Mathematical analysis for detection probability in cognitive radio networks over wireless communication channels

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Abstract: In this study, the authors consider the problem of spectrum sensing based on energy detection method in cognitive radio over wireless communication channels when users experience fading and non-fading effects. The closed-form analytical expressions for the detection probability are derived over non-fading additive white Gaussian noise channel and Rayleigh and log-normal shadowing fading channels. The detection probability involves Marcum-Q function, summations and integrations in the early research papers, which are replaced by closed-form expressions in this study. The probability distribution function of fading channels is used to obtain the expressions for detection probability. The new derived numerical results are simulated under various parameters. The performance of the derived theoretical expressions closely matches with the simulated results.

1 Introduction

Owing to the increasing popularity of wireless devices in recent years, the radio spectrum has been an extremely scarce resource. However, all bands of the spectrum are not fully utilised at specific times or at particular geographic locations according to the report of Federal Communications Commission (FCC) [1]. The underutilisation of spectrum in many of the frequency bands is because of the conventional fixed spectrum allocation policy. To overcome the problem of spectrum scarcity, the license exempted secondary users (SUs) are allowed to exploit the unused spectrum holes over some frequency ranges by using the cognitive radio (CR) technology [2]. The challenge in implementing the CR technology is that the SUs must accurately monitor and be aware of the presence of the primary users (PUs) over a particular spectrum. Several efficient methods have been proposed to address this challenge [3–6]. The detection of unknown deterministic signals with energy detection (ED) is the simplest and most popular method since it does not require any a priori knowledge of the PUs signal and has much lower computational and implementation complexity. The performance of ED over non-fading additive white Gaussian noise (AWGN) channel was first addressed by Urkowitz and comprehensively derived analytic expressions for the probability of detection and the probability of false alarm [7]. Recently, the detection problem over fading channels has been revised in [8, 9]. The expressions derived for the detection probability over fading channels using energy detector in [9–14] depend on the number of terms in the summation and other functions. The implementations of these expressions are susceptible to truncation errors. We summarise our contributions as follows. We provide approximate expressions for the detection probability over non-fading AWGN channel and Rayleigh and log-normal shadowing fading channels. The expressions reported in [9–14] for AWGN channel involves the Marcum-Q function, which is replaced by closed-form expression in our derivation (16). It has much less computational complexity. The derived analytical expressions over fading channels in the recent papers consist of summations and integrations with the limitations. These are susceptible to truncation errors. We compare our new derived expressions (16), (22) and (30) with the expressions reported in [9–14] and we show that our new derived expressions do not involve summations and integrations and they can be used with no limitations. The simulation results of the considered ED method over both AWGN and fading channels are compared with the theoretical evaluation of the derived expressions. The rest of this paper is organised as follows. In Section 2, the system model is discussed. We derive expressions for the detection probability over fading channels in Section 3. The performance of the derived expressions and simulation results are presented in Section 4. Conclusions are drawn in Section 5.

2 Model

The energy-sensing model is as shown in Fig. 1. The received signal $r(t)$ is filtered to the required bandwidth B to reject the noise and adjacent signals. The filtered signal is sampled and quantised using analog-to-digital converter. Next, a square-law device and an integrator measure the received signal energy. The output of the integrator, represented by the decision statistic Y, is compared with a predetermined threshold γ to determine the presence (H_1) or absence (H_0) of a PU. The presence or absence of a PU signal can be modelled as binary hypotheses problem as originally proposed by Urkowitz [7] and Pridham and Urkowitz [15] and latter followed by most researchers [16, 17]. The received signal samples under two hypotheses defined as

$$r(n) = \begin{cases} w(n) & : H_0 \\ hs(n) + w(n) & : H_1 \end{cases} \tag{1}$$

where $n = 1, 2, \ldots, N$. $s(n)$ is the PU signal which is assumed to be an unknown deterministic signal and $w(n)$ is the AWGN with zero mean and variance σ_n^2. h is the channel gain between PU and SU. The energy detector is defined as

$$Y = \sum_{n=1}^{N} |r(n)|^2 \tag{2}$$

The distribution of the decision variable Y will be central chi-square X_N^2 under H_0 and non-central chi-square $\tilde{X}_N^2$ with N degrees of freedom under H_1. The distribution can be expressed as [18]

$$Y \sim \begin{cases} X_N^2 & : H_0 \\ \tilde{X}_N^2 & : H_1 \end{cases} \tag{3}$$

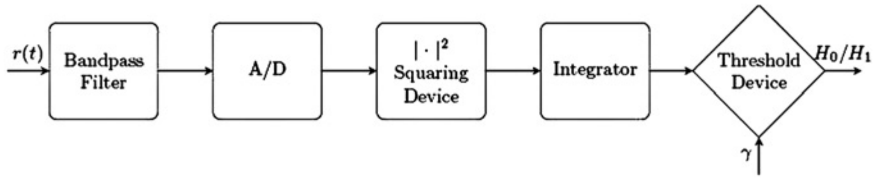

Fig. 1 *Energy detector model*

Its probability density function can be written as

$$f_Y(y) = \begin{cases} \dfrac{1}{\sigma_n^N 2^{N/2}\Gamma(N/2)} y^{(N/2)-1} \exp\left(\dfrac{-y}{2\sigma_n^2}\right) & : H_0 \\ \dfrac{1}{2\sigma_n^2}\left(\dfrac{y}{\zeta}\right)^{(N-2)/4} \exp\left[\dfrac{-1}{2\sigma_n^2}(\zeta + y)\right] I_{(N/2)-1}\left(\dfrac{\sqrt{\zeta y}}{\sigma_n^2}\right) & : H_1 \end{cases} \tag{4}$$

where the non-centrality parameter $\zeta = \sum_{i=1}^{N} \mu_i^2$ and μ_i is the mean of the ith Gaussian random variable of test Y. $I_M(\cdot)$ is the Mth modified Bessel function of the first kind, which has a series representation [19]

$$I_M(Z) = \frac{\sum_{k=0}^{\infty} (Z/2)^{2k+M}}{k!\Gamma(M + k + 1)} \tag{5}$$

and $\Gamma(\cdot)$ is the gamma function $\Gamma(z) = \int_0^\infty t^{z-1} \mathrm{e}^{-t}\, \mathrm{d}t$. The test statistic Y is compared with a predetermined threshold γ, to decide on one of the two hypotheses H_0 and H_1. The probability of detection (P_d) and probability of false alarm (P_fa) can be computed as

$$P_\mathrm{d} = P(Y > \gamma : H_1) = \int_\gamma^\infty f_Y(y)\, \mathrm{d}y \tag{6}$$

$$P_\mathrm{fa} = P(Y > \gamma : H_0) = \int_\gamma^\infty f_Y(y)\, \mathrm{d}y \tag{7}$$

2.1 Probability of false alarm under AWGN channel

To derive the probability of false alarm over non-fading AWGN channel, the probability density function defined under H_0 in (4) is inserted in (7). We define signal-to-noise ratio (SNR) (ρ) as $\rho = h^2\zeta/\sigma_n^2$

$$P_\mathrm{fa} = \int_\gamma^\infty \frac{1}{\sigma_n^N 2^{N/2}\Gamma(N/2)} y^{(N/2)-1} \exp\left(\frac{-y}{2\sigma_n^2}\right) \mathrm{d}y \tag{8}$$

Substituting $t = y/\sigma_n^2$ in (8) and further integrating the P_fa results in

$$P_\mathrm{fa} = \int_{\gamma/\sigma_n^2}^\infty \frac{t^{(N/2)-1}}{2^{N/2}\Gamma(N/2)} \exp\left(\frac{-t}{2}\right) \mathrm{d}t \tag{9}$$

The right-hand side of (9) is the chi-squared density [20, 26.4.2] which can be written as $P_\mathrm{fa} = Q_{X_N^2}(\gamma/\sigma_n^2)$. In the energy detector, the threshold γ can be computed using $\gamma = \sigma_n^2 \left[\sqrt{2/N} Q^{-1}(P_\mathrm{fa}) + 1\right]$, where $Q(x) = \left(1/\sqrt{2\pi}\right)\int_x^\infty \mathrm{e}^{-u^2/2}\, \mathrm{d}u$ is the standard Gaussian tail probability function.

2.2 Probability of detection under AWGN channel

To derive the probability of detection over non-fading AWGN channel, the probability density function defined under H_1 in (4) is inserted in (6). By substituting $t = y/\sigma_n^2$ and using definition of ρ, the probability of detection is given by

$$P_\mathrm{d} = \int_{\gamma/\sigma_n^2}^\infty \frac{1}{2}\left(\frac{t}{\rho}\right)^{(N-2)/4} \exp\left(\frac{-(t+\rho)}{2}\right) I_{(N/2)-1}\left(\sqrt{t\rho}\right) \mathrm{d}t \tag{10}$$

Substituting $t = x^2$, $\rho = a^2$ and with the further simplification, P_d can be written as

$$P_\mathrm{d} = \frac{1}{a^{(N/2)-1}} \int_{\sqrt{\gamma/\sigma_n^2}}^\infty x^{N/2} \exp\left(\frac{-(x^2 + a^2)}{2}\right) I_{(N/2)-1}(ax)\, \mathrm{d}x \tag{11}$$

We can rewrite (11) using the definition of generalised Marcum-Q function [21, 1.2] as

$$P_\mathrm{d} = Q_{N/2}\left(\sqrt{\rho}, \sqrt{\frac{\gamma}{\sigma_n^2}}\right) \tag{12}$$

Since the precise computation of Marcum-Q function is quite difficult, it is represented in the new form as [21, 2.6]

$$P_\mathrm{d} = 1 - \sum_{n \geq 0} (-1)^n \exp\left(\frac{-\rho}{2}\right) \frac{L_n^{(N/2)-1}(\rho/2)}{\Gamma((N/2) + n + 1)} \left(\frac{\gamma}{2\sigma_n^2}\right)^{(N/2)+n} \tag{13}$$

where $L_n^k(x)$ is the generalised Laguerre polynomial of degree n and order k. The absolute convergence of the series in (13) can be easily shown [21, 22] as

$$\left|\sum_{n \geq 0} (-1)^n \exp\left(\frac{-\rho}{2}\right) \frac{L_n^{\frac{N}{2}-1}\left(\frac{\rho}{2}\right)}{\Gamma\left(\frac{N}{2} + n + 1\right)} \left(\frac{\gamma}{2\sigma_n^2}\right)^{\frac{N}{2}+n}\right|$$

$$\leq \exp\left(\frac{-\rho}{2}\right) \sum_{n \geq 0} \frac{1}{\Gamma((N/2)+n+1)} \left(\frac{\gamma}{2\sigma_n^2}\right)^{(N/2)+n} \left|L_n^{(N/2)-1}\left(\frac{\rho}{2}\right)\right| \tag{14}$$

$$= \exp\left(\frac{-\rho}{4}\right) \frac{1}{\Gamma(N/2)} \left(\frac{\gamma}{2\sigma_n^2}\right)^{(N/2)-1} \exp\left(\frac{\gamma}{2\sigma_n^2} - 1\right) \tag{15}$$

Inserting (15) in (13) the P_d for AWGN channel is approximated as

$$P_\mathrm{d} \cong 1 - \exp\left(\frac{-\rho}{4}\right) \frac{1}{\Gamma(N/2)} \left(\frac{\gamma}{2\sigma_n^2}\right)^{(N/2)-1} \exp\left(\frac{\gamma}{2\sigma_n^2} - 1\right) \tag{16}$$

3 Probability of detection under fading channels

We define the average SNR $\bar{\rho}$ as $\bar{\rho} = E[h^2]\zeta/\sigma_n^2$, where $E(\cdot)$ denotes the expectation operator.

3.1 Rayleigh fading channel

The probability distribution function (PDF) of ρ in case of Rayleigh fading channel is given as

$$f(\rho) = \frac{1}{\bar{\rho}} \exp\left(\frac{-\rho}{\bar{\rho}}\right) \tag{17}$$

Using (16), we obtain the probability of detection as (see (18) and (19)).

The first term of (19) is simplified to

$$\int_0^\infty \frac{1}{\bar{\rho}} \exp\left(\frac{-\rho}{\bar{\rho}}\right) \,\mathrm{d}\rho = 1 \tag{20}$$

The second term of (19) is simplified to

$$\int_0^\infty \exp\left(\frac{-\rho}{4}\right)\exp\left(\frac{-\rho}{\bar{\rho}}\right) \,\mathrm{d}\rho = \frac{4\bar{\rho}}{4+\bar{\rho}} \tag{21}$$

After substituting (20) and (21) in (19), the approximate probability of detection for Rayleigh channel is

$$P_{\mathrm{d_{Ray}}} \cong 1 - \frac{4}{4+\bar{\rho}} \frac{1}{\Gamma(N/2)} \left(\frac{\gamma}{2\sigma_n^2}\right)^{(N/2)-1} \exp\left(\frac{\gamma}{2\sigma_n^2} - 1\right) \tag{22}$$

3.2 Log-normal shadowing channel

The link quality in terrestrial and satellite land-mobile systems is affected by slow variation of the mean signal level because of the shadowing from terrain, buildings and trees. The performance of a communication system will depend only on the shadowing if the radio receiver is able to average out the fast multipath fading or if an efficient micro-diversity system is used to eliminate the effects of multipath. Empirical measurements reveal that shadowing can be modelled by a log-normal distribution for various outdoor and indoor environments, in which case the probability distribution of ρ is given by [23]

$$f(\rho) = \frac{4.34}{\sqrt{2\pi}\sigma\rho} \exp\left(-\frac{(10\log_{10}\rho - \mu)^2}{2\sigma^2}\right) \tag{23}$$

where $\mu(\mathrm{d}B)$ and $\sigma(\mathrm{d}B)$ are the mean and the standard deviation of $10\log_{10}\rho$, respectively. To the best of our knowledge, closed-form expression cannot be obtained for the detection probability using (16) and (23). The log-normal distribution can be closely approximated by the Wald distribution [10]. Its PDF is given by

$$f(\rho) = \sqrt{\frac{\eta}{2\pi}}\rho^{-3/2} \exp\left(-\frac{\eta(\rho-\theta)^2}{2\theta^2\rho}\right) \tag{24}$$

where $\theta = E(\rho)$ denotes the expectation of ρ and η is the shape parameter. These parameters are related with μ and σ as given below

$$\begin{aligned} \theta &= \exp\left(\frac{\mu}{4.34} + \frac{\sigma^2}{37.67}\right) \\ \eta &= \frac{\theta}{\exp(\sigma^2/18.84) - 1} \end{aligned} \tag{25}$$

Using (16) and (24), the approximate probability of detection for log-normal shadowing channel is given as (see (26) and (27)).

Using [24, 3.471-9], the first and second terms of (27) are reduced to (see (28) and (29)) where $K_v(x)$ is a modified Bessel function of second kind. By substituting (28) and (29) in (27), the approximate probability of detection over log-normal shadowing channel is given by (see (30) at the bottom of the next page).

4 Results and discussion

In this section, we present the performance of the derived expressions for AWGN and fading channels through simulation.

$$P_{\mathrm{d_{Ray}}} = \int_0^\infty \left[1 - \exp\left(\frac{-\rho}{4}\right) \frac{1}{\Gamma(N/2)} \left(\frac{\gamma}{2\sigma_n^2}\right)^{(N/2)-1} \exp\left(\frac{\gamma}{2\sigma_n^2} - 1\right)\right] \frac{1}{\bar{\rho}} \exp\left(\frac{-\rho}{\bar{\rho}}\right) \,\mathrm{d}\rho \tag{18}$$

$$P_{\mathrm{d_{Ray}}} = \underbrace{\int_0^\infty \frac{1}{\bar{\rho}}\exp\left(\frac{-\rho}{\bar{\rho}}\right) \,\mathrm{d}\rho}_{\text{first term}} - \frac{1}{\bar{\rho}} \frac{1}{\Gamma(N/2)} \left(\frac{\gamma}{2\sigma_n^2}\right)^{(N/2)-1} \exp\left(\frac{\gamma}{2\sigma_n^2} - 1\right) \underbrace{\int_0^\infty \exp\left(\frac{-\rho}{4}\right)\exp\left(\frac{-\rho}{\bar{\rho}}\right) \,\mathrm{d}\rho}_{\text{second term}} \tag{19}$$

$$P_{\mathrm{d_{Log-Normal}}} \cong \int_0^\infty \left[1 - \exp\left(\frac{-\rho}{4}\right) \frac{1}{\Gamma(N/2)} \left(\frac{\gamma}{2\sigma_n^2}\right)^{(N/2)-1} \exp\left(\frac{\gamma}{2\sigma_n^2}\right) - 1\right] \sqrt{\frac{\eta}{2\pi}}\rho^{-3/2} \exp\left(-\frac{\eta(\rho-\theta)^2}{2\theta^2\rho}\right) \,\mathrm{d}\rho \tag{26}$$

$$P_{\mathrm{d_{Log-Normal}}} \cong \sqrt{\frac{\eta}{2\pi}} \left[\underbrace{\int_0^\infty \rho^{-3/2}\exp\left(-\frac{\eta(\rho-\theta)^2}{2\theta^2\rho}\right) \,\mathrm{d}\rho}_{\text{Firstterm}} - \frac{1}{\Gamma(N/2)} \left(\frac{\gamma}{2\sigma_n^2}\right)^{(N/2)-1} \exp\left(\frac{\gamma}{2\sigma_n^2} - 1\right) \underbrace{\int_0^\infty \exp\left(\frac{-\rho}{4}\right)\rho^{-3/2} \exp\left(-\frac{\eta(\rho-\theta)^2}{2\theta^2\rho}\right) \,\mathrm{d}\rho}_{\text{Secondterm}}\right] \tag{27}$$

$$\int_0^\infty \rho^{-3/2}\exp\left(-\frac{\eta(\rho-\theta)^2}{2\theta^2\rho}\right) \,\mathrm{d}\rho = \sqrt{\frac{\eta}{2\pi}}\exp\left(\frac{\eta}{\theta}\right) \int_0^\infty \rho^{-(1/2)-1} \exp\left(-\frac{\eta}{2\rho} - \frac{\eta\rho}{2\theta^2}\right) \,\mathrm{d}\rho = \exp\left(\frac{\eta}{\theta}\right)\sqrt{\frac{2\eta}{\pi\theta}}K_{-1/2}\left(\frac{\eta}{\theta}\right) \tag{28}$$

$$\begin{aligned} \int_0^\infty \rho^{-3/2}\exp\left(\frac{-\rho}{4} - \frac{\eta(\rho-\theta)^2}{2\theta^2\rho}\right)\mathrm{d}\rho &= \sqrt{\frac{\eta}{2\pi}}\exp\left(\frac{\eta}{\theta}\right) \int_0^\infty \rho^{-(1/2)-1} \exp\left(-\frac{\eta}{2\rho} - \left(\frac{\eta}{2\theta^2} + \frac{1}{4}\right)\rho\right) \,\mathrm{d}\rho \\ &= \sqrt{\frac{\eta}{2\pi}}\exp\left(\frac{\eta}{\theta}\right) 2\left(\frac{\eta}{2((\eta/2\theta^2) + (1/4))}\right)^{-1/4} K_{-1/2}\left[2\sqrt{\frac{\eta}{2}\left(\frac{\eta}{2\theta^2} + \frac{1}{4}\right)}\right] \end{aligned} \tag{29}$$

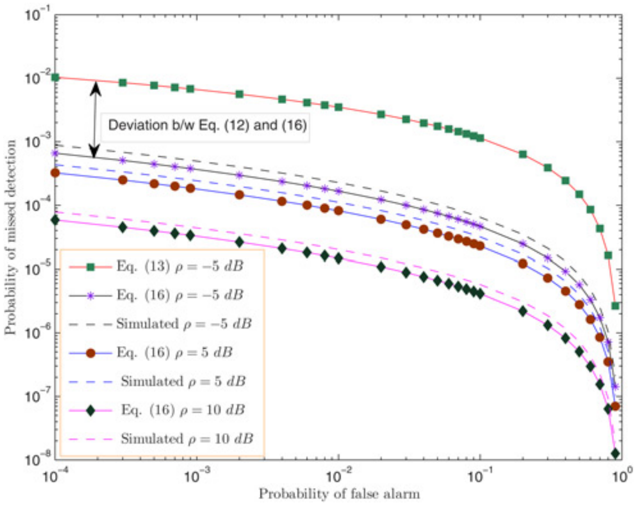

Fig. 2 *Complementary ROC curves for AWGN channel for different values of ρ with $N = 10$ and $\sigma_n^2 = 0.5$*

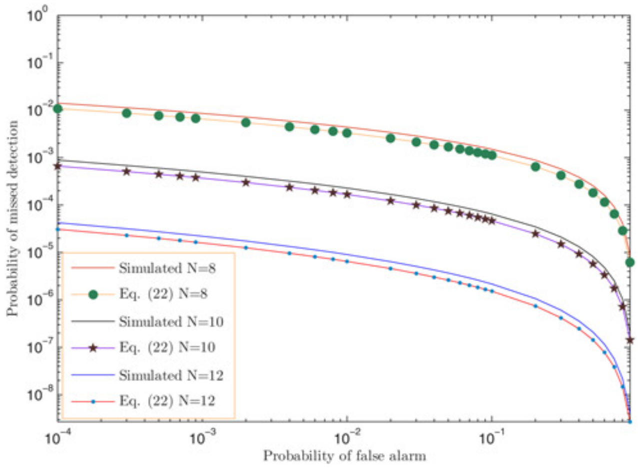

Fig. 4 *Complementary ROC curves for Rayleigh channel for different values of N with $\bar{\rho} = -5$ dB and $\sigma_n^2 = 0.5$*

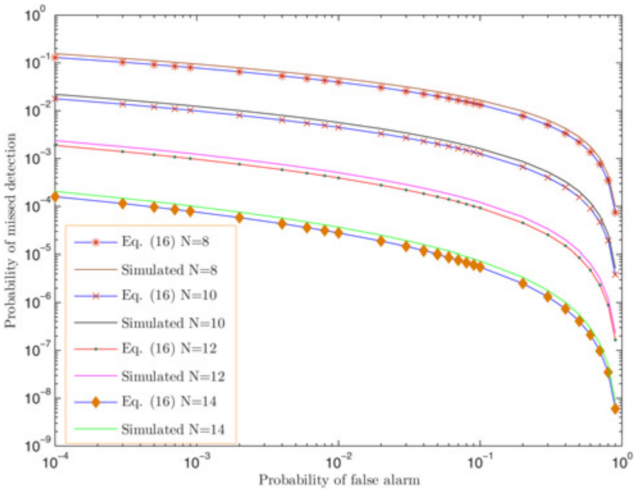

Fig. 3 *Complementary ROC curves for AWGN channel for different values of N with $\rho = -10$ dB and $\sigma_n^2 = 0.75$*

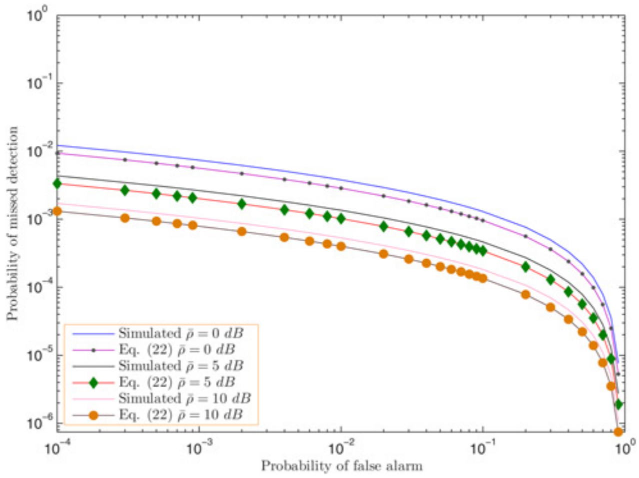

Fig. 5 *Complementary ROC curves for Rayleigh channel for different values of $\bar{\rho}$ with $N = 8$ and $\sigma_n^2 = 0.5$*

Receiver operating characteristic (ROC) has been widely used in the signal detection theory. It is an ideal technique to quantify the trade-off between the probability of detection and probability of false alarm. In the simulation, we use complementary ROC curves ($P_{\rm md}$ against $P_{\rm fa}$) to show the detection performance of ED over non-fading AWGN and fading channels. The noise variance factor $\left(\sigma_n^2\right)$ is considered in each simulation result.

In Fig. 2, we generate the complementary ROC curves for AWGN channel for different values of ρ with $N = 10$ and noise variance $\sigma_n^2 = 0.5$. In this figure, the deviation between (12) which involves generalised Marcum-Q function and the derived closed-form expression (16) is shown. The deviation between (12) and (16) is too small. It is clear that increasing SNR decreases the miss-detection probability and improvement in the detection probability diminishes at low SNR.

Fig. 3 shows the complementary ROC curves for different values of N in AWGN channel for $\rho = -10$ dB and $\sigma_n^2 = 0.75$. Clearly, if the value of N is greater then the network can achieve better performance. From this figure, we can see that the miss-detection probability decreases greatly with a small increase in value of N, which shows the improvement in the sensing performance.

In Fig. 4, we have presented the performance evaluation of ED over Rayleigh channel based on both analytical and simulation for different values of N with $\bar{\rho} = -5$ dB and $\sigma_n^2 = 0.5$. As can be observed, the simulation results well confirm the analytical expression (22). Here, increasing the value of N improves the detection probability. Fig. 5 shows the complementary ROC curves for different values of $\bar{\rho}$ with $N = 8$ and $\sigma_n^2 = 0.5$. We note from the figure that there is an improvement in the performance with several dB's increment in $\bar{\rho}$.

$$P_{\mathrm{d_{Log-Normal}}} \cong \exp\left(\frac{\eta}{\theta}\right)\left\{\sqrt{\frac{2\eta}{\pi\theta}}K_{-1/2}\left(\frac{\eta}{\theta}\right) - \frac{1}{\Gamma(N/2)}\left(\frac{\gamma}{2\sigma_n^2}\right)^{(N/2)-1}\exp\left(\frac{\gamma}{2\sigma_n^2}-1\right)\sqrt{\frac{2\eta}{\pi}}\left(\frac{\eta}{2((\eta/2\theta^2)+(1/4))}\right)^{-1/4}K_{-1/2}\left[2\sqrt{\frac{\eta}{2}\left(\frac{\eta}{2\theta^2}+\frac{1}{4}\right)}\right]\right\} \tag{30}$$

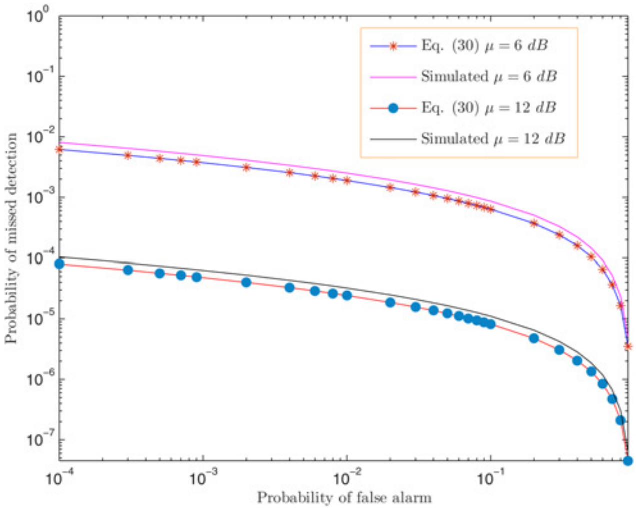

Fig. 6 *Complementary ROC curves for log-normal shadowing channel for different values of* μ *with* $N = 8$, $\sigma = 4$ *and* $\sigma_n^2 = 0.5$

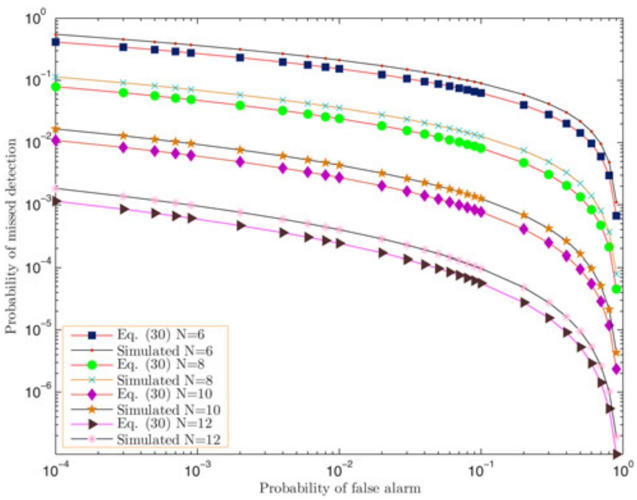

Fig. 7 *Complementary ROC curves for log-normal shadowing channel for different values of N with* $\mu = 5$ dB, $\sigma = 3$ *and* $\sigma_n^2 = 0.6$

In Fig. 6, we plot complementary ROC curves for log-normal shadowing channel for different values of mean (μ) with $N = 8$, $\sigma = 4$ and $\sigma_n^2 = 0.5$. There is a great improvement in the detection probability when μ is increased from 6 to 12 dB. Fig. 7 shows the complementary ROC curves for log-normal shadowing channel for different values of N with $\mu = 5$ dB, $\sigma = 3$ and $\sigma_n^2 = 0.6$. We note that detection probability is low when $N = 6$ and we can achieve very high detection probability for $N = 12$. Here simulation results closely match with analytical results. Finally, we say that all the derived expressions for the detection probability over non-fading and fading channels are simulated in MATLAB under various parameters which concur with the simulation results.

5 Conclusions

In this study, we derived the closed-form expressions for the probability of detection over non-fading AWGN and fading channels. To the best of our knowledge, (16), (22) and (30) are the new numerical results for the detection probability over considered channels. These theoretical expressions are evaluated under various parameters and verified by the simulation results. Our numerical results in Section 3 provide new mathematical analysis for the detection probability over fading channels.

6 References

[1] Kolodzy P., Avoidance I., Models S.: 'Spectrum policy task force'. Federal Communications Commission, Washington, DC, Report ET Docket, 2002

[2] Haykin S.: 'Cognitive radio: brain-empowered wireless communications', *IEEE Trans. Commun.*, 2005, **23**, pp. 201–220

[3] Taherpour A., Norouzi Y., Nasiri-Kenari M., Jamshidi A., Zeinalpour-Yazdi Z.: 'Asymptotically optimum detection of primary user in cognitive radio networks', *IET Commun.*, 2007, **1**, pp. 1138–1145

[4] Ganesan G., Li Y.: 'Cooperative spectrum sensing in cognitive radio part I: two user networks', *IEEE Trans. Wirel. Commun.*, 2007, **6**, pp. 2204–2213

[5] Kim K., Akbar I.A., Bae K.K., Um J.S., Spooner C.M., Reed J.H.: 'Cyclostationary approaches to signal detection and classification in cognitive radio'. DySPAN2007, April 2007, pp. 212–215

[6] Ghasemi A., Sousa E.: 'Opportunistic spectrum access in fading channels through collaborative sensing', *J. Commun.*, 2007, **2**, pp. 71–82

[7] Urkowitz H.: 'Energy detection of unknown deterministic signals', *Proc. IEEE*, 1967, **55**, pp. 523–531

[8] Kostylev V.I.: 'Energy detection of a signal with random amplitude'. Proc. IEEE Int. Conf. Commun., May 2002, pp. 1606–1610

[9] Digham F.F., Alouini M.S., Simon M.K.: 'On the energy detection of unknown signals over fading channels', *IEEE Trans. Commun.*, 2007, **55**, (1), pp. 21–24

[10] Sun H., Laurenson D.I., Wang C.X.: 'Computationally tractable model of energy detection performance over slow fading channels', *IEEE Commun. Lett.*, 2010, **14**, (10), pp. 924–926

[11] Rao A., Alouini M.S.: 'Performance of cooperative spectrum sensing over non identical fading environments', *IEEE Trans. Commun.*, 2011, **59**, (12), pp. 3249–3253

[12] Ruttik K., Koufos K., Jantti R.: 'Detection of unknown signals in a fading environment', *IEEE Commun. Lett.*, 2009, **13**, (7), pp. 498–500

[13] Sofotasios P.C., Rebeiz E., Zhang L., Tsiftis T.A., Freear S., Cabric D.: 'Energy detection based spectrum sensing over *k–μ* and *k–μ* extreme fading channels', *IEEE Trans. Veh. Technol.*, 2013, **62**, (3), pp. 1031–1040

[14] Banjade V.R.S., Rajatheva N., Tellambura C.: 'Performance analysis of energy detection with multiple correlated antenna cognitive radio in Nakagami-*m* fading', *IEEE Commun. Lett.*, 2012, **16**, (4), pp. 502–505

[15] Pridham R., Urkowitz H.: 'Comment on energy detection of unknown deterministic signals', *Proc. IEEE*, 1968, **56**, (8), pp. 1379–1380

[16] Axell E., Larsson E.: 'Optimal and sub-optimal spectrum sensing of OFDM signals in known and unknown noise variance', *IEEE J. Sel. Areas Commun.*, 2011, **29**, (2), pp. 290–304

[17] Gismalla E., Alsusa E.: 'On the performance of energy detection using Bartlett's estimate for spectrum sensing in cognitive radio systems', *IEEE Trans. Signal Process.*, 2012, **60**, (7), pp. 3394–3404

[18] Shnidman D.: 'The calculation of the probability of detection and the generalized Marcum Q-function', *IEEE Trans. Inf. Theory*, 1989, **35**, (2), pp. 389–400

[19] Proakis J.G., Salehi M.: 'Digital communications' (McGraw-Hill, New York, 2008), pp. 46–47

[20] Abramowitz M., Stegun I.A.: 'Handbook of mathematical functions with formulas, graphs, and mathematical tables' (Dover, New York, 1974)

[21] András S., Baricz Á., Sun Y.: 'The generalized Marcum-Q function: an orthogonal polynomial approach', *Acta Univ. Sapientiae Math.*, 2011, **3**, (1), pp. 60–76

[22] Altrad O., Muhaidat S.: 'A new mathematical analysis of the probability of detection in cognitive radio over fading 'channels', *EURASIP J. Wirel. Commun. Netw.* 2013, **2013**, (159), doi: 10.1186/1687-1499-2013-159

[23] Simon M.K., Alouini M.-S.: 'Digital communications over fading channels' (John Wiley and Sons, Inc., 2004, 2nd edn.)

[24] Gradshteyn I.S., Ryzhik I.M.: 'Tables of integrals, series, and products', 7th edn. (Elsevier Academic Press, 2007)