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ANALYTICAL APPROACH TO INVESTIGATION OF DEFLECTION OF CIRCULAR PLATE UNDER UNIFORM LOAD BY HOMOTOPY PERTURBATION METHOD

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Abstract- In this paper Homotopy-Perturbation method (HPM) is introduced to obtain the approximate solution of the governing differential equation of deflection of thin circular plate under uniform loads with two different types of boundary conditions. The edge of the plate is either simply supported or clamped and the plate is assumed to be geometrically perfect. He's Homotopy-Perturbation method is implemented for solving the differential equations. From comparison the results obtained that HPM is very rapid convergence and it can be widely applicable in engineering and especially for the cases have not exact solution, this method can be used as semi-exact solution. The results for both types of boundary conditions were compared with the results obtained by finite element method and exact solution.

Keywords- homotopy perturbation method (HPM), circular plate, clamped edges, simply supported edges, Deflection, uniform load, finite element method (FEM)

1. INTRODUCTION

The circular plate under uniform load is one of the classical problems in elasticity theory, which also is encountered frequently in practice. Analytical solutions of isotropic circular plates with either clamped or simply supported edges under uniform load, which were derived based on thin plate theory with Kirchhoff hypothesis, can be found in [1]. Homotopy perturbation method has been used by many mathematicians and engineers to solve various functional equations. This method was further developed and improved by He and applied to many nonlinear problems [2-12]. In this paper, we consider a solid circular plate with radius a under axisymmetric load, p_0, and set the origin coordinate system at center of plate. It can be shown that the present analysis is very simple and clear. We consider differential equation of deflected surface of circular plate based on elasticity theory as follow [1]:

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$$\frac{d^4w}{dr^4}+\frac{2}{r}\frac{d^3w}{dr^3}-\frac{1}{r^2}\frac{d^2w}{dr^2}+\frac{1}{r^3}\frac{dw}{dr}=\frac{p_0}{D} \tag{1}$$

With two boundary conditions, simply and clamped supports. In this article, HPM as powerful analytical method is employed to solve governing differential equation and find deflection, $w(r)$.

2. APPLICATION OF HOMOTOPY PERTURBATION METHOD

Basic idea of homotopy-perturbation method mentioned in [2-12].Consider equation (1) with below conditions:

2.1. Plate with simply supported edges under a uniform load p_0:

$$r=a\rightarrow\begin{cases} w=0 \\ M_r=-D\left(\dfrac{d^2w}{dr^2}+\dfrac{\nu}{r}\dfrac{dw}{dr}\right)=0\end{cases} \qquad r=0\rightarrow\begin{cases} w\neq\infty \\ \dfrac{dw}{dr}\neq\infty\end{cases} \tag{2}$$

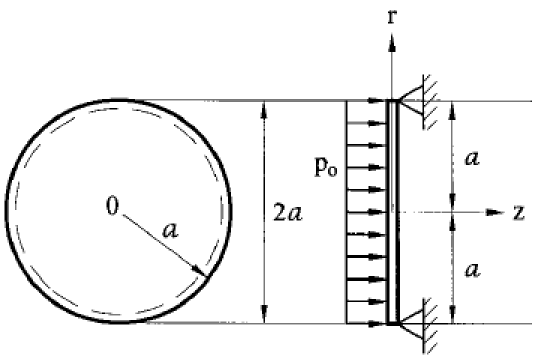

Fig. 1 The circular plate with simply supported edges

Where w is deflection, r is radius, p_0 is uniform load, M_r is radial bending moment and D is the flexural rigidity of plate. The geometry of plate with simply supported edges is shown in Fig.1.To solve Eq. (1) with the boundary condition (2), according to the homotopy perturbation explained in [2-12], we constructed the following homotopy terms:

$$N(v)=0$$

$$L(v)=\frac{d^4v(r)}{dr^4}+\frac{2}{r}\frac{d^3v(r)}{dr^3}-\frac{1}{r^2}\frac{d^2v(r)}{dr^2}+\frac{1}{r^3}\frac{dv(r)}{dr} \tag{3}$$

$$\begin{aligned}H(v,p)=&\frac{d^4v(r)}{dr^4}+\frac{2}{r}\frac{d^3v(r)}{dr^3}-\frac{1}{r^2}\frac{d^2v(r)}{dr^2}+\frac{1}{r^3}\frac{dv(r)}{dr}\\&-\frac{d^4u_0(r)}{dr^4}-\frac{2}{r}\frac{d^3u_0(r)}{dr^3}+\frac{1}{r^2}\frac{d^2u_0(r)}{dr^2}-\frac{1}{r^3}\frac{du_0(r)}{dr}\\&+p\left(\frac{d^4u_0(r)}{dr^4}+\frac{2}{r}\frac{d^3u_0(r)}{dr^3}-\frac{1}{r^2}\frac{d^2u_0(r)}{dr^2}+\frac{1}{r^3}\frac{du_0(r)}{dr}\right)-p\frac{p_0}{D}=0\end{aligned} \tag{4}$$

Based on basic idea of HPM, by substituting $v=v_0+pv_1+p^2v_2+\ldots$ into Eq. (4) and rearranging the equations based on powers of p-terms we will have:

$$\begin{aligned}&p^0:\frac{d^4v_0(r)}{dr^4}+\frac{2}{r}\frac{d^3v_0(r)}{dr^3}-\frac{1}{r^2}\frac{d^2v_0(r)}{dr^2}+\frac{1}{r^3}\frac{dv_0(r)}{dr}-\frac{d^4u_0(r)}{dr^4}\\&-\frac{2}{r}\frac{d^3u_0(r)}{dr^3}+\frac{1}{r^2}\frac{d^2u_0(r)}{dr^2}-\frac{1}{r^3}\frac{du_0(r)}{dr}=0\\&p^1:\frac{d^4v_1(r)}{dr^4}+\frac{2}{r}\frac{d^3v_1(r)}{dr^3}-\frac{1}{r^2}\frac{d^2v_1(r)}{dr^2}+\frac{1}{r^3}\frac{dv_1(r)}{dr}+\frac{d^4u_0(r)}{dr^4}\\&+\frac{2}{r}\frac{d^3u_0(r)}{dr^3}-\frac{1}{r^2}\frac{d^2u_0(r)}{dr^2}+\frac{1}{r^3}\frac{du_0(r)}{dr}-p\frac{p_0}{D}=0\end{aligned} \tag{5}$$

Solution of Eq. (5) may be written as follow:

$$\begin{aligned}&v_0(r)=c_1r^2+c_2r^2\ln(r)+c_3+c_4\ln(r)\\&v_1(r)=\frac{1}{64}\frac{p_0r^4}{D}+\frac{1}{2}c_1r^2+c_2\ln(r)+\frac{1}{2}c_3r^2\ln(r)-\frac{1}{4}c_3r^2+c_4\end{aligned} \tag{6}$$

All the computations obtained by using the Maple package. According to the HPM, we conclude that:

$$w(r)=v_0(r)+v_1(r) \tag{7}$$

Therefore, substituting the values of $v_0(r),v_1(r)$ from Eq. (6) into Eq. (7) yields:

$$w(r)=c_1r^2+c_2r^2\ln(r)+c_3+c_4\ln(r)+\frac{1}{64}\frac{p_0r^4}{D} \tag{8}$$

By applying boundary conditions (2) for Eq. (8) we have:

$$c_1=-\frac{1}{32}\frac{p_0a^2(3+\nu)}{D(1+\nu)},c_2=0,c_3=\frac{1}{64}\frac{p_0a^4(5+\nu)}{D(1+\nu)},c_4=0 \tag{9}$$

And

$$w(r)=-\frac{1}{32}\frac{p_0a^2(3+\nu)}{D(1+\nu)}r^2+\frac{1}{64}\frac{p_0r^4}{D}+\frac{1}{64}\frac{p_0a^4(5+\nu)}{D(1+\nu)} \tag{10}$$

2.2. Plate with clamped edge under a uniform load p_0:

$$r = a \to \begin{cases} w = 0 \\ \dfrac{dw}{dr} = 0 \end{cases} \qquad r = 0 \to \begin{cases} w \neq \infty \\ \dfrac{dw}{dr} \neq \infty \end{cases} \tag{11}$$

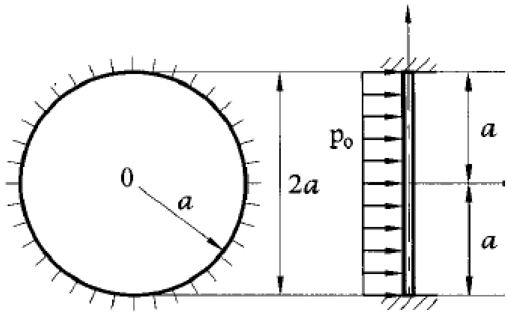

Fig. 2 The circular plate with clamped edges

The geometry of plate with clamped edges is shown in Fig.2. Solution of Eq. (5) may be written as follow:

$$v_0(r) = c_1 r^2 + c_2 r^2 \ln(r) + c_3 + c_4 \ln(r)$$

$$v_1(r) = \frac{1}{64}\frac{p_0 r^4}{D} + \frac{1}{2}c_1 r^2 + c_2 \ln(r) + \frac{1}{2}c_3 r^2 \ln(r) - \frac{1}{4}c_3 r^2 + c_4 \tag{12}$$

Therefore, by substituting the values of $v_0(r)$ and $v_1(r)$ from Eq. (12) in to Eq. (5) yields:

$$w(r) = c_1 r^2 + c_2 r^2 \ln(r) + c_3 + c_4 \ln(r) + \frac{1}{64}\frac{p_0 r^4}{D} \tag{13}$$

By applying boundary conditions (11) for Eq.(13) we have:

$$c_1 = -\frac{1}{32}\frac{p_0 a^2}{D}, c_2 = 0, c_3 = \frac{1}{64}\frac{p_0 a^4}{D}, c_4 = 0 \tag{14}$$

And

$$w(r) = -\frac{1}{32}\frac{p_0 a^2}{D} r^2 + \frac{1}{64}\frac{p_0 r^4}{D} + \frac{1}{64}\frac{p_0 a^4}{D} \tag{15}$$

3. RESULTS AND DISCUSSION

In this paper by using ANSYS package, finite element method is used to compare the results from HPM with numerical solution. In modeling of our problem with ANSYS package, plane elasticity theory was used. Young's modulus and Poisson's ratio of the plate are taken as E=2×10^{11} N/m^2 and ν=0.3, respectively, and the load intensity is p_0=6×10^6 N/m^2. In the calculation, the 10-node tetrahedron element (number

187 in the ANSYS menu) is employed, and for more convenience, the thickness of the plate is taken to be 1 m. The deflections $w(r)$ of the neutral plane are plotted in Figs. (3, 4) for simply supported edge (a = 5, 10) and Figs. (5, 6) for clamped edge (a = 5, 10). All of the figures show that the maximum deflection occur in center of plate ($r=0$). by comparing the exact solution of circular plates with either clamped or simply supported edges under uniform load in [1] and the results of HPM, we can understand that HPM results is adopted with exact solution for this problem and it is verified the accuracy of HPM. The results of FEM is in good agreement with exact solution and HPM .The homotopy perturbation method is extremely simple, easy to apply, and gives a very good accuracy and needs few computations.

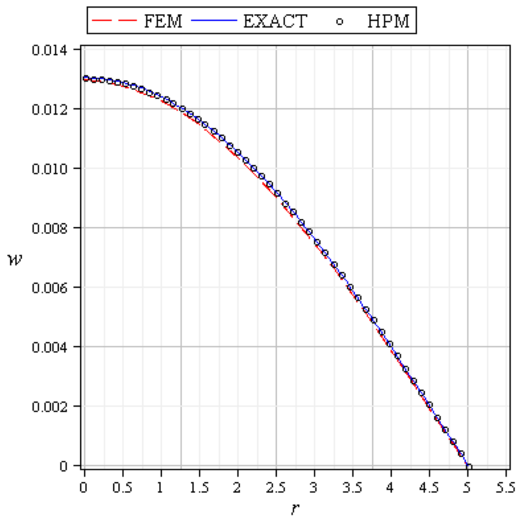

Fig. 3 Results of HPM, FEM and exact solution for simply supported edge and a=5

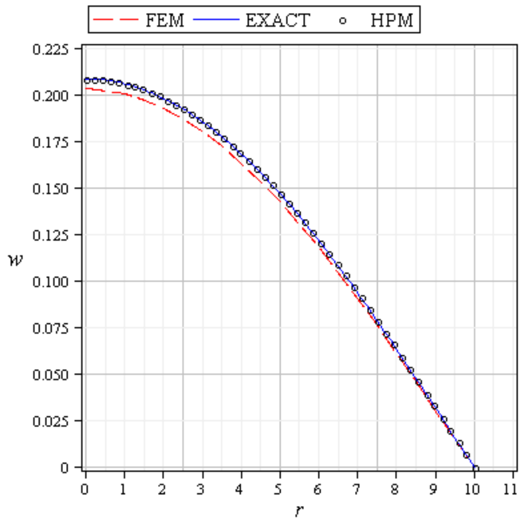

Fig. 4 Results of HPM, FEM and exact solution for simply supported edge and a=10

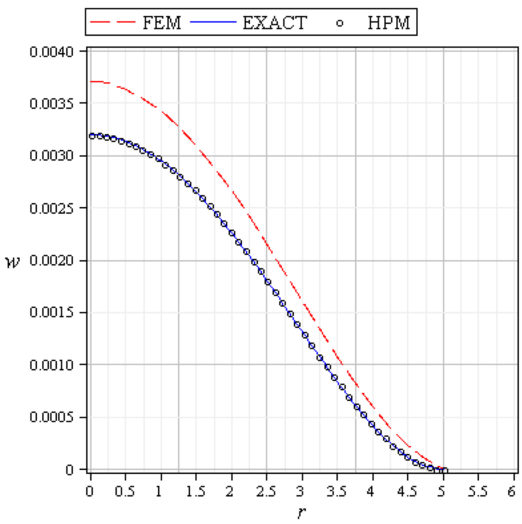

Fig. 5 Results of HPM, FEM and exact solution for clamped edge and a=5

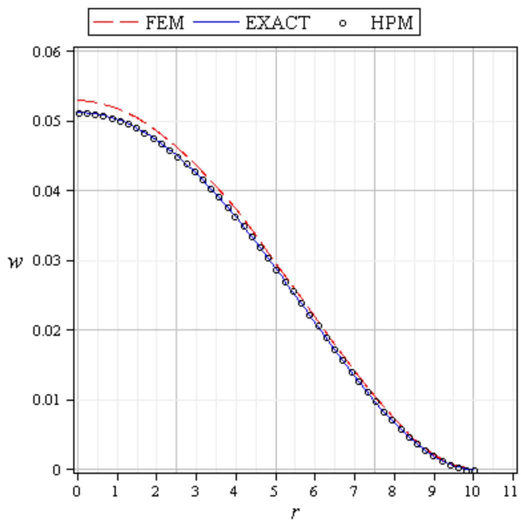

Fig. 6 Results of HPM, FEM and exact solution for clamped edge and a=10

4. CONCLUDING REMARKS

In this paper, the homotopy perturbation method and FEM are applied to solve the governing differential equation of thin circular plate's deflection under uniform loads with two different types of boundary conditions. Comparison of HPM and FEM results with exact solution shown that HPM is very effective and convenient and quite accurate rather than FEM and yields exact solution just in two iterations without complicated.

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