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Solving the logistic problems with optimal resource assignment using fuzzy logic methods

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SUMMARY

Fuzzy approach of optimal resource assignment regarding the given demands in the scope of transportation will be presented in the article. The basis of the research is the crisp solution which is also presented in the article. The basic solution was upgraded in order to be able to handle vaguely (i.e. fuzzily) defined requirements and resource properties. The optimal resource configuration is calculated with the aid of Hungarian algorithm which uses the data calculated with the fuzzy methods for its inputs. The approach described is presented on the example of a military convoy formation. Copyright © 2011 John Wiley & Sons, Ltd.

KEY WORDS: fuzzy logic; fuzzy assignment; Hungarian algorithm; logistics

1. INTRODUCTION

The assignment of resources and their distribution play a major role in logistics, while their direct consequence are among others amount of consumed fuel and traffic density. The benefits of optimal resource assignment and their distribution are therefore vital. Different optimization methods based on mathematics are used in this process. Majority of these methods are facing two problems:

- Many methods used for optimization are based on exhaustive search in the space of solutions (i.e. space of permissible resource assignments). Size of the space is defined with the number of demands and the number of resources available.
- Many methods used operate with crisp values and are unable to cope with vaguely (i.e. fuzzily) defined terms. On the other hand it is much easier to vaguely define most of the parameters when constructing the demands and characterizing the resources.

We directed our research towards the solution of the second issue. A typical example of a demand which cannot be handled with classical methods is *truck with approximately 5 ton load capacity.* The main problem for classical methods in this case is the word *approximately*, which can be easily dealt with when using of *fuzzy logic*. Fuzzy logic has already been proved to be useful on the field of logistic, e.g. in the meaning of modelling complex traffic and transportation processes [1] and in the solving of transshipment problem [2]. However, different approach is required in the problem we are solving.

Hungarian algorithm [3–5] was used as a base of our research. It was extended in order to support the usage of vaguely defined demands and resources. Fuzzy logic methods [6,7,8,9] were used in order to generalize the basic Hungarian algorithm, which allow us to use the terms such as fuzzy, uncertain, ambiguous, etc. in the definitions of demands and resources. We illustrated the functionalities of our method on the example of the military vehicle convoy formation.

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In Section 2, we present the basic principles of resource assignment with the usage of the basic Hungarian algorithm. Section 3 describes the basics of fuzzy logic. In Section 4, steps used in our resource optimization method based on Hungarian algorithm are described. Section 5 explains this method on the practical example of military vehicle convoy formation. Section 6 evaluates our methods with the comparison among it and ordinary, crisp method.

2. SOLVING THE LOGISTIC PROBLEMS WITH OPTIMAL RESOURCE ASSIGNMENT

2.1. Basic concepts

We are frequently facing the terms such as *logistic problems*, *demands* and *resources* in the field of transportation. We can define a logistic problem as a set of demands. Appropriate resource with required properties has to be assigned to each of the demands in order to solve the given logistic problem. Therefore, we are in general operating with a set of demands D which have to be satisfied with resources from another set R where the necessary requirement (but not sufficient) for the existence of the solution is that the number of resources is at least as high as the number of demands $(n = |R|, m = |D|, n \geq m)$. We can presume that each of the demands requires the *assignment* of exactly one *appropriate* resource. Logistic problem can therefore be described as a problem of optimal resource assignment in the meaning of assigning the most appropriate resources to each of the demands.

The most naive approach to the problem would be the exhaustive search of the solution space (i.e. brute force solution). We would therefore have to examine each possible assignment of n resources to m demands. Method described would lead us to the following time complexity

$$T(m,n) = \frac{n!}{(n-m)!};\ n \geq m \Rightarrow T(m,n) = O(n!) \tag{1}$$

It is obvious that the method is unacceptable when the number of resources n is increased over the certain value. This value is exceeded in majority of the real life scenarios. The approach is therefore useless in such cases.

Formal description of the problem is in Table 1 which represents so called *suitability matrix*. Each column presents certain demand and each row presents certain resource. The content of the suitability matrix (s_{ij}) is defined with the suitability of resource with index i to demand with index j, where $1 \leq i \leq n$ and $1 \leq j \leq m$. Resource i is suitable for the demand j if $s_{ij} = 1$ and is not suitable if $s_{ij} = 0$.

2.2. Hungarian algorithm

Hungarian algorithm solves the assignment problem [10] and can be classified as a combinatorial optimization algorithm [4,5]. It is used to optimally assign m of n $(n \geq m)$ resources to m demands, where optimality is defined with minimal cost of assignment. Costs of different assignments are defined within the *cost matrix* (see Table 2).

Table 1. Suitability matrix where the elements of the row i present the suitabilities of resource r_i to each of the demands and where the elements of the column j present the suitabilities of each resource to demand d_j.

	d_1	d_2	$\cdots$	d_m
r_1	s_{11}	s_{12}	$\cdots$	s_{1m}
r_2	s_{21}	s_{22}	$\cdots$	s_{2m}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
r_n	s_{n1}	s_{n2}	$\cdots$	s_{nm}

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Table 2. Cost matrix where c_{ij} represents the cost of the assignment of resource r_i to the demand d_j.

	d_1	d_2	$\cdots$	d_m
r_1	c_{11}	c_{12}	$\cdots$	c_{1m}
r_2	c_{21}	c_{22}	$\cdots$	c_{2m}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
r_n	c_{n1}	c_{n2}	$\cdots$	c_{nm}

Assignment of resource r_i to demand d_j thus costs c_{ij} units. The problem described is called *assignment problem* [10]. We can represent the Hungarian method with the following algorithm, i.e. *Hungarian algorithm*:

1. If number of demands is smaller than number of resources, transfer the cost matrix to square matrix, with the introduction of new fictional demands. Cost of the assignment of certain resource to each fictional demand must be maximal (i.e. bigger or equal to maximal cost in the matrix).
2. Subtract the minimal element of each row from all elements in the same row.
3. Subtract the minimal element of each column from all elements in the same column.
4. Use as few lines as possible to cover all the zeros in the matrix and let l be the number of lines used. If $l < n$ proceed with step 5. If $l < m$ proceed with step 6.
5. Let x be the minimal element, that is not covered. Add x to each element covered by two lines and subtract x from each element that is not covered. Proceed with step 4.
6. Assign the resources to demands starting in the top row. Make only unique assignments-assign a resource only when there is only one zero in a row. As you make the assignment of resource r_i to demand d_j delete the row i and column j. If there is no unique assignment possible, move to the next row. If you reach the bottom of the matrix and all the assignments are made stop. Else proceed to next step.
7. Assign the resources to demands starting in the leftmost column. Make only unique assignments-assign a resource only when there is only one zero in a column. As you make the assignment of resource r_i to demand d_j delete the row i and column j. If there is no unique assignment possible, move to the next column. If you reach the rightmost column of the matrix and all the assignments are made stop. If no assignments were made in step 7 and rightmost column is reached it is legitimate to make assignments that are not unique. Make one arbitrarily assignment and proceed to step 6. Proceed to step 6 also if there was at least one assignment made in step 7.

Hungarian method can also be used in the scenario, where optimality is defined with maximal suitability of assignment. If this is the case we operate with so called *suitability matrix* (see Table 1). In order to find an optimal assignment, cost matrix has to be calculated from the suitability matrix which is used as an input to Hungarian algorithm. Following equation can be used to calculate the cost matrix

$$c_{ij} = M - s_{ij}, \forall i : i \in \mathcal{N}, 1 \leq i \leq n, \forall j : j \in \mathcal{N}, 1 \leq j \leq m \tag{2}$$

where M represents the maximal value in the suitability matrix (in our case 1). Minimal elements therefore become maximal and vice versa.

The time complexity of Hungarian algorithm equals $O(n^3)$, where n stands for number of resources. In fact, Hungarian algorithm time cost can be further reduced [11,12]. For the reasons of simplicity its

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basic version is presented in this article. The time complexity of the naive approach to our problem (see Equation (1)) is therefore drastically decreased.

3. FUZZY LOGIC

The majority of logical operations have been based on the principles of classical (crisp) logic until the mid-sixties of the 20th century, when fuzzy logic was introduced in [6]. The basic principle derives from the estimation of element membership to a certain set. Let us denote the *degree of membership* of element x to a set A with the variable $\mu_A(x)$. Its value is defined by the *membership function*. In the classical (crisp) logic the output of membership function can only take two values. If x is a member of A the membership value equals 1. If x is not a member of A membership value equals 0.

The basic principle of fuzzy logic is in the fact, that element x can be partially included in a set A (element x can be a partial member of A). Output of a fuzzy membership function can therefore be defined as

$$\mu_A(x) \in [0, 1] \tag{3}$$

The output of a membership function is generalized from the set {0,1} to the interval [0,1].

We can illustrate the applicability of fuzzy logic on the field of logistic problems which cope with demands and resources. Let us presume that we are facing an example with three different cargo vehicles ($|R| = 3$), where first vehicle has load capacity of 2000 kg, second 2800 kg and third 4000 kg, which can be formally described as $R = \{2000, 2800, 4000\}$. Now let us presume that we have a demand for a cargo vehicle with load capacity of 3000 kg, which can be formally described as $D = \{3000\}$.

The crisp approach would immediately eliminate the first (r_1) and the second vehicle (r_2) while their load capacities are too small. Demand would be assigned the third vehicle (if the demand is not handled as an exact requirement). Given solution is obviously over dimensioned, while in most cases vehicle with the load capacity of 2800 kg would be appropriate. We can formally described the crisp suitabilities of an example with Table 3.

The main advantage of the fuzzy approach is in a much higher flexibility in the definition of demands. In the meanings of fuzzy logic, given demand (d_1) could be interpreted as a demand *cargo vehicle with load capacity of approximately 3000 kg is needed*. In order to deal with this demand term *approximately* has to be defined. Let us define fuzzy set *suitable* and presume that its membership function is greater than zero in the interval [2500, 3500] kg. The membership function should have the highest output at the exact value 3000 kg (i.e. value 1) and it should linearly decrease towards the values 2500 and 3500 kg where it reaches the lowest output (i.e. value 0). Output equals 0 for all values outside the interval [2500, 3500] kg. Fuzzy set described is presented in Figure 1. Membership values of three cargo trucks at our disposal would equal the values presented in Table 4.

The degree of suitability of each resource to a certain demand can be calculated using the fuzzy rule set in the process of fuzzy inference. Two additional fuzzy sets have to be defined in order to illustrate the fuzzy inference on our example with cargo trucks, i.e. fuzzy set *too small* and fuzzy set *too big*. Input variable *load capacity* can therefore be divided in these three fuzzy sets and is presented in Figure 1. In order to observe the suitability of each cargo truck we also have to define the output fuzzy variable *suitability*, which includes two fuzzy sets, i.e. *true* and *false*. It is presented in Figure 2.

Table 3. Crisp suitabilities for the example, where $R = \{2000, 2800, 4000\}$ and $D = \{3000\}$.

	d_1
r_1	$s_{11} = \mu_{suitable}(r_1) = 0$
r_2	$s_{21} = \mu_{suitable}(r_2) = 0$
r_3	$s_{31} = \mu_{suitable}(r_3) = 1$

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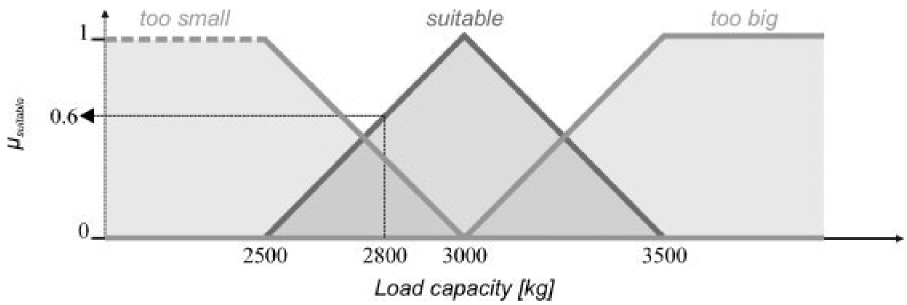

Figure 1. Membership function of fuzzy set *suitable*.

Table 4. Fuzzy membership values of cargo trucks r_1, r_2 and r_3 to fuzzy set *suitable*.

	d_1
r_1	$s_{11} = \mu_{suitable}(r_1) = 0$
r_2	$s_{21} = \mu_{suitable}(r_2) = 0.6$
r_3	$s_{31} = \mu_{suitable}(r_3) = 0$

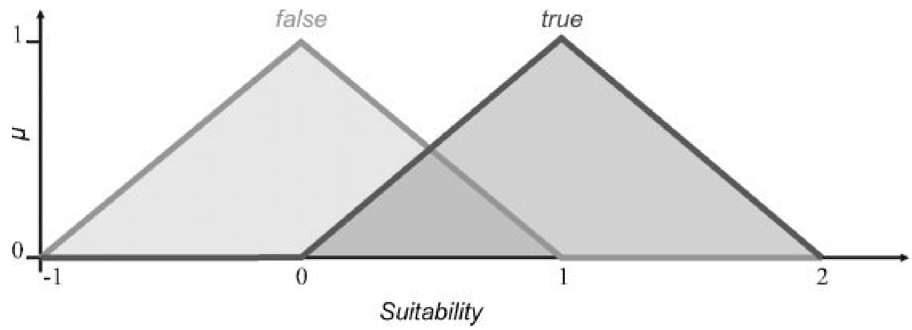

Figure 2. Output fuzzy variable *suitability* which defines the degree of agreement among certain resource and demand.

We can calculate the degree of agreement among certain resource and demand with fuzzy inference using the fuzzy rule set defined as follows

if (*load capacity* **is** *too small*) **then** (*suitability* **is** *false*),
if (*load capacity* **is** *suitable*) **then** (*suitability* **is** *true*),
if (*load capacity* **is** *too big*) **then** (*suitability* **is** *false*).

Output fuzzy variable *suitability* can be calculated using the fuzzy inference. Usually it is necessary to convert the fuzzy output to a crisp number while the other parts of the algorithms and methods are only able to operate with crisp numbers. Therefore the process of conversion, i.e. defuzzification is needed. There exist several defuzzification methods. Usually a method called *Centre of Gravity* (COG) is used. As the name implies centre of gravity of the output fuzzy variable is used as a crisp value. It is calculated with the following equation

$$v_A = \frac{\int_0^{\inf} x \times \mu_A(x)\mathrm{d}x}{\int_0^{\inf} \mu_A(x)\mathrm{d}x} \tag{4}$$

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where $\mu_A(x)$ represents the degree of membership to set A at value x [7,6]. Fuzzy inference is also presented on the example in Section 5.

4. SOLUTION DESCRIPTION

Solution proposed combines the advantages of Hungarian algorithm with the advantages of fuzzy logic methods. In fact, combination of Hungarian algorithm and fuzzy logic methods has already been used [13], but in a different context. Solution proposed can be divided to five phases:

- identification of demands,
- identification of resources,
- fuzzy rule set formation,
- construction of suitability matrix,
- optimal resource assignment establishment.

Each of the phase is explained in details in the remaining of this section. Section 5 explains the solution on the practical example.

4.1. Identification of demands

This phase includes the identification of demands regarding the problem we are facing. Each of the demands can define one or more properties the resource should have. Each demand refers to one resource only. With other words appropriate resource should be assigned to each of the demands *regarding the given properties*. Identification of demands *and their properties* is a procedure performed by a skilled and experienced expert. Exact definition of demands is an extremely difficult, if at all possible, task. Solution proposed allows us to overcome this barrier with fuzzily defined properties of the demands. On the other hand properties that need precise (crisp) fulfilment can also be handled by our algorithm.Demanded properties can be therefore divided in two types:

- Fuzzy properties: properties that are vaguely (imprecisely) defined and to which a resource with an approximately the same property value should be assigned.
- Crisp properties: properties which have an exact definition and to which a resource with an exactly the same property value should be assigned.

Section 4.4 describes the difference in handling of fuzzy and crisp properties.

4.2. Identification of resources

Like former phase, identification of resources and their properties also has to be performed by a skilled and experienced person. Expert has to define the properties which are important for each resource type and has to categorize each specific resource regarding the type and chosen properties. This task is usually straightforward, but in some cases subjective estimations of certain properties are needed (e.g. classification of vehicles regarding their *purpose* – see example in Section 5).

4.3. Fuzzy rule set formation

Fuzzy suitability of each resource to specific demands is calculated with fuzzy inference using fuzzy rule set. Different rules are needed for different resource properties and different demands. Each of the properties that is about to be dealt fuzzily has to be defined as a fuzzy variable. Fuzzy sets (i.e. fuzzy terms) have to be defined in order to map its universe of discourse to their membership values [7,6]. Fuzzy variables are than included in fuzzy rules, which define the suitability of certain resource to each of the demands. Fuzzy rule set formation is a straightforward procedure in most cases.

4.4. Construction of suitability matrix

Formal description of the problem can be presented with the suitability matrix (see Table 1). As already mentioned element on the location i, j (i.e. s_{ij}) represents the suitability degree of resource i to demand j. On one hand in Section 2 the requirement was made that suitability degree is a value from the set

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{0,1}. On the other hand, working with fuzzy values permits the usage of any real number value from the interval [0,1] for the suitability degree.

Suitability matrix is calculated using the fuzzy rule set presented in precedent phase. Suitability degree of each resource property to each demand property has to be calculated. To improve the speed of the procedure fuzzy suitability degrees are calculated only for the resources which are in vague agreement with the specific demands. Resources which do not agree the specific demand in crisp properties (e.g. cargo vehicle is demanded and personal vehicle is at the disposal) can be assigned suitability value 0 without fuzzy inference. Suitability of a specific resource to a given demand can thus be calculated in two phases.

1. Verification whether the resource suits the crisp properties of the demand.
2. If crisp properties are suited, fuzzy degree of suitabilities of each property of resource i to a certain property of demand j is calculated using the fuzzy rule set. The result of the fuzzy set evaluation is defuzzified and latter used as a suitability of resource i to demand j ($s_{ij} = v_{suitability}$). In other case value 0 is assigned to degree of suitability ($s_{ij} = 0$).

4.5. *Optimal resource assignment establishment*

Optimal resource assignment can be calculated using the suitability matrix. We can present a specific assignment with the following equation

$$S = \{(i,j) : i,j \in \mathbb{Z}, 0 \leq i \leq n, 0 \leq j \leq m\} \tag{5}$$

where variable i defines resource index, j demand index, n number of resources and m number of demands. In order to present legitimate assignment one resource can be assigned to only one demand and to each demand exactly one resource has to be assigned. Following criteria have to be fulfilled by S in order to present legitimate assignment

$$(i,j) \in S \Rightarrow \neg\exists l : l \neq j \wedge (i,l) \in S \tag{6}$$

$$(i,j) \in S \Rightarrow \neg\exists k : k \neq i \wedge (k,j) \in S \tag{7}$$

$$|S| = m \tag{8}$$

Assignment S_o is optimal if the value of its suitability function s is higher than the values of suitability functions that belong to any other assignment S, where s is calculated as

$$s = \min_{(i,j) \in S} s_{i,j} \tag{9}$$

where $s_{i,j}$ represents the suitability degree of resource i to demand j. The weakest link in the assignment is observed. The optimality is therefore achieved if the following requirement is fulfilled

$$s_{opt} \geq s \tag{10}$$

where s represents the value of suitability function of any legitimate assignment S.

Optimal resource assignment S_o can be found with the exhaustive search of solution space (brute force solution). Size of the solution space increases exponentially with parameters n and m and is therefore not fit for use. Time cost of this approach was already presented in Equation (1).

We can minimize the time cost of the optimal resource assignment search method with the employment of *Hungarian algorithm* which is in details presented in Section 2.2 and is used to solve the assignment problem.

The similarity between the assignment problem and problem we are solving is obvious. Although, there are minor corrections that have to be made in order to be able to use Hungarian algorithm as an optimal assignment search procedure in our method:

1. In our case, suitabilities have to be maximized, while Hungarian algorithm minimises the cost. Therefore we have to calculate the inversion of suitability matrix – i.e. cost matrix: $\forall i, \forall j \in \mathbb{Z}, 0 \leq i \leq n, 0 \leq j \leq m : c_{ij} = 1 - s_{ij}$.

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2. Suitability matrix has to be converted to square matrix with the introduction of so-called *dummy* demands. We can introduce dummy columns, which are constructed of zeros and exclude the pairs that include dummy demands after the execution. The other approach is to use the modified Hungarian algorithm for rectangular matrice [14].

Optimal resource assignment can thus be achieved in $O(n^3)$ steps instead of exponential complexity time.

5. DEMONSTRATION OF DEVELOPED METHOD

The example of military convoy formation is quite complex. We are dealing with five major groups of resources: vehicles, engineering resources, management resources, communicational resources and staff. For the easier understanding our demonstration will be directed only towards the formation of vehicle resources. Specifically, we will presume that our task is to assign four different vehicles to four different demands.

5.1. Identification of demands

As already said example will be demonstrated on four different demands for four vehicles. Let us presume that we have to find an assignment for the following set of imprecisely defined demands D ($|D| = 4$):

1. d_1: vehicle with approximately 4500 kg *load capacity.*
2. d_2: vehicle with approximately 3.5 m^3 *cargo space*.
3. d_3: vehicle of *type personal vehicle* with arbitrary properties.
4. d_4: vehicle with *long distance* and *urban purpose*.

Vehicle demands are presented in Table 5. Demand properties for vehicle *types* are so called crisp or precise properties and can have three possible values: *working vehicle* (WV), *personal vehicle* (PV) and *cargo vehicle* (CV).

5.2. Identification of resources

In real life problems number of vehicles that can be assigned can easily exceed several hundreds of them. For the meaning of the demonstration we limited their number to four different vehicles. They are all presented in Table 6. Four different properties are being observed:

- *load capacity* in kg,
- *cargo space* size in m^3,
- *type* (*working vehicle* (WV), *personal vehicle* (PV) *and cargo vehicle* (CV)),
- *purpose* (*urban vehicle, long distance vehicle and all terrain vehicle*).

As already mentioned purpose is a subjectively estimated property in predefined boundaries:

$$\mu_{urban}(r_i)\in[0,1], \mu_{long\ distance}(r_i)\in[0,1], \mu_{all\ terain}(r_i)\in[0,1] \tag{11}$$

5.3. Fuzzy rule set formation

Fuzzy rules have to be formed for each resource property, respectively, for each demand property. In our case we can distribute the rules into four sets defined with vehicle properties:

Table 5. Demands for four different vehicles.

ID	Load capacity (kg)	Cargo space (m^3)	Type	Purpose		
				Urban	Long distance	All terrain
1	4500					
2		3.5				
3			PV			
4				1	1	0

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Table 6. Vehicles that are at our disposal.

ID	Load capacity (kg)	Cargo space (m^3)	Type	Purpose		
				Urban	Long distance	All terrain
1	500	1	PV	0.9	0.7	0.1
2	750	2	PV	0.2	0.5	0.9
3	5000	4.2	CV	0.7	0.9	0.3
4	4000	4	CV	0.1	0.6	0.9

- fuzzy rules that define the *load capacity*,
- fuzzy rules that define the *cargo space*,
- fuzzy rules that define the *type*,
- fuzzy rules that define the *purpose*.

In order to define the rules, fuzzy variables which will be used in input (*precedent*) and output (*consequent*) part of each rule have to be defined.

5.3.1. *Definitions of fuzzy variables used*

Rules have to operate with four input fuzzy variables (i.e. *load capacity*, *cargo space*, *type* and *purpose*) and one output fuzzy variable (i.e. *suitability*). All of them are defined in the remaining of this section.

Regarding the parametric notation used in Ref. [7] fuzzy terms belonging to fuzzy variable *load capacity* can be defined as

- *too_small* $(0,0,l-o_l\ ,l)_p$,
- *suitable* $(l\text{–}o_l\ ,l,l,l+o_l)_p$,
- *too_big* $(l,l+o_l,\infty,\infty)_p$,

where variable l defines the crisp value of load capacity demand *property* and o_l the degree of fuzziness and is defined as $o_l = 0.25 \times l$. Fuzzy variable with its belonging terms is presented in Figure 3. It would be possible for each term to have completely different shape like triangular or Gaussian.

If the demand does not specify the load capacity property, fuzzy variable *load capacity* can be defined in a way that its membership function equals 1 for whole universe of discourse.

We can define fuzzy terms *too small*, *suitable* and *too big*, which belong to fuzzy variable *cargo space* in the same manner. Their graphical representation is presented in Figure 4.

Vehicle *type* is a crisp variable, while its value is predefined by the vehicle manufacturer. It is a property that serves as a filter whether a vehicle should be regarded in a process of fuzzy inference, whether its suitability is 0 with no regards to other properties than its type. We can therefore verify before the fuzzy inference, if a certain vehicle is a potential candidate for a certain demand and thus save some processing time.

The definition of a property *purpose* is slightly different from previously defined properties while its values do not derive directly from the technical characteristics of the vehicle. An expert estimates the

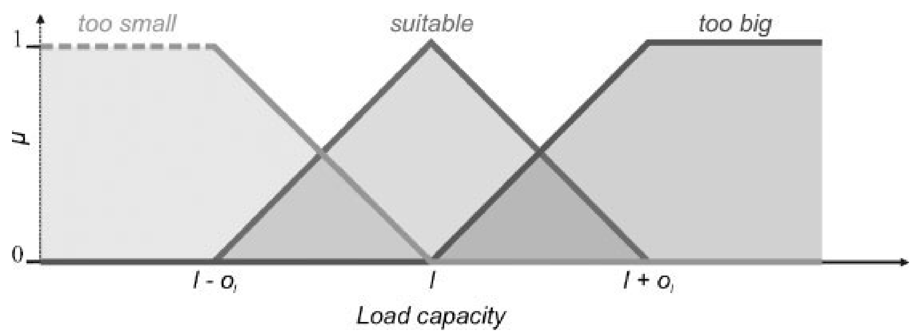

Figure 3. Graphical presentation of terms belonging to fuzzy variable *load capacity*, where l represents the crisp value of load capacity demand property and o_l represents the degree of fuzziness.

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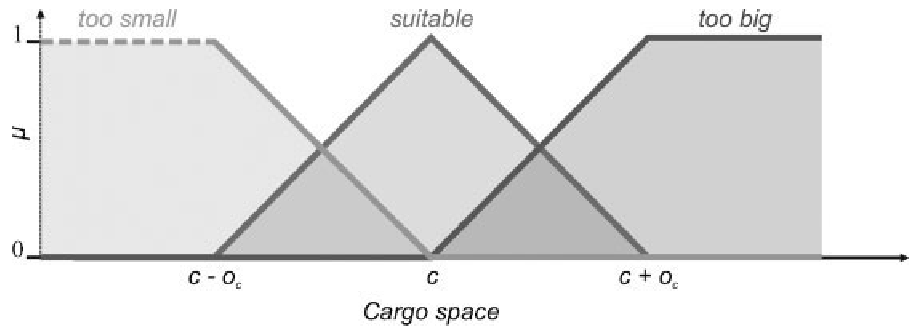

Figure 4. Graphical presentation of terms belonging to fuzzy variable *cargo space*, where *c* represents the crisp value of load capacity demand and o_c represents the degree of fuzziness.

purpose of the vehicle based on his experiences and technical specifications of the vehicle. Purpose can thus be described with three fuzzy variables

- *urban*,
- *long distance*,
- *all terrain*.

Likewise demanded purpose can be described with three similar fuzzy variables

- *demand urban*,
- *demand long distance*,
- *demand all terrain*.

Each of the variables is defined with two fuzzy terms

- *false* $(0,0,0,1)_p$,
- *true* $(0,1,1,1)_p$.

In order to observe the suitability of each vehicle to a specific demand property output (consequent) variable *suitability* has to be defined. Fuzzy terms that define this variable are defined as follows (see Figure 2)

- *false* $(-1,0,0,1)_p$,
- *true* $(0,1,1,2)_p$.

Like in the case of input variables it would be possible for each term to have completely different shape like triangular or Gaussian.

5.3.2. Actual rule formation

Based on given demands of fuzzy variables defined in the preceding section, fuzzy rules used in the process of fuzzy inference can be defined

*r*1: **if** (*load capacity* **is** *too small*) **then** (*suitability* **is** *false*),
*r*2: **if** (*load capacity* **is** *suitable*) **then** (*suitability* **is** *true*),
*r*3: **if** (*load capacity* **is** *too big*) **then** (*suitability* **is** *false*),
*r*4: **if** (*cargo space* **is** *too small*) **then** (*suitability* **is** *false*),
*r*5: **if** (*cargo space* **is** *suitable*) **then** (*suitability* **is** *true*),
*r*6: **if** (*cargo space* **is** *too big*) **then** (*suitability* **is** *false*),
*r*7: **if** (*urban* **is** *false*) and (*demand urban* **is** *false*) *then* (*suitability* **is** *true*),
*r*8: **if** (*urban* **is** *false*) and (*demand urban* **is** *true*) **then** (*suitability* **is** *false*),
*r*9: **if** (*urban* **is** *true*) and (*demand urban* **is** *false*) **then** (*suitability* **is** *false*),
*r*10: **if** (*urban* **is** *true*) and (*demand urban* **is** *true*) **then** (*suitability* **is** *true*),
*r*11: **if** (*long distance* **is** *false*) and (*demand long distance* **is** *false*) **then** (*suitability* **is** *true*),
*r*12: **if** (*long distance* **is** *false*) and (*demand long distance* **is** *true*) **then** (*suitability* **is** *false*),

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$r13$: **if** (*long distance* **is** *true*) and (*demand long distance* **is** *false*) **then** (*suitability* **is** *false*),
$r14$: **if** (*long distance* **is** *true*) and (*demand long distance* **is** *true*) **then** (*suitability* **is** *true*),
$r15$: **if** (*all terrain* **is** *false*) and (*demand all terrain* **is** *false*) **then** (*suitability* **is** *true*),
$r16$: **if** (*all terrain* **is** *false*) and (*demand all terrain* **is** *true*) **then** (*suitability* **is** *false*),
$r17$: **if** (*all terrain* **is** *true*) and (*demand all terrain* **is** *false*) **then** (*suitability* **is** *false*),
$r18$: **if** (*all terrain* **is** *true*) and (*demand all terrain* **is** *true*) **then** (*suitability* **is** *true*).

Fuzzy rules are used in the process of fuzzy inference which calculates the fuzzy suitability of each vehicle to each demand. Crisp suitabilities are calculated from the fuzzy outputs using the COG defuzzification method.

5.4. Construction of suitability matrix

Suitability matrix is constructed of suitabilities of each resource for each of the demands. The construction of the matrix will be demonstrated on the example of calculation of the suitability of resource r_3 (i.e. vehicle with load capacity that equals 5000 kg) to demand d_1 (i.e. demand for vehicle with load capacity 4500 kg). Thus the calculation of an element s_{31} of the suitability matrix will be demonstrated.

5.4.1. Demand fuzzification

Load capacity can be described with the following fuzzy terms:

- *too_small* $(0,0,l\text{–}o_l\ ,l)_p$,
- *suitable* $(l\text{–}o_l\ ,l,l,l+o_l)_p$,
- *too_big* $(l,l+o_l,\infty,\infty)_p$,

where we presume, that l equals 4500 and o_l equals 1125 in our example. Fuzzy variable *load capacity* is presented in Figure 5.

5.4.2. Calculation of resource suitability

Resource membership values to each fuzzy set that define fuzzy variable *load capacity* have to be calculated in the first place-load capacity of the vehicle has to be fuzzified. We do not deal with other resource properties, while only load capacity is specified in the observed demand. The calculation of the membership of crisp value load capacity to each of the fuzzy terms is presented in Figure 6. Membership values obtained are thus $\mu_{too\_small} = 0$, $\mu_{suitable} = 0.555$ and $\mu_{too\_big} = 0.444$.

5.4.3. Fuzzy inference

Fuzzy inference is a procedure that calculates the output fuzzy terms regarding the defined fuzzy rules. We are only dealing with load capacity demand in our example and can therefore limit our fuzzy rule set to the rules that include fuzzy variable *load capacity*, i.e. rule $r1$, $r2$ and $r3$. The evaluation of fuzzy rules truthfulness gives us the results that rule $r1$ is fulfilled with the degree 0, $r2$ with the degree 0.555 and $r3$ with the degree 0.444. The output of fuzzy inference is a fuzzy set *suitability* presented in Figure 7.

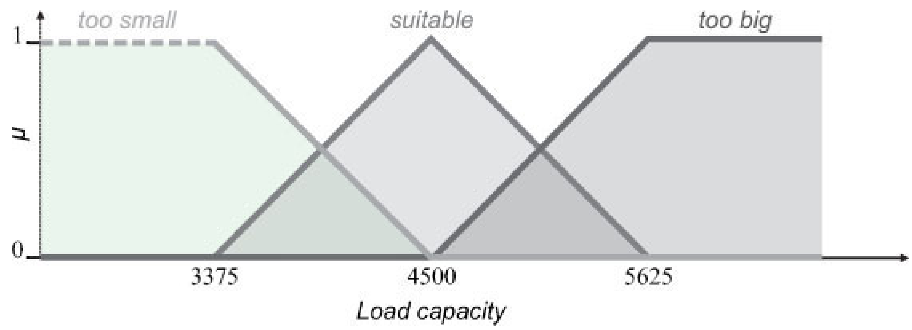

Figure 5. Fuzzy variable, presenting the demand for approximately 4500 kg vehicle load capacity.

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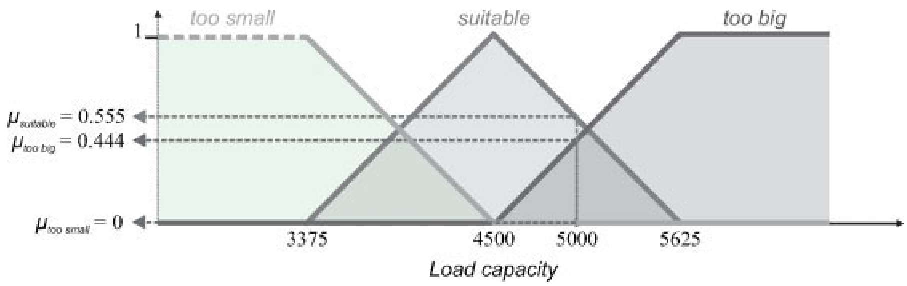

Figure 6. Fuzzifying the crisp property load capacity of the resource r_3.

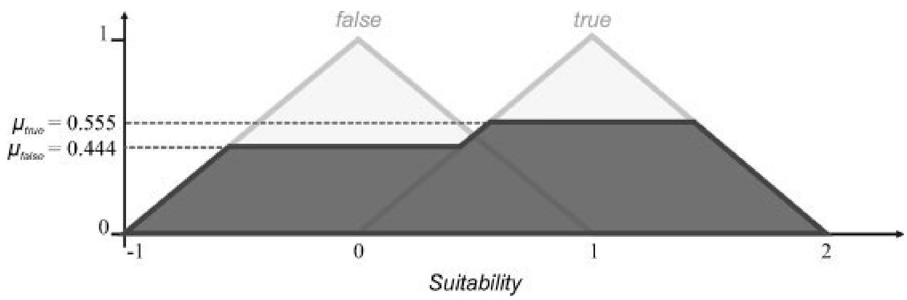

Figure 7. Fuzzy set obtained with the fuzzy inference over the fuzzy rules $r1$, $r2$ in $r3$.

5.4.4. Defuzzification

In order to use the values obtained with the fuzzy inference as an input to Hungarian algorithm, fuzzy values have to be converted to their crisp equivalents, i.e. defuzzified. Fuzzy set presented in Figure 7 is converted to crisp value with already presented COG method, that calculates the centre of gravity of the form presenting fuzzy set *suitability* calculated in preceding step. The actual result of *suitability* defuzzification is a crisp value 0.79.

5.4.5. Suitability matrix construction

In the same manners as presented above all elements of the suitability matrix can be calculated. The final matrix is presented in Table 7.

5.5. Optimal resource assignment establishment

Hungarian algorithm is used in the last step of our procedure in order to find an optimal resource assignment from the suitability matrix calculated in the former steps. The actual optimal assignment is as follows:

$$S_{fuzzy} = \{(1,4),(2,3),(3,1),(4,2)\} \tag{12}$$

Table 7. Suitability matrix containing fuzzy suitabilities of the resources to given demands.

	d_1	d_2	d_3	d_4
r_1	0	0	1	0.76
r_2	0	0.14	1	0.35
r_3	0.79	0.61	0	0.76
r_4	0.79	0.73	0	0.39

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The solution is legitimate, while the conditions presented in Equations (6)–(8) are fulfilled. Exactly one resource is assigned to each demand and none of the resources is assigned to more than one demand at the same time. Suitability can be calculated using Equation (9)

$$s_{fuzzy} = \min(s_{14}, s_{23}, s_{31}, s_{42}) = \min(0.76, 1, 0.79, 0.73) = 0.73 \tag{13}$$

We can reach the conclusion that there is no better solution to our problem in the meaning of the suitability value, while the suitability of any other legitimate assignment is smaller. Our solution is optimal from a suitability viewpoint, therefore following equality is valid

$$s_{opt} = s_{fuzzy} \tag{14}$$

6. EVALUATING THE SOLUTION

6.1. Comparing the fuzzy and the crisp solution

The main difference between the fuzzy and the crisp solution is in the perspective of the suitability, which is defined in the interval [0,1] with the fuzzy solution and in the set {0,1} with the crisp solution. The crisp assignment of the resource to a certain demand is made only when their agreement is perfect. On the other hand it is possible to define the incomplete agreement between the resource and the demand in the fuzzy solution. In our example the only crisp assignment that can be made is the assignment of resource r_3 to a demand d_1. Hungarian algorithm would therefore return the following solution

$$S_{crisp} = \{(1,3), (2,1), (3,2), (4,4)\} \tag{15}$$

All of the assignments except the first one are made in the order from left to the right, while none of the resources is preferred for any of the demands. The suitability of the assignment is therefore

$$s_{crisp} = \min(s_{13}, s_{21}, s_{32}, s_{44}) = \min(1, 0, 0, 0) = 0 \tag{16}$$

6.2. Considering the cost

As presented above the optimal resource assignment was made regarding the suitability of a certain vehicle to a certain demand. Hungarian algorithm is in its original form devoted to optimize (i.e. minimize) the cost of an assignment. With the introduction of fuzzy logic the algorithm was modified in order to optimize (i.e. maximize) the suitability of the assignment. In this scenario the cost of a certain solution could be calculated every time after the assignment is made with the sum of the costs of resources that were assigned. The approach which would also minimize the cost of the assignment would include the effect of vehicle cost in the calculation of suitability matrix in order to increase the priorities of cheaper vehicles. Among two vehicles which have the same suitabilities the cheaper one would therefore be assigned.

7. CONCLUSION

Optimal resource assignment from resource suitability viewpoint with the introduction of fuzzy logic methods and Hungarian algorithm was presented in the article. Method developed is able to cope with vaguely defined resources and demands which are not rare in real life logistic problems. Even more, optimal assignment is made in $O(n^3)$ time. The successful implementation and its results are presented on the example of vehicle assignment. Side-by-side comparison between the crisp and the fuzzy solution is also presented where the benefits of the former are obvious.

8. LIST OF SYMBOLS AND ABBREVIATIONS

cij	cost of the assignment of resource r_i to demand d_j.
COG	centre of gravity defuzzification method.
CV	cargo vehicle.
D	set of demands.

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l number of lines used to cover all the zeros in the suitability matrix.
m number of demands.
M maximal element of the suitability matrix.
n number of resources.
PV personal vehicle.
R set of available resources.
s_{ij} suitability of resource ri to demand d_j.
s minimal suitability within certain assignment.
S specific assignment presented with the set of pairs $(i; j)$ where i presents index of resource and j index of demand.
S_o optimal assignment, regarding the minimal suitability of each assignment.
WV working vehicle.
$\mu_A(x)$ membership value of element x to set A.
ν_A crisp (defuzzified) value of fuzzy value defined over set A.

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