that point. The value of the road ICF at sensor locations is larger than at other locations, that is, the traffic information is more believable at sensor locations than at places with no sensors.

A sensor can fully reflect (without considering the accuracy of the sensor) the traffic information of the point where the sensor is located. For a point p near a sensor location i, specifically, at a certain distance d_p from the sensor, because of the continuity and relativity of traffic information, the sensor at location i can reflect the traffic information credibility at location i with a degree close to 1 and at point p with a degree of $f(d_p)$ $(0 \le f(d_p) \le 1)$. Therefore, throughout the whole domain, $f(d_p)$ can be regarded as the credibility that a sensor represents the traffic information at a distance d_p from the sensor.

Depending on the sensor locations, each sensor has its own sensor ICF. A sensor ICF can be described by $f(d_p)$ and it is defined as follows:

$$\mathrm{ICF}^{sensor} = f(x) \quad x \in (-\infty, +\infty) \tag{5}$$

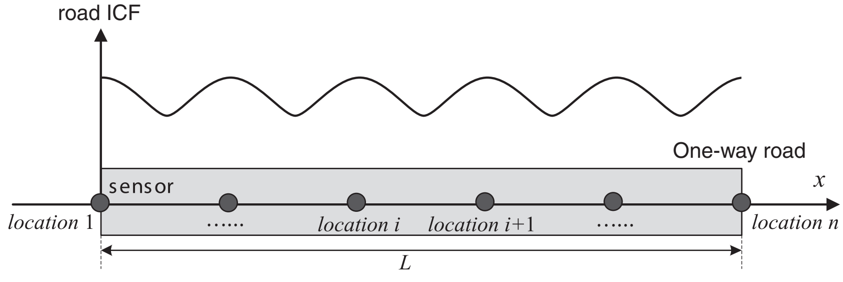

Figure 3. Sketch map of road information credibility function.

where $f(x)$ is a type of function and x is the distance from the sensor. Because of the directivity of distance along a road, the distance x is positive if the direction away from the sensor is on the right-hand side along the one-way road; otherwise, the distance x is negative.

Using Equation (3), a general relationship between SCF and distance can be established using an SCF scatter diagram (Figure 7). Through some function-fitting algorithms, sensor ICF can be calibrated as follows:

$$f(x) = \text{fitt}(\text{SCF}(d_1), \text{SCF}(d_2), \ \ldots, \ \text{SCF}(d_k)) \tag{6}$$

where SCF(d_1), SCF(d_2), …, SCF(d_k) are the SCF values with distances of d_1, d_2, …, d_k in the SCF scatter diagram; fitt is a type of fitting function of the SCF scatter diagram; and $f(x)$ is the sensor ICF.

2.2.2. *Integration cost*

Given the types of sensors and roads for the SLP, the integration cost of a sensor can be calibrated using the following formula:

$$C(i) = C_l(i) + C_d(i) + T_l \times C_m(i) \tag{7}$$

where $C(i)$ is the integration cost of a sensor at location i; $C_l(i)$ is the layout cost of a sensor at location i, including the installation cost and the cost associated with the lane closure; $C_d(i)$ is the device cost, which is the price of a sensor; $C_m(i)$ is the annual maintenance cost of a sensor; and T_l is the lifetime of a sensor in years (a constant).

2.2.3. *Traffic information value*

The traffic information value refers to direct benefits and external benefits provided by a sensor and can be calculated using the following formula.

$$V(i) = B_d(i) + B_e(i) \tag{8}$$

where $V(i)$ is the traffic information value of the sensor at location i; $B_d(i)$ are the direct benefits of the sensor at location i; and $B_e(i)$ are the external benefits of the sensor at location i. $B_d(i)$ can be estimated using the following formula:

$$B_d(i) = T_l \times T_y \times T_p \times C_c \times Q_d(i) \tag{9}$$

where T_l is the lifetime of the sensor in years (a constant); T_y is the days in a year, in days/year (a constant); T_p is the peak hours per day, in hours/day (a constant); C_c is the average congestion cost per hour for one vehicle, which is related to the local average hourly wage, in units/hour/vehicle (a constant); and $Q_d(i)$ is the number of vehicles that do not encounter congestion at the sensor location i, the value of which can be determined from simulation or a field traffic survey. Similarly, $B_e(i)$ can be estimated using the following formula:

$$B_e(i) = T_l \times T_y \times T_p \times C_e \times Q_d(i) \tag{10}$$

where C_e is the external cost because of the reduced traffic congestion per hour for one vehicle. This mainly pertains to fuel savings, reduced emissions, and noise. C_e can be calculated as described in a related research report or estimated from expert evaluation. The other variables are the same as in Equation (9).

2.2.4. *Accuracy*

We denote the accuracy of the sensor at location i by $q(i)$. This accuracy can easily be got from related reports or product specifications (such as those given in Table II).

DOI: 10.1002/atr

2.2.5. Budget

As the number of sensors located in the road increases, the demand of investment increases as well. Denote *IM* as the required investment and *n* as the number of the required sensors, and then the following formula can be given:

$$IM = \sum_{i=1}^{n} C(i) \tag{11}$$

In this paper, the fixed investment is used for convenience. Making use of the fixed investment, every required sensor can be located. The new formula of investment can then be expressed as follows:

$$IM = \sum_{i=1}^{n} (C_l(i) + C_d(i)) \tag{12}$$

To demonstrate the influence of the fixed investment on optimal sensor locations easily, we presume that the layout and device costs of every sensor are equal (such are the facts mostly). C_l and C_d denote the layout cost and device cost respectively, then

$$IM(n) = n(C_l + C_d) \tag{13}$$

As Figure 4 shows, the line *IM*(*n*) represents the relationship between the fixed investment required and the number of sensors required. Denote *B* as the budget (a constant), then there will be two conditions.

Condition 1: $B > IM(n)$ (point p1, Figure 4) or $B = IM(n)$ (point p0, Figure 4). (14)

Condition 2: $B < IM(n)$(point p2, Figure 4). (15)

For condition 1, the budget is adequate, all the required sensors can be placed, and the global optimization of the SLP can be achieved.

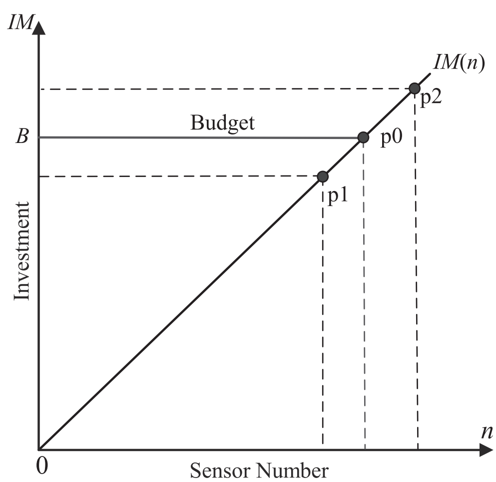

Figure 4. Relationship between budget and fixed investment.

For condition 2, the budget is limited, and we cannot place all the required sensors. Therefore, two strategies should be considered. The first strategy is a global optimization that locates the sensors at optimal locations based on the existing budget. The second strategy is a local optimization that located a certain number of sensors to maximize benefits. If more funds are invested later, the first strategy will work. The layout scheme for first strategy is developed by the way of using the current budget to select some sensor locations (the sensors at those locations should have maximum integration value over other possible locations) from among the optimal locations, which are identified by the model used to select sensor locations. The remaining locations will be selected according to the subsequent budget as it become available. However, if there were no subsequent investment, the second strategy should work and new optimal sensor locations should be determined by some methods

3. MODEL AND ALGORITHM

3.1. Maximum integration value model formulation and algorithm

Based on our prior work [19], the MIVM was selected to analyze the IFs for the SLP.

Sensors are placed densely and spaced equidistantly in a one-way road of freeway corridors, and the location of each sensor is recorded as i ($i = 1,2,\ldots,n$). Taking location 1 as the origin, and a 0–1 program model for the SLP can be established as follows:

$$\max\ z = total\ \text{Value} - total\ \text{Cost} = \sum_{i=1}^{n}\left(X_i \times q(i) \times V(i) \times \frac{\int_{a_{i-}}^{a_{i+}} f_i(x - x_i)dx}{\int_{-\infty}^{+\infty} f_i(x - x_i)dx}\right) - \sum_{i=1}^{n} X_i C(i) \tag{16}$$

$$s.t.\begin{cases} x_i = (i-1)d \\ d = L/(n-1) \\ X_i = 0 \quad or \quad 1 \end{cases} \tag{17}$$

where z is the optimization target, which is the integration value of all the located sensors, namely, the traffic information value of all located sensors minus the integration cost; $q(i)$, $V(i)$, and $C(i)$ are the accuracy, the traffic information value, and the integration cost of the sensor at location i, respectively; x_i is the coordinate of location i along the one-way road, where $x_{i+1} - x_i = d$; f_i is the sensor ICF at location i; X_i is 0 or 1, 1 a sensor is located at location i, 0 indicating that there is not a sensor at that location; a_{i-} and a_{i+} are the coordinates of the crossover point of f_i and f_{i-}, f_{i+}, where $a_{1-} = 0$, $a_{n+} = L$; f_{i-}, f_{i+} are the sensor ICF at location $i-$ and location $i+$, which are the adjacent front location with $X_{i+} = 1$ and the rear location with $X_{i-} = 1$ at location i when $X_i = 1$; and L is the length of the one-way road (Figure 5).

Figure 5 shows the coordinates of a_{i-}, a_{i+}, x_i, x_{i-}, and x_{i+}. The MIVM selects the maximum value from among those sensor ICFs as the final value of the road ICF at the superposition regions.

As described in [19], a reasonable initial placement distance d (to obtain high precision, d should be as short as possible, e.g., 10–50 m) will be set. Given d, the number of sensors n can be obtained as follows:

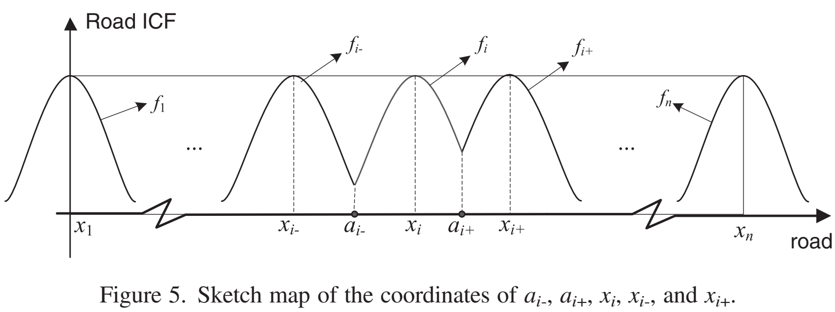

Figure 5. Sketch map of the coordinates of a_{i-}, a_{i+}, x_i, x_{i-}, and x_{i+}.

DOI: 10.1002/atr

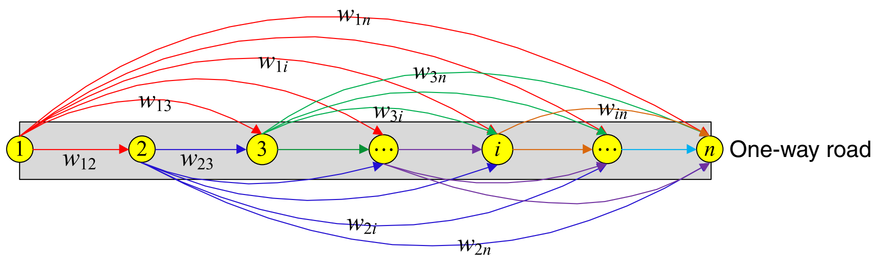

Figure 6. Directed graph D ($w_{ij}(i=1,2,\ldots,n$ and $j=1,2,\ldots,n)$ are the weights of the edges in graph D).

$$n = L/d + 1 \tag{18}$$

A directed graph D (Figure 6) is designed to solve the MIVM. As shown in Figure 6, the vertices (v_1, v_2, …, v_i, …, v_n) of the directed graph are the index of sensor locations (1, 2, …, i, …, n). We let e_{ij} denote the edge from v_i to v_j and let w_{ij} denote the weight of e_{ij}, then the weight w_{ij} of e_{ij} is given by the following formula.

$$w_{ij} = \begin{cases} \dfrac{C(i)+C(j)}{2} - \left(V(i)\times q(i)\times \dfrac{\int_{x_i}^{a_{ij}} f_i(x-x_i)dx}{\int_0^{+\infty} f_i(x-x_i)dx} + V(j)\times q(j)\times \dfrac{\int_{a_{ij}}^{x_j} f_j(x-x_j)dx}{\int_{-\infty}^{0} f_j(x-x_j)dx}\right) & i<j \\ \mathrm{M} & i\geq j \end{cases} \tag{19}$$

where M is a sufficiently large real number; a_{ij} is the coordinate of the crossover point of f_i and f_j; and the other parameters are the same as in Equation (16).

The physical meaning of w_{ij} is the opposite number of the integration value from location i to location j. The aim of the MIVM is to find the maximum integration value (z_m) from location 1 to location n. The shortest path from location 1 to location n for graph D represents the opposite number of z_m. Therefore, the optimal sensor locations on the graph D can be obtained via the shortest path whose vertices are the optimal locations according to the MIVM. Denote Ω as the solution set of the MIVM, and Ω is equivalent to the vertices of the shortest path through graph D. Then, the maximum integration value is given by the following equation:

$$z_m = -\sum_{i,j\in\Omega} w_{ij} \tag{20}$$

3.2. Simplified maximum integration value model and corresponding algorithm

To simplify the SLP and make it easy to analyze the influencing pattern of each factor, it is assumed that the values of each model parameter, such as ICF, integration cost, traffic information value, and accuracy, are equal at all locations. Denote $f(x)$, C, V, and q as the four parameters, respectively, and for $i=1, 2, \ldots, n$, we have the following:

$$f_i(x) = f(x) \tag{21}$$

$$C(i) = C \tag{22}$$

$$V(i) = V \tag{23}$$

$$q(i) = q \tag{24}$$

DOI: 10.1002/atr

Actually, the values of each parameter at different locations for the SLP are always close. Generally, the same types of sensors will be selected for studying the SLP on a single freeway. And the characteristic of the traffic information at different locations along the same road will be similar. Hence, the simplified MIVM described earlier has the practical meaning.

Denote I_{ij} as the integration value between location i and location j, for $1 \le i < j \le n$, the following formula can be obtained:

$$\begin{aligned} I_{ij} &= q(i) \times V(i) \times \frac{\int_{x_i}^{a_{ij}} f_i(x-x_i)dx}{\int_{-\infty}^{+\infty} f_i(x-x_i)dx} + q(j) \times V(j) \times \frac{\int_{a_{ij}}^{x_j} f_j(x-x_j)dx}{\int_{-\infty}^{+\infty} f_j(x-x_j)dx} - \frac{1}{2} \times (C(i)+C(j)) \\ &= qV\left(\frac{\int_0^{a_{ij}-x_i} f(x)dx}{\int_{-\infty}^{+\infty} f(x)dx} + \frac{\int_{a_{ij}-x_j}^{0} f(x)dx}{\int_{-\infty}^{+\infty} f(x)dx}\right) - C = \frac{qV\int_{a_{ij}-x_j}^{a_{ij}-x_i} f(x)dx}{\int_{-\infty}^{+\infty} f(x)dx} - C \end{aligned} \tag{25}$$

Specially, if $f(x)$ is symmetric about the vertical axis, then a_{ij} will be halfway between location i and location j, that is, $a_{ij} = (x_i + x_j)/2$. We denote the distance between location i and location j by d_{ij}, namely, $d_{ij} = x_j - x_i$. Then, for $1 \le i < j \le n$, I_{ij} will be the following:

$$I_{ij} = \frac{qV\int_{-d_{ij}/2}^{d_{ij}/2} f(x)dx}{\int_{-\infty}^{+\infty} f(x)dx} - C = \frac{qV\int_0^{d_{ij}/2} f(x)dx}{\int_0^{+\infty} f(x)dx} - C \tag{26}$$

When n sensors are densely and equidistantly placed in the one-way road (when $n=1$, the sensor will be located in the middle of the road), then $d = L/(n-1) = x_{i+1} - x_i$. We let d_0 denote the distance between x_i and $a_{i,i+1}$ (the coordinates of the crossover point of f_i and f_{i+1}), and I_0 denote the integration value between location i and location $i+1$, then $d_0 = a_{i,i+1} - x_i$. For $i = 1, 2, \ldots, n$, based on Equations (25) and (26), I_0 will be the following:

$$I_0 = I_{i,i+1} = \begin{cases} \dfrac{qV\int_{d_0-d}^{d_0} f(x)dx}{\int_{-\infty}^{+\infty} f(x)dx} - C & f \text{ is not symmetrical} \\ \dfrac{qV\int_0^{d/2} f(x)dx}{\int_0^{+\infty} f(x)dx} - C & f \text{ is symmetrical} \end{cases} \tag{27}$$

It can be concluded that the integration value between two adjacent sensors (location i and location $i+1$) will be equal. Then, the simplified MIVM will be the following formulations.

For $n = 1$:

$$z(n) = \begin{cases} \dfrac{qV\int_{-L/2}^{L/2} f(x)dx}{\int_{-\infty}^{+\infty} f(x)dx} - C & f \text{ is not symmetrical} \\ \dfrac{qV\int_0^{L/2} f(x)dx}{\int_0^{+\infty} f(x)dx} - C & f \text{ is symmetrical} \end{cases} \tag{28}$$

For $n > 1$:

$$\begin{aligned} z(n) &= (n-1)I_0 - \left(\frac{C(0)}{2} + \frac{C(n)}{2}\right) \\ &= \begin{cases} (n-1)\dfrac{qV\int_{d_0-d}^{d_0} f(x)dx}{\int_{-\infty}^{+\infty} f(x)dx} - nC & f \text{ is not symmetrical} \\ (n-1)\dfrac{qV\int_0^{d/2} f(x)dx}{\int_0^{+\infty} f(x)dx} - nC & f \text{ is symmetrical} \end{cases} \end{aligned} \tag{29}$$

DOI: 10.1002/atr

4. A NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

4.1. A numerical example

The urban expressway of the second Ring-road in Beijing (an important freeway corridor in Beijing) was selected as the object for SLP optimization and sensitivity analysis using the simplified MIVM; meanwhile, the magnetic sensors were selected as the sensor type. The length of this urban expressway is 33 km ($L = 33$). The traffic volume data was used to calibrate the sensor ICF. The simplified MIVM parameters of $f(x)$, C, V, and q are calibrated as follows.

4.1.1. Sensor information credibility function

The sensor ICF for the urban expressway is calibrated by the field date for 49 current traffic sensors (the index of each sensor is recorded as i and $i = 1, 2, \ldots, 49$) on the second Ring-road of Beijing, between April 14 and April 18, 2008 [19]. The sampling interval is 2 min. Based on Equations (1)–(3), the SCF of sensor i and sensor j, $\mathrm{SCF}_{i,j}$ ($i = 1,2,\ldots,49$ and $j = 1,2,\ldots,49$) is obtained. As shown in Figure 7, based on the distance between sensor i and sensor j (d_{ij}), the scatter diagram of SCF and the distance are obtained by averaging the $\mathrm{SCF}_{i,j}$s over their distance d_{ij} in a unit interval, namely, $d_{ij} \in [l, l+1)$ ($l = 0, 1, \ldots, L$-1). The fitting curve in Figure 7 illustrates the functional relation of SCF and the distance x. It is a negative exponent curve, namely, $y = e^{-kx}$ (where k is the coefficient of the negative exponent curve and $k \geq 0$). Based on available field data, we obtain $k = 0.1592$. From Equation (6), we have $f(x) = \mathrm{fitt}(\mathrm{SCFs}) = e^{-0.1592x}$. For $|\mathrm{corr}(\boldsymbol{Vo}_{\mathrm{A}}(t), \boldsymbol{Vo}_{\mathrm{B}}(t)| = |\mathrm{corr}(\boldsymbol{Vo}_{\mathrm{B}}(t), \boldsymbol{Vo}_{\mathrm{A}}(t))|$, we have $\mathrm{SCF}_{i,j} = \mathrm{SCF}_{j,i}$. The sensor ICF is formulated as $f(x) = e^{-0.1592|x|}$ $x \in (-\infty, +\infty)$.

4.1.2. Integration cost

The urban expressway described in the numerical example has four lanes in each direction on average (one sensor is required for a lane, thus four sensors are required for a one-way road section). According to Table I, the layout cost is estimated as $C_l = ¥1000$, the device cost is estimated as $C_d = ¥1000$, and the annual maintenance cost is estimated as $C_m = ¥500$. We have $C = 4 \times (C_l + C_d + T_l \times C_m) = ¥20\,000$.

4.1.3. Traffic information value

The sensor lifetime T_l is estimated as 6 years, and the number of days can be taken as 365. The hourly congestion cost and the external cost per vehicle are estimated as ¥8 and ¥2, respectively. There are four peak hours in a day in common. The number of vehicles that do not encounter congestion is assumed to be 170. We have: $T_l = 6$, $T_y = 365$, $T_p = 4$, $C_c = 8$, $C_e = 2$, and $Q_d = 170$. According to Equations (8)–(10), the traffic information value is calculated as follows: $V = 6 \times 365 \times 4 \times (8 + 2) \times 170 = ¥14\,892\,000$.

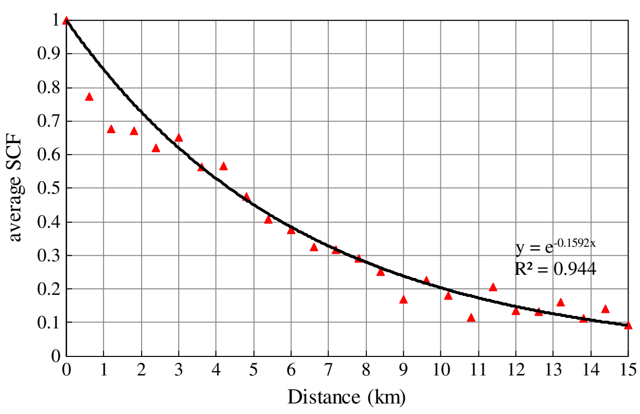

Figure 7. Scatter diagram of spatial correlation function and the distance and its fitting curve.

DOI: 10.1002/atr

4.1.4. Accuracy
Based on the research data given in Table II, q is estimated as 95%.

4.2. Sensitivity analysis

According to Equation (29) and the calibration results described earlier, the following formulas can be obtained:

$$z(n) = \mathit{total}\ \text{Value} - \mathit{total}\ \text{Cost} = (n-1)\frac{qV\int_0^{d/2} f(x)dx}{\int_0^{+\infty} f(x)dx} - nC = (n-1)\frac{qV\int_0^{L/2(n-1)} e^{-kx}dx}{\int_0^{+\infty} e^{-kx}dx} - nC$$
$$= (n-1)qV\left(1 - e^{-\frac{kL}{2(n-1)}}\right) - nC \tag{30}$$

Substituting $L = 33$, $k = 0.1592$, $V = 14\ 892\ 000$, $C = 20\,000$ and $q = 95\%$ into Equation (30) yields the curve of $z(n)$ (Figure 8).

As Figure 8 shows, the optimal sensor number is obtained for this example. When $n = 50$, the maximum z is obtained, and $z(50) = 3.5 \times 10^7$ (¥). Because the second Ring-road in Beijing is circular, the first location and the last location are at the same place. The actual number of required sensors is therefore 49. The optimal sensor spacing for the second Ring-road in Beijing can be calibrated based on Equation (21), which yields a spacing of $d = 33/49 = 0.6735$ km.

Next, the sensitivity analysis that how $z(n)$ changes when one of those parameters changes is studied. By changing the values of the parameters, a series of curves of $z(n)$ versus n can be obtained. Figure 9 shows the curves of $z(n)$ with the model parameters changing. The dotted curves in Figure 9 are based on the estimations of the numerical example. The sensitivity analysis of each parameter is shown as follows.

4.2.1. Sensor information credibility function
Figure 9(a) shows the curves of $z(n)$ with values of the ICF coefficient k from 0.04 to 0.22, and $\Delta k = 0.02$. From Figure 9(a), it can be concluded that along with k increasing, both of the optimal sensor number and the maximum integration value increase. The effect of the ICF coefficient k influencing the integration value is elaborated next. Figure 10 shows three sensor ICFs for different k values. For the same distance (d_0 in Figure 10), the larger k is, the smaller the sensor ICF is. The relationship between SCF and the optimal sensor number can be derived from Equations (3)–(6) and (30). The smaller the spatial correlation coefficient of one kind of traffic flow parameters is, the more traffic sensors will be required to implement

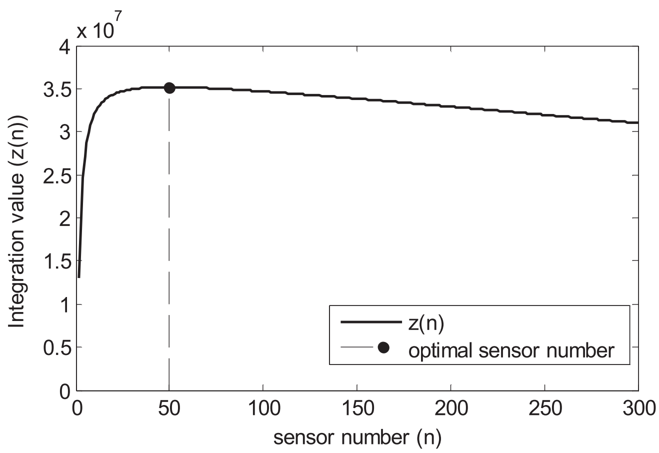

Figure 8. Curve of $z(n)$ and n and its optimal sensor number.

DOI: 10.1002/atr

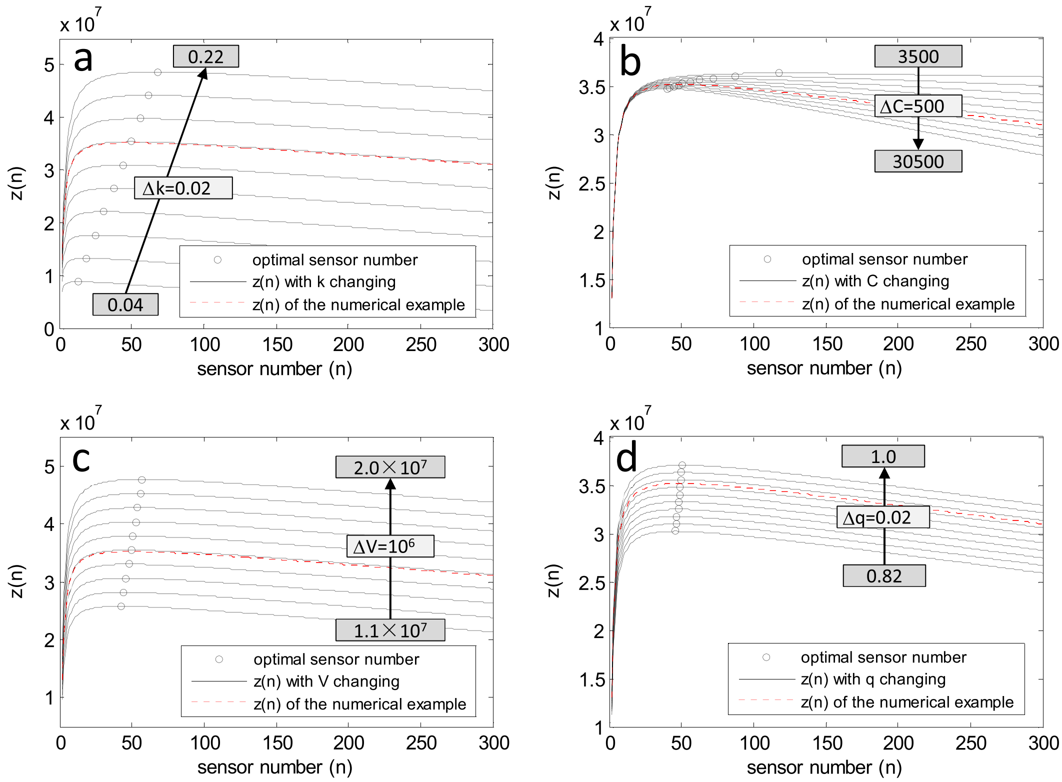

Figure 9. Curves of $z(n)$ with the model parameters changing ((a) sensor information credibility function; (b) integration cost; (c) traffic information value; (d) accuracy).

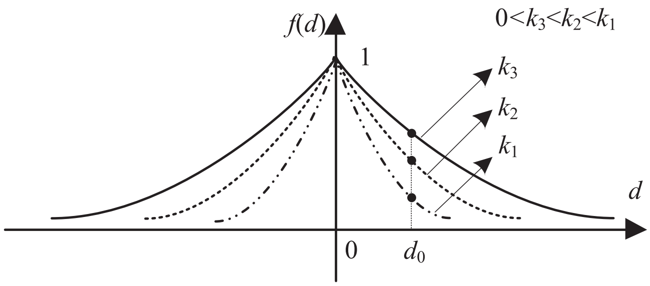

Figure 10. Different sensor ICFs with coefficient k changing ($0 < k_3 < k_2 < k_1$).

the maximum integration value. For freeway ramps, bridge entrances and exits, and road bends, the traffic parameters will change more dramatically; therefore, the SCF for those locations will be smaller than other locations, but k will be larger. Furthermore, according to the trend of optimal sensor number illustrated in Figure 9(a), more sensors should be placed to acquire sufficient traffic information.

4.2.2. *Integration cost*

Figure 9(b) was drawn by varying the parameter C from 3500 to 30 500 with $\Delta C = 3000$. From the trend of optimal sensor number, it can be concluded that the integration cost has a great influence on the optimal sensor number. When sensors with high integration cost are selected for the SLP, the optimal sensor number and the maximum integration value decrease. This shows that it is necessary to reduce the integration cost by using well-developed sensor technologies. In practice, it will be better to employ traffic sensors that are low in cost and easy to install, presuming that they provide the same function and accuracy.

DOI: 10.1002/atr

4.2.3. *Traffic information value*

Based on the estimations of the model parameters in the numerical example, if the traffic information value V changes from 1.1×10^7 to 2.0×10^7, with $\Delta V = 10^6$, the curves of $z(n)$ and n will be those shown in Figure 9(c). Unlike parameter C, a change in the traffic information value has less influence on the optimal sensor number, but it can easily affect the integration value. Therefore, after placing the sensors, the best way to improve the integration value $z(n)$ without placing new sensors is to enhance the effectiveness and efficiency of traffic information derived from those sensors. In addition, making use of data fusion techniques will be significant in improving the accuracy and utilization rate of traffic information. In this way, traffic congestion will be eased, and congestion costs, emissions, fuel consumption, and noise will all be reduced.

4.2.4. *Accuracy*

Similarly, curves of $z(n)$ for q from 0.82 to 1 with $\Delta q = 0.02$ are shown in Figure 9(d). The parameter of q mainly influences the integration value. If the accuracy of a sensor increases, more integration values can be obtained, which is what we want to achieve. The parameter of q has less influence on the optimal sensor number. Using some data fusion algorithms to increase q can not only increase the integration value but also keep the SLP close to the optimal state.

4.2.5. *Budget*

Figure 11 illustrates the variation of $IM(n)$ as n increases. Making use of the estimation of C and Equation (13), the formula for $IM(n)$ is $IM(n) = 8000n$. As shown in Figure 11, n_a is the optimal sensor number for the numerical example, namely, $n_a = 49$. The investment for the example is $IM(n_a) = 8000\times 49 = 392\,000$ (¥). If the budget is not less than $IM(n_a)$ (B_a, B_b in Figure 11), the optimal sensor number and locations will not change. On the contrary, if the budget is less than $IM(n_a)$ (B_c in Figure 11), the maximum number of the placed sensors will be n_c. In that case, the layout strategy should refer to the analyses described in subsection 2.2.5.

4.3. *Discussion*

Figure 8 gives the optimal sensor number of the second Ring-road in Beijing based on the simplified MIVM. Then, the optimal sensor locations and spacing for this freeway corridor will be also figured out. Compared with traditional methods, the method proposed in this paper will be superior to some graph theory-based methods, which cannot address the optimal sensor locations and spacing. Moreover, there is limited literature to address the IFs for the SLP, especially to reveal the influencing mechanisms of those factors. Based on the distribution and trend of those optimal sensor numbers with model parameters changing (Figure 9), we can see clearly how those factors influence the optimal

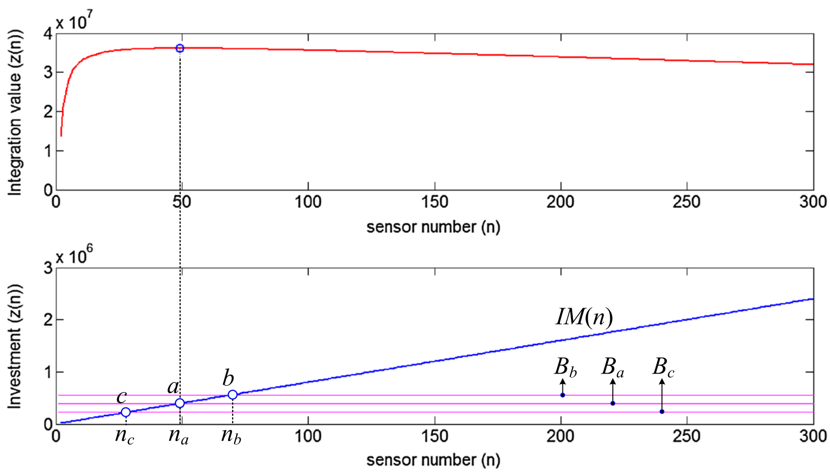

Figure 11. Demand of fixed investment $IM(n)$ for the numerical example.

DOI: 10.1002/atr

SLP. As the ICF coefficient k increases, both of the optimal sensor number and maximum integration value increases (Figure 9(a)). The coefficient k will be larger in the locations of freeway ramps, bridge entrances and exits, and road bends. More sensors will be required in those places. The integration cost C will have a great influence on the optimal SLP (Figure 9(b)), but the influencing patterns of traffic information value V and accuracy q will be similar (Figure 9(c) and (d)). Hence, to develop and select sensors with lower cost and more accuracy will be necessary when we deploy them in road networks. Furthermore, a large traffic information value will result in a large value of $z(n)$, then it is sensible to improve the accuracy and utilization rate of traffic information. The investment is an optional factor that will influence the optimal SLP. As Figure 11 shows, whether the number and locations of the placed sensors of a project change is related to the relationship between the investment and the budget.

5. CONCLUSIONS

This paper addresses the comprehensive analysis of the factors that influence the SLP for freeway corridors. Six factors are identified as important to the optimal sensor number and locations. The mathematical formulations of the model parameters derived from those six IFs are also presented for MIVM. The simplified MIVM is applied to analyze the relationship between the integration value $z(n)$ and the sensor number n, which is related to those IFs. This model is a universal method for solving the traffic SLP, and it can be applied in different situations for different sensor types and traffic information types. A numerical example is presented to analyze how those factors influence the optimal sensor number and locations for the SLP of a freeway corridor. The example results also demonstrate the effectiveness of the proposed method in analyzing the influencing mechanism of those factors. Each factor has its own influencing patterns for the SLP. Given those influencing patterns, we can see clearly how those factors influence the optimal SLP. When one of those factors changes, we will have explicit schemes to keep an optimal SLP solution. When the budget is a constraint of the SLP, some strategies of sensor layout proposed in this paper can be referred in practice.

The main contributions of this paper are the following. (1) It is among the first papers to present an analysis of SLP IFs for freeway corridors, to calibrate every factor with field data, and to address the sensitivity analysis of each factor in detail; (2) the proposed formulation of the SLP was implemented in the context of a traffic information acquisition system that supports multiple scenarios in an ITS, such as different sensor types and different locations; and (3) the proposed sensor ICF can be used to describe the heterogeneity of the spatial distribution of traffic information credibility from point sensor networks. By spatial superposition operation, sensor IDFs on several freeways can be used to describe the spatial heterogeneity of freeway networks and can be used to determine the optimal solution of the SLP for freeway networks.

Although the method proposed in this paper is to address the SLP of a freeway corridor, it could be applied to urban roads with traffic lights. The function for a road ICF could also be calibrated by spatial superposition operation of sensor ICFs. By defining a correlation coefficient matrix of different roads, the method proposed in this paper could be extended to the SLP of an urban road network or other road networks, which will be one direction of our future work. Future research can be conducted in several directions. First, SCF is used to demarcate the sensor ICF in this paper. Other methods must be developed to describe the spatial characteristics of traffic information. Second, a freeway corridor was selected as the study object for the SLP in this paper. Studying the spatial characteristics of other road types would be helpful and meaningful contributions to this research topic. The numerical example presented in this paper assumes that the values of each model parameter are equal at different locations. More complex case studies are more common in the real world and would be more rewarding to study.

6. LIST OF SYMBOLS AND ABBREVIATIONS

6.1. Symbols

$\mathbf{Vo}_{\mathrm{A}}(t)$	the traffic volume time series at point A, at time t
$\mathbf{Vo}_{\mathrm{B}}(t)$	the traffic volume time series at point B, at time t
$Vol_{\mathrm{A}}(t)$	the traffic volumes at point A, at time t

DOI: 10.1002/atr

$Vol_{\mathrm{B}}(t)$	the traffic volumes at point B, at time t
$d_{\mathrm{A,B}}$	the distance between point A and point B
corr	the CC of the two time series
N	the sampling number during a certain period
n	the number of sensor in the one-way road
τ	the sampling interval
$f(i)$, f_i, f	the ICF of the sensor at location i
i, j	the index of sensor location
$\mathrm{C}(i)$, C	the integration cost of the sensor at location i
$V(i)$, V	the traffic information value of the sensor at location i
$q(i)$, q	the accuracy of the sensor at location i
$C_l(i)$	the layout cost of the sensor at location i
$C_d(i)$	the device cost of the sensor at location i
$C_m(i)$	the annual maintenance cost of the sensor at location i
T_l	the lifetime of a sensor
$B_d(i)$	the direct benefits of the sensor at location i
$B_e(i)$	the external benefits of the sensor at location i
T_y	the days in a year
T_p	the peak hours per day
C_c	the average congestion cost per hour for one vehicle
$Q_d(i)$	the number of vehicles that do not encounter congestion at the sensor location i
C_e	the external cost due to the reduced traffic congestion per hour for one vehicle
$IM(n)$, IM	the fixed investment of n sensors
L	the length of a one-way road
B	the budget
z	the integration value of all the located sensors
x_i	the coordinate of location i along the one-way road
X_i	0 or 1, 1 a sensor is located at location i, 0 indicating that there is not a sensor at that location
d	the distance of two adjacent sensor along the one-way road
d_{ij}	the distance between location i and location j
D	the directed graph
v_i	the vertex of directed graph D at location i
e_{ij}	the edge from v_i to v_j
w_{ij}	the weight of e_{ij}
a_{ij}	the coordinate of the crossover point of f_i and f_j
z_m	the maximum integration value
Ω	the solution set
I_{ij}	the integration value between location i and location j
d_0	the distance between x_i and $a_{i,i+1}$
I_0	the integration value between location i and location i+1
$\mathrm{SCF}_{i,j}$	the value of SCF between sensor i and sensor j

6.2. *Abbreviations*

SLP	sensor location problem
MIVM	maximum integration value model
ITS	intelligent transportation systems
IFs	influencing factors
CC	correlation coefficient
SCF	spatial correlation function
ICF	information credibility function

DOI: 10.1002/atr

ACKNOWLEDGEMENT

The authors would like to express their thanks to the editor and anonymous reviewers for their help in revising the manuscript. This work is supported by the National Natural Science Foundation of China (61104164) and the Fundamental Research Funds for the Central Universities of China (2012YJS059), and is also supported by the National 863 program of China (2012AA112401).

REFERENCES

1. Zhou X, List GF. An information-theoretic sensor location model for traffic origin-destination demand estimation applications. *Transportation Science* 2010; **44**(2):254–273.
2. Li R, Jia L. On the layout of fixed urban traffic detectors: an application study, *Intelligent Transportation Systems Magazine* 2009; **1**(2):6–12.
3. Li X, Ouyang Y. Reliable sensor deployment for network traffic surveillance. *Transportation Research Part B: Methodological* 2011; **45**(1):218–231.
4. Li X. Reliable facility location design and traffic sensor deployment under probabilistic disruptions. *PhD thesis*, University of Illinois at Urbana-Champaign, Urbana, 2011.
5. Bai L, Wu QS, Yang M, Wei LX, Li B, Gao R. Highway traffic incident detection and optimal sensor placement layout. *Advanced Materials Research* 2013; **650**:460–464.
6. Ban J, Herring R, Margulici JD, Bayen AM. Optimal sensor placement for freeway travel time estimation. *ISTTT*, 2009; 697–721.
7. Ban J, Chu LY, Herring R, Margulici JD. Sequential modeling framework for optimal sensor placement for multiple intelligent transportation system applications. *Journal of Transportation Engineering-ASCE* 2011; **137**(2):112–120.
8. Bertini RL, Lovell DJ. Impacts of sensor spacing on accurate freeway travel time estimation for traveler information. *Journal of Intelligent Transportation Systems* 2009; **13**(2):97–110.
9. Danczyk A, Liu HX. A mixed-integer linear program for optimizing sensor locations along freeway corridors. *Transportation Research Part B* 2011; **45**(1):208–217.
10. Feng W, Bigazzi AY, Kothuri S, Bertini RL. Freeway sensor spacing and probe vehicle penetration: Impacts on travel time prediction and estimation accuracy. *Transportation Research Record* 2010; **2178**:67–78.
11. Kianfar J, Edara P. Optimizing freeway traffic sensor locations by clustering global-positioning-system-derived speed patterns. *IEEE Transactions on Intelligent Transportation Systems* 2010; **11**(3):738–747.
12. Kim J, Park B, Lee J, Won J. Determining optimal sensor locations in freeway using genetic algorithm-based optimization. *Engineering Applications of Artificial Intelligence* 2011; **24**(2):318–324.
13. Liu HX, Danczyk A. Optimal sensor locations for freeway bottleneck identification. *Computer-Aided Civil and Infrastructure Engineering* 2009; **24**(8):535–550.
14. Tamin OZ, Suyuti R. The impact of location and number of traffic counts in the accuracy of O-D matrices estimated from traffic counts under equilibrium condition: a case study in Bandung (Indonesia). *Journal of the Eastern Asia Society for Transportation Studies* 2003; **5**:1393–1407.
15. Yang H, Zhou J. Optimal traffic counting locations for origin–destination matrix estimation. *Transportation Research Part B: Methodological* 1998; **32**(2):109–126.
16. Bianco L, Confessore G, Reverberi P. A network based model for traffic sensor location with implications on O/D matrix estimates. *Transportation Science* 2001; **35**(1):50–60.
17. Leow WL, Ni D, Pishro-Nik H. A sampling theorem approach to traffic sensor optimization. *Transactions on Intelligent Transportation Systems* 2008; **9**(2):369–374.
18. Hong ZH, Fukuda D. Effects of traffic sensor location on traffic state estimation. *5th Meeting of the Euro-Working-Group-on-Transportation (EWGT)*, Cite Descartes, Paris, France, 2012; 1186–1196.
19. Li HJ, Jia LM, Dong HH, Qin Y, Xu DW, Liu Y. Study on spacing optimization for traffic flow detector. *Proceedings of the 13th Intelligent Transportation Systems Conference*, 2010; 593–598.
20. Martin PT, Feng Y, Wang X. *Detector Technology Evaluation*. University of Utah, Salt Lake City, 2003.

DOI: 10.1002/atr