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# An optimal aircraft fleet management decision model under uncertainty

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## SUMMARY

Decision planning for an efficient fleet management is crucial for airlines to ensure a profit while maintaining a good level of service. Fleet management involves acquisition and leasing of aircraft to meet travelers' demand. Accordingly, the methods used in modeling travelers' demand are crucial as they could affect the robustness and accuracy of the solutions. Compared with most of the existing studies that consider deterministic demand, this study proposes a new methodology to find optimal solutions for a fleet management decision model by considering stochastic demand. The proposed methodology comes in threefold. First, a five-step modeling framework, which is incorporated with a stochastic demand index (SDI), is proposed to capture the occurrence of uncertain events that could affect the travelers' demand. Second, a probabilistic dynamic programming model is developed to optimize the fleet management model. Third, a probable phenomenon indicator is defined to capture the targeted level of service that could be achieved satisfactorily by the airlines under uncertainty. An illustrative case study is presented to evaluate the applicability of the proposed methodology. The results show that it is viable to provide optimal solutions for the aircraft fleet management model. Copyright © 2013 John Wiley & Sons, Ltd.

KEY WORDS: dynamic programming; stochastic demand; aircraft fleet management

## 1. INTRODUCTION

Fleet management determines the optimal number of aircraft needed by an airline to maintain a targeted level of service while maximizing its profit. Two major decisions are to be made, that is, to determine the number of aircraft to be purchased and leased at any point in time (say every year or half a year) to meet the demand. Proper fleet management is important as it would affect the economical efficiency of the airline and it has an influential impact on customer satisfaction 1. An oversized fleet implies an increased cost, whereas an undersized fleet implies an unsatisfied demand and results in a decrease in revenue and profit [2–4.]

In the optimization of the supply of aircraft in fleet management, how the demand is forecasted is important as it could influence the results' robustness. Most of the existing studies, such as those by New 5, Abara 6, Hane *et al*. 7, Desaulniers *et al*. 8, Barnhart *et al*. 9, and Yan *et al*. 10, adopted the deterministic approach in forecasting the travelers' demand level. However, Barnhart *et al*. 11 highlighted that stochastic demand should be considered because the airline's operating environment is stochastic in nature because of the presence of uncertainty. Stochastic demand refers to the demand fluctuation that is uncertain at varying degrees primarily because of the occurrence of uncertain events, which could take place unexpectedly. According to airlines [12,13,] the possible unexpected events

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include the volatility in fuel prices, political instability including terrorist attacks, global economic uncertainties, natural disasters, and others. When these events occur, demand would decrease tremendously. For instance, the volcano eruption in Iceland has caused a decline in demand for about 10 million travelers because of the cancellation of substantial flights 14.

Several studies had attempted to model the stochastic demand in air transportation. List *et al.* 15 used a partial moment measure of risk to inspect the uncertainty of travel demand. Listes and Dekker 16 adopted the scenario aggregation-based approach to determine the best choice of aircraft by assuming that travel demand follows normal distribution. Yan *et al.* 17 captured the demand fluctuations by developing the passenger flow networks and passenger choice model for which the passenger utility and market demand functions are formed to determine the choice probability function of travelers. Pitfield *et al.* 18 employed a simultaneous-equations approach to analyze the demand elasticity and aircraft choice. Hsu *et al.* adopted the grey topological models with Markov chain to capture demand fluctuations 19 and combined the grey topological forecasting results with the Markov chain model to investigate demand fluctuations and to determine the probability of demand 20. They imposed a penalty cost function if the actual demand is more than the forecasted demand. In other areas (not air transportation), stochastic demand is assumed to follow a certain distribution. For example, Berman *et al.* 21 and Batta *et al.* 22 adopted the Poisson distribution to model the stochastic demand for queuing systems. Du and Hall 23 proposed a dynamic model to capture the stochastic demand for port operation. Bojovic 24 modeled the demand of railroad network as a Gaussian probability density function whereas Tan *et al.* 25 assumed that the stochastic demand has a normal distribution in solving a vehicle routing problem.

The proposed methods used in the past studies to capture stochastic demand are interesting, but they have limited applicability. They did not quantify the occurrence of unexpected events in their attempts to model stochastic demand. For example, List *et al.* 15 modeled the demand entirely on the basis of a one-sided risk measure (rather than on demand variation) for which the likelihood of the objective function in meeting the demand is controlled not to exceed the threshold value. Hsu *et al.* 20 adopted the Markov chain model by taking into account one set of transition probability only to model the travel demand. Both studies ignored the possibility of unexpected events that could take place unexpectedly. Instead of demand fluctuations modeling only, the probability of unexpected event occurrence should be quantified systematically as it could affect the stochastic demand to vary differently. Without this element, the level of stochastic demand may not be modeled as accurately close to reality as possible. Moreover, the assumption of fixed types of distribution to quantify the demand fluctuations might be too restrictive. The methodology proposed might not be applicable if the real demand pattern did not follow the types of distribution as assumed. Furthermore, the demand forecasting methods proposed in the existing studies are for short-term periods only. For example, Tan *et al.* 25 and Yan *et al.* 17 modeled the demand fluctuations within a day. Listes and Dekker 16 and Pitfield *et al.* 18 modeled weekly and monthly demand, respectively. Such short-term forecasting methods are not applicable to modeling long-term (more than a year) demand fluctuations, which is required in solving the fleet management problem.

From the operating statistics of the airlines 13,12, it could be found that the travel demand fluctuated from year to year without a specific pattern and trend. Furthermore, the fluctuation is affected by the occurrence of unexpected events. For example, the travel demand reduced tremendously during the volcano eruption in Iceland in 2010 14. Accordingly, we propose a five-step modeling framework to forecast the travelers' demand by considering the occurrence of unpredicted events. In the framework, a stochastic demand index (SDI) is defined to quantify the level of stochastic demand. This is carried out by considering the possible occurrence of unexpected events as one of its inputs. The probability of negative effects (such as a sudden decrease in demand) could be simulated independently. A Monte Carlo simulation is then adopted to predict the probability of the uncertainty and travel demand for each operating period. As such, there is no need to assume a fixed distribution to forecast the demand. By properly integrating the probability of both elements (with the aid of a simulation approach), the fluctuations of demand throughout the planning horizon can be captured precisely. The developed framework relates the current and previous level of demand. The continuous relation of successive operating periods consecutively will quantify the level of stochastic demand over a long-term period. Accordingly, the proposed methodology could address the existing limitations to some extent.

Past studies had adopted various approaches to formulate and optimize the fleet management problem. Wei and Hansen 26 built a nested logit model to inspect the influence of aircraft size, service



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frequency, seat availability, and fare on the airline's demand. They highlighted that aircraft size, which could affect the travelers' choice, needs to be considered for fleet planning. Later, Wei and Hansen 27 developed game-theoretic models to investigate the airlines' decisions on aircraft size and service frequency. They revealed that aircraft size is a significant factor for fleet management decision making, depending on the type of markets. Wei 28 employed the game-theoretical model to investigate how airport landing fees could influence airlines' decisions on aircraft size and service frequency. Kozanidis 29 developed a multiobjective optimization model to maximize the availability of the aircraft. He showed that flight and maintenance requirements are two important factors for fleet planning while Givoni and Rietveld 30 analyzed the environmental impacts of airlines' choice on aircraft size. More recently, Hsu *et al.* 19 formulated a stochastic dynamic programming model to optimize the airline decisions in purchasing, leasing, and disposing aircraft. Hsu *et al.* 20 developed a dynamic programming model that deals with the fleet purchase, dry/wet leases, and disposal of aircraft by considering the impact of a strategic alliance between airlines. These are interesting but posed some limitations. For example, the methods proposed by Hsu *et al.* 19,20 are used to tackle the fleet management problem with stochastic demand, but they did not capture the occurrence of unexpected events in modeling the stochastic demand. In addition, their formulation might be too simplistic by considering the demand as the only constraint. In fact, there are other crucial constraints, such as budget constraint, lead time, and selling time constraint, which are important in fleet management.

Accordingly, there is a need to improve the existing approach. It was found that the inclusion of stochastic demand in fleet management formulation introduces a probabilistic element. Given the current situation, the optimal solution for the next operating period could not be determined because of the possible occurrence of unexpected events that can affect the demand level. As such, we proposed a probabilistic approach in the fleet management decision model to capture the uncertainty. A probable phenomenon indicator is used to capture the likelihood of the airlines (in terms of aircraft supply) in meeting the stochastic demand. It is necessary because there are chances that the travel demand could not be met perfectly because of the uncertainty. With this indicator in place, the airlines could monitor their level of service closely. Furthermore, a dynamic programming model is adopted to formulate the fleet management problem as it has the capability to decompose the proposed model into a series of simpler single-period subproblems. This allows the determination of the optimal solution for more tractable subproblems for each operating period. The objective of the fleet management decision model is to maximize the operational profit of the airlines, subject to several practical constraints. The decision variables are the number and types of aircraft that need to be purchased and leased. An illustrative case study is shown to test the feasibility of the proposed methodology. The findings revealed that the results are sensitive to the modeling parameters, and the proposed methodology is viable in solving the fleet management problem.

This paper is organized as follows. Section 1 introduces the scope of this paper by addressing the related literature review and the significance of and a brief introduction about the proposed methodology. Section 2 lists the notations used throughout the paper, whereas Section 3 explains the modeling framework to model stochastic demand. Section 4 outlines the formulation of the developed model (for fleet management) and the incorporation of probable phenomena in the developed optimization model. Section 5 explains the solution method, and an illustrative case study is then presented in Section 6 to examine the feasibility of the proposed methodology. Subsequently, the computational results of the case study are discussed in Section 7. Section 8 concludes the findings of this paper and suggests prospects for future research.

## 2. NOMENCLATURE

For the operating period $t$, the notations used in this study (applied for $n$ types of aircraft at age $y$) are listed as follows:

### PARAMETERS

| | |
|---|---|
| $T$ | Planning horizon |
| $MAX_{\text{budget}(t)}$ | Allocated budget for aircraft acquisition and leasing |
| $D_t^S$ | Stochastic demand (corresponding to phenomenon $S$) |
| $Index_t$ | Index of stochastic demand |
| $D_f$ | Forecasted demand with mean $\mu_f$ and standard deviation $\sigma_f$ |

| | |
|---|---|
| $D_{f(\mathrm{inc})}$ | Possible increment of forecasted demand |
| $D_0$ | Projected stochastic demand |
| $ORDER_t$ | Total number of aircraft that could be purchased in the market |
| $PARK_t$ | Area of hangar as a geometry limitation |
| $r_t$ | Discount rate for which the discount factor is $(1+r_t)^{-1}$ |
| $\alpha$ | Significance level of the demand constraint |
| $\beta$ | Significance level of the lead time constraint |
| $\gamma$ | Significance level of the selling time constraint |
| $E\left(fare_t^S\right)$ | Expected value of flight fare per passenger |
| $E\left(\cos t_t^S\right)$ | Expected value of flight cost per passenger |
| $p_s$ | Probability of having $I_t^P$ and $I_t^L$ (corresponding to phenomenon $S$) |
| $A_t^n$ | Total operated aircraft |
| $PP_{tc}$ | Product of the probability of uncertainty $c$ |
| $P(hc)$ | Probability that the uncertainty $c$ happens with a probable occurrence of $h$ |

## FUNCTIONS

| | |
|---|---|
| $P\left(I_t^P+I_t^L\right)$ | Function of discounted profit by having $I_t^P$ and $I_t^L$ |
| $f\left(D_t^S,A_t^n\right)$ | Function of the number of flights in terms of $D_t^S$ and $A_t^n$ |
| $gf\left(D_t^S,A_t^n\right)$ | Function of the traveled mileage in terms of $f\left(D_t^S,A_t^n\right)$ |
| $hgf\left(D_t^S,A_t^n\right)$ | Maintenance cost function in terms of $g$ |
| $C(fuel_{tn})$ | Function of fuel expenses |

## SETS

| | |
|---|---|
| $X_t^P=\left(x_{t1}^P,x_{t2}^P,\ldots,x_{tn}^P\right)$ | Number of aircraft to be purchased |
| $X_t^L=\left(x_{t1}^L,x_{t2}^L,\ldots,x_{tn}^L\right)$ | Number of aircraft to be leased |
| $I_t^P=\left(In_{t1y}^P,In_{t2y}^P,\ldots,In_{tny}^P\right)$ | Initial number of purchased aircraft |
| $I_t^L=\left(In_{t1y}^L,In_{t2y}^L,\ldots,In_{tny}^L\right)$ | Initial number of leased aircraft |
| $O_t=(O_{t1},O_{t2},\ldots,O_{tn})$ | Number of aircraft to be ordered |
| $R_t=(R_{t1},R_{t2},\ldots,R_{tn})$ | Number of aircraft to be released for sales |
| $U_t=(u_{t1},u_{t2},\ldots,u_{tn})$ | Setup cost for aircraft acquisition |
| $S=(s_1,s_2,\ldots,s_k)$ | Phenomenon of having $I_t^P$ and $I_t^L$ |
| $PURC_t=(purc_{t1},purc_{t2},\ldots,purc_{tn})$ | Purchase cost of aircraft |
| $LEASE_t=(lease_{t1},lease_{t2},\ldots,lease_{tn})$ | Lease cost of aircraft |
| $DP_t=(dp_{t1},dp_{t2},\ldots,dp_{tn})$ | Payable deposit for aircraft acquisition |
| $DL_t=(dl_{t1},dl_{t2},\ldots,dl_{tn})$ | Payable deposit for aircraft leasing |
| $SEAT_n=(seat_1,seat_2,\ldots,seat_n)$ | Number of seats of aircraft |
| $SOLD_t=(sold_{t1y},sold_{t2y},\ldots,sold_{tny})$ | Number of aircraft sold |
| $RESALE_t=(resale_{t1y},\ldots,resale_{tny})$ | Resale price of aircraft |
| $DEP_t^P=\left(dep_{t1y}^P,dep_{t2y}^P,\ldots,dep_{tny}^P\right)$ | Depreciation value of purchased aircraft |
| $DEP_t^L=\left(dep_{t1y}^L,dep_{t2y}^L,\ldots,dep_{tny}^L\right)$ | Depreciation value of leased aircraft |
| $SIZE=(size_1,size_2,\ldots,size_n)$ | Size of aircraft |
| $RLT_t=(RLT_{t1},RLT_{t2},\ldots,RLT_{tn})$ | Real lead time of aircraft acquisition |
| $DLT_t=(DLT_{t1},DLT_{t2},\ldots,DLT_{tn})$ | Desired lead time of aircraft acquisition |
| $RST_t=(RST_{t1},RST_{t2},\ldots,RST_{tn})$ | Real selling time of aging aircraft |
| $DST_t=(DST_{t1},DST_{t2},\ldots,DST_{tn})$ | Desired selling time of aging aircraft |



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## 3. MODELING OF STOCHASTIC DEMAND

Globally, the airlines forecast the future growth of travelers annually to obtain the latest trend in travel demand. Typically, the forecasting (or prediction) of the growth of demand is found to be positive (i.e., implying positive growth) in accordance with the increase in population size and income level 12,31. However, when there is an occurrence of an unpredicted event that could affect the traveler's decision, there would be a reduction in demand during the period. This is referred to as a negative effect. We develop a five-step modeling framework (as displayed in APPENDIX A) to forecast the demand and its fluctuations during a study horizon. In the framework, an SDI is defined to quantify the probability of the possible occurrence of the demand uncertainty. It is assumed that the value of SDI for the base year (year 0) is 1. A Monte Carlo simulation 32,33 is used to determine the occurrence probability of positive and negative effects with no prior assumption of a fixed distribution. The step-by-step procedure of the proposed framework is elaborated as follows:

Step 1: Determine the possible event's occurrence

Consider a set of uncertain events that could affect the travelers' demand. For example, the occurrence of a biological disease, economic downturn, and natural disaster could take place unexpectedly in real life. The probability of the occurrence of these events is determined. One way to estimate the probability could be from the historical data of the event's occurrence over a period.

Step 2: Determine the probability of the event's occurrence (negative effect)

The probability of the event's occurrence is simulated by using the Monte Carlo simulation, based on the predetermined probability distribution in step 1. The probability of the occurrence is expressed as follows:

$$PP_{tc} = \prod_{c=1}^{C} \sum_{h=1}^{H} P(hc)\Phi \ \text{ for } \ t = 1, 2, \ldots, T, \Phi = \begin{cases} 1, & \text{if } h \text{ happens} \\ 0, & \text{if } h \text{ does not happen} \end{cases} \tag{1}$$

for which $P(hc)$ is the probability that the uncertainty $c$ happens with a possible occurrence of $h$.

Step 3: Determine the possible increment of the forecasted demand $D_{f(\mathrm{inc})}$

The possible increment of the forecasted demand (positive effect) needs to be estimated. The demand growth projection is estimated from past travel trend (through the historical data published by the airlines or air transportation nongovernment organizations) and future travel trend forecasting. The probability distribution that describes the projected growth of demand needs to be modeled as well.

Step 4: Determine the probability of the increment of the forecasted demand $D_{f(\mathrm{inc})}$

From the demand growth projected in step 3, the possible increment of the forecasted demand for each operating period as well as its probability is determined accordingly with the aid of the Monte Carlo simulation.

Step 5: Determine the value of SDI for each operating period

For each operating period, the SDI, $Index_t$, is determined subject to both positive and negative effects. The probabilities of both effects are compiled together to work out the SDI because of the fact that the level of stochastic demand is not only affected by the occurrence of uncertainty (negative effect) but also influenced positively by the demand growth (positive effect). By considering both effects (i.e., to sum up both effects), the SDI could be expressed as



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$$Index_t = \left(PP_{tc} + D_{f(\mathrm{inc})}\right) + 1 \text{ for } t = 1, \ 2, \ldots, T, c = 1, \ 2, \ldots, C \tag{2}$$

for which the constant of 1 is the index value for the base period (year 0). $Index_t > 1$ means that overall (because of both positive and negative effects), the level of stochastic demand in year $t$ is higher than the level of demand in the previous year (i.e., year $t-1$). Similarly, $Index_t < 1$ indicates that the level of stochastic demand in year $t$ overall is lower than the level of demand in year $t-1$. $Index_t = 1$ implies that the demand in year $t$ and its previous year (i.e., year $t-1$) is the same. This is possible because of the nature of uncertainty and the growth of demand, which is stochastic.

By using the SDI, the demand level of each operating year, $D_t$, is determined from the following equation:

$$D_t = \begin{cases} Index_t \times D_0, & \text{for } t = 1 \\ Index_t \times D_{t-1}, & \text{for } t = 2, \ 3, \ldots, \ T \end{cases} \tag{3}$$

For operating year 1, the demand level is determined by using the convolution algorithm as described by Winston 33 (APPENDIX B). According to Winston 33, the convolution algorithm can be adopted to generate normal random variates. Besides, this algorithm incorporates a random number, which is a significant component for simulation to capture vagueness and randomness. The demand level of the operating year 1 could be defined as follows:

$$D_0 = \mu_f + \sigma_f \left( \sum_{r=1}^{12} R_r - 6 \right) \tag{4}$$

Note that for the subsequent operating period, the level of stochastic demand is determined by considering the current SDI and the level of the stochastic demand of the previous operating period.

## 4. FLEET MANAGEMENT OPTIMIZATION MODEL

The fleet management decision model is formulated as a probabilistic dynamic programming model. For a set of the origin–destination pairs, assume that there is a selection of $n$ types of aircraft that could be purchased or leased. The decision variables of the model are the number and types of aircraft to be purchased or leased to maximize the operational profit of the airlines. The stochastic demand from Section 3 is used as one of the inputs to the model.

### *4.1. Stage, state variable, and optimal decision*

The stage of the model is the planning horizon of the fleet management decision model. In this study, the operating period, $t$, in terms of years is the stage variable of the model. The state variable at each stage $t$ consists of various intercorrelated variables, namely the number of aircraft to be purchased or leased (i.e., main decision variable), total operated aircraft, number of aircraft to be sold, number of aircraft to be ordered, and number of aircraft to be released for sales. The optimal decision, that is, the alternative at each stage, is the aircraft acquisition and leasing decision to meet the stochastic demand while making a decision to sell aging aircraft with the goal to maximize the operational profit of the airlines. For a particular operating period, although the state variables and the corresponding optimal solutions could be obtained, the optimal decision for the next operating period is unknown because of uncertainty. The states of the next operating period are uncertain given the current state and current decision because many factors may not be known with certainty in practice 32,33.



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### 4.2. Constraints

The practical constraints considered for the fleet management decision model are as follows:

Budget constraint

The budget constraint ascertains whether or not the solution is financially feasible for the airlines. For this constraint, the sum of the purchase and lease cost of the aircraft should not be more than the allocated budget, which could be expressed as follows:

$$\sum_{i=1}^{n} purc_{ti} x_{ti}^{P} + \sum_{i=1}^{n} lease_{ti} x_{ti}^{L} \leq MAX_{\text{budget}(t)} \text{ for } t = 1, 2, \ldots, T \tag{5}$$

Demand constraint

The stochastic demand derived from Section 3 is used to form the demand constraint. To ensure that travelers' demand could be met satisfactorily, the demand constraint could be expressed as

$$\sum_{i=1}^{n} (SEAT_i)\left(f\left(D_t^S, A_t^i\right)\right) \geq (1-\alpha) D_t^S \text{ for } t = 1, 2, \ldots, T, S = s_1, s_2, \ldots, s_k \tag{6}$$

where $1 - \alpha$ is the confidence level (service level) to meet the stochastic demand.

Parking constraint

When an aircraft is not in operation, it has to be parked at the hangar at the airport. In such a case, the choice of aircraft would sometimes be constrained by the geometry layout of the hangar at the airport. As such, the parking constraint is ought to be considered feasibly. This constraint is shown as follows:

$$\sum_{i=1}^{n} \sum_{y=0}^{m} \left(In_{tiy}^{P} + In_{tiy}^{L} + x_{ti}^{P} + x_{ti}^{L}\right)(SIZE_i) \leq PARK_t \text{ for } t = 1, 2, \ldots, T \tag{7}$$

Sales of aircraft constraint

For some airlines, the aging aircraft, which is less cost-effective, might be sold at the beginning of a certain operating period when the airlines make the decision to purchase a new aircraft. However, the number of aircraft sold should not be more than the aircraft owned by the airlines. It is expressed as follows:

$$sold_{tiy} \leq In_{(t-1)i(y-1)}^{P} \text{ for } t = 1, 2, \ldots, T, i = 1, 2, \ldots, n, y = 1, 2, \ldots, m \tag{8}$$

Order delivery constraint

The delivery of new aircraft depends on the production and the supply of aircraft manufacturers. Sometimes, there might be an availability issue in delivering new aircraft. As such, the aircraft to be purchased should not be more than the number of aircraft available in the market, which is expressed as follows:

$$x_{ti}^{P} \leq ORDER_t \text{ for } t = 1, 2, \ldots, T, i = 1, 2, \ldots, n \tag{9}$$

For the aircraft leasing, it is assumed that the order delivery constraint is not relevant because of its possible availability within 1 year (short-term duration) for some circumstances. In addition,



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the number of leased aircraft is relatively flexible at a certain extent (not really limited to manufacturing constraint).

Lead time constraint

In practice, the airlines would get an agreeable lead time (the period between placing and receiving an order) from the aircraft manufacturer when they place an order for new aircraft. This constraint should be considered as it indicates when the airlines are supposed to order new aircraft. For $n$ types of aircraft, this constraint can be expressed as follows:

$$P(RLT_{ti} \geq DLT_{ti}) \leq \beta \ \text{ for } \ t = 1, \ 2, \ldots, \ T, i = 1, \ 2, \ldots, n \tag{10}$$

Because in real life, there are chances that the targeted lead time would change (say, because of the technical problems of the manufacturer), the lead time should be a random value that could be represented by a certain distribution. In this study, the lead time is assumed to be normally distributed with mean $\mu_{\mathrm{LT}}$ and standard deviation $\sigma_{\mathrm{LT}}$. The constraint could be stated by

$$DLT_{ti} \geq F^{-1}(1-\beta)\sigma_{\mathrm{LT}} + \mu_{\mathrm{LT}} \ \text{ for } \ t = 1, \ 2, \ldots, T, i = 1, \ 2, \ldots, \ n \tag{11}$$

where $F^{-1}(1-\beta)$ is the inverse cumulative probability of $1-\beta$.

Selling time constraint

An aging aircraft, which is considered as less economical, might be sold by the airlines at a certain operating period. In such a case, the airlines need to know the most suitable time to release their aging aircraft for sales particularly to look for prospective buyers in advance. In real practice, the real selling time might be longer than the desired selling time. Therefore, this constraint is formed with the aim to reduce the possibility of this incident as much as possible. This constraint could be defined as follows:

$$P(RST_{ti} \geq DST_{ti}) \leq \gamma \ \text{ for } \ t = 1, \ 2, \ldots, T, \ \ i = 1, \ 2, \ldots, \ n \tag{12}$$

It is assumed that the selling time has a normal distribution with mean $\mu_{\mathrm{ST}}$ and standard deviation $\sigma_{\mathrm{ST}}$,

$$DST_{ti} \geq F^{-1}(1-\gamma)\sigma_{\mathrm{ST}} + \mu_{\mathrm{ST}} \ \text{ for } \ t = 1, \ 2, \ldots, T, i = 1, \ 2, \ldots, n \tag{13}$$

where $F^{-1}(1-\gamma)$ implies the inverse cumulative probability of $1-\gamma$.

### 4.3. *Objective function*

The objective of the fleet management problem is to maximize the operational profit of the airlines by determining the number and types of aircraft that should be purchased or leased to meet the stochastic demand. The operational profit could be derived by subtracting the total operating cost from the total revenue. For an airline, the total revenue is generated from the operational income (i.e., the sales of the flight tickets) and the sales of aging aircraft. The total operating cost is formed by the operational cost of aircraft, purchase and lease cost of aircraft, maintenance cost of aircraft, depreciation expenses of aircraft, payable deposit of aircraft acquisition and leasing, and fuel expenses.

For the operating period $t$, the total revenue, $TR\left(I_t^P + I_t^L\right)$, is expressed as follows:

$$TR\left(I_t^P + I_t^L\right) = E\left(fare_t^S\right)D_t^S + \sum_{i=1}^{n}\sum_{y=1}^{m} sold_{tiy} resale_{tiy} \ \text{ for } \ t = 1, 2, \ldots, T, S = s_1, \ \ s_2, \ldots, s_k \tag{14}$$

The first term on the right-hand side of Equation (14) indicates the expected income from the sales of flight tickets by considering stochastic demand $D_t^S$. The second term signifies the revenue from the sales of aging aircraft.



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The total operating cost for the operating period $t$, $TC\left(I_t^P + I_t^L\right)$, is formed as follows:

$$
\begin{aligned}
TC\left(I_t^P + I_t^L\right) = \; & E\left(\mathrm{cost}_t^S\right) D_t^S + \sum_{i=1}^{n} u_{ti} + (purc_{ti})\left(x_{ti}^P\right) + \sum_{i=1}^{n} lease_{ti}\left(x_{ti}^L\right) + \sum_{i=1}^{n} hgf\left(D_t^S, A_t^i\right) \\
& + \sum_{i=1}^{n}\sum_{y=1}^{m}\left(In_{tiy}^P\right)\left(dep_{tiy}^P\right) + \sum_{i=1}^{n}\sum_{y=1}^{m}\left(In_{tiy}^L\right)\left(dep_{tiy}^L\right) + \sum_{i=1}^{n} dp_{ti}\left(x_{ti}^P\right) + \sum_{i=1}^{n} dl_{ti}\left(x_{ti}^L\right) \\
& + \sum_{i=1}^{n} C(fuel_{ti}) \;\text{ for }\; t = 1,\; 2, \ldots, T, S = s_1,\; s_2, \ldots, s_k
\end{aligned} \tag{15}
$$

The terms on the right-hand side of Equation (15) denote the expected operational cost of aircraft, purchase cost of aircraft, lease cost of aircraft, maintenance cost of aircraft, depreciation expenses of aircraft, payable deposit of aircraft acquisition and leasing, and fuel expenses, respectively.

### *4.4. The probable phenomenon indicator, $s_1, \ldots, s_k$*

Airlines encounter many challenging uncertainties, for instance, the occurrence of natural disaster, economic downturn, and outbreak of diseases, which are unpredictable in nature. In accordance with the occurrence of uncertainties (risks), an efficient risk management is necessary. According to Malaysia Airlines (MAS) 12, a risk management process produces a risk map and likelihood scale for the management to prioritize the action plans in mitigating the possible risks. This highlights that different actions may be required to solve different issues and a particular issue may be handled differently at different times. This signifies that the level of stochastic demand that is relatively influenced by the risks (uncertainties) could be outlined similarly, that is, in terms of the likelihood scale. As such, the probable phenomena, $s_1, \ldots, s_k$ for a total of $k$ phenomena, are defined to describe the possible scenario of aircraft possession in meeting the stochastic demand under uncertainty. The probability of probable phenomena, $p_{s_1}, \ldots, p_{s_k}$, quantifies the likelihood (probability) of meeting the stochastic demand. In other words, they define how well the supply (aircraft of the airline) meets the demand, which in fact measures the level of service of airlines. Preferably, the number of operating aircraft should be available adequately to meet a certain desired service level.

If the probable phenomena and its probability are not defined, it means that the airlines only deal with one possible scenario of stochastic demand, that is, they have perfect confidence that a certain level of stochastic demand will occur for a particular operating period during the planning horizon. However, this should not be the case as there is no perfect forecasting of the future. As such, this indicator is necessary to take into consideration the uncertainty of the forecast. For example, if the probability of probable phenomena is 60% and 40%, it signifies that the aircraft acquisition and leasing decision of the airlines is able to handle the circumstances for which the stochastic demand only happens with the probability of 60% and 40%.

The probable phenomena and its corresponding probability could be estimated on the basis of the decision policy of the airlines, the qualitative judgment of the experts or consultants, and the past operational performance of the airlines. The decision policy primarily refers to the compliant business strategies and corporate framework that have been practiced closely by the decision makers, that is, the managerial board of the airlines. The qualitative judgment of the experts could be obtained by carrying out the questionnaire survey study. The past operational performance includes the records of the number of travelers and the travel trend, which is associated closely to the number of aircraft operated by the airlines.

The number of probable phenomena varies depending on the perception and consideration of the airlines in their decision making. In this study, two probable phenomena are considered for two major aspects. First, the operational aspect refers to the relevant perspectives such as high maintenance cost of the aging aircraft, routes that could be flown with a particular aircraft, ability to secure necessary approvals to fly particular routes, potential risks/operating difficulties of aircraft type, and others. Second, the economy aspect covers the perspectives of the benefits of shareholders, cash balance, debt/lease financing, and economic benefits of new aircraft such as fuel efficiency, high capacity, low maintenance costs, and so on. These are the key considerations of the airlines in fleet management 34,12.



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### 4.5. The optimization model

In summary, the fleet management optimization model is presented as

$$
\text{For } t = 1, 2, \ldots, T
$$

$$
P\left(I_t^P + I_t^L\right) = \max_{X_t} (1+r_t)^{-t} \left\{ \begin{array}{l} p_{s_1} \left( \begin{array}{l} E(fare_t^{s_1}) D_t^{s_1} + \sum_{i=1}^{n}\sum_{y=1}^{m} sold_{tiy} resale_{tiy} - E\left(\cos t_t^{s_1}\right) D_t^{s_1} - \\ \sum_{i=1}^{n} u_{ti} + \left(purc_{ti}\right)\left(x_{ti}^P\right) - \sum_{i=1}^{n} lease_{ti}\left(x_{ti}^L\right) - \sum_{i=1}^{n} hgf\left(D_t^S, A_t^i\right) - \\ \sum_{i=1}^{n}\sum_{y=1}^{m}\left(In_{tiy}^P\right)\left(dep_{tiy}^P\right) - \sum_{i=1}^{n}\left(In_{ti}^L\right)\left(dep_{ti}^L\right) - \sum_{i=1}^{n} dp_{ti}\left(x_{ti}^P\right) - \\ \sum_{i=1}^{n} dl_{ti}\left(x_{ti}^L\right) - \sum_{i=1}^{n} C\left(fuel_{ti}\right) \end{array} \right) + \ldots + \\ p_{s_k} \left( \begin{array}{l} E(fare_t^{s_k}) D_t^{s_k} + \sum_{i=1}^{n}\sum_{y=1}^{m} sold_{tiy} resale_{tiy} - E\left(\cos t_t^{s_k}\right) D_t^{s_k} - \\ \sum_{i=1}^{n} u_{ti} + \left(purc_{ti}\right)\left(x_{ti}^P\right) - \sum_{i=1}^{n} lease_{ti}\left(x_{ti}^L\right) - \sum_{i=1}^{n} hgf\left(D_t^S, A_t^i\right) - \\ \sum_{i=1}^{n}\sum_{y=1}^{m}\left(In_{tiy}^P\right)\left(dep_{tiy}^P\right) - \sum_{i=1}^{n}\left(In_{ti}^L\right)\left(dep_{ti}^L\right) - \sum_{i=1}^{n} dp_{ti}\left(x_{ti}^P\right) - \\ \sum_{i=1}^{n} dl_{ti}\left(x_{ti}^L\right) - \sum_{i=1}^{n} C\left(fuel_{ti}\right) \end{array} \right) + P_{t+1}\left(I_t^P + I_t^L\right) \end{array} \right\} \quad (16)
$$

subject to Equations (5)–(9), (11), and (13), where $D_t^S, X_t^P, X_t^L, I_t^P, I_t^L, SOLD_t, O_t, R_t \in \mathbb{Z}^+ \cup \{0\}$. The term $(1 + r_t)^{-t}$ is used for the discounted value across the planning horizon whereas $k$ indicates the $k$th possible phenomenon for having $I_t^P$ and $I_t^L$ as the aircraft at the beginning of each operating period. The optimal decision (output) of the model, that is, the optimal number of aircraft to be purchased and leased, could be used as the inputs in optimizing other operational decisions of the airlines, such as optimization of fleet routing, flight scheduling, and crew assignment 11.

#### 4.5.1. Lower bound and optimal solutions

The solution for the decision variable in model (16) is found to be influenced by the demand constraint (Equation (6)). In case the change in demand is nonpositive (i.e., no increment of demand), the lower bound of the solution is 0. This is because the decision variable defined is nonnegative, that is, $x_{ti}^P, x_{ti}^L \geq 0$, and the total of $n$ types of aircraft to be purchased and leased is also nonnegative, that is, $\sum_{i=1}^{n} x_{ti}^P + x_{ti}^L \geq 0$, for a particular operating period. In case the change in demand is positive (i.e., demand increases), the lower bound will be governed by the demand constraint (Equation (6)). This is to ensure that the supply of the aircraft (via acquiring or leasing) meets the level of demand at a certain desired service level. Nevertheless, the upper bound (*UB*), that is, the maximum aircraft that could be purchased (or leased), will be subject to the availability of the aircraft in the market, $\Sigma_{\forall n}\mathrm{ORDER_t}$, which is expressed in the order delivery constraint (Equation (9)). To summarize, the lower bound, *LB*, of the optimization model follows

$$
LB = \begin{cases} X_t^P = 0, \ \ X_t^L = 0 & \text{if } \ \Delta D_t^S \leq 0 \\ \left( \sum_{i=1}^{n} (SEAT_i)\left(f\left(D_t^S, A_t^i\right)\right) \geq (1-\alpha) D_t^S \right) \cap \left(x_{ti}^P \leq ORDER_t\right) & \text{if } \ \Delta D_t^S > 0 \end{cases} \quad (17)
$$

where $\Delta D_t^S$ indicates the change of demand from year to year, that is, $\Delta D_t^S = D_t^S - D_{t-1}^S$.

Let $\Omega = \left\{X_{ti}^P, X_{ti}^L : LB \leq X_{ti}^P, X_{ti}^L \leq UB\right\}$ be the set of decision variables for the aircraft fleet management model and the operational profit (i.e., the objective function to be maximized) of the developed model be $P\left(I_t^P + I_t^L\right)$, where $\Omega \subseteq I_t^P \cup I_t^L$. In such a case, the optimal solution of the developed model could be written as $P^*\left(I_t^{P*} + I_t^{L*}\right)$, where $P^*$ is the optimum (maximum) profit of each operating period $t$ for which $I_t^{P*}$ and $I_t^{L*}$ denote the corresponding total of aircraft (including the aircraft to be purchased



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and leased) that maximizes $P\left(I_t^P + I_t^L\right)$. As such, the optimal solution (i.e., maximum operational profit) of the developed model could be written as follows:

$$P^*\left(I_t^{P*} + I_t^{L*}\right) = \underset{I_t}{\mathrm{Max}}\, P\left(I_t^P + I_t^L\right) \tag{18}$$

## 5. SOLUTION METHOD

The proposed probabilistic dynamic programming model can be solved by decomposing it into a series of simpler subproblems. By using the backward working method, the subproblem at the last period of the planning horizon $T$ is solved first. The current optimal solution found for the states at the current stage leads to the problem solving at the period of $T-1$, $T-2$, …, 1. This procedure continues until all the subproblems have been solved optimally so that the decision policy to purchase and/or lease aircraft can be determined strategically. For the developed optimization model, the type of the solution method, that is, linear programming problem or nonlinear programming problem, can be identified clearly from the function of the number of flights, $f\left(D_t^S, A_t^n\right)$; function of the traveled mileage, $gf\left(D_t^S, A_t^n\right)$; function of the maintenance cost, $hgf\left(D_t^S, A_t^n\right)$; function of fuel expenses, $C(fuel_{tn})$; and the practical constraints (5)–(9), (11), and (13). If they are in the form of linear function in terms of the decision variables, then model (16) will be solved as a linear programming model. Otherwise, it is converted as a nonlinear programming model. The linearity of these components is primarily based on the operational data of a particular airline. It shall then be validated by using the regression test with the aid of mathematical software. For the illustrative case study as shown in the following section, a nonlinear relationship was adopted for the aforementioned components as the regression relationship obtained from the published reports 12,13 show nonlinearity. Powell 35 specified that nonlinear programming is the possible solution for the dynamic programming model. Nonetheless, it could not be solved directly with any available conventional methods. The spreadsheet functionality of Excel 2007 coupled with its own developed algorithm was utilized to work out the optimal solutions.

For a larger size of the fleet management decision model, the proposed solution method is still feasible in generating computational results. However, the computational efficiency reduces when the problem size gets larger because of additional modeling parameters and variables. As such, more computational effort is necessary for larger state and stage spaces. There are two major concerns that could affect the computational efficiency, that is, the planning horizon and types of aircraft. The extension of planning horizon $T$ would result in an increment ratio of $1/T$, that is, an additional 10–20% of computational effort for each increment (in year). For each additional type of aircraft, there is $(ORDER_t + 1)$ times more computational time required, where $ORDER_t$ refers to the order delivery constraint. For this study, the computational time required is approximately 50–60 s for each operating period.

## 6. AN ILLUSTRATIVE CASE STUDY

### *6.1. Data description*

This subsection explains the types of data and their values used in the illustrative case study. Most of the values are chosen from published reports and accessible websites of the airlines to design a close-to-reality case study.

#### *6.1.1. Inputs for stochastic demand modeling*

Three types of events, that is, biological disaster (e.g., flu disease), economic recession, and natural disaster (e.g., storm), are assumed to affect the demand level. The modeling of the probability distributions to quantify the uncertainties is carried out on the basis of the published reports. According to the data obtained from the Centre for Research on the Epidemiology of Disasters 36, the occurrence of a biological disaster follows the Poisson distribution and has a mean $\mu$ of 7, that is, Prob (biodisaster) ~ Pois($\mu = 7$). This indicates that the biological disaster happens seven times in average in a year. On the basis of the data from the International Monetary Fund 37,38, it is found that the occurrence of economic downturn also follows the Poisson distribution in which Prob



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(econ-downturn) ~ Pois($\mu = 1/9$). This shows that the economic recession happens once in an average of 9 years. For a natural disaster to occur 36,38, the probability of occurrence is found to follow a normal distribution, that is, Prob(natural disaster) ~ N(64, 8).

For the travel growth projection, historical data show that the growth percentage ranges from 5% to 9% 39,40,12,41. As such, we assume an equal probability for each unit of growth, that is, the percentage growth of 5%, 6%, 7%, 8%, and 9% has a probability of happening during the planning horizon of 0.2. However, there is no restriction if uneven probability is assumed. Besides, the forecasted demand $D_f$ is estimated to follow a normal distribution, that is, $D_f \sim N(1.4141 \times 10^7, 9.04 \times 10^{12})$, according to the data obtained from the published reports from MAS 12. With the aid of the convolution algorithm, the projected demand for base year $D_0$ is then determined.

From the aforementioned data, the level of stochastic demand for each operating period throughout the planning horizon is obtained by applying the modeling framework of stochastic demand as mentioned in Section 3. The detailed output of the stochastic demand is shown in Table I. Table I reveals the fact that the possible occurrence of uncertainty and the predicted growth of travel demand could affect the level of stochastic demand at varying degrees. The SDI value is greater than 1 when the uncertainty does not exist. Conversely, the existence of uncertainty gives the SDI with the value of at most 1.

#### *6.1.2. Inputs for the fleet management model*

In this case study, two types of aircraft, that is, A320-200 ($n = 1$) and A330-300 ($n = 2$), are considered for a set of origin–destination pairs. Only two types of aircraft are considered as many of the low-cost carriers operate their business with few varieties of aircraft types, for example, AirAsia (A320-214 and A320-216), Jetstar Airways (A320-200, A321-200, and A330-200), JAL Express (B737-400 and B737-800), and Tiger Airways (A320-200) (see 42 for more examples). Furthermore, the airlines tend to operate the aircraft from the same aircraft manufacturer (mostly Airbus or Boeing). Therefore, two types of aircraft (both Airbus) as considered in the case study are practical. A320-200 and A330-300 (i.e., the aircraft of Airbus) were chosen as examples as there are more available information for these types of aircraft. The proposed methodology is not restricted to the number and type of aircraft used. In addition, a planning horizon of 8 years is also justified as according to MAS 12 and AirAsia Berhad 13, on average, the acquisition of new aircraft requires a period of 5 years to be completely delivered. Besides, the desired lead time is assumed to have a normal distribution with an average of 3 years and a standard deviation of 1.5, that is, $DLT \sim N(3, 1.5)$. As such, two types of aircraft, which are considered for a planning horizon of 8 years, are reasonably practical to reflect the real operation of the airlines. Tables II and III show the input data of the model.

The capacity of A320-200 and A330-300 is assumed to be 180 (with a total size of 1282 m$^2$) and 295 (with a total size of 3836 m$^2$), respectively 43,44. The expected flight fare and cost as shown in Table II are generated from the available financial reports of MAS 45. In addition, the purchase prices of aircraft as shown in Table III are obtained from the published data of Airbus 46. With the purchase price and the estimated useful life of aircraft (i.e., 5 years), the depreciation values of aircraft are calculated using the straight-line depreciation approach. By considering the residual value as practiced by AirAsia 47, the resale price and the depreciation value (as shown in Table III) are obtained from the assumed residual value (i.e., salvage cost) of aircraft, which is 10% of the purchase cost. For aircraft leasing, the respective lease cost, residual value, and depreciation value are obtained by referring to the finance lease of MAS 48.

### *6.2. Benchmark scenario*

A benchmark scenario is created to test the applicability of the proposed methodology. The data input can be categorized into three categories, that is, by definition, by assumption, or by assumption based on real data. They are shown as follows:

By definition:

- Two possible phenomena are considered, where $k = 2$ for model (16).
- The discount rate is $r_t = 5\%$ for $t = 1, 2, \ldots, T$.
- The significance level of demand constraint is $\alpha = 5\%$.



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Table I. The output of stochastic demand.

| Operating period | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Possible occurrence of uncertainty (Y = it exist, N = it does not exist) | N | N | Y (biological disaster) | N | Y (biological disaster, economic recession) | N | N | N |
| Probability of uncertainty [1] | 0.00 | 0.00 | −0.40 | 0.00 | −0.07 | 0.00 | 0.00 | 0.00 |
| Probability of the possible increment of forecasted demand [2] | 0.09 | 0.09 | 0.08 | 0.05 | 0.07 | 0.08 | 0.08 | 0.06 |
| Total of probability [1] + [2] | 0.09 | 0.09 | −0.32 | 0.05 | 0.00 | 0.08 | 0.08 | 0.06 |
| Index of stochastic demand, SDI | 1.09 | 1.09 | 0.68 | 1.05 | 1.00 | 1.08 | 1.08 | 1.06 |
| Stochastic demand (persons) | 17,332,733 | 18,892,678 | 12,847,021 | 13,489,372 | 13,543,330 | 14,626,796 | 15,796,940 | 16,744,756 |

Table II. The expected value of flight fare and flight cost per passenger.

| | Operating period, $t$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| $E(fare_t^{s_1})$ ($) | 235 | 243 | 254 | 263 | 273 | 284 | 294 | 304 |
| $E(fare_t^{s_2})$ ($) | 205 | 216 | 228 | 237 | 246 | 256 | 265 | 274 |
| $E(cost_t^{s_1})$ ($) | 152 | 158 | 162 | 167 | 171 | 176 | 181 | 186 |
| $E(cost_t^{s_2})$ ($) | 135 | 140 | 146 | 150 | 154 | 158 | 163 | 167 |

Table III. The resale price, depreciation value, purchase cost, lease cost and residual value (million dollars).

| $y$ | $resale_{t1y}$ | $resale_{t2y}$ | $dep_{t1y}^P$ | $dep_{t2y}^P$ | $purc_{t1}^P$ | $purc_{t2}^P$ | $dep_{tny}^L$ | Residual value | $lease_{tn}$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 67.24 | 174.66 | 14.76 | 38.34 | 82 | 213 | 26.66 | 121.43 | 148.09 |
| 2 | 52.48 | 136.32 | 14.76 | 38.34 | | | 26.66 | 94.77 | |
| 3 | 37.72 | 97.98 | 14.76 | 38.34 | | | 26.66 | 68.12 | |
| 4 | 22.96 | 59.64 | 14.76 | 38.34 | | | 26.66 | 41.46 | |
| 5 | 8.2 | 21.3 | 14.76 | 38.34 | | | 26.66 | 14.81 | |
| Average | | | 14.76 | 38.34 | 82 | 213 | 26.66 | 68.12 | 148.09 |

- The significance level of lead time constraint is $\beta = 5\%$.
- The significance level of selling time constraint is $\gamma = 5\%$.

$$D_t^{s_1} = D_t \text{ and } D_t^{s_2} = (1-\alpha)D_t^{s_1} \text{ for } t = 1, 2, \ldots, T \tag{19}$$

By assumption:

- At $t = 1$, the probability to have the aircraft is $p_{s_1} = 0.5$ and $p_{s_2} = 0.5$.
- At $t = 1$, the initial number of A320-200 and A330-300 that is 3 years old is $In_{113}^P = In_{123}^P = 4$.
- The setup cost to acquire $n$ types of new aircraft is $u_{ti} = 0$ for $t = 1, 2, \ldots, T$, $i = 1, 2, \ldots, n$.

By assumption (based on real data):

- At $t = 1$, the initial number of A320-200 and A330-300 is $In_{11}^P = 50$ and $In_{12}^P = 50$, respectively (i.e., $In_{11}^L = In_{12}^L = 0$).
- The allocated budget is $MAX_{\text{budget}(t)} = \$6,500,000,000$.
- The area of hangar is $PARK_t = 500,000\ \text{m}^2$.
- The order delivery constraint is $ORDER_t = 5$.
- The salvage cost of an aircraft is 10% of the purchase cost of the aircraft.
- The deposit of aircraft acquisition, $DP_t$, is 10% of the purchase cost of aircraft for $t = 1, 2, \ldots, T$.
- The deposit of aircraft leasing, $DP_t$, is 10% of the lease cost of aircraft for $t = 1, 2, \ldots, T$.
- For $n$ types of aircraft, the function of the number of flights is

$$f = 22.57\left(A_t^n\right)^2 - 9.776 \times 10^2 A_t^n + 7.83 \times 10^4, \quad t = 1, 2, \ldots, T \quad \left[R^2 = 0.97\right] \tag{20}$$

- The function of the traveled mileage is

$$g = 2,066f - 2,875,383 \quad \left[R^2 = 0.83\right] \tag{21}$$

- The function of the maintenance cost is

$$h = 5.177 \times 10^3 + 7.97 \times 10^{-3} g \quad \left[R^2 = 0.94\right] \tag{22}$$

- For $n$ types of aircraft, the function of fuel expenses is

$$C(fuel_{tn}) = 7.46f + 8.3 \times 10^{-5} f^2 - 98,572 \quad \left[R^2 = 0.88\right] \tag{23}$$



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- The number of aircraft is

$$NA = 10^{-5}NP - 73.6 \quad \left[R^2 = 0.92\right] \tag{24}$$

where $NP$ is the number of travelers.

From the data reported by MAS 12 and AirAsia Berhad 47, Equations (20)–(24) are obtained by conducting a polynomial regression analysis 49. Equations (20)–(22) are anticipated to be correlated with stochastic demand, $D_t^S$, and the total operated aircraft, $A_t^n$. The regression analysis shows that Equations (20)–(22) are fitted fairly well as nonlinear functions in terms of $A_t^n$. Similarly, the analysis reveals that Equation (23) is best fitted as a quadratic function in terms of the number of flights, which could be consequently, expressed as a non-linear function in terms of $A_t^n$ via Equation (20). Besides, the regression analysis exhibits that Equation (24) is best fitted as a linear function in terms of the number of travelers.

Equation (19) implies the proportion of stochastic demand, which corresponds to the phenomenon of $s_1$ and $s_2$. Equation (20) indicates that the number of flights is affected by the total operated aircraft, which is gained from the aircraft acquisition and leasing. Equation (21) denotes that a flight flies 2066 km in average. Equation (22) signifies that a unit cost of 0.00797 is charged as maintenance cost for each additional unit of mileage traveled. For this equation, \$5177 indicates an overall estimated maintenance cost without considering an additional traveled mileage. Equation (23) shows that total of fuel expenses depends on the number of flights, which are operated during the planning horizon. This implies that the fuel expenses associate with the total operated aircraft, $A_t^n$, which depends on the aircraft acquisition and leasing decision. Equation (24) displays that every addition of 100,000 travelers requires one additional aircraft. In other words, one traveler requires 0.00001 aircraft.

According to Meyer and Krueger 49, the intercept of the regression equation carries no practical meaning if the range of the independent variable does not include 0. The number of flights, $f$, in Equation (20) falls within the range of $67{,}460 \le f \le 79{,}927$ (based on the real data). Accordingly, the constant in Equation (20) has no practical interpretation. In addition, it can be shown that the traveled mileage in Equation (21) is always positive. Such an explanation is also applicable to Equations (20), (23), and (24).

For $t = T = 8$, the developed optimization model could be simplified to models (25)–(33) as

$$\begin{aligned} & P(I_8^P + I_8^L) \\ & = \max_{X_8} \frac{1}{(1.05)^8} \left[ \begin{array}{l} p_{s_1} \left( \begin{array}{l} 118 D_8^{s_1} + \left(8.2\text{x}10^6 sold_{815} + 2.13\text{x}10^7 sold_{825}\right) - \left(8.2\text{x}10^7 x_{81}^P + 2.13\text{x}10^8 x_{82}^P\right) - \\ 2.67\text{x}10^7 \left(x_{81}^L + x_{82}^L\right) - \left(5.177\text{x}10^3 + 7.97\text{x}10^{-3} g\right) - \\ \left(1.476\text{x}10^7 In_{81}^P + 3.834\text{x}10^7 In_{82}^P\right) - 2.67\text{x}10^6 \left(In_{81}^L + In_{82}^L\right) - \\ \left(8.2\text{x}10^6 x_{81}^P + 2.13\text{x}10^7 x_{82}^P\right) - 1.48\text{x}10^7 \left(x_{81}^L + x_{82}^L\right) - \left(7.46 f + 8.3\text{x}10^{-5} f^2 - 98{,}572\right) \end{array} \right) + \\ p_{s_2} \left( \begin{array}{l} 101.65 D_8^{s_2} + \left(8.2\text{x}10^6 sold_{815} + 2.13\text{x}10^7 sold_{825}\right) - \left(8.2\text{x}10^7 x_{81}^P + 2.13\text{x}10^8 x_{82}^P\right) - \\ 2.67\text{x}10^7 \left(x_{81}^L + x_{82}^L\right) - \left(5.177\text{x}10^3 + 7.97\text{x}10^{-3} g\right) - \\ \left(1.476\text{x}10^7 In_{81}^P + 3.834\text{x}10^7 In_{82}^P\right) - 2.67\text{x}10^6 \left(In_{81}^L + In_{82}^L\right) - \\ \left(8.2\text{x}10^6 x_{81}^P + 2.13\text{x}10^7 x_{82}^P\right) - 1.48\text{x}10^7 \left(x_{81}^L + x_{82}^L\right) - \left(7.46 f + 8.3\text{x}10^{-5} f^2 - 98{,}572\right) \end{array} \right) \end{array} \right] \end{aligned} \tag{25}$$

subject to

$$82x_{81}^P + 213x_{82}^P + 26.7\left(x_{81}^L + x_{82}^L\right) \le 6500 \tag{26}$$

$$22.57\left(A_8^n\right)^2 - 977.6A_8^n + 11{,}321 \ge 0 \tag{27}$$

$$D_8^{s_1} = 16{,}744{,}756 \text{ and } D_8^{s_2} = 15{,}907{,}518 \tag{28}$$

$$\left(In_{81}^P + In_{81}^L + x_{81}^P + x_{81}^L\right)(1{,}282) + \left(In_{82}^P + In_{82}^L + x_{82}^P + x_{82}^L\right)(3{,}836) \le 500{,}000 \tag{29}$$

$$sold_{815} \le In_{81}^P, \; sold_{825} \le In_{82}^P \tag{30}$$



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$$x_{81}^{P} \leq 5, x_{82}^{P} \leq 5 \tag{31}$$

$$DLT_{81} \geq 32, DLT_{82} \geq 32 \tag{32}$$

$$DST_{81} \geq 24, DST_{82} \geq 24 \tag{33}$$

$D_8^S$, $X_8^P$, $D_8^L, D_8^P, D_8^L, SOLD_8, O_8, R_8 \in Z^+ \cup \{0\}$ and $A_t^n = In_{81}^P + In_{82}^P + In_{81}^L + In_{82}^L + x_{81}^P + x_{82}^P + x_{81}^L + x_{82}^L$. Equation (26) takes the budget constraint of \$6.5 × 10$^9$ to purchase and/or to lease aircraft. If we apply the simulation approach as elaborated earlier, the stochastic demand simulated for $t = 8$ is 16,744,756. With a 95% confidence level, it is found that the total number of aircraft that should be operated for this operating period appears to be a nonlinear function, which is indicated in Equation (27). Equation (28) indicates that the stochastic demand for $t = 8$ is predicted to be 16,744,756 for the probable phenomenon of $s_1$ and 15,907,518 for the probable phenomenon of $s_2$, which is derived by Equation (19). Equation (29) is the parking constraint as a geometry limitation; Equation (30) is the sales of aircraft constraint, which is derived with the assumption that an aircraft of 5 years old or more are considered to be sold; thus, $sold_{815} \leq In_{714}^P$ and $sold_{825} \leq In_{724}^P$. Because $In_{714}^P \leq In_{81}^P$ and $In_{724}^P \leq In_{82}^P$, these expressions subsequently result in $sold_{815} \leq In_{81}^P$ and $sold_{825} \leq In_{82}^P$ as could be seen in Equation (30). Equation (31) indicates the order delivery constraint to purchase new aircraft. With the assumed normal distribution of $RLT_{8n} \sim$ N(2,0.4) and $RST_{8n} \sim$ N(1.5,0.3), Equations (32) and (33) represent the lead time and selling time constraints, respectively, for which the desired period to order new aircraft is at least 32 months (i.e., 2.66 ≈ 3 years) whereas the desired period to release aging aircraft for sales is at least 24 months, that is, 2 years in advance. For model (25), the functions of the number of flights, traveled mileage, maintenance cost, and fuel expenses as depicted by Equations (20)–(23) are found to be nonlinear functions in terms of the total operated aircraft, $A_t^n$. Hence, the developed model (25) is solved as a nonlinear programming model. By working backwards, the procedure can be repeated to formulate the optimization model for the operating period, $t = 7, 6, 5, 4, 3, 2, 1$.

### *6.3. Sensitivity analysis*

To investigate the impact of changes of the inputs to the computational results, six scenarios with variations to some of the modeling parameters used in the benchmark scenario are developed. The following lists the outlined scenarios.

- Scenarios A and B have the confidence level of 90% and 99%, respectively.
- Scenarios C and D have the probable phenomenon indicators (i.e., probability of having the aircraft) of 0.6:0.4 and 0.4:0.6, respectively.
- Scenarios E and F have the order delivery constraints $ORDER_t = 4$ and 6, respectively.

## 7. RESULTS AND DISCUSSIONS

### *7.1. Results for benchmark scenario*

The computational results of the benchmark scenario are shown in Table IV. Table IV shows a consistently increasing trend on the discounted annual profit except where there is a decrease in the stochastic demand or when a cost is charged to purchase new aircraft, to lease aircraft, or to order new aircraft in advance. In particular, the operating period from 1 to 3, which involves aircraft leasing and higher demand, produce a higher operational profit as compared with the subsequent operating periods. For the operating period with aircraft acquisition, that is, the operating period from 4 to 8, the earned profit for the airline increases gradually, mainly because of an increment in stochastic demand. This shows that the proposed methodology is capable of capturing the demand uncertainty in real practice when producing optimal profit. Certainly, this would provide a better insight for the airlines when making a decision to manage their fleet under the inconsistency of demand subject to the operational constraints as elaborated earlier.



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Table IV. The results of benchmark scenario.

| Operating period, *t* | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| Initial number of aircraft owned | A320-200 | 50 | 50 | 46 | 46 | 49 | 52 | 56 | 60 |
| | A330-300 | 50 | 50 | 46 | 46 | 46 | 46 | 47 | 48 |
| Initial number of leased aircraft | A320-200 | 0 | 0 | 5 | 5 | 5 | 5 | 5 | 5 |
| | A330-300 | 0 | 0 | 5 | 5 | 5 | 5 | 5 | 5 |
| Number of aircraft to be ordered | A320-200 | 3 | 3 | 4 | 4 | 5 | 0 | 0 | 0 |
| | A330-300 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| Number of aircraft to be received | A320-200 | 0 | 0 | 0 | 3 | 3 | 4 | 4 | 5 |
| | A330-300 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| Number of aircraft to be leased | A320-200 | 0 | 5 | 0 | 0 | 0 | 0 | 0 | 0 |
| | A330-300 | 0 | 5 | 0 | 0 | 0 | 0 | 0 | 0 |
| Number of aircraft to be released for sales | A320-200 | 4 | 0 | 0 | 0 | 0 | 0 | 3 | 3 |
| | A330-300 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Number of aircraft to be sold | A320-200 | 0 | 0 | 4 | 0 | 0 | 0 | 0 | 0 |
| | A330-300 | 0 | 0 | 4 | 0 | 0 | 0 | 0 | 0 |
| Total operated aircraft | | 100 | 110 | 102 | 105 | 108 | 113 | 118 | 124 |
| Stochastic demand | | 17,332,733 | 18,892,678 | 12,847,021 | 13,489,372 | 13,543,330 | 14,626,796 | 15,796,940 | 16,744,756 |
| Discounted annual profit (million dollars) | | 589 | 224 | 411 | 150 | 158 | 123 | 103 | 267 |



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### 7.2. Results for sensitivity analysis

The graphical results of scenarios A–F are illustrated in Figures 1–3. The results of scenarios A and B (in Figure 1) display that when the confidence level changes, it has an impact on the operational profit. The confidence level signifies the service level (i.e., level of demand) targeted by the airlines, and hence, the profit is affected if the targeted service level changes. Apart from this, the results of scenarios A and B established the fact that a higher profit is gained when the value of confidence level increases, that is, when the level of service rises. The results also show that there is a tendency for the airlines to purchase and/or to lease more aircraft to meet a higher level of demand but subject to operational constraints. In particular, for operating periods 5, 6, and 8, the operational profit of the benchmark problem is higher than that of scenario B because of the aircraft acquisition decision to meet a higher demand. Overall, the sensitivity results show that the airlines have to make their operational decision wisely as well as to set their target properly to maximize operational profit.

Figure 2 shows the sensitivity results in setting the probability of the probable phenomenon. Scenario C has the probability of 0.6:0.4, scenario D has the probability of 0.4:0.6, and the benchmark scenario is 0.5:0.5. The results show that scenario C, which has the highest probability in meeting the demand (i.e., highest level of service) could yield the highest operational profit, which is in average 21% more than scenario D and 11% more compared with the benchmark scenario. Comparatively, the benchmark scenario generates 12% more profit than scenario D. As such, it could be seen that an increment of approximately 1% of the stochastic demand would generate an

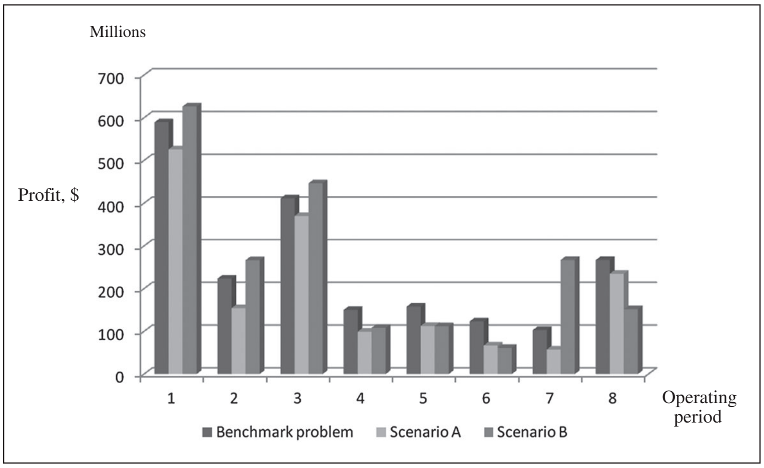


Figure 1. The results of scenarios A and B.

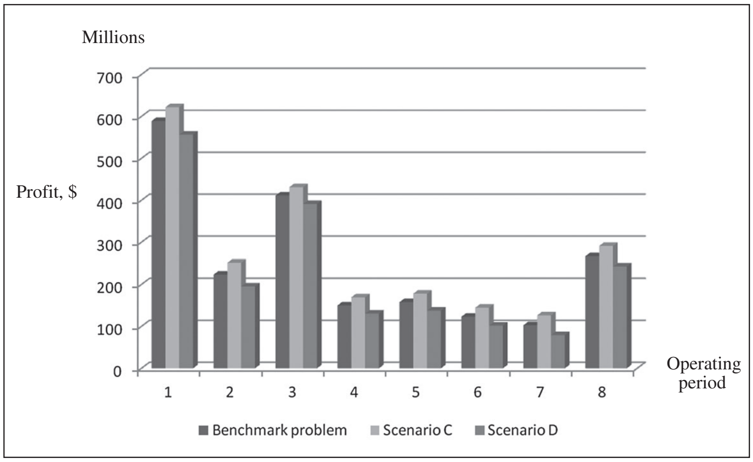


Figure 2. The results of scenarios C and D.



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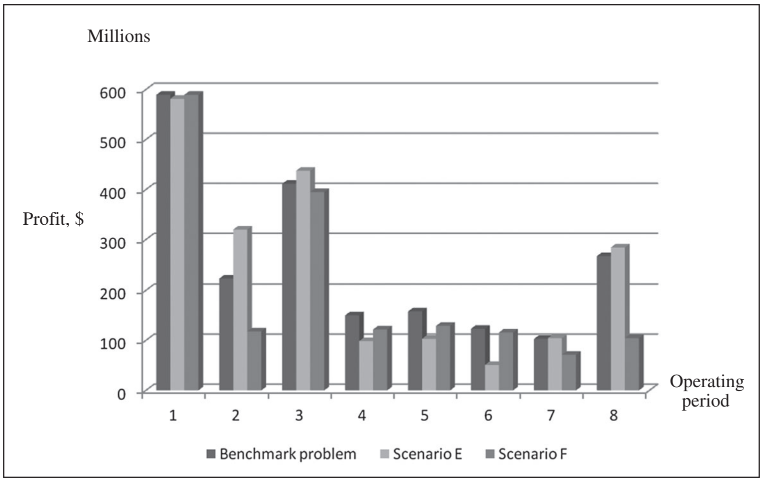


Figure 3. The results of scenarios E and F.

additional 1% of the operational profit. This could be explained by the fact that the service level, which is met at a higher chance (probability) is likely to generate more revenue for the airlines (from the sales of flight tickets). Hence, it could be seen that the probable phenomena and its probability associates closely with the level of stochastic demand, which could affect the operational profit of the airlines.

As displayed in Figure 3, the results of scenarios E and F show that the order delivery constraint could affect the optimal decision and the level of operational profit. The results illustrate that the higher the value of the order delivery constraint is, the lower the profit is. For the operating periods 1, 4, 5, and 6, scenario E produces the lowest profit because of the acquisition deposit and cost that are incurred for aircraft acquisition decision making. Besides, the decision making to purchase and/or to lease aircraft is also affected by the consideration of the airlines in obtaining the least number of aircraft as long as the total number of aircraft is adequate to provide the targeted level of service. Hence, it is important to note that it is not certainly profitable to purchase or lease more aircraft. The decision to purchase (or lease) lesser aircraft probably contributes a higher profit because of the less charged costs.

### 7.3. *Consistency and stability of results*

The consistency and stability of the results could be empirically confirmed by comparing the findings with the actual operational statistics of the airlines 13,12. Table V summarizes the fleet size of the airlines (i.e., AirAsia and MAS) as compiled from their annual reports and the fleet management decision of each operating period as obtained from the model. It could be observed that the fleet size of AirAsia and MAS during the operating years of 2006–2010 falls within the range of 2 standard deviations from its average. The fleet management solutions obtained from the benchmark problem and other scenarios show a similar pattern, that is, the fleet size for the operating periods from 1 to 8 falls within the range of 2 standard deviations from its average. Therefore, the solutions are coherent with the operating performance of the airlines. As such, the findings in this paper are consistent with the actual practice, and hence, the stability of the results (as well as the developed model) could be empirically confirmed.

### 7.4. *Summary*

The results obtained from the model are reasonable and stable when compared with the empirical data. The sensitivity analysis shows that the model and the solutions are sensitive to the choice of parameters. This implies that the values of these parameters need to be chosen with care. In addition, it is important to note that there is no ideal means to obtain a supreme profit as the optimal fleet management decision is affected decisively by several factors, that is, management policy of the airlines (for instance, as reported by MAS 47, as a 100% leased structure is not optimal in the long term, MAS intends to shift to



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Table V. The summary of the fleet management decision.

| | Fleet size | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Empirical (from reports) | | | Model | | | | | | | | |
| | | | | | Scenario | | | | | | | |
| | Year | AirAsia | MAS | $t$ | Benchmark | A | B | C | D | E | F |
| Operating year | 2006 | 42 | 97 | 1 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| | 2007 | 65 | 102 | 2 | 110 | 110 | 110 | 110 | 110 | 108 | 112 |
| | 2008 | 78 | 109 | 3 | 102 | 102 | 102 | 102 | 102 | 100 | 104 |
| | 2009 | 84 | 112 | 4 | 105 | 105 | 106 | 105 | 105 | 104 | 107 |
| | 2010 | 77 | 117 | 5 | 108 | 108 | 110 | 108 | 108 | 108 | 110 |
| | | | | 6 | 113 | 112 | 115 | 113 | 113 | 113 | 115 |
| | | | | 7 | 118 | 117 | 120 | 118 | 118 | 118 | 120 |
| | | | | 8 | 124 | 122 | 125 | 124 | 124 | 123 | 126 |
| Average (AG) | | 69 | 107 | | 110 | 110 | 111 | 110 | 110 | 110 | 112 |
| Standard deviation (SD) | | 17 | 8 | | 8 | 7 | 9 | 8 | 8 | 8 | 8 |
| AG + 2SD | | 103 | 123 | | 126 | 124 | 129 | 126 | 126 | 126 | 128 |
| AG − 2SD | | 35 | 91 | | 94 | 96 | 93 | 94 | 94 | 94 | 96 |

an optimal mix of leased/owned fleet), the desired scenarios to be optimized, and also the occurrence of unpredictable uncertainty. Therefore, to assure an optimal operation and fleet management, the aspects as illustrated earlier should be taken into consideration wisely.

## 8. CONCLUSION

This study proposed a new methodology to solve the fleet management decision model under uncertainty. The methodology comes in threefold. First, a five-step framework, which is incorporated with the SDI, is developed to quantify the demand level under uncertainty for each operating period. Secondly, a probabilistic dynamic programming model is formulated to determine the optimal number and types of aircraft to be purchased and/or leased so that the stochastic demand could be met profitably. Thirdly, a probable phenomenon indicator is defined to ensure that the aircraft possession of the airlines is appropriate at a desired service level. The results of the illustrative case study demonstrated that the proposed methodology is sensitive to the modeling parameters and it is viable in providing an optimal solution for the fleet management decision model.

The proposed study reflects the actual situation of the airline industry, ranging from the challenge of the uncertainty to the practical issues in purchasing and leasing aircraft. Subject to the uncertainty and operational constraints, the proposed study could produce viable solutions for a long-term aircraft acquisition and leasing decision model. For the airlines, this is crucial to ensure the efficiency of management from the operational (by ensuring the supply of aircraft meets the travelers' demand level) and sustainability (by maximizing profits) perspectives.

One of the limitations of this study is that the element of service frequency is partially considered. It could be further extended in the future by incorporating the service frequency as one of the constraints. The computational efforts could be improved by having computational programming, for example, MATLAB, C++, and Visual Basic, for better computational efficiency. In addition, a framework will be derived to illustrate how to quantify the probable phenomena for future study.

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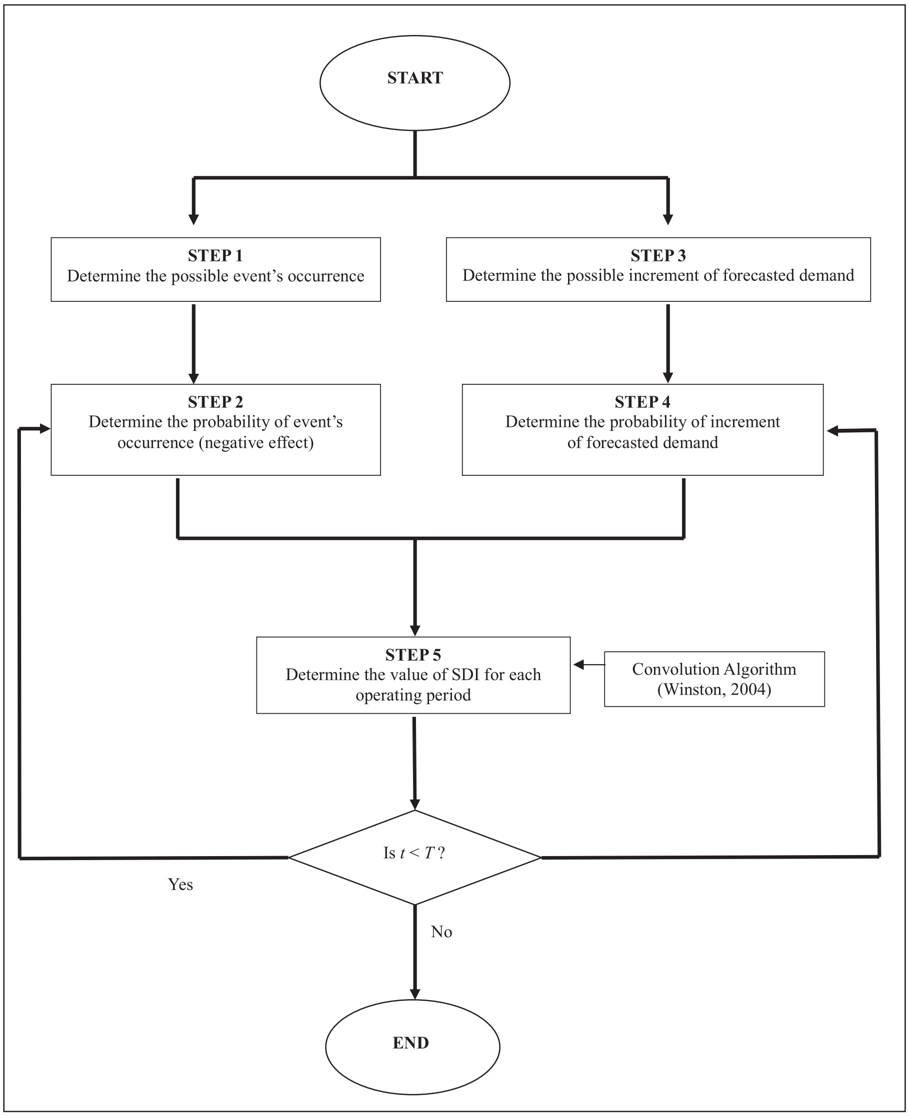
## APPENDIX A: MODELING FRAMEWORK OF STOCHASTIC DEMAND.



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## APPENDIX B: CONVOLUTION ALGORITHM [32].

According to the central limit theorem, the sum $Y$ of $n$ independent and identically distributed random variables (say, $Y_1$, $Y_2$, …, $Y_n$), each with mean $\mu$ and variance $\sigma^2$, is approximately to have a normal distribution with mean $n\mu$ and variance $n\sigma^2$. This implies that the random variable $Y$ could be expressed as

$$Y \tilde{} N\left(n\mu, n\sigma^2\right) \tag{B1}$$

for which $Y = Y_1 + Y_2 + \cdots + Y_n = \sum_{i=1}^{n} Y_i$. With this fact, the normal distribution for the random number $R$ is formed as

$$R \tilde{} N\left(\frac{n}{2}, \frac{n}{12}\right) \tag{B2}$$

for which $R = R_1 + R_2 + \cdots + R_n = \sum_{r=1}^{n} R_r$ is the sum of $n$ random numbers. (Note that each random number has a uniform distribution U(0, 1) with mean 1/2 and variance 1/12).

Corresponding to Equation (B2), in order to generate the standard normal variates for the origin distribution of random number, that is, uniform distribution, the following expression could be formed.

$$z = \frac{\sum_{r=1}^{n} R_r - \frac{n}{2}}{\sqrt{\frac{n}{12}}}. \tag{B3}$$

could be formed, where $z \sim (0, 1)$. To simplify the computational procedure, $n = 12$ is used for Equation (B3), and this results in

$$z = \sum_{r=1}^{12} R_r - 6 \tag{iv}$$

for which $\sum_{r=1}^{12} R_r$ is the sum of 12 random numbers. On the basis of the standard relation of $z = (X - \mu)/\sigma$ for normal distribution, $X = \mu + \sigma z$ is obtained subsequently. As such, for the modeling framework of stochastic demand as elaborated in Section 3, the projected demand, $D_0$, could be formed as

$$D_0 = \mu_f + \sigma_f\left(\sum_{r=1}^{12} R_r - 6\right) \tag{v}$$

for which the forecasted demand, $D_f$, has mean $\mu_f$ and standard deviation $\sigma_f$.

Note that according to Winston 33, $n = 12$ has the advantage to simplify the computational procedure especially the time consumption on a computer. However, there is no problem in using any other value of $n$. In other words, other than $n = 12$, the usage of any other value of $n$ would increase the computational difficulty and hence to avoid the difficulty from this aspect, $n = 12$ is chosen particularly to simplify the computational, by reducing the computational difficulty with the formula.