Friction compensation of servo system based on terminal switching function

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Abstract: Based on the LuGre friction model, a sliding mode adaptive controller is presented to compensate for the friction in the servo system. The terminal switching function is selected as the sliding mode surface, which can make the error of the system converge to the equilibrium point in finite time. The main analytical result is a stability theorem for the proposed controller which can achieve asymptotical stability of the closed-loop system. Furthermore, the transient performance of the system is analytically quantified. To support the theoretical concepts, the authors present dynamic simulations for the proposed control scheme.

1 Introduction

Friction is a complex non-linear phenomenon, which affects the performances of the servo system. If the controller of a system is designed without compensation for friction, it will lead to many bad phenomenon, such as bigger tracking error, limit cycle, stick-slip crawling movement and so on. Hence, it is very necessary to compensate for the friction for high-precision position control.

Up to now, the control schemes proposed to compensate for the non-linear friction in the servo system can be classified into model-free and model-based ones. In numerous friction models, such as Coulomb model, Stribeck model, Dahl model [1], LuGre model [2], Leuven model [3] and so on. The LuGre model is more close to the real friction phenomenon, hence it is widely used [4–7]. Based on the LuGre model, people adopt many compensation methods for designing the controller. For example, in [8], Kwatny *et al*. designed the controller for compensating for friction by the variable structure control. In the literature [9], the controller is designed by the adaptive control and the neural network approximations, based on the LuGre friction model. In [10], Wang *et al.* adopted a neural adaptive control scheme to design the control controller and combined it with the leakages. In [11], Lee *et al.* raised a new method which utilised a proportional-derivative (PD) control structure and an adaptive estimation of the friction based on an observer for compensating for friction. Nakkarat and Kuntanapreeda [12] designed a non-linear controller by using a backstepping approach, which guaranteed the convergence of the tracking error. In [13], a sliding mode control with double boundary layer for robust compensation of payload mass and friction was put forward. In [14], Armstrong *et al*. compensated friction by the non-linear proportional-integral-derivative control for the adaptive control. In [15], a precise friction control was designed by using the friction state observer and sliding mode control with recurrent fuzzy neural networks. In [16], Han and Lee presented an adaptive dynamic surface control scheme combined with sliding mode control to compensate for friction and backlash non-linearities.

In this paper, under some assumptions, a composite friction control system is proposed, which consists of the LuGre model, a dual friction observer, a terminal sliding mode surface and an adaptive controller. The dual friction observer can estimate the unmeasurable friction state variable of the LuGre model, which is used to approximate the non-linear friction efficiently. The terminal sliding mode surface is selected in order to make the error of the system converge to the equilibrium point in finite time and improve the convergence speed of the system. The main analytical result is a stability theorem for the proposed controller, which can achieve asymptotical stability of the closed-loop system by the Lyapunov stability analysis. Furthermore, the transient performance of the system is analytically quantified. The effectiveness of the proposed control scheme and the good robustness of the system are verified by simulation results.

2 Dynamics of the servo system with friction

The dynamic equation of the mechanical and electrical servo system is as follows

$$J\ddot{x} = u - f - d \tag{1}$$

where J is the unknown moment of inertia, x is the displacement of the system, u is the control input, f is the friction force and d is the unknown external disturbance, respectively. Based on the LuGre friction model, the friction force is described as [2]

$$f = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 \dot{x} \tag{2}$$

where σ_0, σ_1 and σ_2 are the stiffness coefficient, the damping coefficient, viscous friction coefficient, respectively. The unmeasured average deflection of the bristles is presented as follows

$$\dot{z} = \dot{x} - \frac{|\dot{x}|}{g(\dot{x})} z \tag{3}$$

where the function $g(\dot{x})$ describes the Stribeck effect and is given by

$$g(\dot{x}) = f_c + (f_s - f_c)\exp(-(\dot{x}/\dot{x}_s)^2) \tag{4}$$

where f_c, f_s and $\dot{x}_s$ are the coulomb friction force, the static friction force and the Stribeck velocity, respectively. Combined with (2)–(4), (1) can be arranged to

$$J\ddot{x} = u - \sigma_0 z + \sigma_1 \frac{|\dot{x}|}{g(\dot{x})} z - \beta \dot{x} - d \tag{5}$$

where $\beta = \sigma_1 + \sigma_2$.

The objective of the proposed control scheme is to design the control law which can ensure that the tracking error converges to zero asymptotically, that is to say, the system output x tracks the expected trajectory x_d.

3 Robust adaptive control

3.1 Controller design

For the development of control laws, some realistic assumptions are as follows:

Assumption 1: the external disturbance d is bounded.

Assumption 2: the expected trajectory x_d and its first second-order derivative are piecewise continuous and bounded.
Assumption 3: the estimated parameters σ_0, σ_1, β and J are unknown, positive and bounded.

To achieve the control objective, the following error variable between the system output x and the expected trajectory x_d is defined

$$e = x - x_d \tag{6}$$

A terminal switching function [17] is selected as the sliding mode surface

$$s = \dot{e} + \beta_1 e^{q/p} \tag{7}$$

where $\beta_1 > 0$, q and p are all positive odd and $q < p$.

The terminal sliding mode can improve the convergence performance of the system and make the tracking error of the system converge to the equilibrium point in finite time.

Since the parameter σ is unknown, the average deflection of the bristles z cannot be measured directly and the two parameters have a bilinear relation, a dual observer [5] is designed

$$\begin{cases} \dot{\hat{z}}_0 = \dot{x} - \dfrac{|\dot{x}|}{g(\dot{x})}\hat{z}_0 + \tau_0 \\ \dot{\hat{z}}_1 = \dot{x} - \dfrac{|\dot{x}|}{g(\dot{x})}\hat{z}_1 + \tau_1 \end{cases} \tag{8}$$

where both z_0 and z_1 are the estimation of the unmeasured friction state z, τ_0 and τ_1 are the dynamic compensation terms which can be designed later. Combined with (3) and (8), the derivative of the dual observer error can be written as

$$\begin{cases} \dot{\tilde{z}}_0 = -\dfrac{|\dot{x}|}{g(\dot{x})}\tilde{z}_0 - \tau_0 \\ \dot{\tilde{z}}_1 = -\dfrac{|\dot{x}|}{g(\dot{x})}\tilde{z}_1 - \tau_1 \end{cases} \tag{9}$$

where $\tilde{z}_0 = z - \hat{z}_0$ and $\tilde{z}_1 = z - \hat{z}_1$. The adaptive law of the system is designed

$$u = \hat{\sigma}_0 \hat{z}_0 - \hat{\sigma}_1 \frac{|\dot{x}|}{g(\dot{x})}\hat{z}_1 + \hat{\beta}\dot{x} + \hat{d} - \hat{J}u_c \tag{10}$$

where $u_c = ks - \ddot{x}_d + \frac{\beta_1 q}{p} e^{(q-p)/p}\dot{e}$; k is a positive constant; $\hat{J}$, $\hat{\sigma}_0$, $\hat{\sigma}_1$, $\hat{\beta}$ and $\hat{d}$ are the estimations of J, σ_0, σ_1 β and d, respectively. That is to say $\widetilde{(\cdot)} = (\cdot) - \widehat{(\cdot)}$. Now, we specify the following update laws

$$\dot{\hat{\sigma}}_0 = -r_0 \hat{z}_0 s \tag{11}$$

$$\dot{\hat{\sigma}}_1 = r_1 \frac{|\dot{x}|}{g(\dot{x})}\hat{z}_1 s \tag{12}$$

$$\dot{\hat{\beta}} = -r_2 \dot{x} s \tag{13}$$

$$\dot{\hat{d}} = -r_3 s \tag{14}$$

$$\dot{\hat{J}} = r_4 u_c s \tag{15}$$

where r_0, r_1, r_2, r_3 and r_4 are positive designed parameters.

3.2 Stability proof

The time derivative of the sliding mode surface s is given by

$$\dot{s} = \ddot{e} + \frac{\beta_1 q}{p} e^{(q-p)/p}\dot{e} \tag{16}$$

On substituting (5) and (10) into (16), the dynamic equation of the close-loop system is

$$\begin{aligned} J\dot{s} = & -\sigma_0 \tilde{z}_0 - \tilde{\sigma}_0 \hat{z}_0 + \sigma_1 \frac{|\dot{x}|}{g(\dot{x})}\tilde{z}_1 + \tilde{\sigma}_1 \frac{|\dot{x}|}{g(\dot{x})}\hat{z}_1 - \tilde{\beta}\dot{x} - \tilde{d} \\ & - J\left(u_c + \ddot{x}_d - \frac{\beta_1 q}{p} e^{(q-p)/p}\dot{e}\right) + \tilde{J}u_c \end{aligned} \tag{17}$$

To prove the stability of the servo system with the proposed scheme, the Lyapunov function is selected

$$\begin{aligned} V = & \frac{J}{2}s^2 + \frac{1}{2r_0}\tilde{\sigma}_0^2 + \frac{1}{2r_1}\tilde{\sigma}_1^2 + \frac{1}{2r_2}\tilde{\beta}^2 + \frac{1}{2r_3}\tilde{d}^2 + \frac{1}{2r_4}\tilde{J}^2 \\ & + \frac{1}{2}\sigma_0 \tilde{z}_0^2 + \frac{1}{2}\sigma_1 \tilde{z}_1^2 \end{aligned} \tag{18}$$

The time derivative of V is

$$\begin{aligned} \dot{V} = & J s\dot{s} - \frac{1}{r_0}\tilde{\sigma}_0 \dot{\hat{\sigma}}_0 - \frac{1}{r_1}\tilde{\sigma}_1 \dot{\hat{\sigma}}_1 - \frac{1}{r_2}\tilde{\beta}\dot{\hat{\beta}} - \frac{1}{r_3}\tilde{d}\dot{\hat{d}} \\ & - \frac{1}{r_4}\tilde{J}\dot{\hat{J}} + \sigma_0 \tilde{z}_0 \dot{\tilde{z}}_0 + \sigma_1 \tilde{z}_1 \dot{\tilde{z}}_1 \end{aligned} \tag{19}$$

By applying (9) and (17) to (19), we have

$$\begin{aligned} \dot{V} = & s\left(-\sigma_0 \tilde{z}_0 - \tilde{\sigma}_0 \hat{z}_0 + \sigma_1 \frac{|\dot{x}|}{g(\dot{x})}\tilde{z}_1 + \tilde{\sigma}_1 \frac{|\dot{x}|}{g(\dot{x})}\hat{z}_1 - \tilde{\beta}\dot{x} - \tilde{d} + \tilde{J}u_c\right) \\ & - Js\left(u_c + \ddot{x}_d - \frac{\beta_1 q}{p} e^{(q-p)/p}\dot{e}\right) - \frac{1}{r_0}\tilde{\sigma}_0 \dot{\hat{\sigma}}_0 - \frac{1}{r_1}\tilde{\sigma}_1 \dot{\hat{\sigma}}_1 \\ & - \frac{1}{r_2}\tilde{\beta}\dot{\hat{\beta}} - \frac{1}{r_3}\tilde{d}\dot{\hat{d}} - \frac{1}{r_4}\tilde{J}\dot{\hat{J}} + \sigma_0 \tilde{z}_0\left(-\frac{|\dot{x}|}{g(\dot{x})}\tilde{z}_0 - \tau_0\right) \\ & + \sigma_1 \tilde{z}_1\left(-\frac{|\dot{x}|}{g(\dot{x})}\tilde{z}_1 - \tau_1\right) \end{aligned} \tag{20}$$

(20) can be rearranged as

$$\begin{aligned} \dot{V} = & -\tilde{\sigma}_0\left(\hat{z}_0 s + \frac{1}{r_0}\dot{\hat{\sigma}}_0\right) + \tilde{\sigma}_1\left(\frac{|\dot{x}|}{g(\dot{x})}\hat{z}_1 s - \frac{1}{r_1}\dot{\hat{\sigma}}_1\right) - \tilde{\beta}\left(\dot{x}s + \frac{\dot{\hat{\beta}}}{r_2}\right) \\ & - \tilde{d}\left(s + \frac{1}{r_3}\dot{\hat{\varepsilon}}\right) + \tilde{J}\left(u_c s - \frac{1}{r_4}\dot{\hat{J}}\right) + \sigma_0 \tilde{z}_0\left(-\frac{|\dot{x}|}{g(\dot{x})}\tilde{z}_0 - \tau_0 - s\right) \\ & + \sigma_1 \tilde{z}_1\left(-\frac{|\dot{x}|}{g(\dot{x})}\tilde{z}_1 - \tau_1 + \frac{|\dot{x}|}{g(\dot{x})}s\right) - Js\left(u_c + \ddot{x}_d - \frac{\beta_1 q}{p} e^{(q-p)/p}\dot{e}\right) \end{aligned} \tag{21}$$

Designing the dynamic compensation terms in (8)

$$\tau_0 = -s \tag{22}$$

$$\tau_1 = \frac{|\dot{x}|}{g(\dot{x})}s \tag{23}$$

On applying (22) and (23) into (8), they are clear that the relation between $\dot{\hat{z}}_0$ and s is linear and the relation between $\dot{\hat{z}}_1$ and s is non-linear. Hence, we can have the first estimation of z which has more accurate estimation error than that of the second one.

Remark 1: in practical application, we can also choose $\tau_0 = -\gamma_0 s$ and $\tau_1 = \gamma_1 \frac{|\dot{x}|}{g(\dot{x})}s$. By adjusting the values of γ_0 and γ_1, we can improve the flexibility of the system, which does not change its overall performance.

By applying the estimation error of the dual observer (9) and the adaptive laws (11)–(15) to (21), we obtain

$$\dot{V} = -kJs^2 - \sigma_0 \frac{|\dot{x}|}{g(\dot{x})}\tilde{z}_0^2 - \sigma_1 \frac{|\dot{x}|}{g(\dot{x})}\tilde{z}_1^2 \tag{24}$$

Since k, J, σ_0, σ_1, $\frac{|\dot{x}|}{g(\dot{x})}$ are all positive, we have

$$\dot{V} \le -kJs^2 \le 0 \tag{25}$$

From (18) and (24), the Lyapunov function V is differentiable and has the finite limit when $t \to \infty$. Owing to the tracking error (6), the terminal sliding mode surface (7), the dual observer error (9) and (24), $\ddot{V}$ exists. Hence, $\dot{V}$ is uniformly continuous. Then, according to (25) and by using the Barbalat lemma, we conclude $\lim_{t\to\infty} |s| = 0$, which further implies that $\lim_{t\to\infty} |e| = 0$ in view of (7).

By enlarging the right side of (24), we have

$$\begin{aligned}\dot{V} \le & -kJs^2 - \sigma_0 \frac{|\dot{x}|}{g(\dot{x})}\tilde{z}_0^2 - \sigma_1 \frac{|\dot{x}|}{g(\dot{x})}\tilde{z}_1^2 + \tilde{\sigma}_0^2 + \tilde{\sigma}_0^2 \\ & + \tilde{\sigma}_1^2 + \hat{\sigma}_1^2 + \tilde{\beta}^2 + \hat{\beta}^2 + \tilde{d}^2 + \hat{d}^2 + \tilde{J}^2 + \hat{J}^2\end{aligned} \tag{26}$$

On substituting equations $\widetilde{(\cdot)} + \widehat{(\cdot)} = (\cdot)$ and $\widetilde{(\cdot)}^2 + \widehat{(\cdot)}^2 = (\cdot)^2 - 2\widetilde{(\cdot)}\widehat{(\cdot)}$ into (26), we have

$$\begin{aligned}\dot{V} \le & -kJs^2 - \sigma_0 \frac{|\dot{x}|}{g(\dot{x})}\tilde{z}_0^2 - \sigma_1 \frac{|\dot{x}|}{g(\dot{x})}\tilde{z}_1^2 + \sigma_0^2 - 2\,\tilde{\sigma}_0\,\hat{\sigma}_0 + {\sigma_1}^2 \\ & - 2\,\tilde{\sigma}_1\,\hat{\sigma}_1 + \beta^2 - 2\tilde{\beta}\hat{\beta} + d^2 - 2\tilde{d}\hat{d} + J^2 - 2\tilde{J}\hat{J}\end{aligned} \tag{27}$$

On using the following inequalities

$$-\tilde{\sigma}_0\,\hat{\sigma}_0 \le -\frac{1}{2}\tilde{\sigma}_0^2 + \frac{1}{2}{\sigma_0}^2 \tag{28}$$

$$-\tilde{\sigma}_1\,\hat{\sigma}_1 \le -\frac{1}{2}{\tilde{\sigma}_1}^2 + \frac{1}{2}{\sigma_1}^2 \tag{29}$$

$$-\tilde{\beta}\hat{\beta} \le -\frac{1}{2}\tilde{\beta}^2 + \frac{1}{2}\beta^2 \tag{30}$$

$$-\tilde{d}\hat{d} \le -\frac{1}{2}\tilde{d}^2 + \frac{1}{2}d^2 \tag{31}$$

$$-\tilde{J}\hat{J} \le -\frac{1}{2}\tilde{J}^2 + \frac{1}{2}J^2 \tag{32}$$

we have

$$\begin{aligned}\dot{V} \le & -kJs^2 - \sigma_0 \frac{|\dot{x}|}{g(\dot{x})}\tilde{z}_0^2 - \sigma_1 \frac{|\dot{x}|}{g(\dot{x})}\tilde{z}_1^2 - \tilde{\sigma}_0^2 - \tilde{\sigma}_1^2 \\ & - \tilde{\beta}^2 - \tilde{d}^2 - \tilde{J}^2 + 2\,{\sigma_0}^2 + 2\,{\sigma_1}^2 + 2\beta^2 + 2d^2 + 2J^2\end{aligned} \tag{33}$$

From (18) and (33), we can obtain

$$\dot{V} \le -\lambda_1 V + \lambda_2 \tag{34}$$

where $\lambda_1 > 0$ and $\lambda_2 > 0$, and they are defined as

$$\lambda_1 = \min\left\{2k,\ 2r_0,\ 2r_1,\ 2r_2,\ 2r_3,\ 2r_4,\ \frac{2|\dot{x}|}{g(\dot{x})}\right\} \tag{35}$$

$$\lambda_2 = 2\sigma_0^2 + 2\sigma_1^2 + 2\beta^2 + 2d^2 + 2J^2 \tag{36}$$

Therefore V satisfies

$$0 \le V(t) \le V(0)\exp(-\lambda_1 t) + \frac{\lambda_2}{\lambda_1}(1 - \exp(-\lambda_1 t)) \tag{37}$$

where

$$\begin{aligned}V(0) = & \frac{J}{2}s(0)^2 + \frac{1}{2r_0}\tilde{\sigma}_0(0)^2 + \frac{1}{2r_1}\tilde{\sigma}_1(0)^2 + \frac{1}{2r_2}\tilde{\beta}(0)^2 \\ & + \frac{1}{2r_3}\tilde{d}(0)^2 + \frac{1}{2r_4}\tilde{J}(0)^2 + \frac{1}{2}\sigma_0\tilde{z}_0(0)^2 + \frac{1}{2}\sigma_1\tilde{z}_1(0)^2\end{aligned} \tag{38}$$

From (18) and (37), we have

$$|s| \le \sqrt{\frac{2}{J}\left(V(0)\exp(-\lambda_1 t) + \frac{\lambda_2}{\lambda_1}(1 - \exp(-\lambda_1 t))\right)} \tag{39}$$

$$|\tilde{\sigma}_0| \le \sqrt{2r_0\left(V(0)\exp(-\lambda_1 t) + \frac{\lambda_2}{\lambda_1}(1 - \exp(-\lambda_1 t))\right)} \tag{40}$$

$$|\tilde{\sigma}_1| \le \sqrt{2r_1\left(V(0)\exp(-\lambda_1 t) + \frac{\lambda_2}{\lambda_1}(1 - \exp(-\lambda_1 t))\right)} \tag{41}$$

$$|\tilde{\beta}| \le \sqrt{2r_2\left(V(0)\exp(-\lambda_1 t) + \frac{\lambda_2}{\lambda_1}(1 - \exp(-\lambda_1 t))\right)} \tag{42}$$

$$|\tilde{d}| \le \sqrt{2r_3\left(V(0)\exp(-\lambda_1 t) + \frac{\lambda_2}{\lambda_1}(1 - \exp(-\lambda_1 t))\right)} \tag{43}$$

$$|\tilde{J}| \le \sqrt{2r_4\left(V(0)\exp(-\lambda_1 t) + \frac{\lambda_2}{\lambda_1}(1 - \exp(-\lambda_1 t))\right)} \tag{44}$$

By using the equation $|\widetilde{(\cdot)}| = \big||(\cdot)| - |\widehat{(\cdot)}|\big|$, we can draw several conclusions about the parameters error

$$\lim_{t\to\infty} |(\hat{\sigma}_0)| \le |\sigma_0| + \sqrt{\frac{2\lambda_2 r_0}{\lambda_1}}\Big) \tag{45}$$

$$\lim_{t\to\infty} |(\hat{\sigma}_1)| \le |\sigma_1| + \sqrt{\frac{2\lambda_2 r_1}{\lambda_1}}\Big) \tag{46}$$

$$\lim_{t\to\infty} |(\hat{\beta})| \le |\beta| + \sqrt{\frac{2\lambda_2 r_2}{\lambda_1}}\Big) \tag{47}$$

$$\lim_{t\to\infty} |(\hat{d})| \le |d| + \sqrt{\frac{2\lambda_2 r_3}{\lambda_1}}\Big) \tag{48}$$

$$\lim_{t\to\infty} |(\hat{J})| \le |J| + \sqrt{\frac{2\lambda_2 r_4}{\lambda_1}}\Big) \tag{49}$$

Remark 2: from (36), the size of λ_2 can be adjusted by σ_0, σ_1, β, d and J which are the fixed arguments of the system. To decrease the convergence region of the estimation error, we can adjust the value of λ_1, which is related to k, r_0, r_1, r_2, r_3, r_4 and $\frac{|\dot{x}|}{g(\dot{x})}$.

In summary, we obtain the following results.

Theorem 1: given the desired trajectory x_d, the terminal sliding mode surface (7), the control law (10) and the adaptive laws (11)–(15) applied to the system (1) with the friction (2) ensure that all the closed-loop signals are uniformly ultimately bounded. Furthermore, the tracking error e converges to a ball, whose radius can be freely adjusted in known form by the design parameters (7) and (39).

Remark 3: According to (45)–(49), the boundary of the parameters can be adjusted by the design parameters J, r_0, r_1, r_2, r_3, r_4, λ_1, λ_2 and the original value $V(0)$. λ_2 and J are the fixed arguments of the system. From a practical perspective, the more prior parameter knowledge we have, the more higher tracking accuracy we can obtain. If we accurately estimated the bounds of z, σ_0, σ_1, β, d and J, we can chose their original value close to z, σ_0, σ_1, β, d and J, respectively. Meanwhile, by choosing appropriate values of k, r_0, r_1, r_2, r_3, r_4 and $\frac{|\dot{x}|}{g(\dot{x})}$, we can have a very accurate tracking precision.

4 Simulation results

In this section, the results of the proposed control scheme applied to the servo system with the non-linear friction via simulation are presented. The parameters of the LuGre model and the motor are as follows $\sigma_0 = 0.5$, $\sigma_1 = 0.3$, $\sigma_2 = 0.1$, $F_c = 0.285$, $F_s = 0.335$, $\dot{x}_s = 0.01$, $T_L = 0.8$, $J = 1.6$ and the given reference signal is $x_d = 2\ \sin\ (\pi/3t)$. To verify the good robustness of the proposed control scheme, we perform some simulation experiments under the absence of disturbance. Figs. 1 and 2 show the tracking performance of the system with disturbance, which prove that the tracking error with disturbance is bounded by the presented

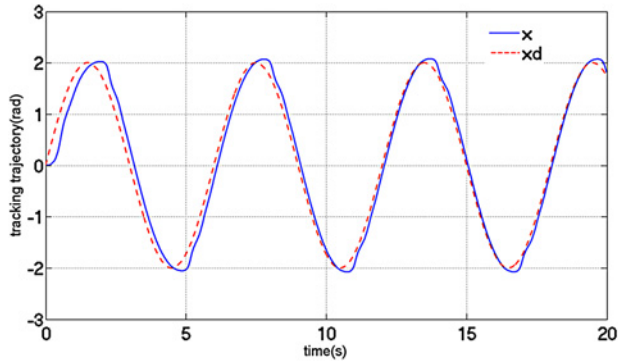

Fig. 1 *Tracking performance with disturbance*

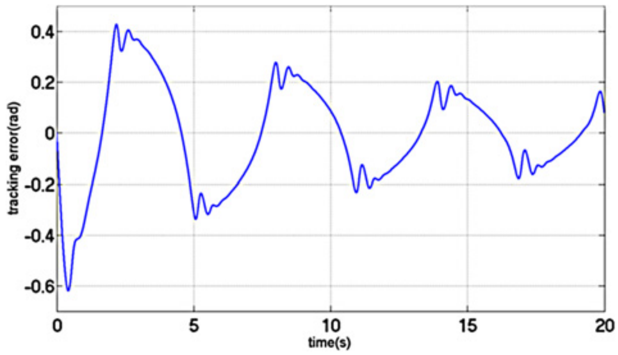

Fig. 2 *Tracking error with disturbance*

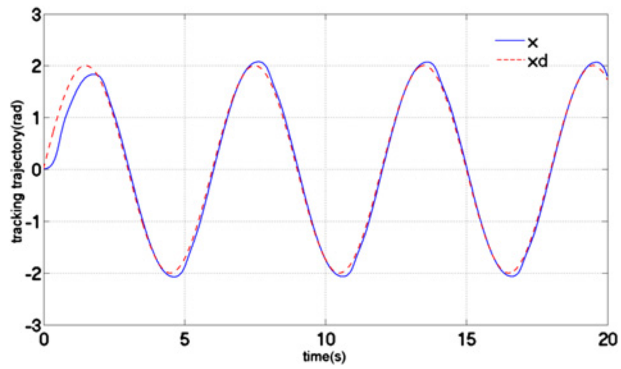

Fig. 3 *Tracking performance without disturbance*

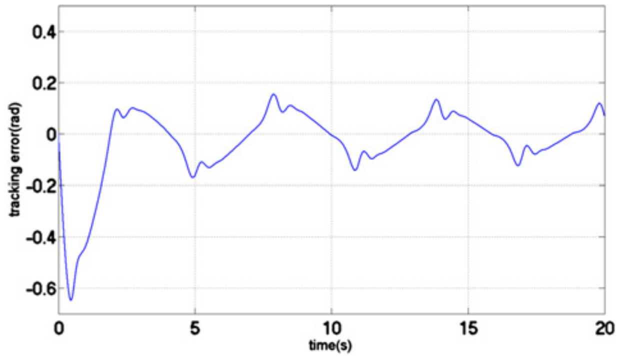

Fig. 4 *Tracking error without disturbance*

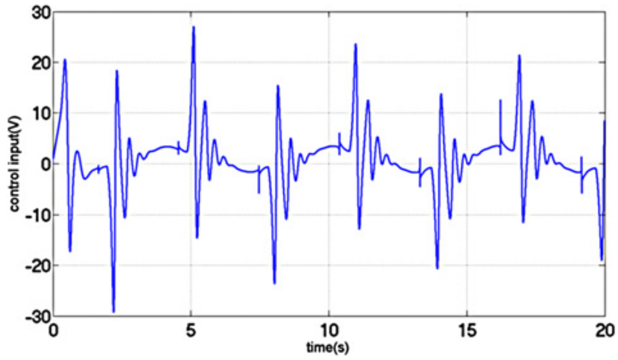

Fig. 5 *Control input with disturbance*

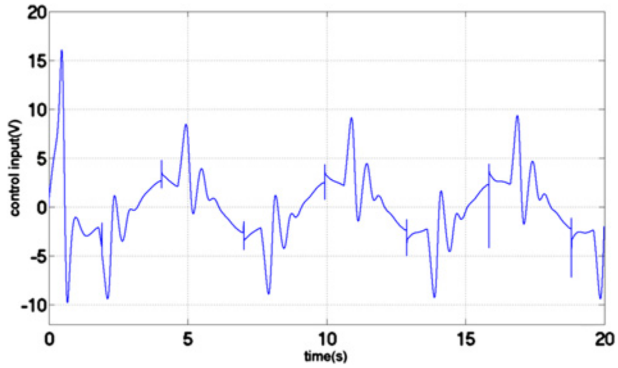

Fig. 6 *Control input without disturbance*

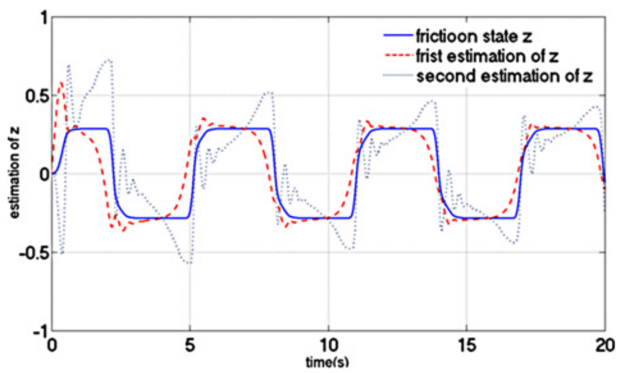

Fig. 7 *Estimation of the dual observer with disturbance*

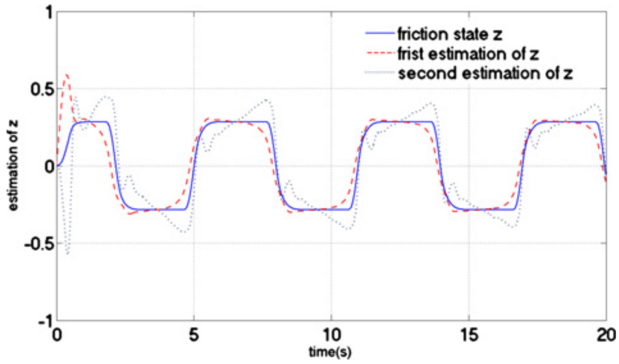

Fig. 8 *Estimation of the dual observer without disturbance*

scheme. Figs. 3 and 4 show the tracking performance of the system without disturbance, which are better than the ones with disturbance. Figs. 5 and 6 show the control input of the system with disturbance and without disturbance, respectively. Figs. 7 and 8 show the estimation of the dual observer with disturbance and without disturbance, respectively. Figs. 6 and 8 also prove that the first estimation of z has more accurate estimation error than that of the second one. In conclusion, the simulation results prove the feasibility and the good robustness of the controller.

5 Conclusion

This paper has presented an adaptive control solution for non-linear friction in the servo system, which is under some realistic assumptions. The good steady-state performance and transient performance are proved by the Lyapunov theory. The simulation results show the good robustness and precise position tracking performance of the system.

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