

Capacity investment model for airport facilities under demand uncertainty

Yanshuo Sun and Paul M. Schonfeld*

Department of Civil and Environmental Engineering, University of Maryland, College Park, MD 20742, U.S.A.

SUMMARY

This paper addresses strategic airport facility planning under demand uncertainty. Existing studies are improved by (1) allowing capacity contraction and (2) adopting more flexible delay functions. A mixed-integer nonlinear program, which incorporates scale economies in construction, time value of money, nonlinear congestion effect, and other factors, is proposed for optimizing the capacity expansion/contraction decisions over time for multiple airport components. The stochastic problem is converted into its deterministic equivalent because the number of demand scenarios considered is finite. A discrete approximation technique is used to remove the nonlinearities. Numerical studies are presented to demonstrate the capability of the proposed model and the computational efficiency of the solution method. The “Flaw of Averages” due to faulty decisions based on the average future condition is illustrated, and trade-offs among various costs are discussed in the numerical analyses. Copyright © 2016 John Wiley & Sons, Ltd.

KEY WORDS: airport development; uncertainty; discrete approximation; stochastic optimization

1. INTRODUCTION

The aviation sector fosters a nation’s prosperity by generating a tremendous amount of economic activity and creating millions of jobs. However, infrastructure aging and congestion at major airports greatly concern economists, policymakers, and transportation engineers. Airport development decisions are critical as well as difficult because (1) substantial capital investments are needed and they are usually irreversible; (2) airport facilities have rather complex performance functions, for example, delay level as a nonlinear function of the capacity utilization rate; (3) several practical factors, such as the scale economies and lumpiness of construction, are essential in sizing the expansion and determining the frequency; and (4) the development of an airport can have significant social, economic, and environmental impacts. The dynamic and uncertain nature of future changes further complicates the decision making process by transforming the problem into a stochastic one. To address the pressing and challenging issue of airport facility development, this paper presents a method for managing capacities of airport facilities in light of expected demand changes and uncertainties, with considerations of the interactions between the supply and demand sides and various cost characteristics.

Quite a few studies on airport development can be found, which can be divided into macro and micro ones. These macro studies, such as [1, 2], are quite useful for the preliminary evaluation of various airport development plans. Nonetheless, they cannot systematically generate detailed plans of interest by their design purpose. When we turn to the micro analyses, only few relevant studies [3–5] are found and thus reviewed. In the first one, Solak *et al.* [3] consider an airport terminal capacity planning problem; only expansions of gates are studied in [4]; Yoon and Jeong [5] present a study on

*Correspondence to: Paul M. Schonfeld, Department of Civil and Environmental Engineering, University of Maryland, College Park, MD 20742, U.S.A. E-mail: pschon@umd.edu

planning baggage carousel capacity. By noting that most previous studies focus on one specific component of the airport system, Sun and Schonfeld [6] provide a global planning model considering multiple airport components and several essential factors in the development process, such as economies of scale in expanding facilities, time value of money, and nonlinear congestion effects. Generally, the paper presented here follows the modeling framework of [6]; however, the following two major assumptions are relaxed:

- (1) While demands might drop, capacity never decreases in any facility of an airport;
- (2) The delay cost function is continuously differentiable.

The first assumption reduces the capacity planning model into a capacity expansion model, which disallows potential capacity contractions. If both capacity expansion and contraction are allowed, the model is called a capacity investment model. The term “investment” is used to indicate the change of capacity in both directions [7, 8]. The consideration of capacity contraction is quite important, especially when demand changes sharply. For example, when US Airways moved its hub operations from the Baltimore–Washington Airport (BWI) to nearby Philadelphia, it left a large underutilized passenger terminal.

Delay functions pertaining to airport facilities are inherently nonlinear, with average delays rising very rapidly and nonlinearly as delays approach ultimate capacities. The desired differentiable property of the delay curve enables the design of a specialized linear approximation technique as described in [6]. We would like to explore what methods can be used if this property is missing, especially noting that most delay curves are obtained from computer simulations and empirical studies [9, 10], meaning that delay functions are likely non-differentiable.

The remainder of the paper is structured as follows. The effect of uncertainty on the development of airport facilities is analyzed, and various methods addressing uncertainty are reviewed. Then, a deterministic capacity investment model is proposed and extended into a stochastic model. The resulting nonlinear model is linearized and solved with proposed techniques. Numerical studies are presented to demonstrate the capability of the proposed method. The last section concludes with a summary of current work and extensions.

2. LITERATURE REVIEW

2.1. *Effect of uncertainty on airport development*

Many sources of uncertainties are inherent in the airport development process. These uncertainties can affect the overall volume of air traffic levels (e.g., annual enplanements, total number of aircraft operations, and cargo volumes) as well as the mix of traffic (e.g., domestic versus international passengers and wide-body versus narrow-body aircrafts). A fairly complete list of uncertainties involved in the airport long-term planning is provided in Airport Cooperative Research Program (ACRP) Report 76 [11]. Because of various uncertainties, traffic level can build up quickly to create severe congestion and it can also drop even faster, resulting in underutilized facilities.

The standard master planning process has been criticized by de Neufville and Scholtes [1], de Neufville and Odoni [12], and Kincaid *et al.* [11]. The motivation of [11] is that traditional approaches for incorporating uncertainty in the airport planning, such as what-if analysis and sensitivity analysis, are insufficient and can only provide a cursory understanding of future risks. The master plan based on what is most likely to happen, without considering what can happen, tends to be wrong because the aviation industry is becoming more volatile and unpredictable [12].

2.2. *Trigger point approach*

One widespread method for determining the timing of a new expansion project in practice is called threshold or trigger point approach. For instance, when traffic reaches 80% of the current capacity, predefined facility expansion projects are initiated. Such a timing approach is helpful when traffic keeps increasing steadily, while projects might be wrongly launched when traffic fluctuates greatly. No theoretical studies have been found to justify the optimality of such a capacity expansion policy; even assuming that such a threshold-based approach can produce optimal results, the trigger point

seems arbitrary without sufficient quantitative support. For example, how can we determine these trigger points (e.g., 85%, 90%, or 95% of the existing capacity) and the size of capacity increments for various facilities?

One additional problem arises when demand drops, which means capacity might be reduced as well. What models can be used to derive the trigger for capacity contraction?

2.3. Macro approaches

As pioneers advocating explicit treatment of uncertainties in the airport master planning, de Neufville and Odoni [12] developed the concept of dynamic strategic planning in airports. They argue that airport traffic forecasts are “always wrong” and the unreliability of forecasts has crucial implications for airport planning. However, the traditional process for developing an airport master plan is essentially reactive. To make the plan proactive and flexible, they propose a modified form of master planning by considering several possible levels and types of future traffic. A five-step framework for addressing uncertainty about future airport activity levels is provided in [11]. Kwakkel *et al.* [2] propose another concept called dynamic adaptive planning.

In addition to the conceptual planning concepts reviewed earlier, option valuations [13] are also used in the literature. The concept of “real options” is adapted from financial options. Investors have the right rather than the obligation to buy or sell a security or other assets at an agreed price during a certain period of time.

2.4. Micro approaches

Macro studies address the airport as a whole, while micro studies focus on individual components, for example, baggage handling. There are very few micro analyses of the airport facility development problem in the literature. Table I provides a brief comparison of these relevant studies at the micro level. Several ACRP reports cover the planning methodologies for some specific airport components. For example, ACRP Report 25 [14] deals with passenger terminals; ACRP Report 96 [15] is for apron; ACRP Report 143 [16] is for cargo facility. Javid *et al.* [17] present a method for estimating the required capacity of parking facilities at airports. A series of studies examine how the airport passenger departure lounge [18], the airside [19], the terminal configuration [20], and the baggage claim area [21] should be redesigned when the new large aircraft is introduced. In De Barros and Wirasinghe [22], the concept of “stage construction” is proposed and an analytical model is used, which allows only part of the final airport terminal to be built initially. Such an approach is shown to reduce interest and delay costs. The trade-off between operational delays and construction costs is also analyzed. This study does not incorporate the effect of demand uncertainty on expansion decisions. Because most existing studies address the design of a specific component in one period, Sun and Schonfeld [6] provide a global planning model that optimizes the capacities of multiple components over a multi-period horizon. Several major factors, such as nonlinear congestion effect, scale economies in expansion, and future cost discounting, are included. The study adopts most of the modeling techniques in [6]; however, additional contributions are made by relaxing two major assumptions in [6]. The out-approximation technique relies on the desired differentiable property of the delay function, which might be unavailable in practice. A more general method that does not require the differential property is proposed. The second is to allow capacity contraction. If capacity can never be reduced, the capacity

Table I. Summary of micro analyses of airport facility development.

Study	Component	Facility Performance	Uncertainty	Optimization Method
Solak <i>et al.</i> (2009)	Passenger terminal	Analytical approximation	Only demand	Multi-stage stochastic program
Chen and Schonfeld (2013)	Boarding gates	Analytical function	Demand and construction time	Analytical
Yoon and Jeong (2015)	Baggage carousels	Simulation evaluations	N/A	Heuristics

investment problem would be reduced into only an expansion problem. Unfortunately, for airports, capacity reductions must often be considered, especially in response to major changes in airline operations at some airports.

In summary, to augment existing airport master plan, various macro and micro modeling tools are developed. The so-called trigger point approach might be promising; however, it has not yet been examined rigorously. Quite a few macro procedures can be useful for the preliminary evaluation of various airport development plans at the airport level. However, these macro-level methods cannot systematically generate detailed plans of interest by their design purpose. Therefore, existing micro models are extended by allowing capacity contraction and adopting more flexible delay functions.

3. MODEL

The methodological development is presented in the following steps:

- (1) The notation list is provided in Table II, and various cost functions, such as capacity adjustment, operating, and delay costs, are defined.
- (2) A base model assuming deterministic demand is formulated. It is a mixed-integer nonlinear program (MINLP).
- (3) A discrete approximation technique is used for linearizing the base model, and multiple demand scenarios are then considered, transforming the deterministic problem into a stochastic one.
- (4) The model's structure is analyzed, based on which a decomposition solution approach is proposed.

3.1. Assumptions

Although some assumptions made in [6] are relaxed in this study, other assumptions still apply, as described as follows:

- (1) The economic life of new infrastructure exceeds the planning horizon (typically 20 or 30 years); that is, infrastructure replacement or demolition are not considered;
- (2) Various demand measures (e.g., enplanements, number of aircraft operations, and tons of cargo shipped) are estimated for each individual airport component, and capacity is analyzed separately for each component.

Table II. Notation list.

Sets and Indices	
i	Component of airport system $I = \{1, 2, \dots, l\}$, $i \in I$
j or t	Time period within the planning horizon $J = \{0, 1, 2, \dots, m\}$, $j, t \in J$
k	Step of a discrete delay function $K = \{1, 2, \dots, n\}$, $k \in K$
a	Capacity management choice $a \in A = \{1, 0\}$. When $a = 1$, additional capacities are purchased; that is, facilities are expanded; otherwise, existing capacities are salvaged
Parameters	
f_{ij}^a	Fixed capital cost of adjusting capacity of component i in period j when the capacity management choice is a
v_{ij}^a	Variable capital cost of adjusting capacity of component i in period j when the capacity management choice is a
o_{ij}	Unit operating cost of component i in period j
q_{ij}	Demand on component i in period j
b_i^k	Stepping points of the delay function of component i
c_i^k	Delay level in interval k of delay function i
δ	Discount factor
M_i , $i \in I$	The maximum capacity change (expansion or contraction) to component i in any single period
N_i , $i \in I$	The maximum supplied capacity of component i
Decision variables	
x_{ij}^a	The amount of capacity adjusted to component i in period j when the capacity management choice is a
y_{ij}^a	Whether to adjust capacity to component i in period j when the capacity management choice is a

3.2. Cost functions

3.2.1. Capacity adjustment costs

Capacity adjustment costs are capital costs, which include fixed and variable parts. The variable costs are functions of the magnitude of the capacity increment. The capital cost of component i in period j can be written as follows:

$$C_{ij} = \delta^j \sum_{a \in A} \left(f_{i0}^a v_{ij}^a + v_{i0}^a x_{ij}^a \right), \quad \forall i \in I, j \in J \quad (1)$$

where coefficient $\delta = 1/(1 + \Delta)$ is used to discount future values and Δ is the periodical discount rate.

The capacity adjustment cost is more compactly written as follows:

$$C_{ij} = \sum_{a \in A} \left(f_{ij}^a v_{ij}^a + v_{ij}^a x_{ij}^a \right), \quad \forall i \in I, j \in J \quad (2)$$

When capacities are salvaged ($a=0$), the decision maker can receive a variable value $|v_{ij}^0|$ per unit capacity salvaged, but at a one-time fixed cost f_{ij}^0 . Because v_{ij}^a is defined from the perspective of cost, v_{ij}^a should be negative when $a=0$, meaning that some positive values will be received because of the salvage. When $v_{ij}^0 + v_{ij}^1 = 0$, $\forall i \in I, j \in J$, the variable capital cost is reversible; when $v_{ij}^0 + v_{ij}^1 > 0$, $\forall i \in I, j \in J$, investments can only be partially recovered. Particularly, when $v_{ij}^0 = 0$, $\forall i \in I, j \in J$, no investments can be reversed. Note that, in practice, it is impossible to have $v_{ij}^0 + v_{ij}^1 < 0$, $\forall i \in I, j \in J$; that is, the salvage value should not exceed the purchase price.

The capacity of component i in period j is the initial capacity s_{i0} plus net adjusted capacity in each period, as shown in Equation (3).

$$s_{it} = s_{i0} + \sum_{j=1}^t x_{ij}^1 - \sum_{j=1}^t x_{ij}^0, \quad \forall i \in I, t \in J \quad (3)$$

3.2.2. Operating Costs

The operating cost of component i in period j is the discounted unit operating cost o_{ij} multiplied by the supplied capacity s_{ij} :

$$O_{ij} = o_{ij} s_{ij}, \quad \forall i \in I, j \in J \quad (4)$$

Unit operating costs are discounted in the same way as capital costs with the coefficient of δ^j .

3.2.3. Delay Costs

Airport congestion constitutes a major problem for airport authorities and their customers. Because of the dynamic characteristics of demands, that is, daily pattern, day-of-the-week pattern, and seasonal pattern, airport delays are difficult to estimate. In practice, advanced computer-based tools (simulation or analytical) are needed to obtain good approximations of airport facility delays. Although complex short-term behaviors of delays exist, in the long run major airport components experience increased delay costs when demands grow, especially when the demands approach the capacity limits.

For a specific airport component, its operating characteristics can be described with the delay level as a function of the capacity utilization, that is, the ratio of the demand q divided by the capacity s .

For component i , we denote the delay function as follows:

$$d_{ij} = F_i \left(\frac{q_{ij}}{s_{ij}} \right) \quad (5)$$

where d_{ij} is the delay level of component i in period j and $F_i(\cdot)$ is the delay function of component i in period j .

Delay costs are demands multiplied by the delay level, which can be written as follows:

$$D_{ij} = F_i \left(\frac{q_{ij}}{s_{ij}} \right) q_{ij} \quad (6)$$

Note that delay level is measured in dollars per unit of demand, while delay cost is measured in dollars.

In airports, various airside and landside facilities have different operating characteristics. Simulation is the dominant method for quantifying the capacity and performance of airside facilities, especially runway systems; queueing theory is sometimes used to assess the demand needs and estimate the delay level for terminal facilities, such as boarding gates. Whatever specific function forms the delay function for a facility assumes, delay costs are essentially nonlinear, which creates difficulties in utilizing linear programming techniques to solve airport capacity planning problems. In [6], the convexity of the delay function is identified and the function is assumed to be continuously differentiable. However, the delay function estimated from either empirical studies or computer simulations does not necessarily have such desired properties as continuous differentiation. Even the closed-form expression of the delay curve is unavailable in some cases. Therefore, another method that does not rely on such properties should be developed.

3.3. Mixed-integer nonlinear program

After definitions of various costs, the airport capacity investment model, which is a total cost minimization problem, can be written as follows:

$$\min_{\{x_{ij}^a \geq 0, y_{ij}^a \in \{0, 1\}\}} \sum_i \sum_j \sum_a \left(f_{ij}^a y_{ij}^a + v_{ij}^a x_{ij}^a \right) + \sum_i \sum_j o_{ij} s_{ij} + \sum_i \sum_j \delta^j F_i \left(\frac{q_{ij}}{s_{ij}} \right) q_{ij} \quad (7.0)$$

subject to

$$x_{ij}^a \leq M_i y_{ij}^a, \quad \forall i \in I, j \in J, a \in A \quad (7.1)$$

$$s_{it} = s_{i0} + \sum_{j=1}^t x_{ij}^1 - \sum_{j=1}^t x_{ij}^0, \quad \forall i \in I, t \in J \quad (7.2)$$

$$q_{ij} \leq s_{ij}, \quad \forall i \in I, j \in J \quad (7.3)$$

The objective function is the net present value of total cost, which includes capital costs, operating costs, and delay costs. While the first two are incurred by the airport authority, delay costs are borne by airport users, for example, aircraft operators, passengers, and cargo shippers. Constraint (7.1) guarantees that no capacities can be added (i.e., $x_{ij}^1 = 0$) unless the capacity expansion decision is made ($y_{ij}^1 = 1$) or no capacities can be reduced ($x_{ij}^0 = 0$) unless the salvage decision is made ($y_{ij}^0 = 0$), where M_i is the maximum capacity change (expansion or contraction) to component i in any single period.

Constraint (7.2) defines the supplied capacity of component i in period j . Constraint (7.3) specifies that demands cannot exceed capacities, at least over extended periods.

Because of the two-sided fixed costs, that is, because there are fixed costs when the capacity is either increased or decreased, we have the following valid inequality:

$$\sum_{a \in A} y_{ij}^a \leq 1, \quad \forall i \in I, j \in J \quad (8)$$

Intuitively, we need to show that, at most, one capacity management choice can be selected.

If both choices are selected for component i in period j , that is, $y_{ij}^0 = 1$ and $y_{ij}^1 = 1$, the capital cost C_{ij} is as follows:

$$C_{ij} = f_{ij}^1 + f_{ij}^0 + v_{ij}^1 x_{ij}^1 - |v_{ij}^0| x_{ij}^0, \quad \forall i \in I, j \in J \quad (9)$$

Depending on the relation between x_{ij}^1 and x_{ij}^0 , we have the following three cases.

When $x_{ij}^1 > x_{ij}^0$, we can drop f_{ij}^0 from C_{ij} and then $C_{ij} > f_{ij}^1 + v_{ij}^1 x_{ij}^1 - |v_{ij}^0| x_{ij}^0$. Noting $v_{ij}^1 \geq |v_{ij}^0|$, we have $f_{ij}^1 + v_{ij}^1 x_{ij}^1 - |v_{ij}^0| x_{ij}^0 > f_{ij}^1 + v_{ij}^1 (x_{ij}^1 - x_{ij}^0)$, which further leads to $C_{ij} > f_{ij}^1 + v_{ij}^1 (x_{ij}^1 - x_{ij}^0)$. $f_{ij}^1 + v_{ij}^1 (x_{ij}^1 - x_{ij}^0)$ is the capital cost of only increasing the capacity by $(x_{ij}^1 - x_{ij}^0)$.

When $x_{ij}^1 < x_{ij}^0$, we can obtain $C_{ij} > f_{ij}^0 - |v_{ij}^0| (x_{ij}^0 - x_{ij}^1)$, whose right-hand side represents the capital cost of only decreasing the capacity by $(x_{ij}^0 - x_{ij}^1)$.

When $x_{ij}^1 = x_{ij}^0$, we have $C_{ij} > 0$.

Combining the earlier cases, we conclude that, at most, one management choice is needed, that is, $\sum_{a \in A} y_{ij}^a \leq 1, \quad \forall i \in I, j \in J$. Such an equality can be added to the original problem, which significantly reduces the solution space.

4. REFORMULATION

4.1. Model linearization

A discrete approximation technique is proposed to convert the MINLP into a linear program, so that the problem can be solved much more efficiently. A general delay function, which is not necessarily differentiable or continuous, is approximated by a step function, as shown in Figure 1.

For sufficiently small step sizes, we assume that if the capacity utilization rate is within the interval k , that is, $b^{k-1} \leq \frac{q}{s} \leq b^k$, the delay level is approximated by c^k . To find the corresponding approximated delay level r at a given demand level, we formulate the following mathematical program:

$$\min_{\{r, c^k \geq 0, u^k \in \{0, 1\}\}} r \quad (10.0)$$

subject to

$$r \geq u^k c^k, \quad \forall k \in K \quad (10.1)$$

$$(b^k - b^{k-1}) u^{k+1} \leq \frac{z^k}{s} \leq (b^k - b^{k-1}) u^k, \quad \forall k \in K \quad (10.2)$$

$$\sum_k z^k = q \quad (10.3)$$

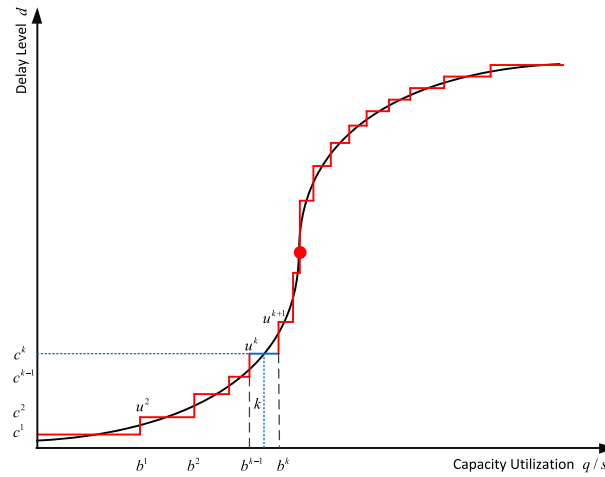


Figure 1. Discrete approximation of delay level.

where z^k is the increment of demand corresponding to the k th interval of the capacity utilization rate in Figure 1 and u^k is 1, if $\frac{q}{s} > b^{k-1}$, $u^k = 0$ otherwise.

Note that constraint (10.2) is still nonlinear and capacity s as an auxiliary decision variable appears as the denominator. If we multiply both sides by s , constraint (10.2) is transformed into

$$(b^k - b^{k-1})u^{k+1}s \leq z^k \leq (b^k - b^{k-1})u^k s, \forall k \in K \quad (11)$$

To remove the product of two auxiliary decision variables, we substitute $u^k s$ with nonnegative w^k . We add three additional constraints to ensure the equivalence of this substitution.

$$w^k \leq N u^k \quad (12.1)$$

$$0 \leq w^k \leq s \quad (12.2)$$

$$w^k \geq s - (1 - u^k)N \quad (12.3)$$

where N is a sufficient large limit on s .

If u^k is 1, constraint (12.1) is inactive, constraints (12.2) $w^k \leq s$ and (12.3) $w^k \geq s$ together restrict w^k to be s . If u^k is 0, from constraint (12.1) we have $w^k \leq 0$, that is, $w^k = 0$ because of the nonnegativity.

4.2. Mixed-integer linear program

The deterministic version of airport capacity investment problem in linear form can be written as follows:

$$\min_{\{x_{ij}^a, r_{ij}, z_{ij}^k, w_{ij}^k \geq 0, y_{ij}^a, u_{ij}^k \in \{0, 1\}\}} \sum_i \sum_j \sum_a \left(f_{ij}^a y_{ij}^a + v_{ij}^a x_{ij}^a \right) + \sum_i \sum_j o_{ij} s_{ij} + \sum_i \sum_j \delta^j r_{ij} q_{ij} \quad (13.0)$$

subject to constraints (7.1, 7.2, 7.3), (8), and

$$r_{ij} \geq u_{ij}^k c_i^k, \quad \forall i \in I, j \in J, k \in K \quad (13.1)$$

$$(b_i^k - b_i^{k-1})w_{ij}^{k+1} \leq z_{ij}^k (b_i^k - b_i^{k-1})w_{ij}^k, \quad \forall i \in I, j \in J, k \in K \quad (13.2)$$

$$\sum_k z_{ij}^k = q_{ij}, \quad \forall i \in I, j \in J \quad (13.3)$$

$$w_{ij}^k \leq N_i u_{ij}^k, \quad \forall i \in I, j \in J, k \in K \quad (13.4)$$

$$w_{ij}^k \leq s_{ij}, \quad \forall i \in I, j \in J, k \in K \quad (13.5)$$

$$w_{ij}^k \geq s_{ij} - (1 - u_{ij}^k)N_i, \quad \forall i \in I, j \in J, k \in K \quad (13.6)$$

The first group of constraints (i.e., constraints (7.1, 7.2, 7.3) and (8)) duplicate those used in the MINLP to restrict capacity expansion variables. The second group of constraints (i.e., constraints (13.1)–(13.3)) is used to approximate the delay costs. Other constraints are auxiliary constraints to preserve the linear property of this program.

4.3. Two-stage stochastic program

Future air traffic demands are quite difficult to predict accurately due to various economic fluctuations, technology innovations, competition among airports, and competition among transportation modes. For long-term forecasts, aviation analysts develop a range of plausible scenarios after considering all the factors relevant to airport facility development.

To account for the uncertainties in demand forecasts, multiple discrete demand patterns are considered. The stochastic version of airport capacity expansion problem in linear form can be written as follows:

$$\min_{\{x_{ij}^a \geq 0, y_{ij}^a \in \{0, 1\}\}} \sum_i \sum_j \sum_a \left(f_{ij}^a v_{ij}^a + v_{ij}^a x_{ij}^a \right) + \sum_i \sum_j E_{\xi} Q_{ij}(S, \xi) \quad (14.0)$$

subject to Constraints (7.1), and (8), where $Q_{ij}(S, \xi(\omega))$ is the optimal value of the second-stage program:

$$\min_{\{r_{ij}^k, z_{ij}^k, w_{ij}^k, s_{ij} \geq 0, u_{ij}^k \in \{0, 1\}\}} o_{ij}s_{ij} + \delta^j q_{ij}(\omega) r_{ij}(\omega) \quad (14.1)$$

subject to Constraint (7.2) and

$$q_{ij}(\omega) \leq s_{ij}, \quad \forall i \in I, j \in J, \omega \in \Omega \quad (14.2)$$

$$r_{ij}(\omega) \geq u_{ij}^k(\omega) c_i^k, \quad \forall i \in I, j \in J, k \in K, \omega \in \Omega \quad (14.3)$$

$$(b_i^k - b_i^{k-1})w_{ij}^{k+1}(\omega) \leq z_{ij}^k(\omega) \leq (b_i^k - b_i^{k-1})w_{ij}^k(\omega), \quad \forall i \in I, j \in J, k \in K, \omega \in \Omega \quad (14.4)$$

$$\sum_k z_{ij}^k(\omega) = q_{ij}(\omega), \quad \forall i \in I, j \in J, \omega \in \Omega \quad (14.5)$$

$$w_{ij}^k(\omega) \leq N_i u_{ij}^k(\omega), \quad \forall i \in I, j \in J, k \in K, \quad \omega \in \Omega \quad (14.6)$$

$$w_{ij}^k(\omega) \leq s_{ij}, \quad \forall i \in I, j \in J, k \in K, \quad \omega \in \Omega \quad (14.7)$$

$$w_{ij}^k(\omega) \geq s_{ij} - \left(1 - u_{ij}^k(\omega)\right) N_i, \quad \forall i \in I, j \in J, k \in K, \quad \omega \in \Omega \quad (14.8)$$

In this program, $\omega \in \Omega = \{\omega_1, \dots, \omega_R\}$ is the realization of the random demand ζ . R is the number of scenarios considered. In objective (14.1), $o_{ij} s_{ij}$ is the discounted operating cost of component i in period j . $\delta^j q_{ij}(\omega) r_{ij}(\omega)$ represents the discounted delay cost of component i in period j in a demand scenario w . $q_{ij}(\omega)$ is the demand volume and $r_{ij}(\omega)$ is the delay level. Their product is the delay cost. Capacity management decisions, that is, x_{ij}^a and y_{ij}^a , have to be determined before the random demand is observed. Other decision variables, for example, z_{ij}^k and w_{ij}^k , are second-stage variables, which depend on ω .

Usually, these demand scenarios are discrete; for example, pessimistic or optimistic, we can replace $E_{\zeta} Q_{ij}(S, \zeta)$ with $\sum_{r=1}^R p_r Q_{ij}(S, \zeta(\omega_r))$, $\forall i \in I, j \in J$, where p_r is the probability associated with scenario ω_r . Uncertainty is usually characterized by a probability distribution, which either is assumed to be known or can be estimated. In cases where such a distribution is difficult to obtain, for example, uncertainty parameters are known within certain bounds, robust optimization can be used to protect the system against the worst-case scenario, which may be considered overly conservative for airport capacity decisions. After such a replacement in objective (14.0), the stochastic program is converted to its deterministic equivalent.

4.4. Decomposition

Although multiple airport components are considered in the optimization problem, the resulting large-scale mixed-integer program can be partitioned into manageable subproblems, which are also independent. Note that each constraint is defined separately for each component and the objective includes three separable cost components. The structure of the large-scale program can be illustrated with Figure 2.

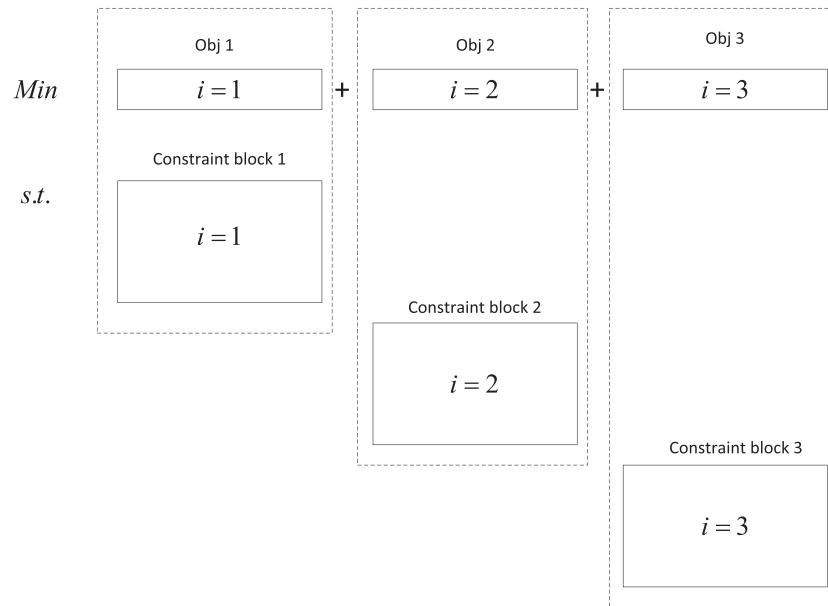


Figure 2. Problem structure.

Assuming that three airport components are considered, we can thus partition the overall problem into three subproblems, each of which is for the development of one airport component. Such a transformation provides significant savings in computational time because the solution time for linear programs grows quickly with the number of constraints. In general, according to Bradley *et al.* [23], if the number of subproblems were k , the solution time would be $1/k^2$ times that required for an unstructured problem of comparable size.

Such a decomposition is possible only because the interactions between airport components are not considered, which renders these subproblems independent of each other.

5. NUMERICAL EXAMPLES

5.1. Data inputs

Three major components of airports, namely, the airfield system, passenger terminal, and cargo facility, are considered. Demand measures for these three facilities are aircraft operations, enplanements, and cargo tons per period, respectively. Four plausible demand growth patterns are considered, as shown in Figures 4, 7, and 8.

The probabilities associated with those scenarios are 0.2, 0.2, 0.3, and 0.3, respectively. The proposed model can take any scenario probability distribution and accommodate any demand growth pattern, as long as the demand can be estimated for each component in a certain period. Table III presents cost-related data for each airport component.

The discrete delay functions shown in Figure 3 are used in numerical analyses. The number of steps in the delay function is 30, that is, $n=30$. A larger number can provide better approximations at the cost of additional computations.

Table III. Cost parameters (million dollars).

Component	Fixed capital		Variable capital		Operating
	Purchase	Salvage	Purchase	Salvage	
Airfield system	720	216	3.2	−1.28	0.8
Passenger terminal	220	66	6.5	−2.6	1.0
Cargo facilities	16	4.8	6.4	−2.56	1.2

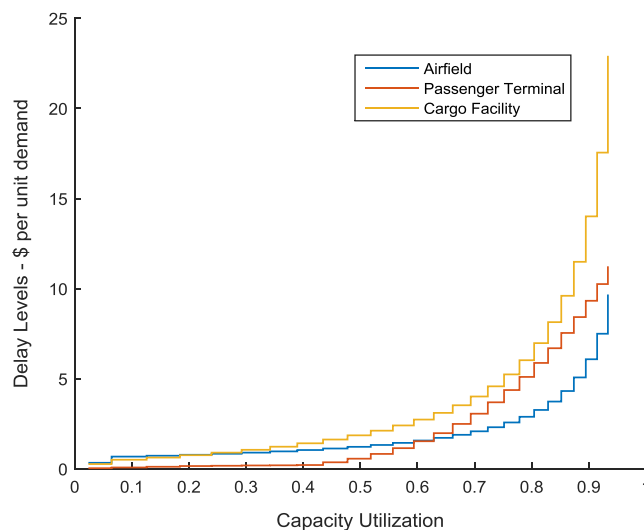


Figure 3. Discrete delay levels.

The discounting rate $\delta=0.97$. The planning horizon $m=30$. In implementation, $\{M_i=2 * \max\{q_{ij}(\omega)\}\}$ and $N_i=4 * \max\{q_{ij}(\omega)\}$, where $\max\{q_{ij}(\omega)\}$ is the maximum possible demand on component $i \in I$. The model is implemented in GAMS v24.7.1, and the MIP solver is FICO-Xpress 28.01.

5.2. Optimization results

The development plan for cargo facilities is shown in Figure 4. It can be observed that two capacity expansions are planned, first in period 6 and then period 13, with the magnitude of each capacity addition specified in Figure 4. Because of the expected demand drop, the capacity is reduced in period 20.

The resulting costs in each period for the cargo facility are plotted in Figure 5. The figure can clearly show the trade-offs among capital costs, operating costs, and delay costs. As traffic grows, delays increase, which produces capacity expansion decisions (thus capital cost expenditures). As capacities increase, operating costs jump to new levels because operating cost is linear with respect to the capacity provided. Then delays start to grow again, leading to the next cycle of expansion. When

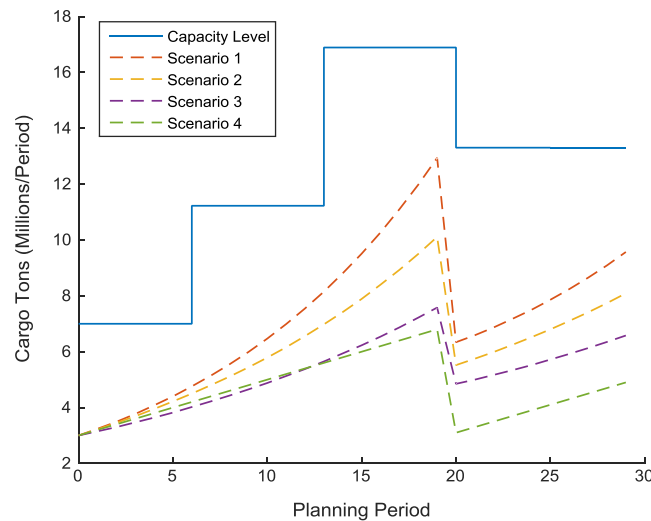


Figure 4. Capacity over time for cargo facilities.

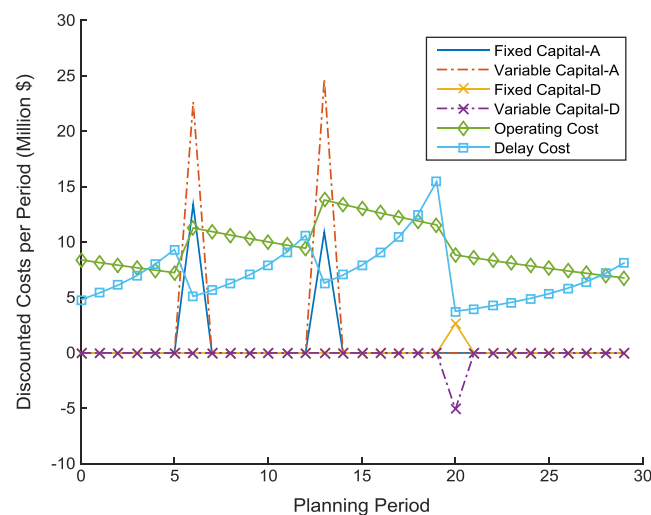


Figure 5. Resulting costs in each planning period for cargo facilities.

demand drops suddenly, capacity is reduced in order to (i) obtain the salvage value and (ii) reduce the operating cost.

Without using the stochastic program, we can formulate a deterministic version with the average value of demand forecasts. The resulting capacity level over time based on the expected scenario is shown in Figure 6. Clearly, when the first scenario occurs, demand will exceed capacity in some periods, which implies enormous delay costs. The term “Flaw of Averages” [1] refers to the value loss from the practice of making decisions based on average future conditions. The potential saving obtained from considering a range of future scenarios and solving a stochastic program is also called the value of stochastic solution in [24].

The resulting capacity provision level versus each possible demand growth scenario for each of two other airport components is plotted in Figures 7 and 8. Two capacity expansions are planned for both the airfield and passenger terminal. The computation time for solving the passenger terminal subproblem on a desktop computer (Intel Core Quad CPU 2.83 GHz, 3.25-GB RAM) is shown in Figure 9. When the optimality gap is 0.01 (meaning near-optimal), it takes less than 1 h to solve the

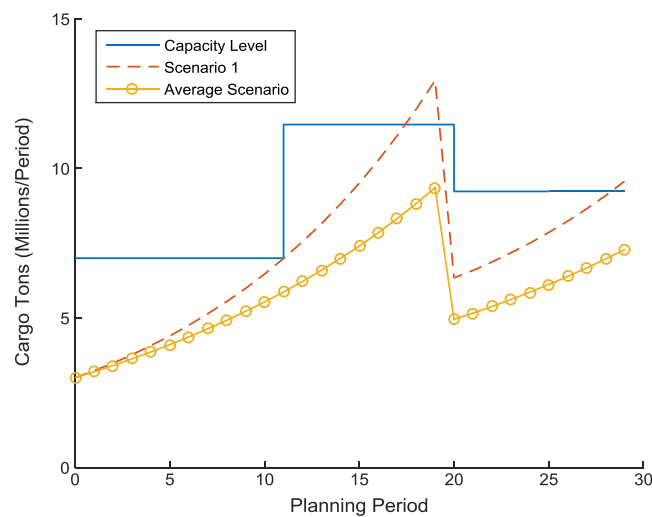


Figure 6. Capacity decisions for cargo facilities based on the average scenario.

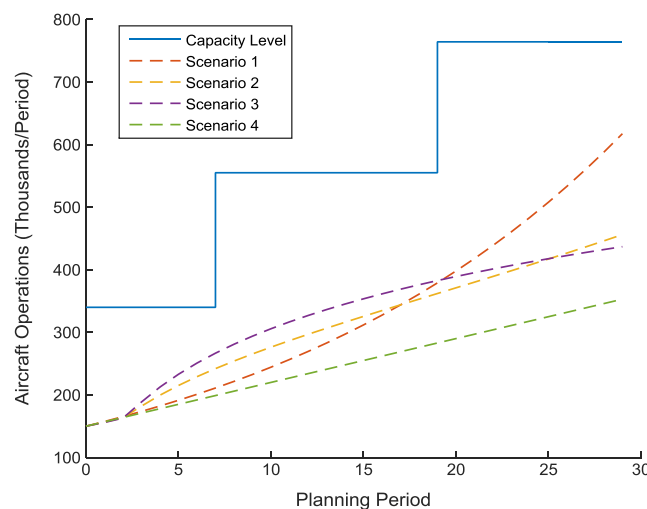


Figure 7. Capacity over time for the airfield.

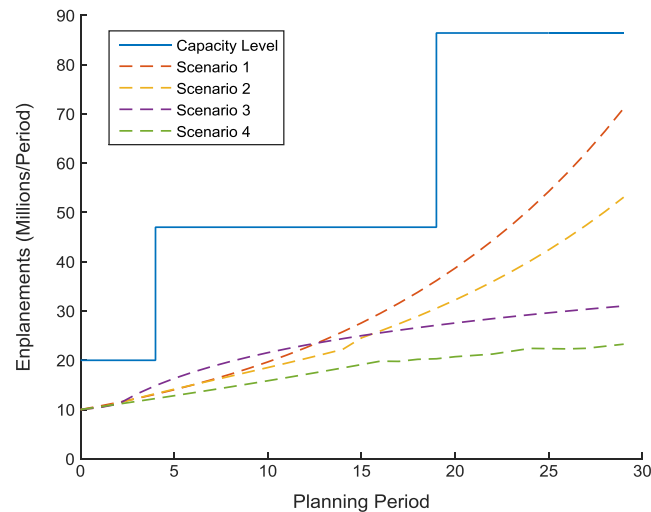


Figure 8. Capacity over time for the passenger terminal.

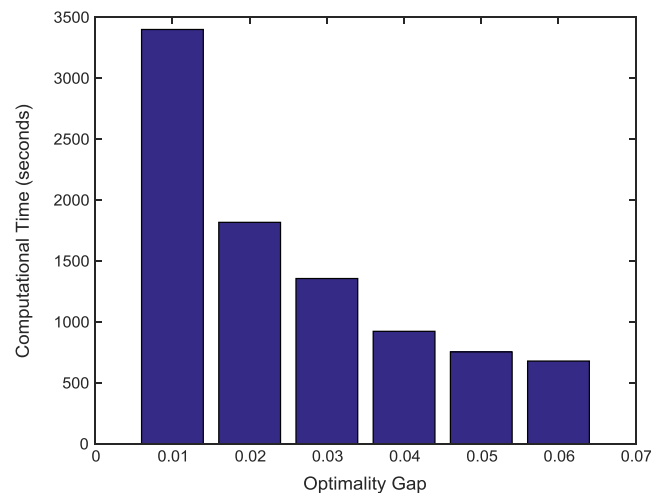


Figure 9. Computation time for solving the passenger terminal subproblem.

subproblem. Since the evaluation of long-term capacity decisions for airport facilities occurs once every few years, the computational time is acceptable, even without refinements of solution methods.

6. CONCLUSIONS

This paper presents a stochastic model for optimizing the strategic facility development decisions for airport systems in the presence of demand uncertainty. An MINLP is proposed as a starting model and then reformulated into a mixed-integer program after the delay costs are approximated and linearized. The linearization removes nonlinear properties of this program, enabling the design of efficient and reliable solution methods. After the inclusion of demand uncertainties, the deterministic model is extended into a stochastic program, which can yield better results than deterministic ones using average values of demand estimates. The stochastic program is solved in its deterministic equivalent form because discrete demand scenarios are assumed. Numerical case studies are conducted to demonstrate the capabilities of the proposed model, and trade-offs among various costs are also analyzed. In addition, the “Flaw of Averages” due to decisions based on averages is identified in the illustrative example.

Compared with the outer-approximation technique, which required differentiability [6], the discrete approximation is more widely applicable. As a result, the computation time is longer than that in [6]. Without further improvements to the solution methods, the near-optimal solution (e.g., optimality gap of 1%) can be found on a standard desktop computer within 1 h, which is acceptable considering that it is a long-term planning problem. The method proposed in this study is also more flexible than that in [6] because demand decreases as well as increases are allowed.

This study might also be improved in the following ways:

1 Rolling horizon approach.

In practice, the planning horizon in an airport master plan is typically 20 years, and the plan is updated every few years, such as 5 years. In addition, long-term forecasts tend to be less reliable than near-term ones [25]. Therefore, capacity decisions in the near term are more crucial for airport planners. To this end, a rolling horizon approach seems desirable.

2 Coordinated development.

Although several airport components are considered and each has different operating and construction characteristics, the interactions among them are ignored in this study, which is why the model can be decomposed into independent subproblems. Methods that coordinate the development of various interrelated facilities are highly desirable.

3 Other uncertainties.

Only demand uncertainty is considered. Additional factors, such as aircraft characteristics and traffic mix, might also be uncertain and should be included.

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