

Integrated smart feeder/shuttle transit service: simulation of new routing strategies

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SUMMARY

The idea of designing an integrated smart feeder/shuttle service stemmed from the need to overcome the problem of using an excessive number of cars arriving and parking at a train station within the same time span. This problem results in high parking demand around the train station. Moreover, some potential train riders will, instead, use their cars and hence become a party to increasing the traffic congestion. This work develops a new idea of an integrated and innovative feeder/shuttle system with new operating and routing concepts. The fulfilled objectives are as follows: (i) to construct and examine different operating strategies from both the user and operator perspectives; (ii) to examine different routing models and scenarios; and (iii) to construct a simulation tool for (i) and (ii). Ten different routing strategies are examined, with all the combinations of fixed/flexible routes, fixed/flexible schedules, a unidirectional or bidirectional concept, and shortcut (shortest path) and/or short-turn (turnaround) concepts. These strategies are investigated by employing a simulation model specifically developed and constructed for this purpose. This simulation model is used in a case study of Castro Valley in California in which the feeder/shuttle service is coordinated with the Bay Area Rapid Transit service, and the 10 routing strategies are compared in regard to four fleet-size scenarios. One of the interesting results found is that the fixed-route and flexible-route concepts are comparable in performance measures when applying a combination of operating strategies. Copyright © 2011 John Wiley & Sons, Ltd.

KEY WORDS: feeder/shuttle transit service; advanced public transit systems; demand-responsive transit; simulation; case study; California

1. INTRODUCTION

The known transportation objectives for any residential community are as follows: (i) to find a reasonable-cost approach in order to ease traffic congestion; (ii) to eliminate parking problems; (iii) to reduce road accidents; and (iv) to improve the pollution level. Such a reasonable-cost approach must rely on an attractive, well-distributed, and comfortable public transit system [1].

Service improvements will require spatial changes in public transit that may lead to unfeasible solutions because of the high cost involved. However, by creating pilot projects on certain segments with an attractive transit service, shifts from private cars to public transit can gradually be made. The pilot projects (if successfully implemented) can then become elements of a community plan, along with some complementary measures (higher parking prices, road pricing, fuel taxes, etc.). We cannot change the direction of the wind (evolution of lifestyles, land-use patterns), but we can adjust the sails (create four-star and five-star transit services that eventually will pay off their expenses).

The European Commission perspective expressed in Viegas [2] indicates that the mission of transit has changed: “Whereas until the 70’s its main function was to satisfy the individual needs of the less

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affluent members of society, progressively the policy discourse has been changing, pointing instead to the necessary contribution of public transport for congestion relief and environmental preservation. This represents a fundamental change of emphasis, in the sense that public transport now would be a role geared more to the satisfaction of *collective well-being* than to the direct *individual needs* of those who use it.”

The choice between public and private transport is an individual decision that is influenced by government/community decisions. These decisions often send mixed signals (e.g., “use transit” and “we’re trying to reduce traffic congestion to ease your automobile driving”) to the transit and potential transit passengers while failing to recognize more system-wide, integrated implications. Generally speaking, the majority of large cities have encouraged the use of the private car through planning (dispersed land use in the suburbs), infrastructure (available parking and circulation traffic flow), pricing, and financial decisions. Consequently, there is a growing confusion in many of those cities about what to do. One way to handle the known decline in transit use is to retain high level of satisfaction among transit users, while fully retaining the protection of accessibility to the less affluent travelers. Some research projects in Europe—ISOTOPE [3], QUATTRO [4], and the one done by Kottenhoff [5]—attempt to show a way to overcome this decline in transit patronage.

This work sets out to construct a new idea for designing an integrated, smart feeder/shuttle service. The feeder-service idea stems from the need to overcome the problem of people driving their cars to train or express-bus stations (e.g., Bay Area Rapid Transit (BART)), with consequent high parking demand around these stations. Moreover, some potential train riders, instead, will drive their cars to the workplace and, hence, assist in increasing the traffic congestion. The purpose of this study is to design an innovative feeder/shuttle system that will (i) meet the needs and desires of end users and (ii) increase operational efficiency, with the use of intelligent-transportation technologies.

Ideally, this smart shuttle/bus system will provide advanced, attractive feeder and distributor services that operate reliably and relatively rapidly, forming part of the door-to-door passenger chain, aided by smooth, synchronized transfers. In order to arrive at the design of this type of innovative bus system, a simulation model first had to be constructed and tested. The objectives, therefore, for a smart feeder/shuttle service are as follows: (i) to construct and examine different operating strategies from both the user and operator perspectives; (ii) to examine different routing models and scenarios; and (iii) to construct a simulation tool for (i) and (ii).

This work consists of five parts. First, an overview of the literature is presented. Second, the concepts of transit integration and routing strategies for a feeder/shuttle bus system are described. Third, a simulation model is developed for a smart feeder/shuttle service. Fourth, a case study of Castro Valley is presented; that is, how and on what basis should a smart shuttle service in Castro Valley be designed to coordinate with the BART service. Finally, the conclusions of the study are presented.

2. BACKGROUND

The key issue in providing a smart transit service is finding a good match between users’ needs/desires and the service offered. The term “good match” does not mean relying on known concepts; rather, it signifies that the transit service has been provided with a full understanding of the needs and desires of both existing and potential transit riders [1]. The present section presents an overview of various known, relevant operational transit concepts and related research.

In order to alleviate the problems encountered in traditional transit service, several flexible services were studied. In North America, dial-a-ride and door-to-door paratransit have played a vital role in providing equitable transportation service to elderly and handicapped persons who have difficulty in accessing regular public transit systems [6]. Such a demand-responsive transit (DRT) system can be investigated from different perspectives [7,8], but it does not fulfill the need of the entire transit population. An interesting study [9] distinguishes between two classes of users, so-called *passive users* and *active users*. The *passive users* make use of traditional transit, that is, boarding and alighting at compulsory stops. No reservation is necessary because vehicles are guaranteed to serve each compulsory stop within a given time window. The *active users*, who board or alight at an optional stop, must issue a *service request* and specify pickup and drop-off stops, as well as earliest departure and latest

arrival times. In that study [9], transit vehicles had to be rerouted and rescheduled to satisfy as many requests as possible, complying with passage-time constraints at compulsory stops, while activating optional stops, between two compulsory stops, on demand. The method used in the present study integrates mathematical programming tools into a tabu search framework, taking advantage of the particular structure of the problem formulation.

Dial-a-ride problems usually are solved using classic vehicle-routing heuristics [10,11], based on arc-and-node manipulation, which in turn is generally based on insertion, deletion, and exchange of stops in and out of a current tour. The computation of a lower bound in finding the optimal dial-a-ride solution is not a trivial issue. The linear relaxation of any arc-based integer linear programming model provides a loose bound to some extent. Therefore, heuristics are necessary to cope with practical routing problems. Exact approaches of dial-a-ride problems have been also extensively investigated. Parragh *et al.* [13] indicate that dial-a-ride problems are generalizations of pickup and delivery problems with time windows, with people instead of goods; they considered service quality in the dial-a-ride objective function. A good review of these approaches can be found in Cordeau and Laporte [14].

A few studies make use of simulation as a tool to arrive at satisfactory routing and scheduling DRT solutions. Three waves of simulation studies can be traced in the literature. The first wave consists of the research conducted by Wilson *et al.* [15,16] for evaluating various heuristic routing rules and algorithms used in a computer-aided routing system. These studies, developed for mainframe computers, have limitations in their handling of large-size road networks with different routing strategies. The second wave of research was conducted by Fu [17–19] and Fu and Xu [20], who considered the use of advanced technologies. Their studies present a simulation model, Sim-Paratransit, which was developed for evaluating advanced paratransit systems, such as automatic vehicle location (AVL) and computer-aided dispatch (CAD) systems. The simulation model is described in Fu [17,18], and the evaluation of AVL and CAD systems in Fu and Xu [20]. The ability to track the location of a transit vehicle continuously allows for the introduction of intelligent paratransit systems, which will naturally lead to operating the paratransit systems at a significantly improved level of productivity and reliability [17,18,20–22]. Fu [19] incorporated also a time-dependent and stochastic travel time model in the problem formulation with the consideration of computational efficiency.

The third wave of studies involved simulation based on a mobility allowance shuttle transit (MAST) system in which the transit vehicles can deviate from the fixed routes to allow for the users to be picked up or dropped off at desired locations. This wave is related to the work of Daganzo [23] in which pickup and drop-off points are at centralized locations called checkpoints. The MAST system is described extensively by Quadrifoglio *et al.* [24–27]. In their studies, the authors brought up the point that scheduling and dispatching flex-route transit can raise a significant challenge because of the demand at fixed stops, the requests for deviations, and the dynamics of transit-vehicle schedule adherence. This challenge has led to many proposed heuristics on scheduling flex-route transit. Quadrifoglio *et al.* [26] developed an insertion algorithm to schedule the MAST system. Associated with the third wave, an additional study on vehicle routing and scheduling of a DRT system is made by Dessouky *et al.* [28]. The authors demonstrated through simulation that it is possible to reduce environmental impact substantially, while increasing operating costs and service delays only slightly for the joint optimization of cost, service, and life-cycle environment.

Another perspective of providing paratransit systems is the zonal arrangement. Often, DRT designers divide the service area into zones to better manage for the operation. Recent research concerning optimal zones was reported by Quadrifoglio and Li [29–31]. Quadrifoglio and Li [29] developed a model to determine the best operating policy to be adopted in one residential zone to maximize the level of service; they defined and derived the “critical demand density” representing the switching point between the fixed-route and DRT competing policies. The same authors [31] added to their study a tool to choose between fixed route and DRT and when to switch from one to the other during the day. Li and Quadrifoglio [30] developed a model to determine the optimal number of zones for the feeder transit systems, for which a one-vehicle operation is adopted in each zone.

According to the American Public Transit Association [32], the total operating expenses of paratransit service in the U.S. exceeded \$4.84 billion in 2008, whereas only \$498.6 million were collected in fares. Introduction to advanced technologies opens a window of opportunity to reduce the operating costs and to increase the ridership of these types of transit services. At the same time, there is also a

need to investigate the users' willingness to use and pay for an advanced transit system. Employing the computer-assisted telephone-interview method, the Partners for Advanced Transportation Technology (PATH) program carried out a consumer-response study of DRT systems in the San Francisco Bay Area in 1999 [33] to address the users' perspective. This study examined the factors that are likely to influence the decision to take an "on-demand" rather than a "fixed-schedule" DRT system, in which the pickups and drop-offs are made at fixed but convenient locations. The researchers' survey indicates that the DRT idea appeals to both commuters and non-commuters. About 15% of those surveyed were considered "very likely" to use the DRT service, whereas about 48% were willing to consider it as an option. A majority of the DRT pre-disposed were willing to pay from \$5–\$10 for a 30-minute trip using either a fixed-schedule service (62%) or an on-demand service (73%). Overall, the results of that study [33] show that a reasonably priced DRT service that is reliable and meets customer expectations (of cost, travel time, and waiting time) can be successful.

There exists, then, a recognition that the demand for public transit will grow in the future, given the provision of attractive, advanced transit systems. Galileo wrote: "Science proceeds more by what it has learned to ignore than by what it takes into account." In order to proceed toward a better transit system, more attention should be given to reducing the gap between users' needs and desires and the transit service offered, and less to subordinate issues such as various personal complaints. The previous studies cited do not contain analyses of the strategic-operational side of potential smart systems. This side, however, cannot be ignored. It is the intention of this work to map, explore, and analyze all possible operational strategies that may appeal to potential transit users. Certainly these strategies assume the use of advanced technologies, and therefore, they will be investigated using a simulation tool, which is also reported by Ceder [1].

3. SMART FEEDER/SHUTTLE: INTEGRATION AND ROUTING CONCEPTS

3.1. System integration

It is known [1] that the basis for attracting more transit patronage is to allow for the following: (i) comfort, (ii) perceived low out-of-pocket cost, and (iii) flexibility (always there when needed, allowing its user to enjoy door-to-door services; with a low level of information required for its use). One essential item for increasing system attractiveness is to have good integration, which can be variously interpreted: (i) good information on the available options; (ii) stability of perception of service; (iii) network integration; (iv) ticketing integration using smart cards; and (v) maximum synchronization.

Good information on the user's options should cover all transit modes and needs to be tailored to the user's needs. The information should be based on simplicity and accuracy, while taking into account the exact way to reach B from A for any A–B points on the transit network. The information should consist of all transit modes and all operators in a single system. The *stability of perception of service* implies infrequent schedule changes because frequent changes may introduce confusion among the users. In other words, "stability" refers to long validity periods for transit timetables. *Network integration* implies smooth transfers and comfortable interchanges, that is, easy change on routes in a single trip, no matter if routes are traversed by more than one mode and/or operator, and available interchanges to allow for smooth transfers. *Ticketing integration* is based on a combined tariff using the same payment method, such as the same smart card (e.g., the Octopus card in Hong Kong, which is used by heavy rail, metro, bus, and ferry modes alike). Finally, *maximum synchronization* is for better coordination among the routes and transit modes and for minimization of the transfer and waiting times. This synchronization of the users' timetables should be carried out both off-line (planning stage) and online (considering actual situations involving a transit vehicle being behind or ahead of schedule).

Based on European Union Papers and Studies [2–5], successful integration requires a hierarchical road-network design that integrates surface transit, private cars, bicycles, and pedestrians. Physical integration is attained by means of the optimal arrangement of individual motorized transportation plus public transit and transfers from individual motorized transportation to public transit. These arrangements include Park+Ride (P+R), Kiss+Ride (K+R), Bike+Ride (B+R), and taxis. Interconnections of different types of transit systems (rail, bus, taxi, and ferry) take place, using many architectural

forms in transferring passengers from one transit mode to another. The number of cities considering these schemes continues to grow. There is also an increase in the number and types of interchanges developed for different transit systems.

One benefit that emerges from P+R systems is that they enable economic and environmental enhancement. A successful P+R scheme can help pedestrianization, which might otherwise be resisted. A successful transit practice at interchanges is the combined activities of different transit systems, for example, feeder buses and local train services using the same platform. In studies of interchanges [3], emphasis is normally given to short-distance transport facilities, which include continuous systems (pedestrian corridors, constant-speed and accelerating conveyors, and escalators), semi-continuous systems (vehicles slowing down in stations), and discontinuous systems (shuttles). A recently developed innovative system is an accelerating conveyor, called "Walkway," produced by Mitsubishi, Japan.

3.2. Transfers

Transit passengers usually perceive a transfer (vehicle to vehicle using either the same transit mode or different modes) as one of the most inconvenient attributes of a transit system. Such a transfer involves walking and waiting (often in a queue), the two elements that usually are not part of using a car. In existing transit systems, the recommendation is to minimize this type of transfer or, at least, to minimize one (or both) of its intermediate elements, walking and/or waiting [1].

Whenever a transit-development alternative is under consideration, there is usually need to evaluate the adverse effects of transfers inherent in the alternative plan. However, one might, instead, think of how to avoid an inconvenient transfer by introducing the idea of smooth, synchronized transfers. These smooth transfers rely on new technologies (e.g., moving walkways, escalators, elevators, using carts, and electrical slow-speed vehicles). The synchronization is based on an exact arrival/departure timing that can be handled by a real-time intelligent control system and the use of certain algorithms to create the transfer connections listed in the timetables. Therefore, any alternative that contains large walking +waiting transfers should be eliminated or revised.

3.3. Smart feeder/shuttle

Here is one definition [1] of a smart feeder and/or shuttle transit service: *An advanced and attractive feeder/shuttle transit system that operates reliably and relatively rapidly, with smooth (ease of), synchronized transfers, part of the door-to-door passenger chain.* The interpretation of each component in this definition is as follows:

Attractiveness: available information (telephone center, Internet, newspaper, radio, TV, mail leaflets), simple communication (abbreviated telephone number, automatic storage of users' telephone number and address), clear user/service meeting points characteristics (smart-vehicle color and logo, user waving smart-vehicle card) boarding/alighting/riding comfort (low-floor, extra space next to driver, comfortable seats, possible features for physically challenged people, low noise), onboard service (newspapers, magazines, free coffee/tea, TV/video display of timetable, weather, etc.), simple payment (electronic ticketing, pre-paid, transfer and smart-card ticketing).

Reliability: small variance of measures of concerns to users (total travel time, waiting time, in-vehicle time, seat availability), small variance of measures of concerns to smart vehicles (schedule adherence, headways, on-time pullouts, missed trips, breakdowns, load counts, late reports), small variance of measures related to pre-trip information using telephone communication (online timetable, travel time to caller, suggested time interval for second call from or to the user).

Rapidness: local authority permission for smart vehicles to stop along the route (fixed stops with shelters and information, bus bays at timepoints, with an extra approach lane at signalized intersection, flexible stops along the route, with the smart vehicle equipped with flashing lights), smart-vehicle preference at unsignalized intersections ("yield" or "stop" not according to traffic procedures, special bypass arrangements at strategic points), smart-vehicle preference at signalized intersections (passive priority by extending or shortening green, active priority using AVL-actuated smart-vehicle signals—e.g., radio, inductive loop), purchase and validate tickets (electronically, ordinary) on smart vehicles (one way, round trip, transfer, daily, weekly, monthly).

Smoothness (ease of): comfortable routing (min/max criterion on walking distance, round-trip deviation from designated route in bad weather, evolution of flexible routing and scheduling), special train-station entrance (smart-vehicle special gate, smart-vehicle entrance door, with comfortable stairs/escalator to the train platform), special train exit (exit door next to the train platform for smart-vehicle ticket holders, smart vehicle waiting at exit or under shelter, with vehicle-arrival announcement on variable message signs (VMS)).

Synchronizes: online communication between the train service and the smart vehicle (vehicle equipped with arrival information for the relevant station(s), and time difference, positive or negative, for synchronization), smart vehicle subscription with serial numbers (adding a variable scheduling element to suit subscribers, planning the fixed scheduling component with subscribers' information), short turn or turnaround and shortcut routing strategies (computerized suggestion for the smart-vehicle driver on short turn and shortcut, VMS onboard information on meeting time with another or the same transit mode).

3.4. Routing strategies

Once the major elements of the smart feeder/shuttle transit service are defined, attention should be given to smart routing strategies. These strategies represent the flexibility and, to some extent, part of the attractiveness of the transit system. Ten routing strategies are investigated in this work:

- (1) Fixed route with a fixed schedule (timetable) and fixed direction.
- (2) Fixed route with a flexible (demand driven) schedule, fixed direction.
- (3) Fixed route with a flexible schedule, bidirectional.
- (4) Fixed route, flexible schedule, fixed direction, with a possible short turn.
- (5) Fixed route, flexible schedule, bidirectional, with a possible short turn.
- (6) Fixed route, flexible schedule, fixed direction, with a possible shortcut.
- (7) Fixed route, flexible schedule, bidirectional, with a possible shortcut.
- (8) Fixed route, flexible schedule, fixed direction, with possible short turn and shortcut.
- (9) Fixed route, flexible schedule, bidirectional, with possible short turn and shortcut.
- (10) Flexible (demand responsive) route with a flexible schedule.

It is worth mentioning that a previous study [34] provides optimal procedures to design the fixed route(s) of a shuttle service; thus, in this work, it is assumed as given. Fixed direction means that the shuttle will always maintain the same direction of travel (same sequence of stops), whereas bidirectional allows for having the flexibility to select the direction based on real-time demand information. The term "short turn" means that based on certain loading threshold and synchronization criteria and given that no more passengers are waiting in the remaining part of the fixed route, the shuttle will not continue on its route. Instead, it will turn around and arrive at the train station in the opposite direction, with the possibility of picking up passengers who were too late to be picked up when the shuttle passed through the station previously. The loading threshold is a given (input) number of passengers on board the shuttle. The synchronization criterion means matching the shuttle's new (short turn) arrival time with an earlier train than that originally planned if the entire route is completed. The term "shortcut" means that, based on certain loading threshold and synchronization criteria and given that no more passengers are waiting in the remaining part of the fixed route, the shuttle will not continue its route and, instead, will use the shortest path (minimum travel time) to arrive at the train station. Each strategy allows the flexibility of the other; that is, the loading threshold of the shortcut strategy is higher than the loading threshold of the short-turn strategy. If the latter is reached and there is the possibility of picking up x passengers (after turning around), where x is equal to or greater than the difference between the two loading thresholds, then the shortcut strategy (if feasible) is recommended after picking up these passengers.

Figure 1 represents the 10 strategies on a small network with two shuttle routes, one with a dashed line and one with a dotted line. The clock on the upper-right-hand side exhibits the fixed schedule (in only one strategy); when crossed with X, it means a flexible schedule situation. Arrows in both directions of the route means a bidirectional situation. It can be seen in Figure 1 that the lines with the

arrows deviate from the fixed route in the shortcut strategy. The arrows turn around at a certain point of the network in the short-turn strategy, whereas both representations appear in the strategy involving a possible combination of shortcut and short-turn runs. The last strategy is for a DRT type of service, allowing for the creation of a new route every time, based on the trip bookings.

The idea of covering almost all possible practical routing strategies stemmed from the need to arrive at user desires and understandings. Certainly, there is no intention that all strategies be used at the same time; rather, the idea is to examine which strategy is best for a given demand pattern and magnitude while taking into consideration the real-time traffic situation in the area of the shuttle's trips. A simulation model was devised for that purpose. This simulation tool, to be explained in the next section, enables a comparison of the various strategies, based on the following measures:

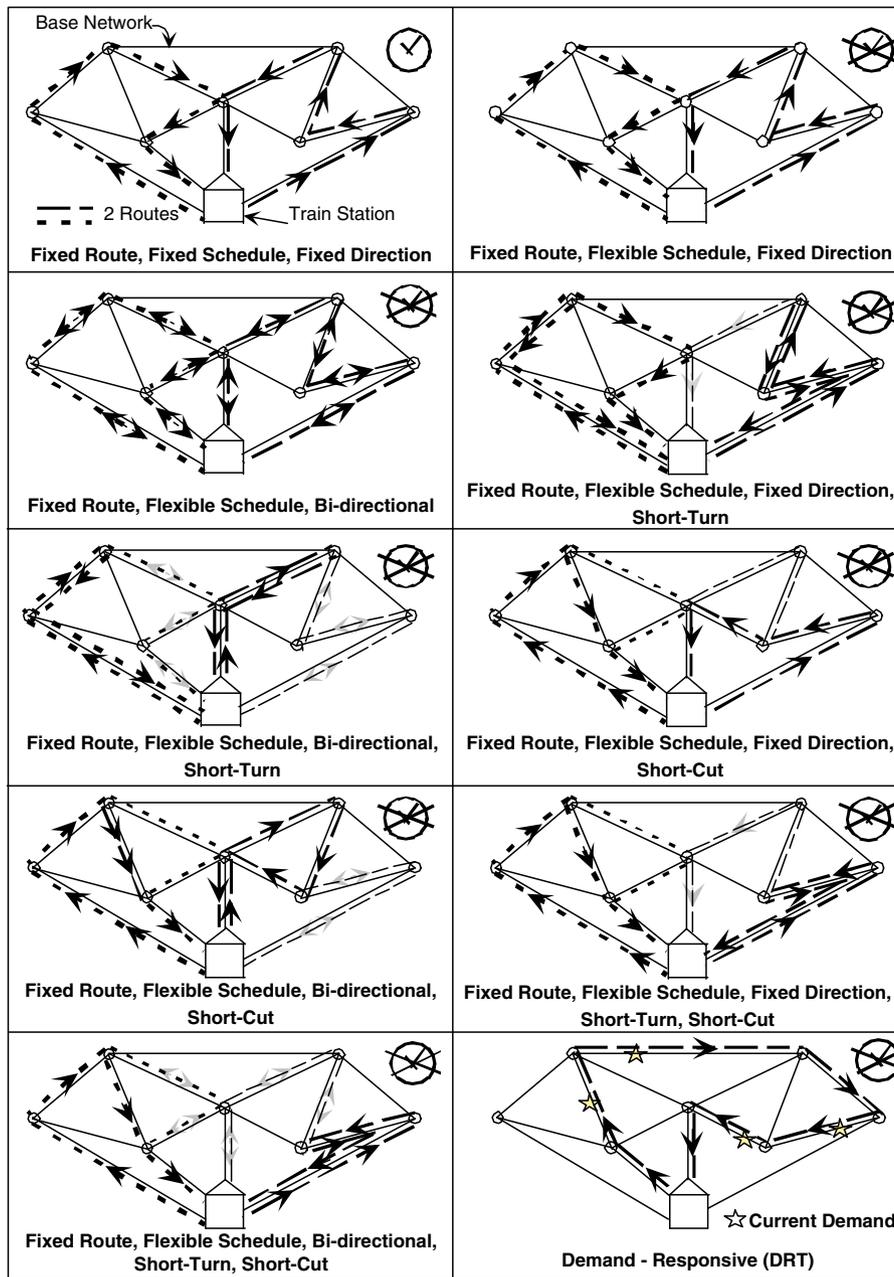


Figure 1. Routing strategies considered for a small network.

- (1) Sum of total time (in passenger-hours) from passenger pickup to train-departure times.
- (2) Sum of total time (in passenger-hours) riding the shuttle vehicle.
- (3) Sum of total waiting time (in passenger-hours) for the train.
- (4) Sum of total waiting time (in passenger-hours) for the shuttle vehicle.
- (5) Total number of transit vehicles (by number of seats) required to meet the demand.

These measures of travel and waiting times and number of vehicles characterize the effectiveness and efficiency of each strategy. Principally, a strategy selected for a given demand, combines the aforementioned measures (1) to (5). However, for the comparison between the different scenarios for a given number of vehicles, the most dominant measure to impact the decision between the use of car or the transit vehicle is the waiting time per passenger [1]. User's perception of this waiting-time measure outscores by far other measures, in terms of value-of-time and importance level based on surveys made by Wardman [35].

4. SIMULATION MODEL

The simulation was written in C++ language and can be run on a PC using Windows. The demand can either be inserted as part of the input or generated randomly on the network. Ten different routing strategies, as outlined in the previous section, can be examined.

4.1. Simulation input variables

Following are the input variables in the simulation model. Each variable is presented by its simulation name and an explanation and interpretation of its substance. What is referred to here as "bus" can be applied to any feeder/shuttle vehicle.

Bus2Train	= Time in seconds that bus must be at the station before train arrives to ensure an efficient meeting
Train2Bus	= Time in seconds that bus must wait after train arrival to ensure pickup
SizeType	= Number of seats in this bus type
Quantity	= Number of vehicles of this SizeType
FixPick	= Fixed time in seconds for one passenger pickup including bus slow down
FixDrop	= Fixed time in seconds for one passenger drop-off including bus slow down
FixBoard	= Fixed (additional) boarding time per passenger
FixAlight	= Fixed (additional) alighting time per passenger
NodeNo	= Node index at section end points
SectionNo	= Section number between two nodes
StopNo	= Stop number starts with SectionNo representing an intersection, not a node
MeanDemand	= Mean number of potential travel requests per given hour and SectionNo to train
MeanDestin	= Mean number of potential travel requests per given hour and SectionNo from train
MeanTime	= Mean section travel time in seconds
StDevTime	= Standard deviation of section travel time in seconds
Min4Turn	= Minimum number of onboard passengers to allow a short turn
Min4Cut	= Minimum number of onboard passengers to allow a shortcut
Min4Trip	= Minimum number of travel requests by calls to allow a non-scheduled trip
Min4Dep	= Minimum of number of waiting passengers to allow a non-scheduled trip
RouteNo	= Unique route index
RouteDir	= Direction of RouteNo (start westbound or eastbound)
TTimeTable	= Fixed Train timetable in hhhmss form hh is from 00 to 24
BTimeTable	= Fixed Bus timetable in hhhmss form hh is from 00 to 24
Layover	= Fixed time for driver rest at the end of each trip

All the aforementioned variables interact in each simulation iteration while using the various strategies and other internal features.

It is to note that the input variables Min4turn, Min4cut, Min4Trip, and Min4Dep are all from the operator perspective; in addition, to construct the strategies with a fixed-route concept is derived also from an operator perspective, and not only from the user perspective.

4.2. Simulation procedures

The simulation model is based on events. The simulation starts with reading the input data and proceeds by arranging the train arrival events and the passenger arrival events. Figure 2 presents the basic event-oriented simulation logic.

There are eight main events, classified in Figure 3. Event 1 represents passengers walking to the stop to wait for the next shuttle in order to arrive at the train station. Event 2 represents passengers arriving on the train and then waiting for the next shuttle. Event 3 occurs when a vehicle becomes available for the next trip. Event 4 is when the shuttle arrives at a node (intersection) on the road network being considered. Event 5 represents the arrival of passengers who want to ride the shuttle from its stop to the train station. Event 6 represents passengers who are about to arrive at the train station (but not yet) and will seek to ride the shuttle. Event 7 is the arrival of the train at the station, including the time for the passengers to arrive at Event 2. Lastly, Event 8 is the time when the shuttle departs, in accordance with the timetable.

The actions taken for Events 1, 2, and 3 appear in Figure 4. This starts with enquiring whether the number of passengers who want the service has reached the minimum required for dispatching a vehicle. For the DRT strategy (no fixed route), the procedure in Figure 4 identifies the section with current (booked) demand and the application of a simple shortest-path algorithm, which employs a known dynamic-programming process called Bellman–Ford [36] to do so. The dynamic-routing procedure is to move with the shuttle from the train station across all the demand points to the first demand point that is within the shortest path from the station. From the last point, the shortest-path algorithm is used again for all the other demand points that were not visited, until all the points have been included in the dynamic route. This DRT routing procedure has been found to be effective and convenient to use in the simulation [37], although more advanced procedures are known.

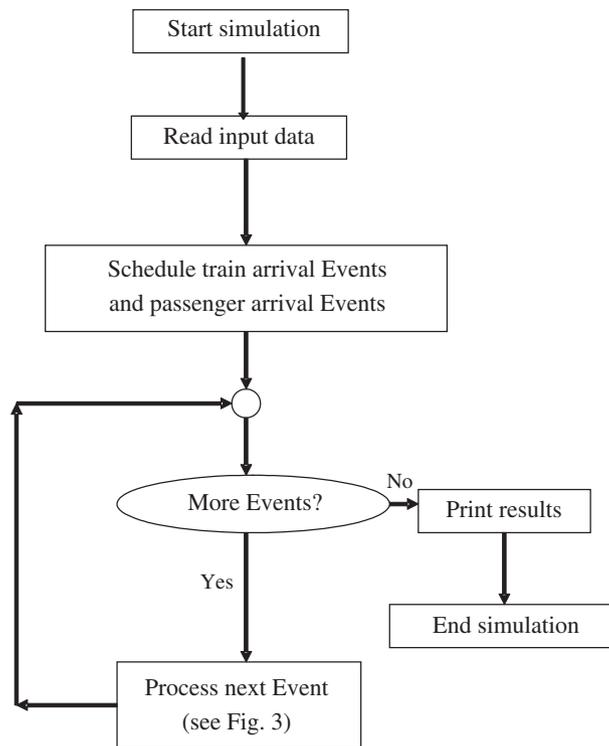


Figure 2. Basic simulation logic.

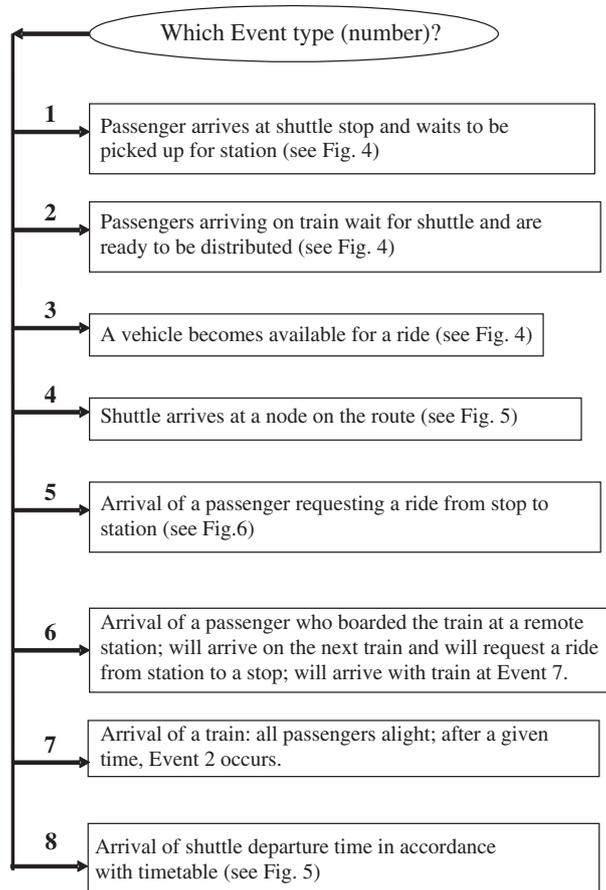


Figure 3. Event classification.

In Figure 4, once a vehicle is available, the next event is of type 4, described in Figure 5. Thus, Event 4, in Figure 5, starts either with a station node (train station), whether at the end or at the beginning of the shuttle ride or at an intermediate node. For any intermediate node, the procedure checks whether the minimum number of passengers onboard the shuttle reaches the threshold for either a short turn or a shortcut. The procedure then checks to see whether creating a short turn or shortcut enables arriving at an earlier train than that which would be met by completing the entire route. Finally, Figure 6 describes the actions taken in the simulation for Event 5, which is the process of informing the user of the available online service. There are two alternatives: (i) the user will be able to reach his/her adjacent stop on time, and (ii) the user will be notified by a callback of the reliable arrival time. It is assumed that users will either call or look at the shuttle Web site to ascertain the arrival time. The user will then be asked to click, for instance, “1” for wanting to use the service or “2” for not wanting to use it (following the announcement of the expected arrival time, which might be inappropriate for his/her use). Only those who click “1” (OK) are taken into account in the simulation process. The simulation model can either consider a given demand figure or be used to generate a random demand, based on the residential density of each section of the network. In the fixed-route case, the users reach their closest stop in the network. The travel time is a random variable with a normal distribution, and the simulation model calculates the probability of being on time. If this probability is below 90%, the user is notified to wait for a callback. In this way, the system uses the philosophy of advanced technologies and maintains a highly reliable service. Finally, the results of the simulation runs exhibit, for each set of input and chosen strategy, the expected values and their magnitudes (e.g., of the wait time). What follows is a detailed example of the simulation model and its components.

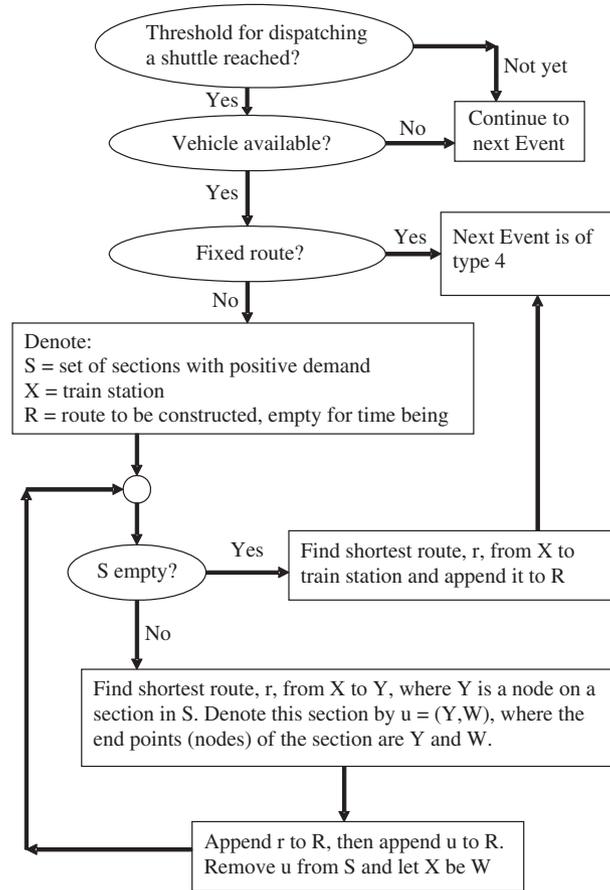


Figure 4. Actions taken for Events 1, 2, and 3.

4.3. Detailed example

A small example is depicted in Figure 7. In this example, there are six sections; of the six, five are two-way sections, and one is a one-way section. There are also four stops, none in a node. That is, the shuttle can make pickups only in the four stops.

What follows are the input parameters and data of the example problem. First, is the information on sections (called SECTION GEOMETRY in the simulation model) represented by six numbers separated by comma in the following order: starting node, ending node, mean travel time in seconds without stopping, standard deviation of travel time without stopping, number of stops, and directions (1 = one way, 2 = two way). The train station is node 0. In this example, the travel time is deterministic (standard deviation = 0).

Thus, in the example:

- SECTION GEOMETRY: 0, 1, 120, 0, 0, 2;
- SECTION GEOMETRY: 0, 4, 120, 0, 0, 2;
- SECTION GEOMETRY: 1, 2, 420, 0, 1, 2;
- SECTION GEOMETRY: 2, 3, 240, 0, 2, 2;
- SECTION GEOMETRY: 2, 4, 120, 0, 0, 1;
- SECTION GEOMETRY: 3, 4, 420, 0, 1, 2.

Table I contains the parameters of the simulation model. The input of Table I is expressed, in the simulation, as follows.

- BUS: 1, 2, 27;
- T DELAYS: 240, 180;

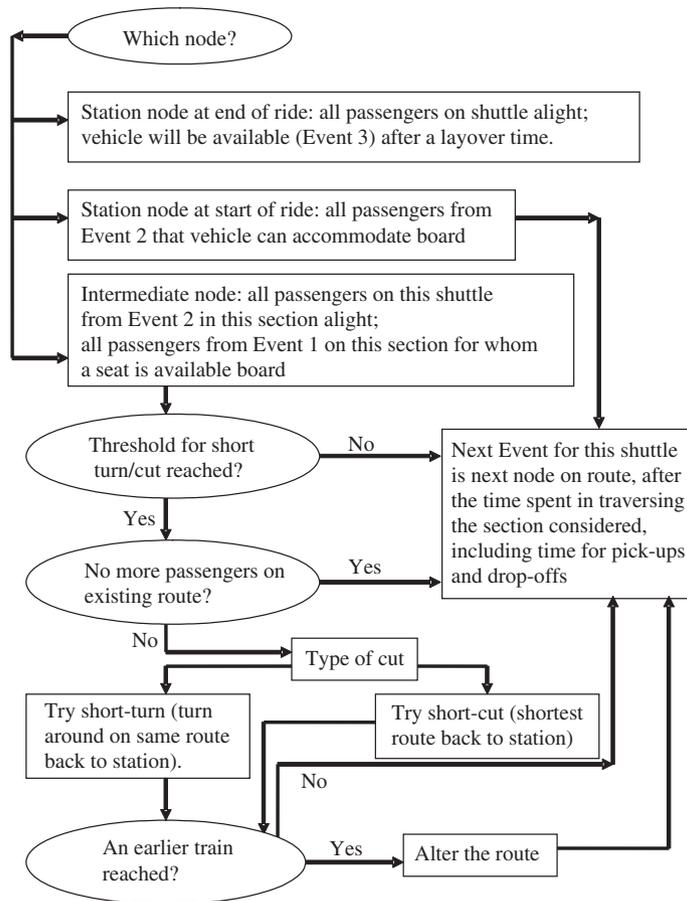


Figure 5. Actions for Events 4 and 8.

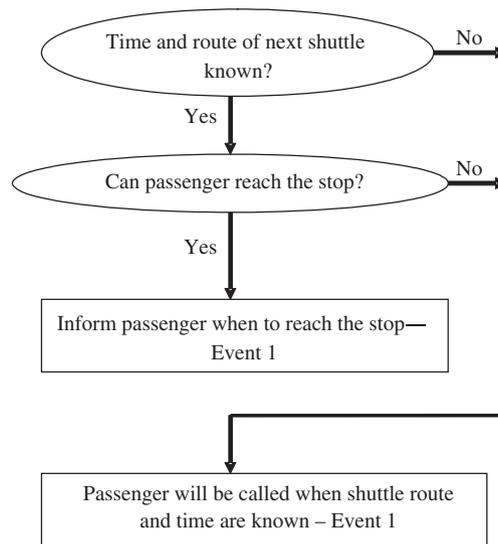


Figure 6. Actions taken for Event 5.

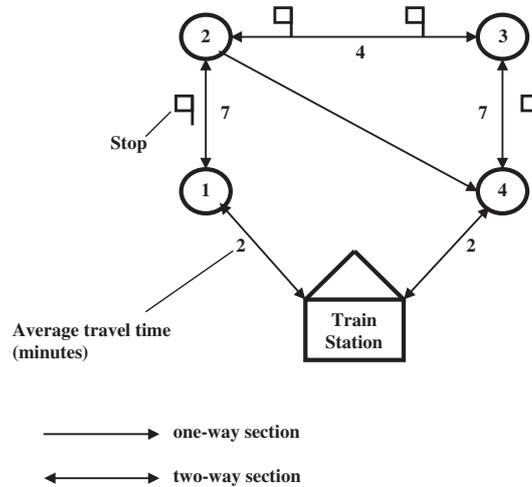


Figure 7. Small network example.

B DELAYS: 3, 5, 20, 25;
 LAYOVER: 10;
 START ROUTE: 20, 19;
 STOP ROUTE: 18, 17.

Passenger demand is assumed to be 50 for both train station’s pickup and drop-off per hour. This demand is distributed among the sections by 10% (Sections 1–2), 65% (Sections 2–3), and 25% (Sections 3–4).

In the simulation model, the demand is expressed as follows:

- (1) T2B DEMAND LIMITS: 630, 50, 1000; that is, between 06:30 and 10:00, the average number of passengers to be distributed by the shuttle service is 50 (following the arrival of the train).
- (2) B2T DEMAND LIMITS: 630, 50, 1000; that is, between 06:30 and 10:00, the average number of passengers to be picked up by the shuttle service is 50.

Table I. Simulation parameters and their values for the detailed example.

Name	Explanation	Value	Remark
Quantity	Number of vehicles	2	Only one type of shuttle
Size	No. of seats in each vehicle	27	
Bus2Train	Time that a vehicle must be before train arrival to ensure meeting	240	Time in seconds
Train2Bus	Time that a vehicle must wait after train arrival to ensure pickup	180	Time in seconds
FixAlight	Alighting time per passenger	3	Time in seconds, starting with 2 nd passenger
FixBoard	Boarding time per passenger	5	Time in seconds, starting with the second passenger
FixDrop	Time for first passenger drop-off	20	Time in seconds
FixPick	Time in seconds for first passenger pickup	25	Time in seconds
Layover	Time for driver rest at the end of each trip	10	Time in minutes
Min4Dep	Minimal number of waiting passengers to allow a non-scheduled trip	20	Infinity for all trips in a timetable
Min4Trip	Minimal number of travel requests by calls to allow a non-scheduled trip	19	Infinity for all trips in a timetable
Min4Cut	Minimal number of onboard passengers to allow a shortcut	18	Infinity for no shortcut
Min4Turn	Minimal number of onboard passengers to allow a short turn	17	Infinity for no short turn

Times are in hhmm (without colons) and start just after midnight. Thus, 6:00pm is 1800 and quarter past midnight next day is 2415. Next, the demand's distribution is inserted (for each direction) as follows:

SECTION DEMAND: 10, 10;
SECTION DEMAND: 65, 65;
SECTION DEMAND: 25, 25.

In the simulation input, each demand's proportion appears after its corresponding SECTION GEOMETRY. Fixed routes such as 0–1–2–3–4–0 are inserted simply as ROUTE: 1, 0, 1, 2, 3, 4, 0. Note that each route must start and end at node 0. However, the first number in this input is the route's index starting with 1. This index is served as a reference for possible (if any) shuttle timetables.

Train timetable (arrival times to the station) is given as TRAIN TIMES: 700, 800, 900, and 1000; that is, trains arrive at each round hour from 7:00 to 10:00 inclusive. Finally, the shuttle timetable is defined for the example as FIXED BUS TIMES: 1, 630, 720, 735, 820, 825, 920, and 925. The first number is the index of route followed by departure times in hhmm format.

The simulation output files appear best on Excel while reading them as tab-delimited text files (the default for text files within Excel). For instance, one file summarizes what occurred to each passenger; an example of such is reproduced in Table II.

The first four columns of Table II are interpreted as follows.

- *Index*: ID for each passenger.
- *Phoned*: The time of demand for a pickup arises (via Tel or arrival at the station).
- *Stop*: Stops are numbered, –1 is the train station, 0, 1, . . . , are the shuttle stops.
- *Bus no.*: ID for each vehicle.

Statistics on the results can be easily obtained using Excel. For example, the histogram for Phone-RideStart of passengers seeking a ride to the train station can be depicted as is shown in Figure 8. The *Y*-axis represents the number of passengers that experienced wait time from call to pickup between $5 \times (I - 1)$ and $5 \times I$ minutes, where *I* is the number on the *X*-axis.

More statistics of the exact simulation runs are available such as in-vehicle time and operator cost. Analysis of the results may shed light on the operational conclusions to be drawn.

5. CASE STUDY: CASTRO VALLEY

In order to test a real-life situation, the area of Castro Valley in California was selected for data collection and simulation runs. The BART station in Castro Valley is on the "blue" line, Dublin/Pleasanton–Daly City. Currently there is one bus line (AC Transit, line 87) within the Castro Valley neighborhood that provides a transit service to the BART station. However, line 87 is not effective and has a low level of passenger use.

A site observation was conducted in the Castro Valley area from which the base network and stops were created; they appear in Figure 9. One route on this base network was considered as beginning at the BART station. The single fixed-route and the two fixed-route systems were determined after a site visit that allowed for the consideration of detailed actual features of the area. That is, width of streets,

Table II. An example summary of each-passenger activities.

Index	Phoned to order a ride	Stop number	Got on the bus at	Bus no.	Got off the bus at stop	Arrival time at destination	Wait time in minutes	Time between phone and pickup (minutes)
15	6:39	3	6:46	0	–1	6:52		7
13	6:38	2	6:41	0	–1	6:52		4
...
6	7:03	–1	7:20	1	0	7:22	17	
35	7:03	–1	7:20	1	1	7:29	17	

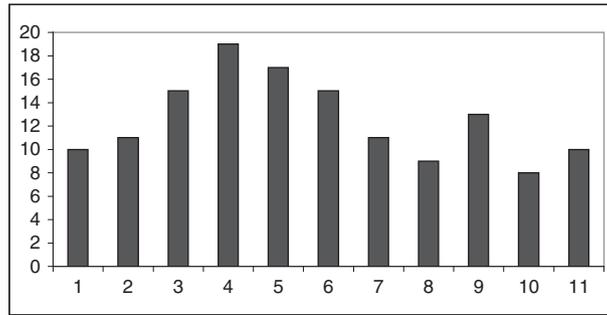


Figure 8. Frequency (no. of passengers in the Y-axis) experiencing wait for the shuttle (in units of 5 minutes) of the example problem.

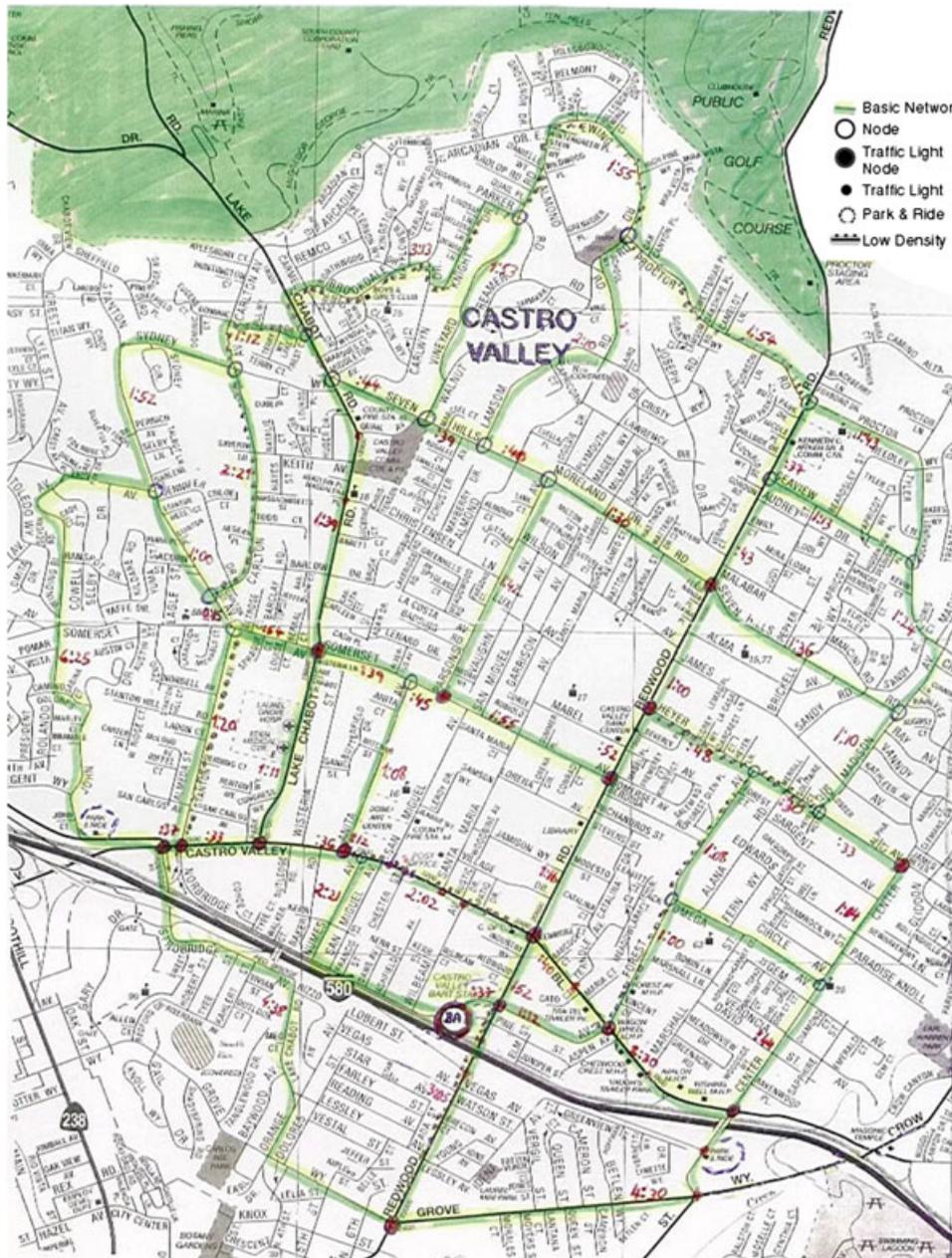


Figure 9. The basic network configuration overlaid on a map of Castro Valley.

sidewalk condition, terrain-specific elements, convenience and distance of accessing the stops and the routes from the residential houses, and safety features. The rationale behind the two fixed routes was to allow for a better coverage of the single fixed route with less walking distance given the topographical features of the site.

Stops were considered at all intersection points among the fixed route(s), other streets, and footpaths.

Based on the 24 input variables specified in Table III, hundreds of simulation runs were executed across the 10 routing strategies described and for the available four different shuttle bus numbers: buses 1, 2, 3, and 4. The input values for these runs appear in Table III. All the scenarios of one, two, three, or four same-size buses assumed the same type of operation, start at the BART station and making a loop of one of the fixed routes. Certainly, with more buses, the respond time and headway will be shorter.

In these runs, with the input of Table III, only one level of demand was considered for Castro Valley: a survey-based current demand of 400 passengers daily, with demand generated randomly (see explanation of the simulation components earlier). The evaluation of the 10 strategies is performed by the measure of waiting time per passenger for each scenario considered so as to impact the decision between the use of car or of the transit vehicle.

The single fixed-route and the two fixed-route systems in Castro Valley appear in Figures 10 and 11, respectively. Table IV summarizes the results obtained for the waiting time per passenger in 40 cases (10 strategies with four different fleet sizes) relating to the single route depicted in Figure 10. The minimum (best) passenger-waiting-time results in Table IV are indicated by an asterisk for each

Table III. Input values for the Castro Valley runs.

Bus2Train	3:00minutes
Train2Bus	4:00minutes
SizeType	30feet bus, with 27 seats
Quantity	1,2,3, or 4 buses
FixPick	25seconds
FixDrop	20seconds
FixBoard	5seconds
FixAlight	3seconds
NodeNo	One-route system
SectionNo	Network input
StopNo	Network input (intersections)
MeanDemand	Use of a random demand on each segment between two nodes (the highest demand is between the stops and the train station) with a total passengers per day of 400 (as per a current survey)
MeanDestin	Same as the demand to the train but in opposite direction.
MeanTime	Given in seconds
StDevTime	5%–10% of the MeanTime
Min4Turn	6, 8, 10, 12, 14, or 16 passengers, given that by turning around no one is waiting downstream, that there are five requests waiting and, of course, that the short turn will reach an earlier train
Min4Cut	8, 10, 15, 18, or 22 passengers given that no one is waiting downstream and, of course, that the shortcut will reach an earlier train
Min4Trip	8 passengers (can be changed from 8 to 10, 14, and 18, and to see the effect of results)
Min4Dep	6, 8, 10, or 14 passengers
RouteNo	S1
RouteDir	Given for one and bidirectional
TTimeTable	BART timetable
BTimeTable	The number of bus-departure times is same or similar to the maximum number of departures determined, where during rush hours (5–8AM), the bus headway (time between two departures) is shorter by 25% than in the other hours of the day (similar to the train-BART schedule)
Layover	15, 10, or 20minutes minimum

BART, Bay Area Rapid Transit.

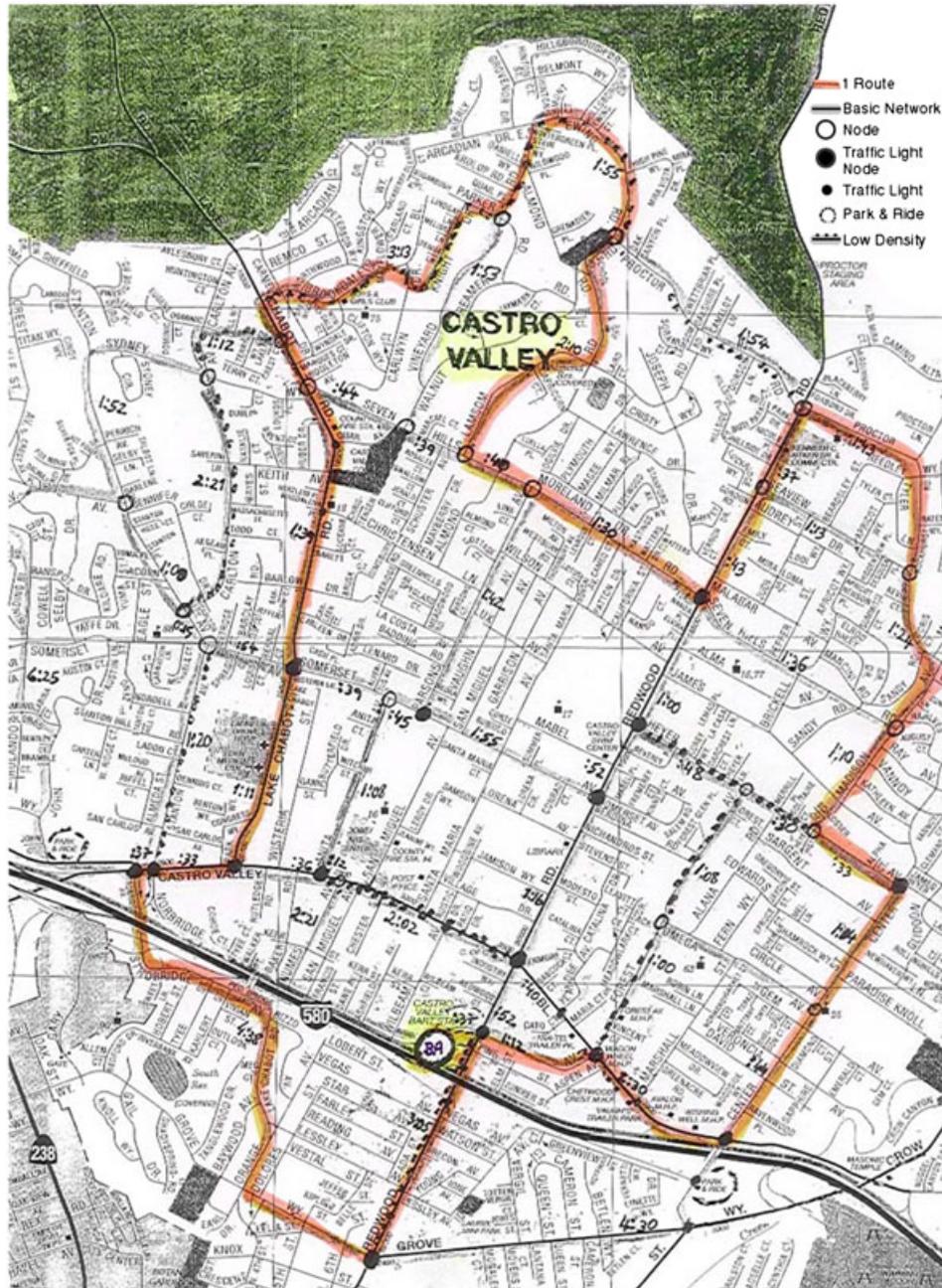


Figure 10. The single route determined for the basic network.

bus-number scenario; the second-best results are marked by a “✓”. The waiting time in Table IV is the average time per passenger, in minutes, that elapses from the time he/she calls the feeder/shuttle bus-information center until he/she boards the bus. It includes the walking time from the place of the call (e.g., one’s home) to the bus stop (assuming the shortest walk time) and the waiting time until the bus arrives. In addition, Table IV contains the percent difference from the best result.

From Table IV, one can see that the fixed-route fixed-schedule strategy (#1) results in the highest (longest) waiting times. It may also be observed that the flexible-route flexible-schedule (demand responsive) strategy (#10) does not always provide the best results; hence, it cannot be *a priori* superior to the other strategies. In fact, the best routing strategies observed in this simple, real-life test are those with two asterisks and two “✓” symbols: fixed route-flexible schedule and bidirectional, with

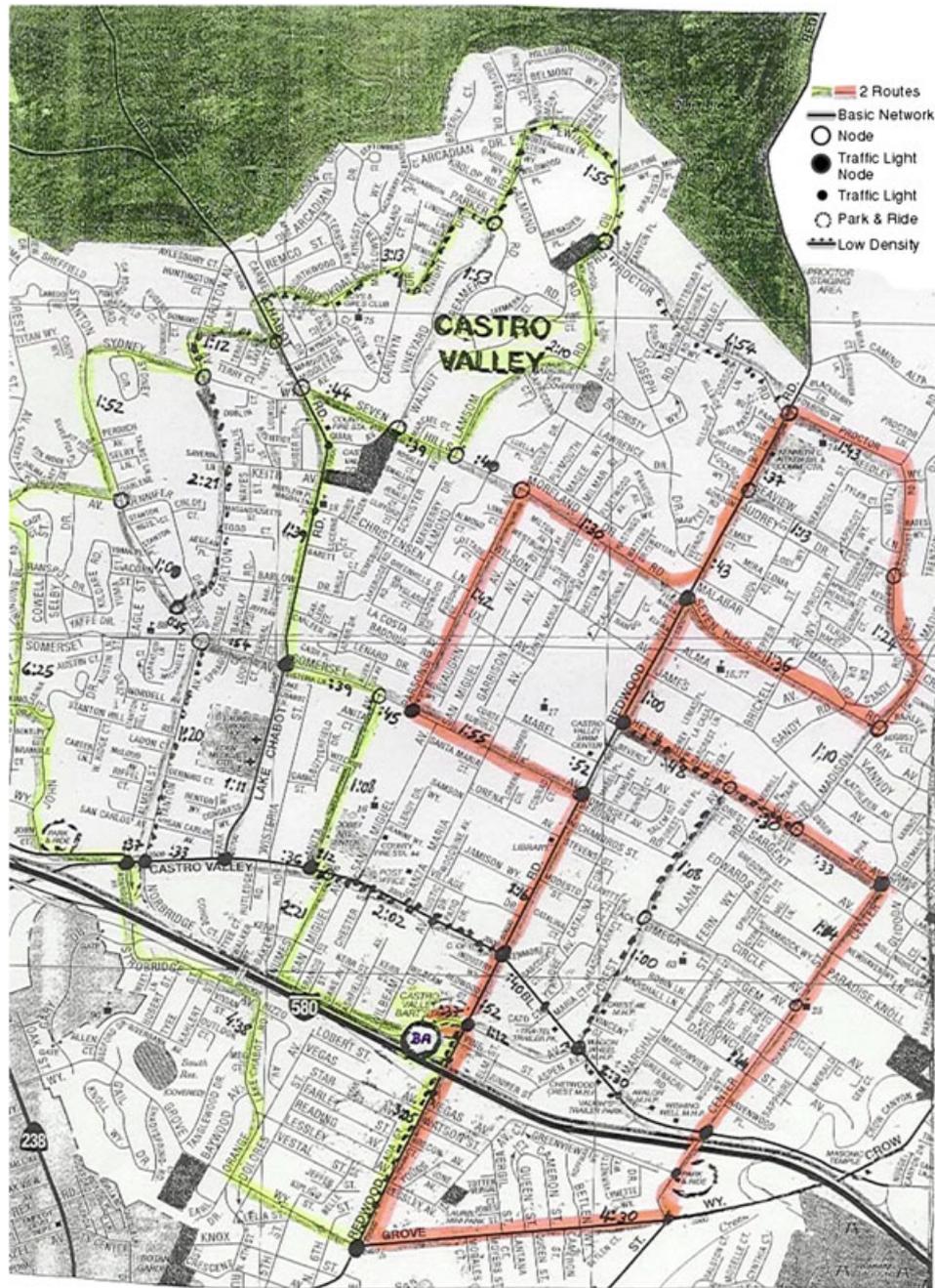


Figure 11. The two-route system determined for the basic network.

possible short turn (see Strategy #5 row in Table IV); fixed route-flexible schedule and bidirectional, with possible shortcut (#7); and fixed route-flexible schedule and bidirectional, with possible short turn and shortcut (#9). The short turn, shortcut, and bidirectional-based routing strategies indeed proved worthwhile to consider. These three uncommon strategies reflect the current availability of online information and communication systems that enable determining when and how to adopt each of these strategies. It is interesting to note that in Table IV, sort of a Pareto frontier is obtained between the level of service (represented by the waiting time) and operational cost (represented by the number of buses used); it consists of 22minutes waiting, 1 bus; 17minutes waiting, 2 buses; 15minutes waiting, 3 buses; and 12minutes waiting, 4 buses. It exhibits a trade-off between cost and service.

Table IV. Simulation results for waiting time per passenger (in minutes), using different combinations of strategies and numbers of buses for the Castro Valley one-route case study (given demand: 400 daily passengers).

Strategy	1 bus		2 buses		3 buses		4 buses	
	Min	%	Min	%	Min	%	Min	%
1	51	132	22	30	20	25	20	67
2	25	14	22	30	17	10	15	25
3	24 [✓]	9	23	35	15 [*]	0	14 [✓]	17
4	25	14	17 [*]	0	16 [✓]	5	15	25
5	24 [✓]	9	18 [✓]	6	15 [*]	0	12 [*]	0
6	24 [✓]	9	17 [*]	0	16 [✓]	5	15	25
7	24 [✓]	9	18 [✓]	6	15 [*]	0	12 [*]	0
8	24 [✓]	9	23	35	16 [✓]	5	15	25
9	24 [✓]	9	18 [✓]	6	15 [*]	0	12 [*]	0
10	22 [*]	0	18 [✓]	6	15 [*]	0	15	25

^{*}Best result.

[✓]Second-best result.

%=Percent deviation from best result.

In addition, six more simulation runs were performed for the two-route system depicted in Figure 11, using the most different strategies #1, #8, and #10, with four buses, for both picking up and taking passengers to the train station and distributing them from the station. In these six runs, an additional criterion was established for the maximum waiting time while at the phone location (e.g., home and work). That is, a criterion of 20minutes was used (this figure can be changed in the simulation runs) to reflect the fact that the caller will not actually wait if the announced waiting time for the shuttle/feeder bus is more than 20minutes but will cancel his/her request or do something else rather than “really” wait. Table V summarizes the average waiting time per passenger for these two pickup and drop-off cases, including the standard deviation determined for each simulation run.

The cumulative curves for the waiting times in these six situations appear in Figures 12–14. Table V shows that the average waiting time for distributing passengers in the fixed-schedule case is much higher than that in the flexible schedule cases. Furthermore, the standard deviations are lower in the pickup cases than in the drop-off cases. More precise configurations for these results are shown in Figures 12–14.

In these figures, the upper cumulative curve refers to the pickup case, and the lower curve refers to the drop-off case. It is certain that the waiting time in the drop-off case depends on the bus-departure time from the train station. The reason is that the cumulative curves for waiting at the train station have the shape of large step functions. It should also be mentioned that the X-axis scale is not the same in all cases; it simply reflects the resultant waiting-time range. In strategy 1 (fixed two route, fixed schedule), the waiting time at the train station is heavily distributed between low values (short waits) and waiting for the next bus departure after 20minutes. In strategies 8 and 10 (with flexible schedules), the waiting time is sharply reduced, indicating that once a train arrives and deposits passengers, the bus will depart immediately. A comparison between strategies 1 and 2 (Figures 12 and 13) for the pickup case reveals that the wait ranges from 5 to 20minutes in the fixed schedule, whereas it ranges from 3 to 18minutes

Table V. Simulation analysis for two-route, four-bus case, with 20-minute criterion (maximum waiting after call).

Strategy =>	Fixed-route fixed-schedule (#1)	Fixed-route flex-schedule (#8)	Flex-route flex-schedule (#10)
Average wait from phone call to bus arrival (minute)	13.3	8.9	6.8
Standard deviation (minute)	3.0	3.7	2.5
Average wait at train station (minute)	13.6	1.6	1.4
Standard deviation (minute)	15.3	8.5	5.0

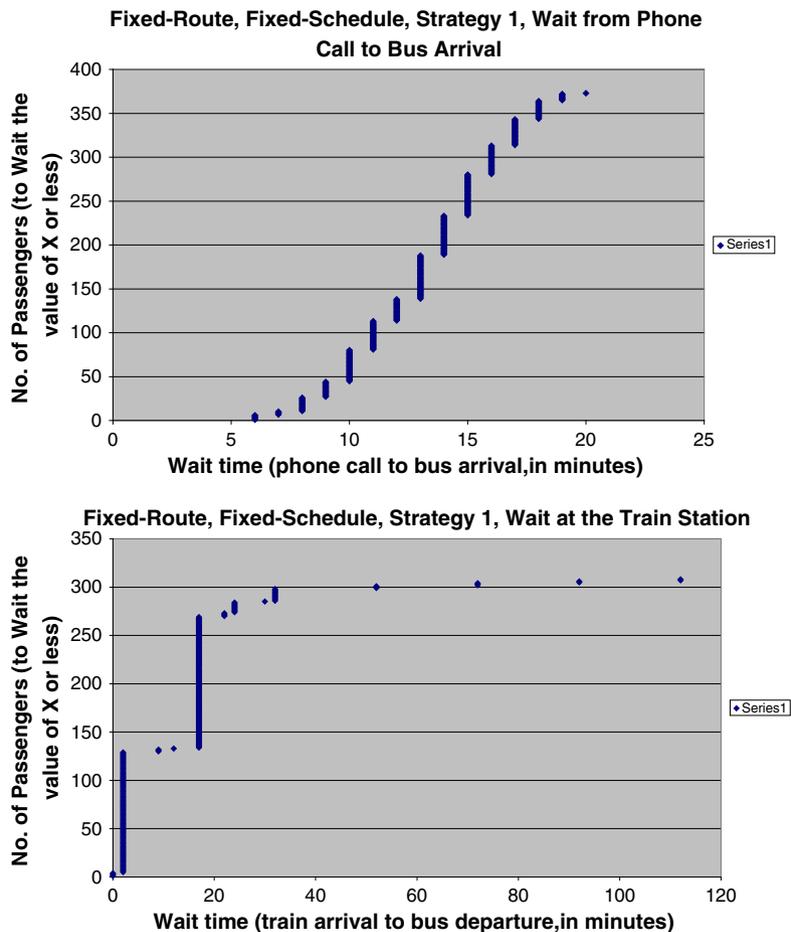


Figure 12. Waiting time (phone call to bus arrival in upper curve, and at the train station in lower curve) in minutes, for Strategy 1 (fixed two-route and fixed schedule).

in the flexible schedule. In the demand-responsive case (strategy 10, Figure 14), the waiting time for the pickup case ranges from 3 to 13 minutes.

Overall, the analysis shows that the fixed-route(s) system combined with operating strategies (strategies 3–9) is comparable with the DRT type of service. This is clearly shown in Table I and to some extent provides surprising results. Thus, strategies 3–9 are recommended to be tested.

A final comment is about the demand level used. As noted, hundreds of simulation runs were performed including a sensitivity analysis of the demand level, although the 400-passenger demand analyzed is based on a survey made by the California PATH program in Castro Valley. The demand (generated randomly) analyzed between 400 and 2000 passengers/day, and results in different waiting times but with the same conclusion reached above. It is worth noting that these simulation runs are only preliminary steps toward the examination of a smart feeder/shuttle service in actual pilot operation discussed in the next section.

6. CONCLUDING AND FUTURE-RESEARCH REMARKS

This work portrays a methodology of examining different operational strategies and routing scenarios for an implementation of a smart feeder/shuttle service. The contribution of this work is threefold: (i) construction of innovative operating strategies from both the user and operator perspectives; (ii) examination of different routing models and operating strategy-based scenarios; and (iii) construction of a simulation tool enabling (ii). In addition, one of the interesting results found is that the fixed-route and flexible-route concepts are comparable in performance measures when applying

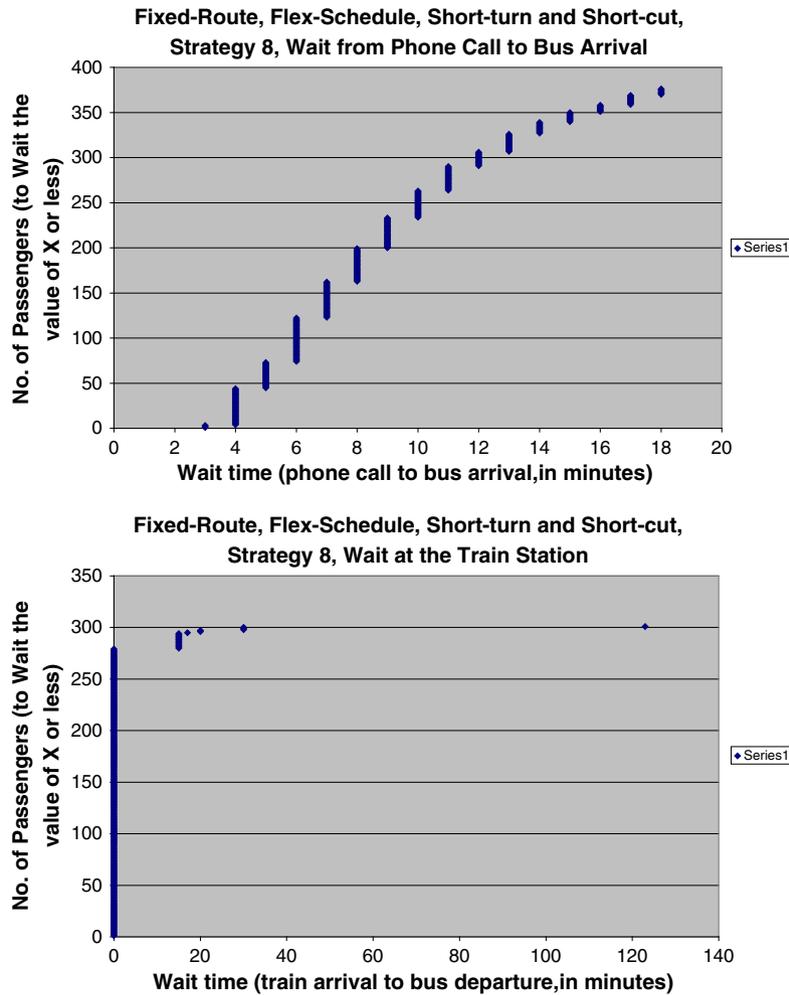


Figure 13. Waiting time (phone call to bus arrival in upper curve, and at the train station in lower curve) in minutes, for Strategy 8 (fixed two-route, flex schedule, with short turn and shortcut).

a combination of operating strategies. The advantage of having a fixed route over a flexible route are obvious from the user perspective; convenience stops can be located, waiting for the feeder/shuttle service without a booking, clarity in understanding the pickup and drop-off locations, and more.

Once the analysis of the feeder/shuttle service is completed, it is recommended that a pilot study be conducted. The study can adopt the 12 steps detailed below in order to explore the possibility of a feeder/shuttle service. These steps can serve as a framework for the master plan of a pilot study, in which each outcome of a previous step becomes an additional input to the next step, except for Step 6.

The pilot master plan starts, in Step 1, with a demand analysis, according to time of day and day of week, in order to ascertain the origin–destination pattern of the site and its consumer-oriented features. Step 2 is to design the fixed-routing and stop system. Step 3 is to determine the base frequencies and timetable for each route. Step 4 is to determine the number and size of the feeder/shuttle vehicles and to create the trip chains (vehicle schedules) that will serve Step 5, which is constructing the crew schedules.

The pilot plan continues, in Step 6, with the establishment of effective information channels and instruments (e.g., telephone center, internet, newspapers, radio, TV, and mail leaflets), which will lead, in Step 7, to the development of user-friendly communication procedures between the users and the operator. Step 8 constructs the DRT operational strategies, without the use of the fixed-route/stop/schedule system. Step 9 determines the testing scenarios for the pilot plan, whereas Step 10 presents the process of how to select an adequate operator. Step 11 uses proper advertisement tools to approach

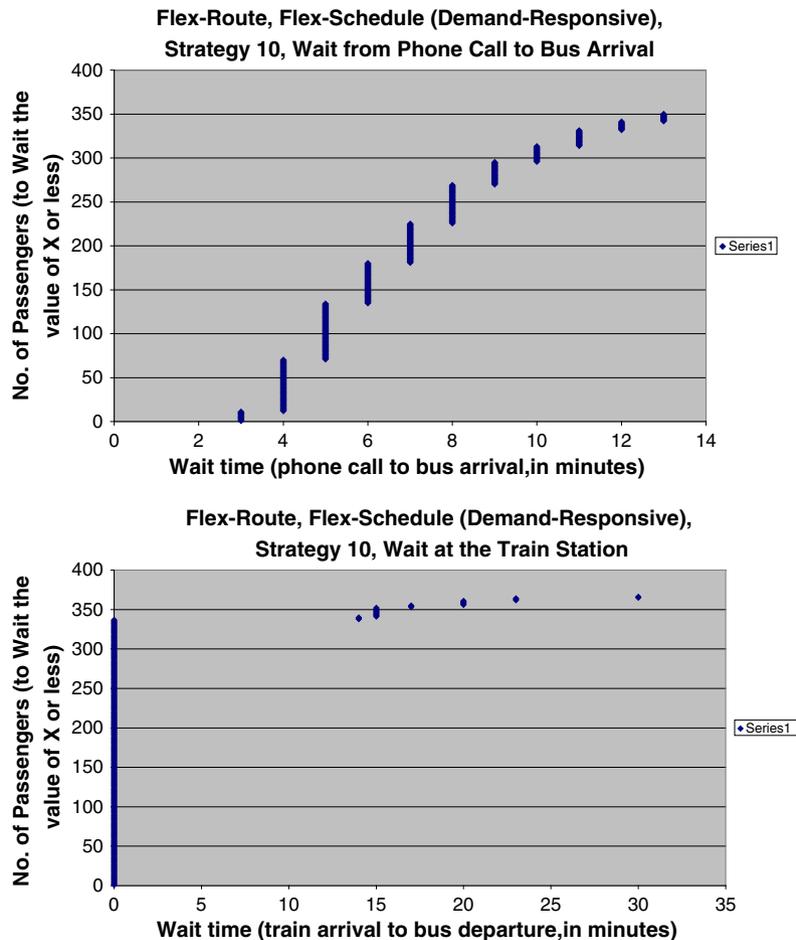


Figure 14. Waiting time (phone call to bus arrival in upper curve, and at the train station in lower curve) in minutes, for Strategy 10 (demand responsive).

the people in the pilot's area. Finally, Step 12 aims at improving the plan's instruments, procedures, and strategies with the use of innovative intelligent-transportation systems elements.

To sum up, this work introduces new ideas for designing an integrated, smart feeder/shuttle service. Ideally, this smart bus system will provide advanced, attractive feeder and distributor services that operate reliably and relatively rapidly, with smooth and synchronized transfers, as part of the door-to-door passenger chain. The new concepts and tools developed in this study, practically speaking, will help meet the needs and desires of end users, allow for utilizing intelligent-transportation technologies, and will increase operational efficiency.

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