

# Analysis of full-duplex relay networks with opportunistic scheduling

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**Abstract:** This Letter addresses a two-hop decode-and-forward relay system with full-duplex relaying and opportunistic scheduling. Exact expressions for outage probability, average capacity and symbol error rate are presented in an independent identically distributed Rayleigh fading environment. Numerical and simulated results show the validity of the analytical results.

## 1 Introduction

Wireless cooperative technique has recently attracted the attention of many scholars from all over the world because of their key features of coverage expansion and increasing capacity without increasing the transmitter power. The performance of the single-user relay network has been researched widely [1]. Recently, the multi-user relay network has gained more and more attention [2], where the cooperative diversity was combined with multi-user diversity to further improve the network performance. In [3], performance analysis of opportunistic scheduling was investigated for a multi-user relay system equipped with one amplify-and-forward relay in Rayleigh fading environments. However, their research is performed based on assuming of half-duplex relaying. Compared with half-duplex mode, the full-duplex relay is allowed to receive and send the signals at the same time and frequency [4, 5]. This Letter focuses on the full-duplex relay networks with opportunistic scheduling. Some expressions for outage probability, average capacity and symbol error rate (SER) of the multi-user full-duplex relay network with decode-and-forward (DF) protocols are derived to obtain the network performance easily.

## 2 Performance analysis

Consider a relay system in which a source node ( $S$ ) communicates with  $N$  destination nodes  $D_n$  ( $1 \leq n \leq N$ ) with the help of a single DF relay node ( $R$ ). We assume that there is no direct link between the nodes  $S$  and  $D_n$ , and the transmission is executed through the relays. The nodes  $S$  and  $D_n$  working in half-duplex mode are equipped with an antenna, but the relay terminal with a receiving antenna and a transmitting antenna operates in full-duplex working mode. Let  $h_{AB}$  be the channel response between nodes  $A$  and  $B$ , where  $A, B \in \{S, D_n, R\}$ . It is assumed that all the links are independent identically distributed (IID) Rayleigh distributed. For the first hop link, received signal can be expressed by

$$y_1 = h_{SR}x_S + h_1x_R + n \quad (1)$$

where  $x_S$  and  $n$  represent the transmitting signal with the power  $P_s$  at node  $S$  and additive white Gaussian noise with zero mean and unit variance ( $N_0$ ). On the right side of equal sign in (1), the second item represents the loop interference between the relay input and output [4, 5]. The instantaneous signal-to-interference plus noise ratio for the one-hop link is expressed as  $\gamma_S = \gamma_{SR}/(\gamma_1 + 1)$  where  $\gamma_{SR} = |h_{SR}|^2 P_s / N_0 = \gamma_1 |h_{SR}|^2$  and  $\gamma_1 = |h_1|^2 P_R N_0$ . For simplicity, it is assumed that relay with spatially separated transmitting and receiving antennas is fixed, and furthermore it is non-fading interference channel, where the instantaneous interference-to-noise ratio (INR) is equal to corresponding average INR,  $\gamma_1 = \bar{\gamma}_1 = \varepsilon \left\{ |h_1|^2 \right\} P_R / N_0$ . Thus,

cumulative distribution function (CDF) of  $\gamma_S$  can be given as  $F_{\gamma_S}(\gamma) = 1 - e^{-(\bar{\gamma}_1+1)\gamma/\gamma_1}$  where  $\gamma_1$  is the average signal-to-noise ratio (SNR) of the  $S$ - $R$  link.

In the second hop, opportunistic scheduling is employed and the relay selects the destination node (mobile user) with the strongest channel to schedule data transmission. Thus, the SNR for the second hop can be written as  $\gamma_D = \max_{n=1, \dots, N} \{\gamma_D^n\}$  where  $\gamma_D^n$  is the instantaneous SNR for the  $R$ - $D_n$  link. On the basis of order statistics, the CDF of  $\gamma_D$  can be provided as  $F_{\gamma_D}(\gamma) = [F_{\gamma_D^n}(\gamma)]^N$ , where  $F_{\gamma_D^n}(\gamma) = 1 - e^{-\gamma/\gamma_2}$ .

For a DF protocol, the instantaneous received SNR at node  $D$  is approximated in the high SNR level as [1]:  $\gamma_{eq} = \min\{\gamma_S, \gamma_D\}$ . Then, its CDF equals

$$F(\gamma) = \Pr\{\min\{\gamma_S, \gamma_D\} < \gamma\} = 1 - e^{-(\gamma_1+1)\gamma/\gamma_1} (1 - e^{-\gamma/\gamma_2})^N \quad (2)$$

On the basis of binomial theorem, the probability density function of  $\gamma_{eq}$  is written as

$$f(\gamma) = \sum_{n=1}^N \binom{N}{n} (-1)^{n-1} [(\gamma_1 + 1)/\gamma_1 + n/\gamma_2] e^{-(\gamma_1+1)\gamma/\gamma_1 - n\gamma/\gamma_2} \quad (3)$$

### 2.1 Outage probability

As a communication system, outage probability is a key performance measure and can be defined as the probability  $P_{out}$  that the end-to-end received SNR at node  $D$  is smaller than a specific value  $\gamma_{th}$ . Accordingly, a concise closed-form outage probability expression is written as

$$P_{out} = \Pr\{\gamma_{eq} < \gamma_{th}\} = F(\gamma_{th}) \quad (4)$$

### 2.2 Average capacity

The average capacity of the full-duplex relay system with opportunistic scheduling can be expressed by

$$C = \int_0^{+\infty} \log_2(1 + \gamma) f(\gamma) d\gamma \quad (5)$$

Substituting (3) into (5), an exact expression for the average capacity can be given by

$$C = \log_2(e) \sum_{n=1}^N \binom{N}{n} (-1)^{n-1} \Gamma(0, (\gamma_1 + 1)/\gamma_1 + n/\gamma_2) e^{(\gamma_1+1)/\gamma_1 + n/\gamma_2} \quad (6)$$

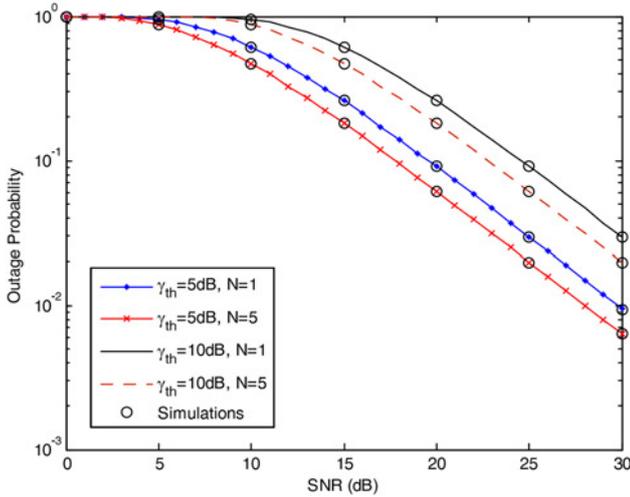


Fig. 1 Outage probability against SNR  $\gamma_0$

where  $\Gamma()$  denotes the incomplete Gamma function [6, eq. (8.350.1)].

### 2.3 Symbol error rate

The average SER can be determined by

$$P_e = \int_0^{+\infty} aQ(\sqrt{2bx})f(\gamma)d\gamma \quad (7)$$

where  $Q()$  is the Gaussian Q-function,  $a$  and  $b$  denote the modulation-specific constants. For example, when binary phase-shift keying (BPSK) modulation is employed,  $a = b = 1$ ; when  $M$ -ary pulse amplitude modulation is used,  $a = 2(M-1)/M$  and  $b = 3/(M^2 - 1)$ . Our results also provide the approximate SER for  $M$ -ary PSK with  $a = 2$  and  $b = \sin^2(\pi/M)$ . Substituting (3) into (7), we can provide the concise expression for average SER

$$P_e = \frac{a}{2} - \frac{a\sqrt{b}}{2} \sum_{n=1}^N \binom{N}{n} \frac{(-1)^{n-1}}{\sqrt{(\gamma_1 + 1)/\gamma_1 + n/\gamma_2 + b}} \quad (8)$$

## 3 Numerical and simulated results

In this section, it is assumed that all links are Rayleigh distributed and transmit SNR  $\gamma_1 = \gamma_2 = \gamma_0$ . Fig. 1 plots outage probability as a function of  $\gamma_0$  in (7) when  $\gamma_1 = 0$  dB. As expected, the multi-user scenario ( $N=5$ ) outperforms the single-user scenario ( $N=1$ ), and the appearance of opportunistic scheduling improves the system performance.

Fig. 2 provides average capacity against  $\gamma_0$  for various INR  $\gamma_1$  and  $N=2$ . And then, we can find that the average capacity gain will increase with the decrease of interference level between relay input and output. Therefore a valid technique to eliminate the interference is the key for a full-duplex relay system. Fig. 3 plots the average SER with various  $N$  and BPSK ( $a = b = 1$ ) modulation, when  $\gamma_1 = \gamma_2 = 15$  dB. For the small INR  $\gamma_1$ , increasing  $N$  can improve the error performance; however, it disappears with the increase of  $\gamma_1$  because excessive interference leads to the poor first hop SNR which determines the system performance. In Figs. 1–3, it can be seen that the analytical results are in good agreement with the results obtained from Monte Carlo Simulations.

## 4 Conclusion

In this Letter, a DF-based relay system with opportunistic scheduling and full-duplex relaying in IID Rayleigh fading environment is

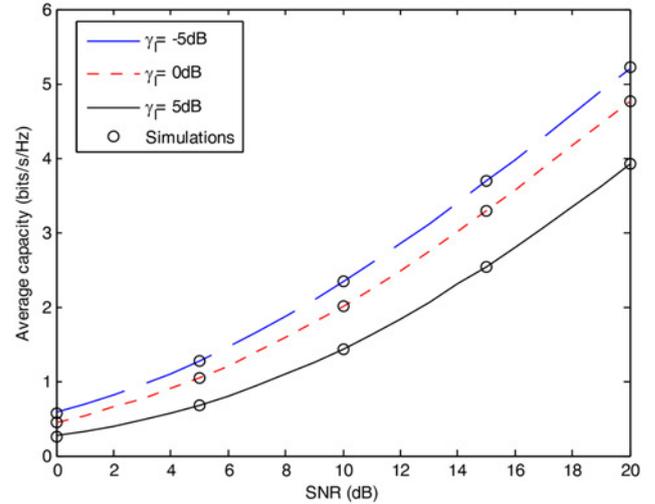


Fig. 2 Average capacity against SNR  $\gamma_0$

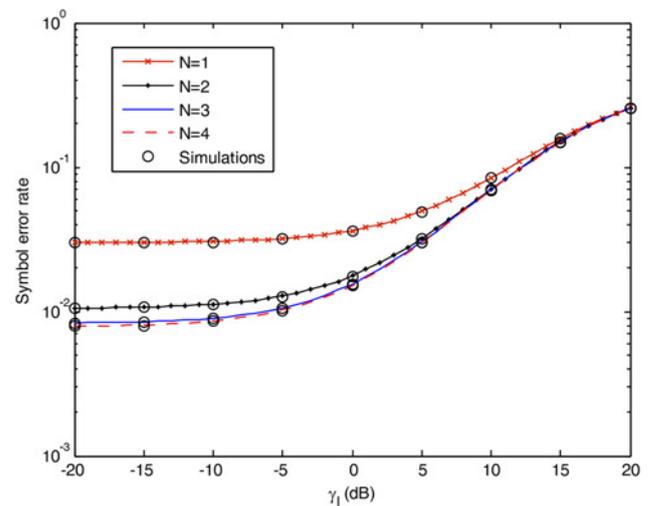


Fig. 3 Average SER of BPSK against INR  $\gamma_1$

analysed, and concise expressions are derived to provide the error and capacity performance easily.

## 5 Acknowledgment

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