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Optimal transit fare in a bimodal network under demand uncertainty and bounded rationality

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SUMMARY

This paper investigates the optimal transit fare in a simple bimodal transportation system that comprises public transport and private car. We consider two new factors: demand uncertainty and bounded rationality. With demand uncertainty, travelers are assumed to consider both the mean travel cost and travel cost variability in their mode choice decision. Under bounded rationality, travelers do not necessarily choose the travel mode of which perceived travel cost is absolutely lower than the one of the other mode. To determine the optimal transit fare, a bi-level programming is proposed. The upper-level objective function is to minimize the mean of total travel cost, whereas the lower-level programming adopts the logit-based model to describe users' mode choice behaviors. Then a heuristic algorithm based on a sensitivity analysis approach is designed to solve the bi-level programming. Numerical examples are presented to illustrate the effect of demand uncertainty and bounded rationality on the modal share, optimal transit fare and system performance. Copyright © 2013 John Wiley & Sons, Ltd.

KEY WORDS: transit fare; demand uncertainty; bounded rationality; bi-level programming

1. INTRODUCTION

In most cities, people have a choice between using a private car and using a public transportation, and in making a choice, people would consider not only direct monetary costs but also travel times, comfort, and so on. Therefore, demands for car and buses have cross-elasticities not only with respect to price but also with respect to time and quality of service. As is clear from many theoretical and empirical researches, congestion tolls for private car, transit prices, subsidies, and service variables—such as frequency—for public transportation are closely interrelated [1–4]. Moreover, their optimal values and the modal split should strongly depend on the way mode choice occurs.

This paper focuses on the optimal transit fare and its effect on the optimal modal split and transport equilibrium. In the previous work on this topic, a bimodal transportation system has been considered, such as public transport (including the bus and subway) and private transport. Huang [5] dealt with transit pricing and modal split in a competitive mass transit/highway system with heterogeneous commuters. They compared three pricing schemes: the marginal cost-based transit fare with no toll, the average cost-based fare with no toll, and the marginal cost-based fare with time-invariant toll for subsidizing transit, and they derived a socially optimal combination of transit fare and road toll that minimizes the total social cost of the competitive system mean while ensuring no deficit to the transit side. Huang [6] studied the transport pricing mechanism and the corresponding mode choice behavior in a simple bimodal transportation system with elastic demand. They derived

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and compared three pricing schemes: the arbitrarily fixed pricing, the first best pricing for a social optimum of the system, and the second best pricing in the case of incapability of road toll. It was shown that the first best pricing requires to implement a road toll and a transit fare simultaneously, and the optimal transit fare for the second best solution should be set to a weighted sum of the marginal external costs between auto and transit commuters. Proost and Dender [7] calculated the optimal transport price and its effects on the transport equilibrium and on welfare by using a numerical model of the urban transportation sector. They found that optimal prices are higher than current prices in most transport markets so that optimal transport demand is below current demand. Calculations of optimal public transport prices for unchanged reference car taxes indicated that only limited welfare gains can be obtained by charging near-zero transit fares in peak hours. Basso and Jara-Diaz [8] modeled and analyzed optimal (welfare maximizing) prices and design of transport services in a bimodal context. They obtained some main results: the optimal car transit split is generally different from the total cost-minimizing one; optimal congestion and transit price are interdependent and have an optimal frequency attached, and the optimal money price difference together with the optimal frequency yield the optimal modal split.

The literatures earlier typically assumed that the origin–destination (O–D) travel demand is fixed or elastic. However, the travel demand is often subject to stochastic variations in reality. Because of demand uncertainty, the transport system may become unreliable, which can also change traveler's behavior, for example, change of mode, route diversion, or trip rescheduling. Watling [9] assumed a Poisson distribution of travel demand and derived the equilibrium route choice condition considering the mean travel time. Shao *et al*. [10] used normal distribution to represent the travel demand in which the drivers trade-off between the mean travel time and safety margin. Zhou and Chen [11] assumed the travel demand to follow log-normal distribution and proposed different risk-based assignment frameworks. Sumalee *et al*. [12] proposed a stochastic network model for multimodal transport network that considers auto, bus, underground, and walking modes. The stochastic demand was assumed to follow the Poisson distribution. The risk-averse travelers were assumed to consider both the mean and variance of the random perceived travel time on each multimodal path in their path choice decisions.

Another assumption in the literatures earlier is that users are “perfectly or unboundedly rational”, that is, they always choose the travel mode of which perceived travel cost is absolutely lower than the one of other modes. However, doing so only leads to a small or negligible improvement in their travel cost comparing with traveling by other modes, some users may not be sufficiently encouraged to change mode in practice. Under the circumstance, users are said to be “boundedly rational”. The literature in psychology and economics has provided a wide range of evidence that bounded rationality is important in many contexts, particularly in the context of day-to-day choices [13]. In the context of transportation (in particular, route choice behavior research), Nakayama *et al*. [14] concluded that their experimental study indicates a need to evaluate the validity of the “perfectly rational” assumption in the traffic equilibrium analysis. Although referred to as “tolerance-based,” Szeto and Lo [15] were “forced” to consider bounded rationality in their dynamic traffic assignment because the problem may be infeasible when travelers are perfectly rational. Lou *et al*. [16] investigated congestion pricing strategies in static networks with “boundedly rational” route choice behavior and presented “boundedly rational” user equilibrium.

The aforementioned researches addressed bounded rationality in route choice behaviors in static networks where user equilibrium with bounded rationality arises. In this paper, we try to incorporate bounded rationality into travelers' mode choice behavior that is similar to route choice behavior in some ways. Generally, travelers have no perfect information on traffic conditions when choosing the travel mode so that they are not “perfectly rational” in minimizing their own travel costs. If the travel cost for one mode is acceptable to users, the travel mode will be considered. Instead of the traditional equilibrium with perfect rationality for modal split, the equilibrium state with bounded rationality arises whenever users' perceived travel cost of the chosen mode is not sufficiently larger than the one of other mode, and no user has an incentive to switch his or her travel mode. Such a mode choice behavior with bounded rationality may affect the effectiveness of traditional transport pricing mechanism. For example, the transit fare scheme commonly advocated in the aforementioned literatures may not necessarily reduce the total

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system travel cost to its minimum level because users may not switch to modes with the least generalized cost.

This paper attempts to investigate the optimal transit fare in a simple bimodal transportation system that comprises public transport and private car. We relax the two aforementioned assumptions and consider two new factors: demand uncertainty and bounded rationality. The impact of demand uncertainty on the traveler occurs through the induced uncertainty of travel cost. Travelers are assumed to consider both the mean travel cost and travel cost variability in their mode choice decision. Then at equilibrium, the modal split is governed by a logit-based model. We thus define travelers with bounded rationality as those who may choose the mode when the difference between its perceived travel cost and that of the other mode is no greater than a prespecified threshold value. The system manager can set the transit fare that affects the distribution of travel demand between two modes. He or she aims to obtain a transport pricing pattern that minimizes the total social cost among all the possible equilibrium flow patterns for each travel mode. A bi-level model of optimizing the transit fare is constructed. The upper-level objective seeks to minimize the mean total system travel cost, whereas the lower-level programming adopts a logit-based mode choice model.

The outline of this paper is as follows: Section 2 introduces the formulation of perceived travel cost of each mode and presents the mode choice model with stochastic demand and bounded rationality. In Section 3, the bi-level model for optimizing transit fare is developed. In Section 4, numerical experiments are conducted to demonstrate the model and solution algorithm. Finally, Section 5 contains conclusion and future research.

2. MODE CHOICE MODEL

2.1. Formulation of travel costs

Consider a simplified network that comprises two types of modes to provide transportation service between O (a residential area or home) and D (a workplace). Mode 1 represents the public transport (e.g., transit mode, railway, or bus with bus-only lane), and mode 2 represents the private transport (e.g., auto mode, one person per car). It is assumed that the O–D demand X is a random variable and follows a normal distribution:

$$X \sim N\left(x, (cv \cdot x)^2\right) \tag{1}$$

where x and cv are the mean and the coefficient of variance (CV) of the O–D demand, respectively. Let X_T (with mean x_T) and X_A (with mean x_A) denote the stochastic numbers of transit and auto travelers, and $x_T + x_A = x$ holds. It is assumed that (i) the numbers of travelers using transit mode and auto mode follow the same type of probability distribution as the O–D demand; and (ii) the CV of the numbers of travelers using each mode is equal to that of O–D demand.

The travel cost experienced by a transit passenger depends on the travel time, the transit fare, and the discomfort generated by body congestion at stations and in carriages. The transit mode represents railway or bus with bus-only lane, so the in-vehicle travel time t_T is constant. The waiting time for a train or bus that relates to the transit service frequency is assumed to be a constant and incorporated into t_T. The time for walking to a station and from the station to final destination is omitted for simplicity.

Introducing crowding congestion to a certain extent may relax infinite capacity assumption on transit in other literatures [5]. We assume that public transport has enough capacity to absorb the increase in passengers. However, it still generates congestion discomfort due to the increasing number of transit users. It is obvious that the factor of body congestion affects users' travel behavior greatly. A remarkable fact is that when the body congestion at railway or bus stations and in carriages reaches a certain level of discomfort, some people will abandon mass transit model for auto mode or give up their travels [17]. The body congestion discomfort experienced by a transit user can be described

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through an increasing function of the number of users selecting this mode [5,6] The function takes the form as follows,

$$g(X_T) = u(X_T)^2 + vX_T \tag{2}$$

where u and v are parameters in the function describing body congestion discomfort. The introduction of body congestion function in Equation (2) makes the travel costs incurred on transit users become mode-usage dependent, although users' travel times by transit are constants.

Then the generalized travel cost of a transit passenger is

$$\widetilde{C}_T(X_T) = \alpha t_T + \pi g(X_T) + \tau_T \tag{3}$$

where α is the unit cost of travel time, π is the unit cost of discomfort, and τ_T is the transit fare.

Similarly, the generalized travel cost of an auto user is

$$\widetilde{C}_A(X_A) = \alpha T_A(X_A) + oc_A \tag{4}$$

where $T_A(X_A)$ is the stochastic travel time by private car and oc_A is the operating cost of an auto user. We do not consider the interaction between the models of public transit and private car in this paper. The road network in this paper is reduced to two links; one representing the transit network and the other the auto network, which connect a single origin to a single destination. The travel time experienced by an auto user can be described through an increasing function of the number of users selecting this mode. And the commonly adopted auto users' travel time function is used.

$$T_A(X_A) = t_A^0\left[1 + \beta\left(\frac{X_A}{b}\right)^n\right] \tag{5}$$

where t_A^0 and b are auto free-flow travel time and notional road capacity, respectively; β and n are deterministic parameters of the travel time function. When the value of parameter n is equal to 1, the travel time function becomes a linear function.

2.2. *Stochastic travel cost distribution*

The distribution of two mode travel cost can be characterized by their mean and variance that can be defined as

$$E\left(\widetilde{C}_T(X_T)\right) = \alpha t_T + \pi E(g(X_T)) + \tau_T \tag{6}$$

$$Var\left(\widetilde{C}_T(X_T)\right) = \pi^2 Var(g(X_T)) \tag{7}$$

$$E\left(\widetilde{C}_A(X_A)\right) = \alpha E(T_A(X_A)) + oc_A \tag{8}$$

$$Var\left(\widetilde{C}_A(X_A)\right) = \alpha^2 Var(T_A(X_A)) \tag{9}$$

It is further assumed that the numbers of transit passengers and auto travelers also follow normal distribution and have the same CV with the O–D demand. Thus, the mean and variance of body congestion cost for transit mode can be expressed as (see Appendix A):

$$E(g(X_T)) = E\left(u(X_T)^2 + vX_T\right) = uE\left(X_T^2\right) + vE(X_T) = u\left(x_T^2 + (cv{\cdot}x_T)^2\right) + vx_T \tag{10}$$

$$Var(g(X_T)) = (cv \cdot x_T)^2 \left(4u^2(x_T)^2 + 2u^2(cv \cdot x_T)^2 + 4uvx_T + v^2\right) \tag{11}$$

By using the method proposed by Shao *et al*. [10], we have the mean and variance of travel time for auto mode.

$$E(T_A(X_A)) = t_A^0 + t_A^0 \frac{\beta}{b^n} \sum_{i=0, i=even}^{n} \binom{n}{i} (cv \cdot x_A)^i (x_A)^{n-i} (i-1)!! \tag{12}$$

$$Var(T_A(X_A)) = \left(t_A^0 \frac{\beta}{b^n}\right)^2 \left(\begin{array}{l} \sum\limits_{i=0, i=even}^{2n} \binom{2n}{i} (cv \cdot x_A)^i (x_A)^{2n-i} (i-1)!! - \\ \left(\sum\limits_{i=0, i=even}^{n} \binom{n}{i} (cv \cdot x_A)^i (x_A)^{n-i} (i-1)!! \right)^2 \end{array} \right) \tag{13}$$

Substituting Equations (10)–(13) into Equations (6)–(9), we can obtain the mean and variance of travel cost for each mode.

2.3. Perceived travel cost of each mode

Travelers are assumed to consider both average travel cost and travel cost variability in their mode choice decision [18]. Hence, they choose their travel mode by trade-off between the mean and standard deviation of travel cost on each mode. The effective generalized travel cost for each mode can be defined as

$$\overline{C}_i = \mathrm{E}\left[\widetilde{C}_i(X_i)\right] + \lambda \cdot \sqrt{Var\left[\widetilde{C}_i(X_i)\right]} \tag{14}$$

where $i = T \; or \; A$, T represents public transit mode and A represents private car mode; λ is the risk-aversion parameter [19–21].

The traveler's attitude towards risk is modeled in the form of effective generalized travel cost for each mode. The travelers' perception errors due to the imperfect knowledge of network characteristics are modeled separately as the perceived effective generalized travel cost on each mode

$$C_i = \overline{C}_i + \epsilon_i \tag{15}$$

where ϵ_i is the additive random term that represents the uncertainty in specifying the cost of selecting mode i. Suppose the random terms be identically and independently distributed Gumbel variables with mean zero.

2.4. Mode split under bounded rationality

Generally, travelers always choose the travel mode with a minimum total travel cost. The perceived costs of different modes are random variables, so the traveler's mode choice problem can be modeled by a probability with which traffic modes will be chosen. Then, for example, the choice probability that public transit are chosen is the probability that the travel cost of transit mode C_T is lower than that of private car mode C_A, which can be written as

$$p_T = pr\{C_T \le C_A\} \tag{16}$$

where p_T is the probability that the public transit are chosen. Let p_A denote the probability that the private cars are chosen, and $p_T + p_A = 1$ holds.

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Equation (16) is the conventional and "perfectly rational" general mode choice model. As mentioned earlier, a mode choice behavior with bounded rationality is likely to occur. The general mode choice model with bounded rationality can be written as

$$p_T = pr\{C_T \leq C_A + \delta\} \tag{17}$$

$$p_A = pr\{C_A \leq C_T + \delta\} \tag{18}$$

The probability that a mode is chosen is the probability that its perceived effective travel cost is perceived no greater than a threshold value of the other, which represents the choice behavior under bounded rationality. Travelers with bounded rationality still follow the behavior that exhibits a tendency toward cost minimization but not necessarily to the absolute minimum level. Travelers do not necessarily switch to the mode of which travel cost is absolutely lower than the ones of other modes. They may choose one mode even if the cost of this mode is larger, as long as the excess is smaller than a threshold value. Assuming that users of the same O–D pair have the same threshold value denoted as δ, the threshold value can be obtained from empirical research that is one of our topics for further investigation.

However, the Equations (17)–(18) make $p_T + p_A \neq 1$. To avoid the problem, we look at the difference of the generalized costs of the two modes, $C_T - C_A$.

When $C_T - C_A \leq -\delta$, travelers choose transit mode for sure; when $C_T - C_A \geq \delta$, travelers choose auto mode for sure; when $-\delta \leq C_T - C_A \leq \delta$, travelers may choose either of them, as is defined by the choice behavior with bounded rationality. That is to say, there are three classes of travelers (see Figure 1). The Class 1 travelers believe that $C_T - C_A \leq -\delta$, whereas the Class 2 travelers consider that $C_T - C_A \geq \delta$, and the Class 3 travelers think that $-\delta \leq C_T - C_A \leq \delta$.

Nevertheless, how the travelers make choice, transit mode or auto mode, when the cost difference falls into the interval $-\delta \leq C_T - C_A \leq \delta$, is not defined. To resolve this problem, we assume that users are indifferent within this entire interval and take random chance to either modal choice. If we make this assumption,

$$p'_T = pr\{C_T - C_A \leq -\delta\} + \omega pr\{-\delta \leq C_T - C_A \leq \delta\} \tag{19}$$

$$p'_A = pr\{C_T - C_A \geq \delta\} + (1 - \omega) pr\{-\delta \leq C_T - C_A \leq \delta\} \tag{20}$$

where ω $(0 \leq \omega \leq 1)$ is a parameter that reflects free mode choice for travelers when there is no significant cost difference between the two modes. In this way, the summation of two probabilities would be

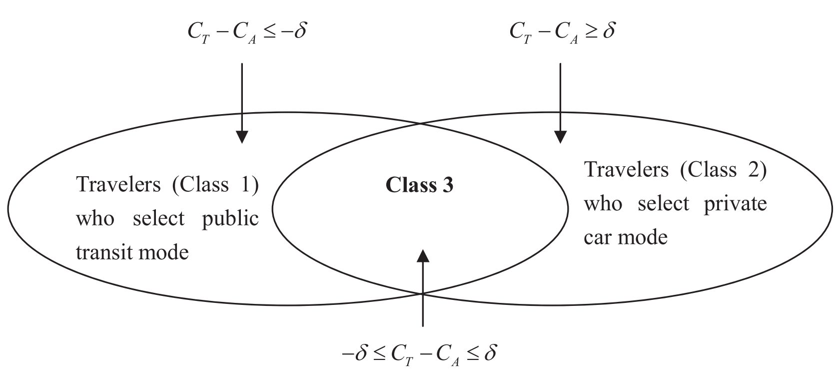

Figure 1. The three classes of travelers.

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equal to 1. Then we can determine the flow x_T and x_A. If $\delta=0$, Equations (19)–(20) reduce to the classical and "perfectly rational" general modal split models.

Depending on the classical binary logit model (see Appendix B), we have

$$p'_T = (1-\omega)\frac{e^{-\overline{C}_T-\delta}}{e^{-\overline{C}_T-\delta}+e^{-\overline{C}_A}} + \omega\left(1-\frac{e^{-\overline{C}_A-\delta}}{e^{-\overline{C}_A-\delta}+e^{-\overline{C}_T}}\right) \tag{21}$$

$$p'_A = \omega\frac{e^{-\overline{C}_A-\delta}}{e^{-\overline{C}_A-\delta}+e^{-\overline{C}_T}} + (1-\omega)\left(1-\frac{e^{-\overline{C}_T-\delta}}{e^{-\overline{C}_T-\delta}+e^{-\overline{C}_A}}\right) \tag{22}$$

In practice, the modal split model based on perfect rationality typically provide a unique equilibrium solution. In other words, such model typically provide a point estimate of traffic flow distribution. On the other hand, from Equations (21)–(22), the modal split model based on bounded rationality provide an interval estimate instead and equilibrium solution may not be unique.

3. THE BI-LEVEL MODEL FOR OPTIMIZING TRANSIT FARE

As transit fare is the unique tool to adjust traffic flow distribution between two modes, the transit fare optimization problem can be represented as a leader–follower game where the system manager corresponds to the leader and the travelers to the followers. It is assumed that the system manager can influence but cannot control the travelers' mode choice through fare policy. With each transit fare structure, travelers choose their travel mode in a stochastic user equilibrium manner.

3.1. The bi-level model

From the view of system manager, the upper-level objective function is to minimize the mean of the total travel cost.

$$\begin{aligned}\min_{\tau_T} Z &= E[X_T(\alpha t_T + \pi g(X_T)) + \alpha X_A T_A(X_A)] \\ &= \alpha x_T t_T + \pi u\left(x_T^3 + 3x_T(cv\cdot x_T)^2\right) + \pi v\left(x_T^2 + (cv\cdot x_T)^2\right) + \alpha x_A t_A^0 \\ &\quad + \frac{t_A^0\alpha\beta}{b^n}\sum_{i=0,i=even}^{n+1}\begin{pmatrix} n+1 \\ i \end{pmatrix}(cv\cdot x_A)^i (x_A)^{n+1-i}(i-1)!!\end{aligned} \tag{23}$$

s.t.

$$\underline{\tau}_T \le \tau_T \le \overline{\tau}_T \tag{24}$$

where $\underline{\tau}_T$ and $\overline{\tau}_T$ are the minimum and maximal transit fare constrains, respectively. The derivation of the objective functions of Equation (23) is provided in Appendix C.

The lower-level problem can be described as the logit-based mode choice model.

$$X_i = X\cdot p'_i \quad i = T, A \tag{25}$$

By taking the expectation of both sides of Equation (25), we obtain

$$x_i = x\cdot p'_i \quad i = T, A \tag{26}$$

where p'_T and p'_A are obtained by Equations (21) and (22), respectively.

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3.2. Solution algorithm

The key to solve the bi-level model is to find the form of response function, that is, the changes in equilibrium share of traffic mode caused by the price of public transport. It is difficult to evaluate the changes in the equilibrium modal split volume directly because of the implicit, nonlinear function form of equilibrium share of traffic mode. A good idea is to use the linear function to approximate the nonlinear function of equilibrium travel volume by mode 1.

$$x_T(\tau_T) \approx x_T\left(\tau_T^k\right) + \left.\frac{\partial x_T(\tau_T)}{\partial \tau_T}\right|_{\tau_T=\tau_T^k}\left(\tau_T - \tau_T^k\right) \tag{27}$$

Although some common methods, such as sensitivity analysis based method, can be used to obtain the response function under certain strong assumptions approximately, its computational expense usually becomes unendurable or even impossible as the problem's scale increases. Here, we use the difference $\frac{\Delta x_T(\tau_T)}{\Delta \tau_T}$ approximate the differential $\frac{\partial x_T(\tau_T)}{\partial \tau_T}$ in each iteration.

When Equation (27) is put into the objective function, the models (23) and (24) are changed into nonlinear optimization problems, which can be solved by the known methods. Then according to the optimal price τ_T, we can obtain new equilibrium x_T again. Then a new optimal price discount can be obtained by repeating the aforementioned basic idea. After computing with some times, the optimal solutions for programming model can be obtained. The solution algorithm can be stated as follows:

Step 1. Initialization. Determine an initial value τ_T^0 and x_T^0, and set $k=0$.
Step 2. Solve the models (23) and (24) based on the given τ_T^k and obtain the corresponding optimal x_T^k.
Step 3. Derivative calculation. Calculating $\frac{\Delta x_T^k}{\Delta \tau_T^k}$.
Step 4. Solve the programming model. Put (27) into the objective function and then obtain a new x_T^{k+1}.
Step 5. Convergence check. If $\left|x_T^{k+1} - x_T^k\right| \le \theta$ (where θ is a convergence tolerance), stop and let $x_T^* = x_T^{k+1}$; otherwise, let $k=k+1$ and go to Step 2.

4. NUMERICAL EXAMPLES

In this section, we present four numerical examples to calculate the transit modal share, optimal transport price and total (mean) travel cost that are affected by the users' characteristics of bounded rationality and risk preference. The parameters δ and ω reflect the level of rationality in travelers' mode choice decision making, which can be estimated by conducting a stated preference survey. However, given the fact that the estimate of δ and ω can be biased or inaccurate, it makes sense to examine how the mode share, optimal transit fare and total travel cost change with different levels of rationality. The other parameters of our numerical examples are $t_T = 20$(*minute*), $t_A^0 = 15$(*minute*), $b = 60$(*person/minute*), $u = 0.05$, $v = 0.25$, $\beta = 0.15$, $n = 4$, $x = 2000$(*person*), $cv = 0.10$, $\alpha = 1.00$(¥/*minute*), $\pi = 0.01$ (¥/*discomfort*), $oc_A = 30$(¥), $\underline{\tau}_T = 10$(¥), $\overline{\tau}_T = 20$(¥).

4.1. Example 1: effect of bounded rationality on the modal split

It is assumed that the transit fare is arbitrarily fixed (i.e., $\tau_T = 15$(¥)), and travelers are risk neutral ($\lambda = 0$) in this example. Figure 2 shows the number of transit users with ω varying from 0 to 1 and δ varying from 1 to 5. As we can see, the larger ω is the one where more travelers choose transit mode. The number of transit users depends on the ω value intuitively from Equation (19). Interestingly, as the value of δ increases, the number of transit users will decrease when ω value is small but increase when ω value is relatively large ($\omega \ge 0.7$). According to Equation (19), we obtain that the number of transit users depend on the number of Class 1, the number of Class 3, and the value of ω. According to Equations (33)–(35), we know that the number of Class 1 and 2 decrease but the number of Class 3 raises with increase of δ value. When the ω value is small and the δ value is increasing, the magnitude of decrease of the first term in Equation (19) is larger than the magnitude of increase of the second term in Equation (19). Although the ω value is relatively large, the situation is the opposite.

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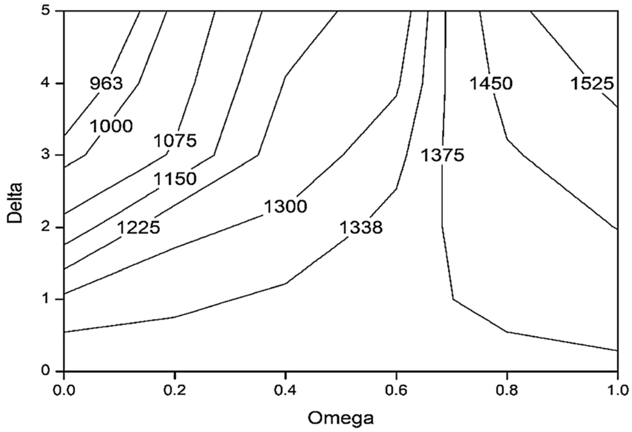

Figure 2. Number of transit users under different levels of bounded rationality.

Figure 3 shows the total travel cost with ω varying from 0 to 1 and δ varying from 1 to 5. From Figures 2 and 3, we can see that the total travel cost does not fall with the increase of the number of transit users. Considering the discomfort generated by body congestion in the bus or subway, the increasing share of transit mode may not reduce the total travel cost. The government should make the optimal design of scheduled public transport services, such as frequencies, vehicle sizes, spacing of bus stops, and so on.

4.2. *Example 2: effect of risk preference on the modal split*

Figure 4 shows the number of transit users with different risk preference. It can be seen that the number of transit users decreases with an increasing risk averse preference. The result is universal under all levels of bounded rationality. The number of users of transit mode diminishes smoothly as its generalized cost grows farther away from that of the auto mode. Travelers with high risk averse preference choose private car over transit. They prefer the risk of congestion on road over the risk of discomfort in the bus or subway. It is obvious that travelers' risk attitude has sizeable impacts on the modal split.

4.3. *Example 3: effect of bounded rationality on the optimal transit fare*

In the following, we assumed that all travelers are risk-averse ($\lambda = 0.1$). Figure 5 depicts the optimal transit fare versus the level of bounded rationality. We can see that the optimal transit fare decreases with the increasing parameter δ when the ω value is small, while it increases when the ω value is large.

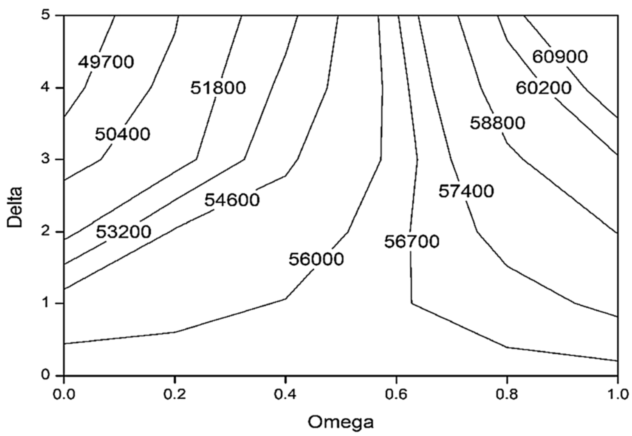

Figure 3. The total travel cost under different levels of bounded rationality.

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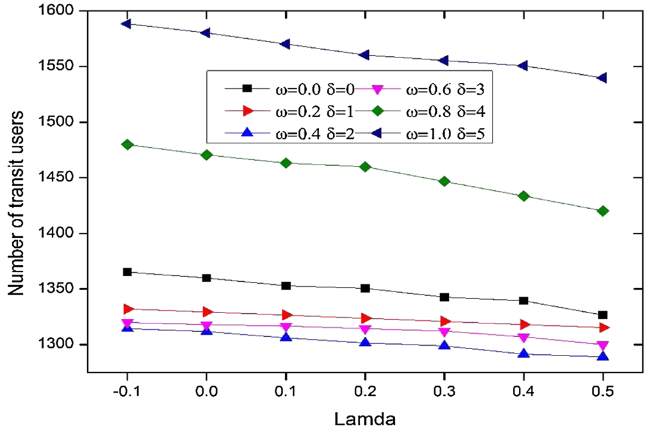

Figure 4. Number of transit users with different risk preference.

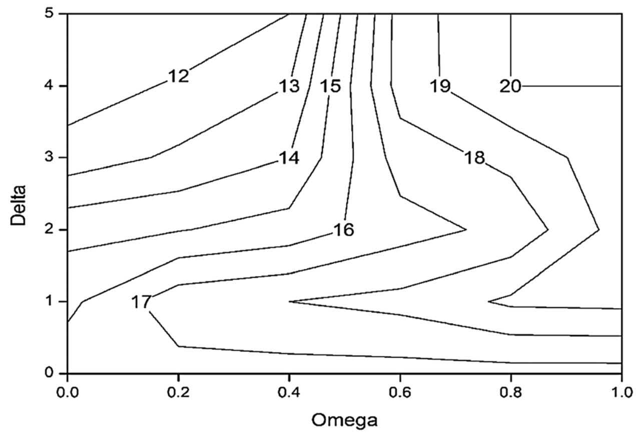

Figure 5. The optimal transit fare under different levels of bounded rationality.

The lower transit fare attracts more transit users but causes more discomfort generated by body congestion at stations and in carriages. On the other hand, the larger transit price attracts less transit users but leads to more private car users. Therefore, the lower or larger transit pricing generates the higher total system travel cost. It is necessary to optimize modal split and minimize the total system travel cost by optimal transit fare.

4.4. Example 4: effect of transit fare on the system performance

Figure 6 shows the comparison of the mean of total travel cost with fixed and optimal transit fare. It is clear that the optimal transit fare can reduce the total travel cost under all levels of bounded rationality. There is less variability in total travel cost under optimal transit fares. The optimal transit fare is more effective at reducing the total travel cost, especially when the ω value is relatively large ($\omega \geq 0.6$).

4.5. Example 5: effect of different bounded rationality on the modal split and optimal transit fare

We study travelers' mode choice behavior with the same tolerance parameters in Equations (17) and (18). Users of the same O–D are assumed to have the same threshold value for different travel modes

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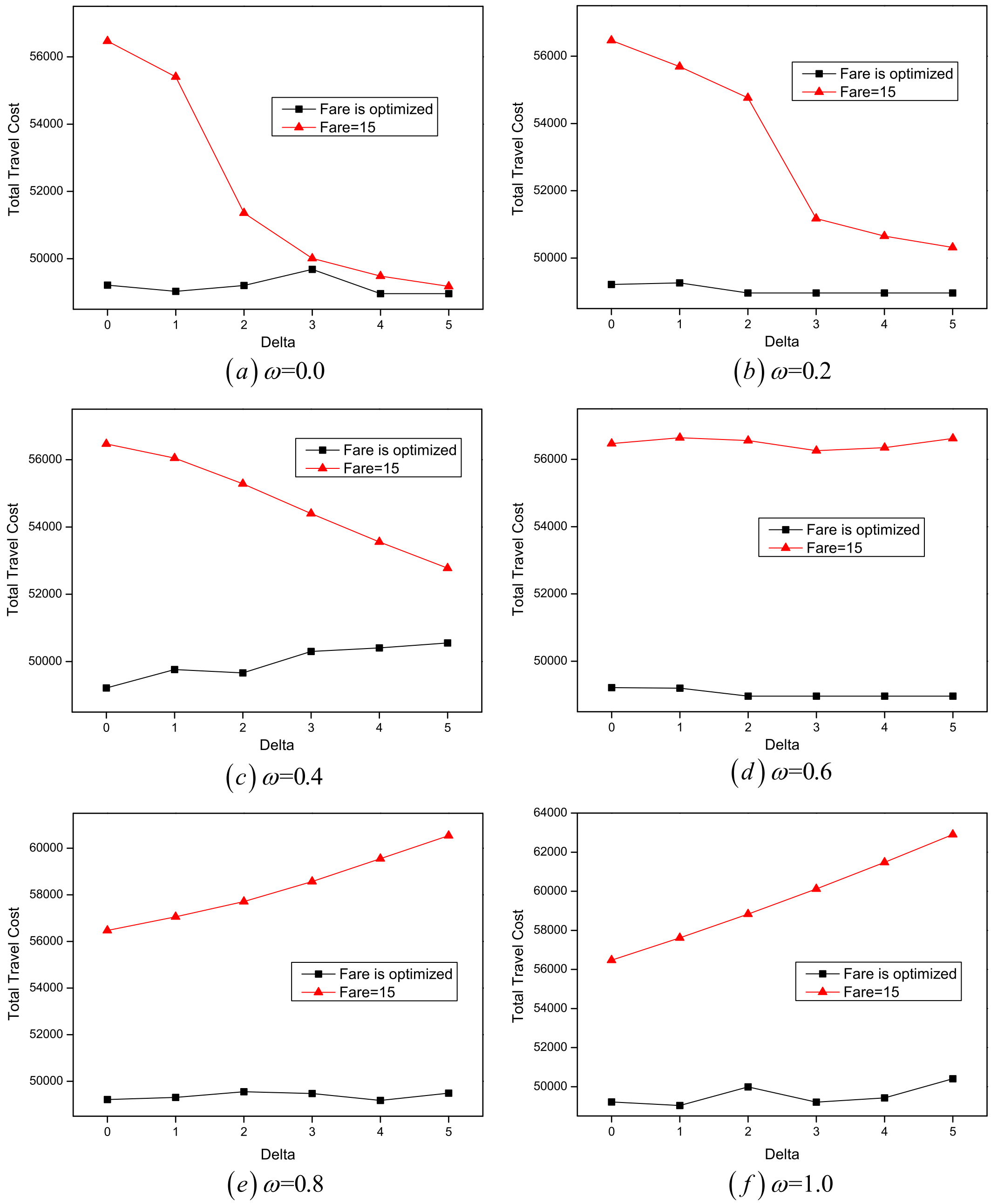

Figure 6. Comparison of total travel cost with fixed and optimal transit fare.

in the preceding examples. In this example, we relax this assumption and indicate that travelers of different modes perceive travel cost with different bounded rationality.

The general mode choice model with different bounded rationality could be written as follows,

$$p_T = pr\{C_T \le C_A + \delta_T\}$$
$$p_A = pr\{C_A \le C_T + \delta_A\}$$

Transit users and auto users of the same O–D pair have different threshold values, denoted as δ_T and δ_A ($\delta_T \neq \delta_A$), respectively. Without loss of generality, we assume that $\delta_T > \delta_A$. When $C_T - C_A \le -\delta_A$,

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travelers choose transit mode for sure; when $C_T - C_A \geq \delta_T$, travelers choose auto mode for sure; when $-\delta_A \leq C_T - C_A \leq \delta_T$, travelers may choose either of them, as is defined by the choice behavior with bounded rationality. We can obtain

$$p'_T = pr\{C_T - C_A \leq -\delta_A\} + \omega pr\{-\delta_A \leq C_T - C_A \leq \delta_T\}$$
$$p'_A = pr\{C_T - C_A \geq \delta_T\} + (1-\omega) pr\{-\delta_A \leq C_T - C_A \leq \delta_T\}$$

Then we have

$$p'_T = (1-\omega)\frac{e^{-\overline{C}_T-\delta_A}}{e^{-\overline{C}_T-\delta_A}+e^{-\overline{C}_A}} + \omega\left(1-\frac{e^{-\overline{C}_A-\delta_T}}{e^{-\overline{C}_A-\delta_T}+e^{-\overline{C}_T}}\right)$$
$$p'_A = \omega\frac{e^{-\overline{C}_A-\delta_T}}{e^{-\overline{C}_A-\delta_T}+e^{-\overline{C}_T}} + (1-\omega)\left(1-\frac{e^{-\overline{C}_T-\delta_A}}{e^{-\overline{C}_T-\delta_A}+e^{-\overline{C}_A}}\right)$$

The transit fare is arbitrarily fixed (i.e., $\tau_T = 15$(¥)), and travelers are risk neutral ($\lambda = 0$) in this example. Figure 7 shows the number of transit users with ω varying from 0 to 1 and δ_T varying from 1 to 5 when δ_A is fixed ($\delta_A = 1$). It can be seen that, as the value of δ_T largens, the number of transit users will increase. The reason for the change is that the number of Class 2 decreases but the number of Class 3 raises with increase of δ_T value, whereas the number of Class 1 is constant. Thus, the number of transit users goes up with increasing number of Class 3.

Figure 8 shows the number of transit users with ω varying from 0 to 1 and δ_A varying from 1 to 5 when δ_T is fixed ($\delta_T = 5$). It can be shown that, as the value of δ_A increases, the number of transit users will fall. The reasonable explanation is that the number of Class 1 decreases but the number of Class 3 rises with increase in δ_A value, whereas the number of Class 2 is constant. Then the number of auto users becomes higher with increasing number of Class 3, which leads to the decrease in transit users.

Figures 9 and 10 show changes in optimal transit fare with respect to the different levels of bounded rationality. Compared with the Example 3, the similar results can be obtained under the condition that travelers choose the mode with different tolerance parameters reflecting bounded rationality. From Examples 3 and 5, we can determine the effect of bounded rationality in mode choice behavior on the evaluation of optimal transit fare.

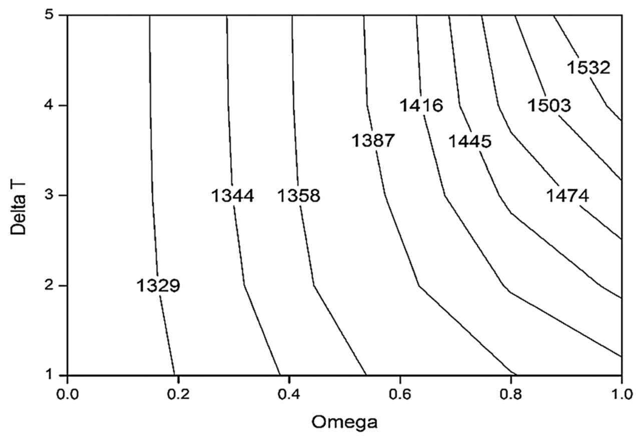

Figure 7. Number of transit users under different levels of bounded rationality ($\delta_A = 1$).

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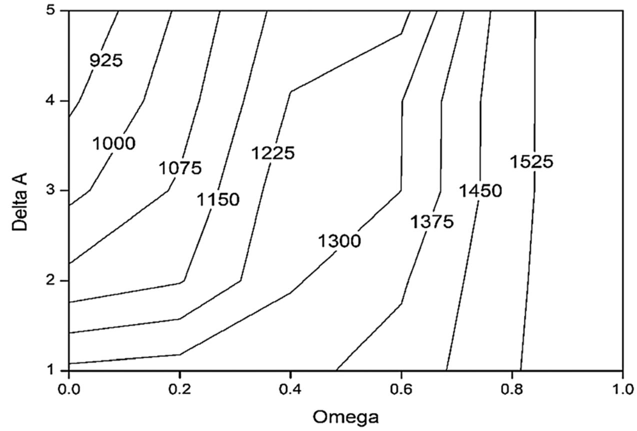

Figure 8. Number of transit users under different levels of bounded rationality ($\delta_T = 5$).

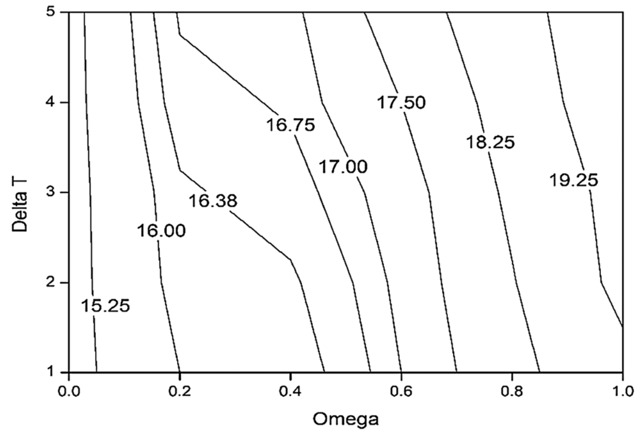

Figure 9. The optimal transit fare under different levels of bounded rationality ($\delta_A = 1$).

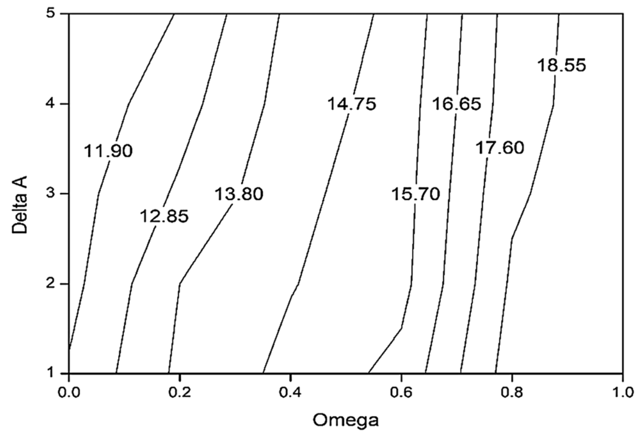

Figure 10. The optimal transit fare under different levels of bounded rationality ($\delta_T = 5$).

5. CONCLUSIONS

This paper developed a bi-level model in a simple bimodal transportation system under demand uncertainty and bounded rationality to investigate the optimal transit pricing. The transportation

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system comprised two modes of mass transit and private car. With travel demand uncertainty, the travel costs of public and private mode were also uncertain. Travelers were assumed to consider both the mean and the variance of travel cost in their mode choice decision. Under bounded rationality, users did not necessarily choose the travel mode of which perceived travel cost is absolutely lower than the one of the other mode. The reason was that doing so did not reduce their travel cost by a significant amount. Some numerical examples were presented to illustrate the effect of demand uncertainty and bounded rationality on the modal share, optimal transit fare, and system performance.

We found that the equilibrium solution of the modal split model based on bounded rationality was not unique. This was different from the conventional modal-split model. The number of travelers selecting mass transit or private car mode depended not only on the travel cost (including fare, time, and comfort) but also on travel psychology and behavior characteristics based on bounded rationality. Considering the discomfort generated by body congestion for public transportation, the total travel cost is not decreasing with the number of transit users. Travelers with risk-averse attitude would make a trade-off between the risk of congestion in bus or subway and congestion on road. And they tend to choose the private car over mass transit. The optimal transit fare could reduce the total travel cost significantly, especially when the level of bounded rationality in the travelers' mode choice decision making is high.

It should be pointed out that the transportation system and the models studied in this paper are simple from the viewpoint of practice. Therefore, further studies may focus on the more general traffic network. We plan to carry out further work on a combined model based on bounded rationality for modal split and flow assignment in multimode network. And it is interesting but challenging to extend the proposed model to the situation considering congestion pricing for private car.

6. LIST OF NOTATION

X	stochastic O-D demand
x	mean O-D demand
cv	coefficient of variance of the O-D demand
X_T	stochastic numbers of transit travelers
X_A	stochastic numbers of auto travelers
x_T	mean numbers of transit travelers
x_A	mean numbers of auto travelers
t_T	in-vehicle travel time by transit
α	unit cost of travel time
π	unit cost of discomfort
τ_T	transit fare
T_A	stochastic travel time by private car
oc_A	operating cost of an auto user
$\widetilde{C}_T$	generalized travel cost of a transit passenger
$\widetilde{C}_A$	generalized travel cost of an auto user
$\overline{C}_i$	effective generalized travel cost for mode $i(i=T,A)$
C_i	perceived effective generalized travel cost for mode $i(i=T,A)$
ϵ_i	additive random term which represents the uncertainty in specifying the cost of selecting mode $i(i=T,A)$
p_i, p'_i	probability that mode $i(i=T,A)$ are chosen
$\underline{\tau}$	the minimum transit fare constraint
$\overline{\tau}_T^T$	the maximal transit fare constraint
Z	mean total travel cost
u, v	parameters in the body congestion function
t_A^0	auto free-flow travel time

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b notional road capacity
β, n parameters of the travel time function
λ risk-aversion parameter
δ pre-specified threshold value
δ_i pre-specified threshold value for mode $i(i=T, A)$ user
ω parameter which reflects free mode choice

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APPENDIX A
DERIVATION OF THE VARIANCE OF BODY CONGESTION COST FOR TRANSIT MODE

The variance of body congestion cost can be calculated by the following formulae:

$$\begin{aligned} Var(g(X_T)) &= Var(uX_T{}^2 + vX_T) \\ &= E\left[\left(uX_T{}^2 + vX_T\right)^2\right] - \left[E(uX_T{}^2 + vX_T)\right]^2 \\ &= E[u^2X_T{}^4 + 2uvX_T{}^3 + v^2X_T{}^2] - \left[E(uX_T{}^2) + E(vX_T)\right]^2 \\ &= u^2E(X_T{}^4) + 2uvE(X_T{}^3) + v^2E(X_T{}^2) - \left[uE(X_T{}^2) + vE(X_T)\right]^2 \end{aligned} \tag{28}$$

By using the moment-generating function of normal distribution, we obtain

$$E\left(X_T^2\right) = x_T^2 + (cv \cdot x_T)^2 \tag{29}$$

$$E\left(X_T^3\right) = x_T^3 + 3x_T(cv \cdot x_T)^2 \tag{30}$$

$$E\left(X_T^4\right) = x_T^4 + 6x_T^2(cv \cdot x_T)^2 + 3(cv \cdot x_T)^4 \tag{31}$$

Substituting Equations (29)–(31) into (28), we can derive Equation (11).

APPENDIX B
THE PROBABILITY THAT EACH MODE IS CHOSEN BY USERS WITH BOUNDED RATIONALITY

From Equation (19), we have

$$pr\{C_T - C_A \leq -\delta\} = pr\left\{\overline{C}_T + \epsilon_T \leq \overline{C}_A + \epsilon_A - \delta\right\} = pr\left\{\epsilon_T - \epsilon_A \leq \overline{C}_A - \overline{C}_T - \delta\right\} \tag{32}$$

By using the cumulative distribution function and probability density function of Gumbel variates, we found that Equation (32) implies that

$$pr\{C_T - C_A \leq -\delta\} = \frac{e^{-\overline{C}_T - \delta}}{e^{-\overline{C}_T - \delta} + e^{-\overline{C}_A}} \tag{33}$$

Similarly, the first term in Equation (20) can be defined as

$$pr\{C_T - C_A \geq \delta\} = \frac{e^{-\overline{C}_A - \delta}}{e^{-\overline{C}_A - \delta} + e^{-\overline{C}_T}} \tag{34}$$

Obviously, the following equation of probability holds

$$pr\{-\delta \leq C_T - C_A \leq \delta\} = 1 - pr\{C_T - C_A \geq \delta\} - pr\{C_T - C_A \leq -\delta\} \tag{35}$$

Then it follows that

$$pr\{-\delta \leq C_T - C_A \leq \delta\} = 1 - \frac{e^{-\overline{C}_T-\delta}}{e^{-\overline{C}_T-\delta} + e^{-\overline{C}_A}} - \frac{e^{-\overline{C}_A-\delta}}{e^{-\overline{C}_A-\delta} + e^{-\overline{C}_T}} \tag{36}$$

Substituting Equations (33)–(36) into Equations (19) and (20), Equations (21) and (22) can be derived.

APPENDIX C
DERIVATION OF THE MEAN AND VARIANCE OF TOTAL TRAVEL COST

The mean of total travel cost can be calculated by the following formulae:

$$Z = E[X_T(t_T + \pi g(X_T)) + X_A T_A(X_A)] \tag{37}$$

By substituting Eqsuations (2) and (5) into Equation (37), we obtain

$$Z = x_T t_T + \pi u E\left(X_T^3\right) + \pi v E\left(X_T^2\right) + t_A^0 x_A + \frac{\beta t_A^0}{b^n} E\left(X_A^{n+1}\right) \tag{38}$$

By using the moment-generating function of normal distribution, we also obtain

$$E\left(X_A^{n+1}\right) = \sum_{i=0, i=even}^{n+1} \binom{n+1}{i} (cv \cdot x_A)^i (x_A)^{n+1-i} (i-1)!! \tag{39}$$

Substituting Eqsuations (29)–(30) and (39) into Equation (37), we can derive Equation (23).

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