A NOVEL METHOD FOR MULTIPLE ATTRIBUTE DECISION-MAKING OF CONTINUOUS RANDOM VARIABLE UNDER RISK WITH ATTRIBUTE WEIGHT UNKNOWN

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Abstract- The extension of the fuzzy TOPSIS method based on the combination weight is presented to deal with multiple attribute decision-making problems under risk where the attribute value takes the form of the continuous random variable on the bounded intervals. First, the risk decision matrix is normalized by the transformation of the density function, and the variation coefficient method is used to determine the objective weights based on the expectations of the random variables. Subsequently, according to the maximizing rule of the weighted synthetic value of alternatives, the synthetic weight model is established. Then, the ideal solution and negative ideal solution is defined, the distances between the alternatives and the ideal/negative ideal solutions, and the relative closeness coefficients are calculated. In addition, the alternatives are ranked by the relative closeness coefficient of the alternatives. Finally, an illustrative example with the interval number is given to demonstrate the steps and the effectiveness of the proposed method.

Key Words- Risk decision, Density function, TOPSIS, Multiple attribute decision-making

1. INTRODUCTION

The multi-attribute decision making is widely used in the areas of society, economy, management, military affairs and engineering technology, such as the investment decision making, the alternative evaluation, the economic benefit evaluation and the staff evaluation. So far, some methods have been proposed to solve the multi-attribute decision making problems [1] [2] [3]. But most of them must know the attribute values of the alternatives beforehand. In the realistic decision making, the decision makers sometimes face the uncertain condition. The attribute values are the random variables and they change with natural state. The decision makers do not know the real state in the future, but they can know the possible natural states, and they can qualify the randomness according to the probability distribution. This decision making problem is called the multi-attribute decision making problem under risk [4]. So, the research of the multi-attribute decision making problems under risk possesses the important theory and real meaning.

At present, less research on the multi-attribute decision making problem under risk have been done. Literature [5] studied the definition and the characteristic of the solution, but it did not establish the systematic decision making system; Literature [6],

proposed a random multi-rules decision making method where the attribute values were the continuous variable on the finite interval, based on the satisfaction closeness coefficients of the alternatives. This method established the non-linear programming model whose objective function is more complex. So it must be solved by the Genetic Algorithms, and the calculation steps are more complex. Literature [7] constructed the TOPSIS method to deal with the multi-attribute decision making problem under risk where the attribute weight information is known and the attribute values take the form of the continuous random variable. This method has some limitations. It can not solve this problem where the attribute value is unknown. Literature [8] proposed a multi-rules decision making method based on the WC-OWA operator where the weight information was incomplete certain and the value of the rules were the normal distribution. This method transformed the rules value of the normal distribution into the interval number based on the 3 rule of normal distribution, and it utilized the WC-OWA operator to aggregate the interval number. Obviously, this method is applied in this condition that the rule values take the form of the random variable of the normal distribution, which has some limitation. This method also established the non-linear programming model to solve the weight value and the calculation steps were more complex. Literature [9] studied the multi-attribute decision making problems under risk where the attribute value is the continuous random variable on the finite interval. The attribute weight was calculated based on the maximizing deviations method, and the objective subjective weight model was constructed, and the order of the alternatives was ranked based on the weighted method. The advantage of this method is that the subjective and the objective weight are comprehensively considered. But the simple method of the expectations weighted is not used to rank the order of the alternatives where the expectation values are the same.

On the foundation of the literature [9], based on the multi-attribute decision making problem where the attribute value is the continuous random variable on the finite interval, this paper firstly uses the variation coefficient method to determine the objective weights. Then the model is constructed to determine the synthetic weight according to the maximizing rule of the weighted synthetic value of the alternatives. This model is the linear model, and the calculation steps are simple. At last, the order of the alternatives is ranked by the TOPSIS method, which overcomes the disadvantage of the literature [9].

2. DECISION PROBLEM DESCRIPTION

Suppose that: $A = (a_1, a_2, \cdots, a_m)$ represents the set of the alternatives, and $C = (c_1, c_2, \cdots, c_n)$ represents the set of the attributes (indexes) in the decision making problem under risk, and the weight of the attribute c_j is w_j where $0 \leq w_j \leq 1$ and $\sum_{j=1}^{n} w_j = 1$. The weight vector $W = (w_1, w_2, \ldots, w_n)$ is composed of the objective weight ω_j and the subjective weight λ_j. The attribute value X_{ij} $(1 \leq i \leq m; 1 \leq j \leq n)$ of the alternative a_i under the attribute c_j is the random variable. The density function

$f_{ij}(x_{ij})$ of X_{ij} is known and the value of X_{ij} is on the closed interval $[x_{ij}^L, x_{ij}^U]$ where x_{ij}^L is the minimum value and x_{ij}^U is the maximum value. This multi-attribute decision making alternatives under risk can be solved based on these conditions.

3. EVALUATION METHOD

3.1 The initialization of the decision-making data

The decision making matrix should be standardized in order to avoid the influence to the decision making result of the different physical dimensions. The most familiar index styles are the benefit index (I_1) and the cost index (I_2). Suppose that R_{ij} is the standardization attribute value after the attribute value X_{ij} is standardized and its value range is $[r_{ij}^L, r_{ij}^U]$, and $g_{ij}(r_{ij})$ represents the probability density function of R_{ij}. The following is the method of standardization [8]:

$$\begin{cases} r_{ij}^L = \dfrac{x_{ij}^L}{\sqrt{\sum\limits_{i=1}^{m}(x_{ij}^U)^2}} \\[4mm] r_{ij}^U = \dfrac{x_{ij}^U}{\sqrt{\sum\limits_{i=1}^{m}(x_{ij}^L)^2}} \end{cases} \qquad (1 \le i \le m; 1 \le j \le n) \quad j \in I_1 \tag{1}$$

$$\begin{cases} r_{ij}^L = \dfrac{1/x_{ij}^U}{\sqrt{\sum\limits_{i=1}^{m}(1/x_{ij}^L)^2}} \\[4mm] r_{ij}^U = \dfrac{1/x_{ij}^L}{\sqrt{\sum\limits_{i=1}^{m}(1/x_{ij}^U)^2}} \end{cases} \qquad (1 \le i \le m; 1 \le j \le n) \quad j \in I_2 \tag{2}$$

The density function $f_{ij}(x_{ij})$ of the random variable X_{ij} on the closed interval $[x_{ij}^L, x_{ij}^U]$ $(1 \le i \le m; 1 \le j \le n)$ is linear transformed into the density function $g_{ij}(r_{ij})$ of the random variable R_{ij} on the interval $[r_{ij}^L, r_{ij}^U]$:

$$R_{ij} = K_{ij} \times X_{ij} + B_{ij}$$

(3)

where K_{ij} and B_{ij} are the linear transformation constant of random variable R_{ij} and X_{ij}, respectively.

If c_j is the benefit index, the following function can be constructed based on the relationship between r_{ij}^L and x_{ij}^L and between r_{ij}^U and x_{ij}^U:

$$
\begin{cases}
r_{ij}^{L} = k_{ij} x_{ij}^{L} + b_{ij} \\
r_{ij}^{U} = k_{ij} x_{ij}^{U} + b_{ij}
\end{cases}
\tag{4}
$$

Calculated the above equations, the following result can be obtained:

$$
\begin{cases}
k_{ij} = \dfrac{r_{ij}^{U} - r_{ij}^{L}}{x_{ij}^{U} - x_{ij}^{L}} \\
b_{ij} = \dfrac{r_{ij}^{L} x_{ij}^{U} - r_{ij}^{U} x_{ij}^{L}}{x_{ij}^{U} - x_{ij}^{L}}
\end{cases}
\tag{5}
$$

The probability density function of R_{ij} can be calculated by the linear transformation $R_{ij} = K_{ij} \times X_{ij} + B_{ij}$ (c_j is benefit index and $K_{ij} > 0$) according to the knowledge of probability theory,:

$$
g_{ij}(r_{ij}) = \frac{1}{k_{ij}} f_{ij}(\frac{r_{ij} - b_{ij}}{k_{ij}}) \quad (1 \le i \le m; 1 \le j \le n)
\tag{6}
$$

If c_j is the cost index, the following function can be constructed based on the relationship between r_{ij}^{L} and x_{ij}^{L} and between r_{ij}^{U} and x_{ij}^{U} ,:

$$
\begin{cases}
k_{ij} = \dfrac{r_{ij}^{U} - r_{ij}^{L}}{x_{ij}^{L} - x_{ij}^{U}} \\
b_{ij} = \dfrac{r_{ij}^{U} x_{ij}^{U} - r_{ij}^{L} x_{ij}^{L}}{x_{ij}^{U} - x_{ij}^{L}}
\end{cases}
\tag{7}
$$

Because of $K_{ij} < 0$, the probability density function is:

$$
g_{ij}(r_{ij}) = -\frac{1}{k_{ij}} f_{ij}(\frac{r_{ij} - b_{ij}}{k_{ij}}) \quad (1 \le i \le m; 1 \le j \le n)
\tag{8}
$$

3.2 Calculating the objective weight of the evaluation index by the variation coefficient method

(1) Because R_{ij} is the random variable, the expectation value ER_{ij} is shown as follows:

$$
ER_{ij} = \int_{-\infty}^{+\infty} r_{ij} g_{ij}(r_{ij}) dr_{ij} = \int_{r_{ij}^{L}}^{r_{ij}^{U}} r_{ij} g_{ij}(r_{ij}) dr_{ij}
\tag{9}
$$

(2) Calculate the average value of the *jth* index (attribute):

$$
\overline{ER_j} = \frac{1}{m} \sum_{i=1}^{m} ER_{ij} = \frac{1}{m} \sum_{i=1}^{m} \int_{r_{ij}^{L}}^{r_{ij}^{U}} r_{ij} g_{ij}(r_{ij}) dr_{ij}, \quad j = 1, 2, \cdots, n
\tag{10}
$$

(3) Calculate the mean square deviation of the *jth* evaluation index (attribute):

$$
D_j = \sqrt{\frac{1}{m-1} \sum_{i=1}^{m} (ER_{ij} - \overline{ER_j})^2}, \, j = 1, 2, \cdots, n
\tag{11}
$$

(4) Calculate the variation coefficient of the *jth* evaluation index (attribute):

$$
E_j = \frac{D_j}{\overline{ER_j}}, \, j = 1, 2, \cdots, n
\tag{12}
$$

(5) Calculate the weight of the indexes (attributes):

$$\omega_j = \frac{E_j}{\sum\limits_{j=1}^{n} E_j}, j = 1, 2, \cdots, n \qquad (13)$$

3.3 The synthetic weight

The objective weight of the decision making matrix can be obtained based on the above model. But the obtained objective weight ignores the knowledge, the experience, and the preference of the decision makers. So it does not satisfy the need of the objective reality. This paper proposes a method which can satisfy the objective and the subjective needs to determine the synthetic weight of the multi-attribute decision making problem under risk.

Suppose that the subjective weight vector is $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_n)$ determined by AHP method, where $\sum\limits_{j=1}^{n} \lambda_j = 1$ and $0 \le \lambda_j \le 1$. Let the synthetic weight is $W = (w_1, w_2, \cdots, w_n)$, where $W = \alpha \times \omega + \beta \times \lambda$, $\alpha^2 + \beta^2 = 1, \alpha \ge 0, \beta \ge 0$.

To the synthetic weight $W = (w_1, w_2, \cdots, w_n)$, the weighted expectation value of each alternative is:

$$ER_i = \sum_{j=1}^{n} (w_j \times ER_{ij}) = \sum_{j=1}^{n} [(\alpha \times \omega_j + \beta \times \lambda_j) \times ER_{ij}] \qquad (14)$$

The selection of W (the same as the selection of α and β) should make the weighted expectation value ER_i of each alternative to be the maximum value:

$$\max \quad E = (ER_1, ER_2, \cdots, ER_m).$$

Because there is not preference relation, the above multi-objective programming can be integrated to the single-objective programming:

$$\max \quad E = \sum_{i=1}^{m} \sum_{j=1}^{n} [(\alpha \times \omega_j + \beta \times \lambda_j) \times ER_{ij}]$$

$$s.t. \quad \alpha^2 + \beta^2 = 1, \qquad (15)$$

$$\alpha \ge 0, \beta \ge 0$$

Construct the Lagrange multiplier function:

$$L(\alpha, \beta, \eta) = \sum_{i=1}^{m} \sum_{j=1}^{n} [(\alpha \times \omega_j + \beta \times \lambda_j) \times ER_{ij}] + \eta(\alpha^2 + \beta^2 - 1)$$

Subject to
$$\begin{cases} \dfrac{\partial L(\alpha, \beta, \eta)}{\partial \alpha} = \sum\limits_{i=1}^{m} \sum\limits_{j=1}^{n} (\omega_j \times ER_{ij}) + 2\eta\alpha = 0 \\[3mm] \dfrac{\partial L(\alpha, \beta, \eta)}{\partial \beta} = \sum\limits_{i=1}^{m} \sum\limits_{j=1}^{n} (\lambda_j \times ER_{ij}) + 2\eta\beta = 0 \\[3mm] \dfrac{\partial L(\alpha, \beta, \eta)}{\partial \eta} = \alpha^2 + \beta^2 - 1 = 0 \end{cases} \qquad (16)$$

Calculate the equation:

$$\begin{cases} \alpha = \dfrac{\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n}(\omega_j \times ER_{ij})}{\sqrt{[\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n}(\omega_j \times ER_{ij})]^2 + [\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n}(\lambda_j \times ER_{ij})]^2}} \\[4ex] \beta = \dfrac{\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n}(\lambda_j \times ER_{ij})}{\sqrt{[\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n}(\omega_j \times ER_{ij})]^2 + [\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n}(\lambda_j \times ER_{ij})]^2}} \end{cases} \qquad (17)$$

α and β should be normalized, in order to normalize W, and it must be satisfy $\sum\limits_{j=1}^{n} w_j = 1$:

$$\alpha = \frac{\alpha}{\alpha + \beta} = \frac{\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n}(\omega_j \times ER_{ij})}{\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n}(\omega_j \times ER_{ij}) + \sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n}(\lambda_j \times ER_{ij})} \qquad (18)$$

$$\beta = \frac{\beta}{\alpha + \beta} = \frac{\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n}(\lambda_j \times ER_{ij})}{\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n}(\omega_j \times ER_{ij}) + \sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n}(\lambda_j \times ER_{ij})} \qquad (19)$$

The synthetic weight is:
$$W = \alpha \times \omega + \beta \times \lambda \qquad (20).$$

3.4 The decision making Steps

(1) The definition of the ideal solution and the negative ideal solution

Let the ideal solution is $V^+ = (v_1^+, v_2^+, \cdots, v_n^+)$, and the negative ideal solution is $V^- = (v_1^-, v_2^-, \cdots, v_n^-)$. The definitions are represented as follows:

$$\begin{cases} v_j^+ = \max_i(r_{ij}^U) \\ v_j^- = \min_i(r_{ij}^L) \end{cases} \qquad (21)$$

(2) Calculate the weighted distance between each alternative, the ideal solution and the negative ideal solution:

Suppose that:

$$Ed_{ij}^+ = E\left|r_{ij} - v_j^+\right| = \int_{-\infty}^{+\infty}\left|r_{ij} - v_j^+\right| g_{ij}(r_{ij})dr_{ij} , \; Ed_{ij}^- = E\left|r_{ij} - v_j^-\right| = \int_{-\infty}^{+\infty}\left|r_{ij} - v_j^-\right| g_{ij}(r_{ij})dr_{ij} ,$$

then the weighted distance is :

$$\begin{cases} Ed_i^+ = \left[\sum_{j=1}^{n} (w_j Ed_{ij}^+)^2 \right]^{1/2} = \left[\sum_{j=1}^{n} (w_j E|r_{ij} - v_j^+|)^2 \right]^{1/2} \\ Ed_i^- = \left[\sum_{j=1}^{n} (w_j Ed_{ij}^-)^2 \right]^{1/2} = \left[\sum_{j=1}^{n} (w_j E|r_{ij} - v_j^-|)^2 \right]^{1/2} \end{cases}$$ (22)

(3) Determine the relative approach degree. The relative approach degree of each alternative to the ideal solution is:

$$C_i = \frac{Ed_i^-}{Ed_i^+ + Ed_i^-} \cdot (i = 1, 2, \cdots, m)$$ (23)

(4) Rank the order of the alternatives:

The alternatives can be ranked according to the relative approach degree. The larger the relative approach degree is, the better the alternative is.

4. APPLICATION EXAMPLES

Five invest alternatives $a_i (i = 1, 2, 3, 4, 5)$ are carried out in order to develop new products. The attributes are: the expected net present value, the risk profit value, the investment amount and the risk loss value, where the expected net present value and risk profit value are the attributes of benefit type and the investment amount and risk loss value are the attributes of cost type. The attribute values are shown in Table 1(unit 1000yuan). Suppose that the subject weight given by the decision maker is $\lambda = (0.2, 0.25, 0.25, 0.3)$. Please choose the best decision making alternative.

Table 1 Product investment decision data[10]

	investment amount	expected net present value	venture profit value	risk loss value
a_1	[5,7]	[4,5]	[4,6]	[0.4,0.6]
a_2	[10,11]	[6,7]	[5,6]	[1.5,2.0]
a_3	[5,6]	[4,5]	[3,4]	[0.4,0.7]
a_4	[9,11]	[5,6]	[5,7]	[1.3,1.5]
a_5	[6,8]	[3,5]	[3,4]	[0.8,1]

Because the attribute values take the form of the interval numbers, when we can not get more attribute information, we think the attribute values on the interval are the uniformly distributed random variable. The decision making steps are shown as follows.
(1) Based on the formula (1) or (2), standardize the interval $[x_{ij}^L, x_{ij}^U]$ to the interval $[r_{ij}^L, r_{ij}^U]$:

$$R = \begin{bmatrix} [0.3960,0.7056] & [0.3162,0.4951] & [0.3234,0.6547] & [0.4289,0.9796] \\ [0.2520,0.3528] & [0.4743,0.6931] & [0.4042,0.6547] & [0.1287,0.2612] \\ [0.4620,0.7056] & [0.3162,0.4951] & [0.2425,0.4364] & [0.3676,0.9796] \\ [0.2520,0.3920] & [0.3953,0.5941] & [0.4042,0.7638] & [0.1716,0.3014] \\ [0.3465,0.5880] & [0.2372,0.4951] & [0.2425,0.4364] & [0.2574,0.4898] \end{bmatrix}$$

(2) Based on the formula (5) or (7), calculate the linear transform coefficient, we can get:

$$K = \begin{bmatrix} -0.1548 & 0.1788 & 0.1656 & -2.7534 \\ -0.1008 & 0.2188 & 0.2504 & -0.2651 \\ -0.2436 & 0.1788 & 0.1939 & -2.0398 \\ -0.0700 & 0.1988 & 0.1798 & -0.6492 \\ -0.1207 & 0.1290 & 0.1939 & -1.1622 \end{bmatrix}, B = \begin{bmatrix} 1.4796 & -0.3992 & -0.3392 & 2.0809 \\ 1.3607 & -0.8382 & -0.8479 & 0.6589 \\ 1.9235 & -0.3992 & -0.3392 & 1.7955 \\ 1.0220 & -0.5987 & -0.4946 & 1.1454 \\ 1.3125 & -0.1497 & -0.3392 & 1.4196 \end{bmatrix}$$

(3) Based on the formula (6) or (8), the density function is transformed:

Because the probability density is constant, the matrix is shown as follows:

$$g = \begin{bmatrix} 3.2302 & 5.5914 & 3.0187 & 1.8160 \\ 9.9218 & 4.5712 & 3.9932 & 7.5444 \\ 4.1054 & 5.5914 & 5.1573 & 1.6341 \\ 7.1434 & 5.0301 & 2.7814 & 7.7014 \\ 4.1410 & 3.8774 & 5.1573 & 4.3021 \end{bmatrix}$$

(4) Based on the formula (9), the expectation value of the random variable R_{ij} is calculated:

$$ER = \begin{bmatrix} 0.5508 & 0.4057 & 0.4890 & 0.7043 \\ 0.3024 & 0.5837 & 0.5294 & 0.1950 \\ 0.5838 & 0.4057 & 0.3395 & 0.6736 \\ 0.3220 & 0.4947 & 0.5840 & 0.2365 \\ 0.4673 & 0.3661 & 0.3395 & 0.3736 \end{bmatrix}$$

(5) Based on the formula (10) to (13), the objective weight is calculated:

$\omega = (0.2263, 0.1522, 0.1917, 0.4298)$

(6)Based on the formula (18) and (19), the integrated weight coefficients α and β are calculated, respectively:

$\alpha = 0.4073 \quad \beta = 0.5927$

(7) Based on the formula (20), the objective and subjective weight is combined to the synthetic weight:

$W = (0.2107, 0.2102, 0.2262, 0.3529)$

(8) Based on the formula (21), the ideal solution and the negative ideal solution are calculated:

$V^+ = (0.7056, 0.6931, 0.7638, 0.9796)$

$V^- = (0.2520, 0.2372, 0.2425, 0.1287)$

(9) Based on the formula (22), the weighted distances between each alternative and the ideal and negative ideal solution are calculated, respectively:

$$Ed^+ = \left(0.1342,\ 0.2953,\ 0.1587,\ 0.2805,\ 0.2494\right)$$

$$Ed^- = \left(0.2227,\ 0.1009,\ 0.2088,\ 0.1028,\ 0.1036\right)$$

(10) Based on the formula (23), the relative approach degree is calculated:

$$C = \left(0.6239,\ 0.2546,\ 0.5682,\ 0.2681,\ 0.2936\ \right)$$

(11) Rank the order:

According to the relative approach degree, the order of the alternatives is ranked:

$$a_1 \succ a_3 \succ a_5 \succ a_4 \succ a_2.$$

(12) Analysis:

The order of the alternatives in this paper is the same as the order of the decision making method based on the interval numbers in literature [10], and it is also the same as the result of the decision making in literature [9]. So, it is demonstrated that this decision making method in this paper is very effective.

5. CONCLUSION

The multi-attribute making problem under risk is wildly used in the real situation. This paper assumes that the probability density function of the indexes which valued in the closed interval is known, which may be some differences in the practical application. But the attribute values can be approximated to obey a certain probability distribution in the real decision problem, such as the range uniform distribution, the Normal distribution or the Gaussian distribution. For the Normal distribution, the Gaussian distribution and so on, the range of the attribute value is $[-\infty, +\infty]$. We can transform the infinite closed interval into the finite closed interval according to the 3 rule, which may meet our assumption condition proposed in this paper. Therefore, this paper proposes a TOPSIS method with the synthetic weight based on the multi-attribute decision making problem where the attribute values take the form of the continuous random variables on the finite interval. And the decision making steps are given and it can be satisfied the real needs of decision making. This method is easy to understand. It enriches and develops the decision making theory and method under risk and proposes a new idea to solve the multi-attribute decision making problem under risk. In the future, the certain multi-attribute decision making method, however, is further applied to the process of risk decision-making, such as the projection method, the gray correlation degree method and the OWA integrated operator method, due to the research of multi-attribute decision making problems under risk are being developed, In addition, we can further study the multi-attribute decision making problems under risk where the attribute values take the form of the discrete random variables, and how to determine the weight and rank the order of the alternatives.

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