Mutual capacitor and its applications

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Abstract: This study presents a new ac circuit element – the mutual capacitor, being a dual of the mutual inductor, which is also a new ac transformer. This element is characteristic of the mutual-capacitance coupling of a multi-capacitance system. A unity-coupled mutual capacitor works as an ideal current or voltage transformer, and incidentally acts as waveform separating when inductor employed or waveform converting from square-wave to quasi-sine or waveform filtering, between ports. As a transformer, the mutual capacitor is easy to design, easy for heat cooling, more accurate for current or voltage transformation, dissipating less energy as well as saving materials, suitable for high-power and high-voltage applications. Experiments to demonstrate performances of unity-coupled mutual capacitors are also given.

1 Multi-capacitance systems and mutual capacitor

Of an ac network, a linear capacitor C, when supplied with an ac voltage source across its terminals, is described as $i = C(\mathrm{d}v/\mathrm{d}t)$ [1], which characterises the $i - v$ relationship between its two terminals or between a single conductor and somewhere at infinity in system; where C is its symbol and also denotes its capacitance value, i is the current flowing through it, v is the voltage across its terminals and we assume that it is non-dissipative (Fig. 1*a*).

As well, there exist multi-capacitance ac systems in engineering, such as transmission lines in electric power transmission, multi-conductor systems in complicated circuital environment and issues on distributions of stray capacitances over adjacent circuits investigated in solid-state electronics etc. A multi-capacitance ac system consists of $K+1$ conductors, where assuming the potential of the $K+1$st conductor is referenced [Note: a multi-capacitance ac system in an infinite space may consist of K conductors, where the potential at infinity is referenced.], potentials of the other conductors are in turn $v_1, v_2, \ldots, v_n, \ldots, v_K$. For an independent, linear and lossless multi-capacitance ac system, such as the multi-conductor system in Fig. 1*b*, the current flowing into conductor m accordingly is

$$\begin{aligned} i_m = C_{m1}\frac{\mathrm{d}v_1}{\mathrm{d}t} + C_{m2}\frac{\mathrm{d}v_2}{\mathrm{d}t} + \cdots + C_{mn}\frac{\mathrm{d}v_n}{\mathrm{d}t} \\ + \cdots + C_{mK}\frac{\mathrm{d}v_K}{\mathrm{d}t}, \quad \begin{pmatrix} m = 1,\ 2,\ \ldots,\ K \\ n = 1,\ 2,\ \ldots,\ K \end{pmatrix} \end{aligned} \tag{1A}$$

Or, if only the performances in sinusoidal steady-state operation are concerned, the current has its phasor-domain notation as

$$\begin{aligned} I_m &= \mathrm{j}\omega C_{m1}V_1 + \mathrm{j}\omega C_{m2}V_2 \\ &\quad + \cdots + \mathrm{j}\omega C_{mn}V_n \\ &\quad + \cdots + \mathrm{j}\omega C_{mK}V_K, \quad \begin{pmatrix} m = 1,\ 2,\ \ldots,\ K \\ n = 1,\ 2,\ \ldots,\ K \end{pmatrix} \end{aligned} \tag{1B}$$

where C_{mn}, when $m = n$, is termed self-capacitance, the relationship between conductor m and the reference potential; C_{mn} when $m \neq n$, is termed mutual-capacitance [2], the relationship between conductors m and n; and they both contribute to producing the current i_m or I_m.

It must be noted that the C_{mn} in multi-capacitance system (1) (*A* or *B*) (Fig. 1*b*) differs obviously from the C discussed in Fig. 1*a*, for in (1) there exists not only the self-capacitance $C_{mn}(m = n)$ in self-excitation — a performance on its current by potential of its own conductor, but also the mutual-capacitance $C_{mn}(m \neq n)$ in mutual excitation — a performance on the current by potentials of the other conductors, whereas the C means only the performance on its current by potential of its own conductor.

It is easy to find or imagine that behaviours of a multi-capacitance system described in (1) are quite similar to those of a well-known multi-inductance system. A physical phenomenon of the mutual inductance is a behaviour of magnetic-field coupling; and a physical phenomenon of the mutual-capacitance falls in the category of electric-field coupling. That the coupling of mutual inductance of multi-inductance systems was timely discovered and fully exploited benefited from the effect of ferromagnetic media, which allows mankind to have had the mutual inductor widely used for long [Note: a mutual inductor, or coupling inductors as popularly called, normally means a device of two or more coils, but in this paper we emphatically mean those of two coils, being magnetically coupled between each other, including the conventional inductance transformer.]. Although the coupling of mutual-capacitance of multi-capacitance systems scatters much sparser in nature and both magnitudes and coupling levels of its behaviours are far less than those of the coupling of mutual inductance of any iron-cored mutual inductor. It is why for so long no such practical device — a capacitance transformer, which may or should be termed mutual capacitor, exists appearing and performing similar to the mutual inductor. Even so, the author thinks, it will not prevent us from getting to know of and exploit the principle of multi-capacitance systems.

Anyone that expects a mutual capacitor may ideally have thought that a mutual capacitor should be the same as or similar thoroughly to a mutual inductor both in appearance and in performance; it should work ideally dual to a mutual inductor or inductance transformer. However, it is not true and there is no such device! Maxwell's electromagnetic-field equations ($\nabla D = \rho$ and $\nabla B = 0$) reveal that electric field acts mostly analogously to magnetic field, but not all. Therefore an ideal mutual capacitor built up in an integrated structure and working ideally dual to a mutual inductor under any condition does not exist! However if we examine and analyse this issue in a point for practical use, we may set this device as such a model: (i) it can be a dual of the mutual inductor acting well enough in most behaviours, especially for practical use although not ideally in every aspect and (ii) it can succeed in performance as a transformer in engineering. In this paper, we also call such a device the mutual capacitor.

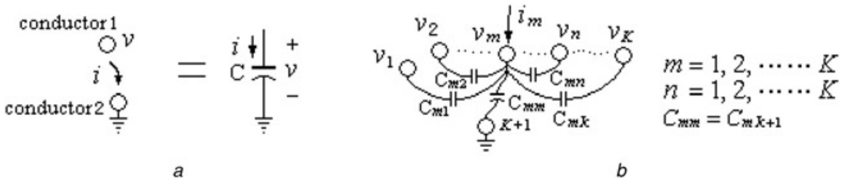

Fig. 1 *Capacitance relationship of ac networks*
a Between two conductors
b For conductor *m* in a multi-conductor system

For better understanding of multi-capacitance systems and for a purpose of practical use, we designate $K = 2$ for (1), the mutual capacitor will be put forth and discussed in this paper. Accordingly, its theory and feasibility will be discussed as well which include circuit configurations of the mutual capacitor and its definition and schematic symbols; concepts of its unity coupling and principle of its transformation of voltage or current – its prerequisite conditions for unity coupling, methods and formulae; its characteristics of waveform separation; and the experimental demonstrations to show its performances in practical uses. Owing to limited space, more and further investigations will be presented following this publication.

2 Basics of the mutual capacitor

2.1 Mutual capacitor

A mutual capacitor is a two-port network element with no power loss and coupled by electric field between ports of input and output.

To investigate the new network component more clearly, we use Figs. 2 and 3 to be its schematic symbols or circuit models for two types of the mutual capacitor, in which the position of circles or dots represents the polarity notation of the port voltages, and circles or dots mean different types of the mutual capacitor as well. As seen in both figures, v_1 and v_2 are port voltages, respectively, for input and output and i_1 and i_2 each are port currents for them.

The mutual capacitor shown in Fig. 2 may be configured in its simplest schematic diagrams as in Fig. 4. Its equation is expressed in phasor domain as (2), wherein it is presented by port currents, termed the ‘current type of mutual capacitor’; also referred to as the Δ (or π) mutual capacitor. [Note: (2) is obtained from (1B) by designating $K = 2$ and making notations in accordance with those illustrated in Fig. 4; or it also can be a dual brought out directly, by the principle of duality, from the characteristic equation of the mutual inductor or lossless voltage inductance transformer.]

$$\begin{cases} I_1 = \mathrm{j}\omega C_{\mathrm{I}} V_1 - \mathrm{j}\omega C_M V_2 = \mathrm{j}\omega (C_A + C_M) V_1 - \mathrm{j}\omega C_M V_2 \\ I_2 = \mathrm{j}\omega C_M V_1 - \mathrm{j}\omega C_{\mathrm{II}} V_2 = \mathrm{j}\omega C_M V_1 - \mathrm{j}\omega (C_B + C_M) V_2 \end{cases} \quad (2)$$

where the second step of both formulae is in accordance with Fig. 4*a*, described with the structural parameters C_A, C_B and C_M which can exist all over the three-dimensional (3D) space, that is, $\{|C_A| < \infty,\ |C_B| < \infty,\ |C_M| < \infty\}$; C_{I} and C_{II} are termed the ‘self-capacitance coefficients’ and C_M termed the ‘mutual-capacitance coefficient’ of the current type mutual capacitor; and they are, respectively, defined in the following

$$I_1 = \mathrm{j}\omega C_{\mathrm{I}} V_1 \Big|_{V_2 = 0} \quad (3)$$

$$I_2 = -\mathrm{j}\omega C_{\mathrm{II}} V_2 \Big|_{V_1 = 0} \quad (4)$$

$$\begin{cases} I_1 = -\mathrm{j}\omega C_M V_2 \Big|_{V_1 = 0} \\ I_2 = \mathrm{j}\omega C_M V_1 \Big|_{V_2 = 0} \end{cases} \quad (5)$$

Obviously there are

$$\begin{cases} C_{\mathrm{I}} = C_A + C_M = C_A // C_M \\ C_{\mathrm{II}} = C_B + C_M = C_B // C_M \end{cases} \quad (6)$$

The coupling coefficient of the current type mutual capacitor is defined as

$$k_{\mathrm{c}} = \frac{C_M}{\sqrt{C_{\mathrm{I}} C_{\mathrm{II}}}} \quad (7)$$

and when $|k_{\mathrm{c}}| = 1$, the current type mutual capacitor is known as unity-coupled or in unity coupling, that is, fully coupled or in full coupling.

The mutual capacitor shown in Fig. 3 may be configured in its simplest schematic diagrams as in Fig. 5. Its equation is expressed

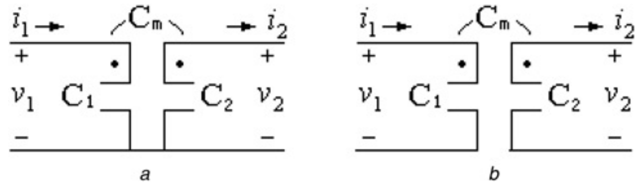

Fig. 3 *Circuit symbols for voltage type of mutual capacitor*
a Referenced in common
b Referenced separately

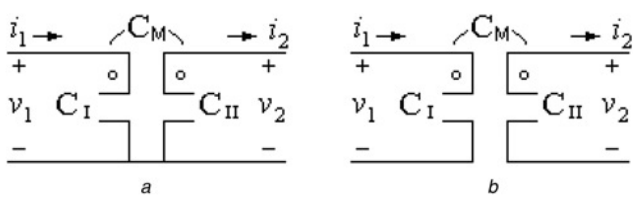

Fig. 2 *Circuit symbols for current type of mutual capacitor*
a Referenced in common
b Referenced separately

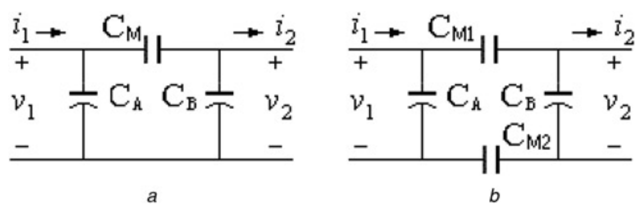

Fig. 4 *Circuit implementation for current type of mutual capacitor*
a Referenced in common
b Referenced separately

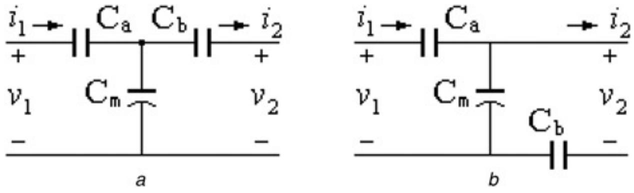

Fig. 5 *Circuit implementation for voltage type of mutual capacitor*
a Referenced in common
b Referenced separately

in phasor domain as (8), wherein it is presented by port voltages, termed the 'voltage type of mutual capacitor'; also referred to as the *Y (*or *T)* 'mutual capacitor'. Equation (8) is also a dual from (2) (in a similarity of operational rules in mathematics) [Note: (8) can also be obtained from the characteristic equation of the mutual inductor by substitutions of a relationship $L = -1/(\omega^2 C)$.]

$$\begin{cases} V_1 = \dfrac{1}{\mathrm{j}\omega C_1} I_1 - \dfrac{1}{\mathrm{j}\omega C_m} I_2 = \dfrac{1}{\mathrm{j}\omega}\left(\dfrac{1}{C_a} + \dfrac{1}{C_m}\right) I_1 - \dfrac{1}{\mathrm{j}\omega C_m} I_2 \\ V_2 = \dfrac{1}{\mathrm{j}\omega C_m} I_1 - \dfrac{1}{\mathrm{j}\omega C_2} I_2 = \dfrac{1}{\mathrm{j}\omega C_m} I_1 - \dfrac{1}{\mathrm{j}\omega}\left(\dfrac{1}{C_b} + \dfrac{1}{C_m}\right) I_2 \end{cases} \tag{8}$$

where the second step of both formulae in (8) is in accordance with Fig. 5*a*, described with the structural parameters C_a, C_b and C_m which exist in 3D space but not all, that is, $\{C_a \neq 0,\ C_b \neq 0,\ C_m \neq 0\}$; C_1 and C_2 are termed the 'self-capacitance coefficients' and C_m termed the 'mutual-capacitance coefficient' of the voltage type mutual capacitor; and they are, respectively, defined in the following

$$V_1 = \frac{1}{\mathrm{j}\,\omega C_1} I_1 \Big|_{I_2 = 0} \tag{9}$$

$$V_2 = -\frac{1}{\mathrm{j}\omega C_2} I_2 \Big|_{I_1 = 0} \tag{10}$$

$$\begin{cases} V_1 = -\dfrac{1}{\mathrm{j}\omega C_m} I_2 \Big|_{I_1 = 0} \\ V_2 = \dfrac{1}{\mathrm{j}\omega C_m} I_1 \Big|_{I_2 = 0} \end{cases} \tag{11}$$

Obviously there are

$$\begin{cases} C_1 = \dfrac{C_a C_m}{C_a + C_m} = C_a \perp C_m \\ C_2 = \dfrac{C_b C_m}{C_b + C_m} = C_b \perp C_m \end{cases} \tag{12}$$

The coupling coefficient of the voltage type mutual capacitor is defined as

$$k_\mathrm{v} = \frac{C_m}{\sqrt{C_1 C_2}} \tag{13}$$

and when $|k_\mathrm{v}| = 1$, also the voltage type mutual capacitor is known as unity-coupled or in unity coupling, that is, fully coupled or in full coupling.

Although both the denomination and the definition of the mutual capacitor are described with capacitances, they are mostly implemented with capacitors as well as inductors, for, at a fixed frequency ω, a positive inductance functions exactly as a negative capacitance does, namely $C = -1/(\omega^2 L)$. [Note: even an arrangement of three inductors in a Δ (or Y) configuration is a mutual capacitor, other than a mutual inductor, unless there exists magnetic coupling between ports.] Besides, for electronic circuits, a negative capacitance value can also be achieved through a negative impedance converter (NIC) [3] of integrated circuits (ICs); in terms of which a mutual capacitor constituted operates theoretically at any frequency.

2.2 Unity-coupled mutual capacitor

2.2.1 Unity coupling of the current type mutual capacitor and its current transformation property: For the current type mutual capacitor shown in Fig. 4 to be unity-coupled, manipulating (6) and (7) by letting $|k_\mathrm{c}| = 1$, establishes its prerequisite condition for unity coupling as

$$C_\mathrm{I} C_\mathrm{II} - C_M^2 = 0 \text{ or } \frac{1}{C_A} + \frac{1}{C_B} + \frac{1}{C_M} = 0 \tag{14}$$

In a sense of being unity-coupled, its definition (2) will become

$$\begin{cases} I_1 = \dfrac{C_A}{C_A + C_B}\left(\mathrm{j}\omega C_A V_1 + \mathrm{j}\omega C_B V_2\right) \\ I_2 = -\dfrac{C_B}{C_A + C_B}\left(\mathrm{j}\omega C_A V_1 + \mathrm{j}\omega C_B V_2\right) \end{cases} \tag{15}$$

Note that the algebraic sum of both current terms in above parentheses is not zero, for $\mathrm{j}\omega C_A V_1 + \mathrm{j}\omega C_B V_2 = I_{CA} + I_{CB} \neq 0$, meaning physically that the net current flowing into reference potential keeps not being zero [Note: according to geometrical symmetry of its structure, it means that the current type mutual capacitor cannot be designed as a current transformer of a ratio equal to one.]. When this remains true, a unity-coupled current type mutual capacitor will have its current transforming ratio between ports as

$$\begin{aligned} n_\mathrm{c} = \frac{I_1}{I_2} &= -\frac{C_A}{C_B} = \frac{C_\mathrm{I}}{C_M} \\ &= \mathrm{sgn}(C_M C_\mathrm{I})\sqrt{\frac{C_\mathrm{I}}{C_\mathrm{II}}} \\ &= -\mathrm{sgn}(C_A C_B)\sqrt{\frac{C_\mathrm{I}}{C_\mathrm{II}}} \end{aligned} \tag{16}$$

This is also called the ratio of the unity-coupled current type mutual capacitor. It indicates that this ratio is set up only when it is unity-coupled, and determined only by its structural parameters C_A and C_B, or C_I and C_II, independent of its operating frequency or/and its load across output port. The unity-coupled current type mutual capacitor is an ideal current transformer.

2.2.2 Unity coupling of the voltage type mutual capacitor and its voltage transformation property: For the voltage type mutual capacitor shown in Fig. 5 to be unity-coupled, manipulating (12) and (13) by letting $|k_\mathrm{v}| = 1$, establishes its prerequisite condition for unity coupling as

$$C_1 C_2 - C_m^2 = 0 \text{ or } C_a + C_b + C_m = 0 \tag{17}$$

In a sense of being unity-coupled, its definition (8) will become

$$\begin{cases} V_1 = \dfrac{C_b}{(C_a + C_b)}\left(\dfrac{I_1}{\mathrm{j}\omega C_a} + \dfrac{I_2}{\mathrm{j}\omega C_b}\right) \\ V_2 = -\dfrac{C_a}{(C_a + C_b)}\left(\dfrac{I_1}{\mathrm{j}\omega C_a} + \dfrac{I_2}{\mathrm{j}\omega C_b}\right) \end{cases} \tag{18}$$

Let the algebraic sum of both voltage terms in above parentheses keep not being zero, that is, $(I_1/\mathrm{j}\omega C_a) \neq -(I_2/\mathrm{j}\omega C_b)$, meaning physically to keep it true as $V_{Ca} \neq -V_{Cb}$ [Note: according to geometrical symmetry of its structure, it means that the voltage type mutual capacitor cannot be designed as a voltage transformer of a ratio equal to one.]. When this keeps true, a unity-coupled voltage type mutual capacitor will have its voltage transforming ratio between ports as

$$
\begin{aligned}
n_v = \frac{V_1}{V_2} = -\frac{C_b}{C_a} = \frac{C_m}{C_1} \\
= \operatorname{sgn}(C_m C_1)\sqrt{\frac{C_2}{C_1}} \\
= -\operatorname{sgn}(C_a C_b)\sqrt{\frac{C_2}{C_1}}
\end{aligned}
\tag{19}
$$

This is also called the ratio of the unity-coupled voltage type mutual capacitor. It indicates that this ratio is set up only when it is unity-coupled, and determined only by its structural parameters C_a and C_b, or C_1 and C_2, independent of its operating frequency or/and its load across output port. The unity-coupled voltage type mutual capacitor is an ideal voltage transformer.

2.3 *Characteristics of waveform conversion of the unity-coupled mutual capacitor*

The mutual capacitor, particularly a unity-coupled mutual capacitor, is constructed by combining capacitors of positive and negative capacitances. In a case of power application, based on today's technology, a negative capacitance can be implemented only by an inductor, for $C = -1/(\omega^2 L)$, leading to an LC network (where L is the inductance and C is capacitance) formed as a mutual capacitor. It is a known fact that an LC network is a harmonic filter; but what if it does a great attenuation of high harmonics while normally transforming the fundamental current and voltage between ports? Or, what if a current or voltage source of ac square-wave is applied onto its input and its output supplies a current or voltage whose magnitude has been transformed as needed but with its waveform shaped of a quasi-sine? The unity-coupled mutual capacitor or capacitance transformer performs a transformation of current or voltage while converting the waveforms from square-wave to quasi-sine, which has never been in such a case for the inductance transformer.

We take the voltage type mutual-capacitance transformer for an example to investigate the performance of waveform conversion from square-wave to quasi-sine. In Fig. 5*a*, we choose $C_a = -1/(\omega^2 L_a)$, $C_b > 0$ and $C_m > 0$ to form a unity-coupled mutual capacitor, where ω is the electric angular frequency of the fundamental of the square-wave at a fixed frequency applied to the input. Thus, the phasor expression for (17) is $\omega^2 L_a (C_b + C_m) = 1$, which is the base to design a unity-coupled voltage type mutual capacitor. This transformer is a step-up ($0 < n_v < 1$); and the frequency spectrum analysis of its responses to a square-wave voltage should be derived out from Fig. 5*a* and (19) in the phasor domain. Assuming its input being applied a voltage of symmetrical square-wave and its harmonic has an order number as $k(=1, 2, 3, \ldots)$, we have a voltage amplitude relationship between the kth harmonic and its fundamental at the output as (20), where R is the load at the output. It is provable that a unity-coupled voltage type mutual capacitor has a good performance of removing the forward voltage harmonics, either by chart, list or even by experiments as long as suitable parameters L_a, C_b and C_m of the mutual capacitor are given. Meanwhile, we can also prove that it has an excellent capacity of removing the backward current harmonics. Assuming its output is supplied a current of symmetrical square-wave, we derive out the current amplitude relationship of the kth harmonic reflected to the input side over its fundamental as

$$
\left|\frac{I_{1k}}{I_{11}}\right| \le \frac{1}{\left|(k/n_v)\left[1 - k^2(1 - n_v)\right]\right|}, \quad k = 1, 2, 3, \ldots \tag{21}
$$

If setting the circuit parameters in Fig. 5*a* as $C_a = -1/(\omega^2 L_a)$, $C_b = -1/(\omega^2 L_b)$ and $C_m > 0$, we can design a voltage transformer either step-up or step-down, but it must be noted that the voltages on both ports in this situation are anti-phased from each other ($n_v < 0$). Based on almost the same as above phasor-domain analysis, a voltage amplitude relationship between the kth harmonic and its fundamental at the output and a current amplitude relationship of the kth harmonic reflected to the input side over its fundamental are, respectively, derived out as (22), where R is the load at the output. It is provable that it has a better performance of removing/attenuating the forward voltage harmonics than that of the transformer of (20), either by chart, list or even by experiments, whereas they both have the same capacity of removing/attenuating the backward current harmonics.

2.4 *Unity-coupled mutual capacitor against the circuits in resonance*

Generally speaking, by taking a tap from the joint of two capacitors in series, we can also obtain a voltage transformation, which is the concept of a voltage divider. However the voltage divider can only step-down a voltage, together with its voltage ratio changing dependently on both the power angular frequency ω and the load. A mutual capacitor, however, when unity-coupled, transforms a current or voltage independently, either step-up or step-down, which completely is its own internal mechanism. This mechanism is essentially analogous to the circuit resonance, but it is controllable and it is safe at any situation, or we may say, it is a special resonance set with three independent elements under a condition of (15) or (18), namely (2) and (14) or (8) and (17). As seen in (2) and (8), there exist some zeros and poles, which are actually those values satisfying equations $C_{\mathrm{I}}\, C_{\mathrm{II}} = (C_A + C_M)\,(C_B + C_M) = 0$ or $C_1\, C_2 = (C_a + C_m)\,(C_b + C_m) = 0$. Among these zeros or poles, anywhere a sum of two structural parameters equals zero is where a circuit resonance occurs, which interprets that the circuit resonance is not defined in the unity-coupled mutual capacitor, as well as not a safe circuital phenomenon.

$$
\begin{cases}
\left|\dfrac{V_{2k}}{V_{21}}\right| \le \dfrac{1}{\left|\left(k/n_v\right)\left[1 - k^2(1 - n_v)\right] + \left(\mathrm{j}\omega L_a/n_v^2 R\right)(k^2 - 1)\right|}, \quad k = 1, 2, 3, \ldots \\
n_v = -\dfrac{C_b}{C_a} = \omega^2 L_a C_b = \dfrac{C_b}{C_b + C_m}
\end{cases}
\tag{20}
$$

$$
\begin{cases}
\left|\dfrac{V_{2k}}{V_{21}}\right| \le \dfrac{1}{\left|(k/n_v)\left[1 - k^2\left(1 - n_v\right)\right] + \left(\left(\mathrm{j}k^2\omega(L_a + L_b)\left(1 - k^2\right)\right)/n_v\right)(1/R)\right|}, \quad k = 1, 2, 3, \ldots \\
\left|\dfrac{I_{1k}}{I_{11}}\right| \le \dfrac{1}{\left|(k/n_v)\left[1 - k^2\left(1 - n_v\right)\right]\right|}, \quad n_v = -\dfrac{L_a}{L_b}
\end{cases}
\tag{22}
$$

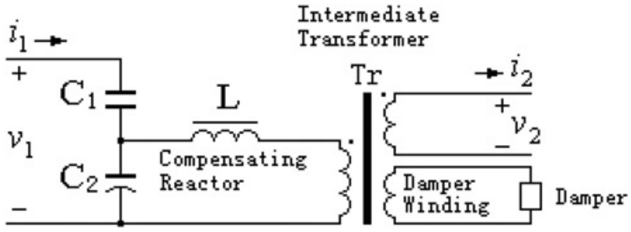

Fig. 6 *Capacitor voltage transformer*

2.5 *Mutual capacitor against the capacitor voltage transformer*

The idea of coupled capacitor including its two-port networks was presented by Maxwell in his Treatise of the mid 1800s. The theory for them had also been much developed by followers once for some time, especially in circuit synthesis community. In 1936, being an expectation or proposition, the CVT being only a term was presented [4], with no claim made for presenting any new principles of design. One of its earliest practical devices was given and published in 1949 [5]. Today the CVT has been widely used as an instrument transformer in power systems (Fig. 6). In operation, the CVT works in almost the same way as a unity-coupled voltage type mutual capacitor does; however, it is much different from the mutual capacitor in many aspects: (i) In forming principle, the CVT employs an inductor or compensating reactor L connected to the joint of a capacitor voltage divider to compensate the circuit into resonance, whereas the mutual capacitor is built of independent capacitors of positive and negative capacitances under unity-coupled conditions. (ii) In practice, the CVT has a damper winding added to its Tr plus a damper which are a must to suppress harmonics (see Fig. 6), being configured more complicated than the mutual capacitor of just three components working in linear status. (iii) The CVT has only one arrangement, whereas the mutual capacitor in Fig. 5 has more – any one or two of its three capacitors being given negative capacitance will make an arrangement. (iv) The CVT came out of a technique being a device only while the mutual capacitor originates from a new theory being a circuit element dual to the mutual inductor as well as a new transformer complementarily with the conventional transformer for engineering uses; which is why, although another half century has already passed, the idea of coupled capacitor had still to be stopped at the CVT which so far can serve only as instrument voltage transformers (of output quite small to lessen interference of harmonics) while the mutual capacitor can and will be used, with its various configurations, either for instruments or for power applications, not only in voltage – but also in current – transformation, plus functioning waveform conversion. [Note: owing to space limitation, only the basics of the principle of the mutual capacitor are presented in this paper.]

3 Experiments

The above presentation of the mutual capacitor is proposed under a condition of linearity. As a matter of fact, a practical component or device in engineering is normally non-linear, and as its operating state as well as surroundings – especially the temperature change, its non-linearity keeps unstable or its performing property goes non-linear, leading errors to be in existence. The detailed analyses on errors of the mutual capacitor, approaches to minimise them, formulae for more accurate calculations etc. will be given by author in another publication. Fortunately, practical non-polar capacitors, particularly polypropylene-dielectric, in engineering have high linearity within their operating regions, normally being treated as linear components which can meet most engineering's requirements. A negative capacitance can only be obtained by employing an inductor for power application; however, iron-cored inductors normally are non-linear, and will bring in a great deal of harmonics when directly used, as well as spoil the prerequisite condition of the mutual capacitor. The author completed the following experiments by employing an approach 'the linearisation processing of inductors', that is, by employing a structure of magnetic circuit of air-gapped cores or magnetic powder cores, so as to have the inductors operate within the required regions of linearity, resulting in that the mutual capacitor works in linearity. The inductors for negative capacitance, after cores being air-gapped, will have much lower inductances although, which really is not a bad thing since mutual capacitors for power applications, especially in low frequency, are made up of large value capacitors, accordingly by (17), the values of negative capacitance are large as well by $C = -1/(\omega^2 L)$; thus extremely, in the case of huge power applications, air-cored coils may be used to lower the cost, raise the efficiency (because of lower loss of iron and copper) and also improve the heat-cooling condition of the mutual capacitor. The following three experiments, although not of big power output (which may be calculated from the data given), were completed at high efficiency ($P_{\mathrm{out}}/P_{\mathrm{in}} > 90\%$) under very limited laboratory conditions of the author.

3.1 *Unity-coupled current type mutual capacitor*

An experiment for the unity-coupled current type mutual capacitor in Fig. 4*a* was completed, at a frequency $f = 50$ Hz, with parameters of $C_A = -1/(\omega^2 L_A)$, $C_B > 0$ and $C_M = -1/(\omega^2 L_M)$. Its specific circuit parameter values, experimental conditions, measurement results and one of its waveform records are given in Fig. 7, where the records of (V_{11}, V_{21}), (V_{12}, V_{22}) and (V_{13}, V_{23}), respectively, represent three pairs of the waveform data (V_1:V_2) measured at low, middle and high sections of operating range of the mutual capacitor (the photo displays the second pair). The current ratios calculated from the waveforms measured are lined in turn as $I_1/I_2 = 4.2607$, 4.2353, 4.2250, whereas the ratio calculated by applying (16) yields $n_c = I_1/I_2 = 4.2412$. This unity-coupled current mutual capacitor has a relative error of $|\delta_c| \simeq 0.33\%$, which is a reasonable result.

3.2 *Unity-coupled voltage type mutual capacitor*

An experiment for the unity-coupled voltage type mutual capacitor in Fig. 5*a* was done, at a frequency $f = 50$ Hz, with parameters of $C_a = -1/(\omega^2 L_a)$, $C_b > 0$ and $C_m > 0$. And its specific circuit

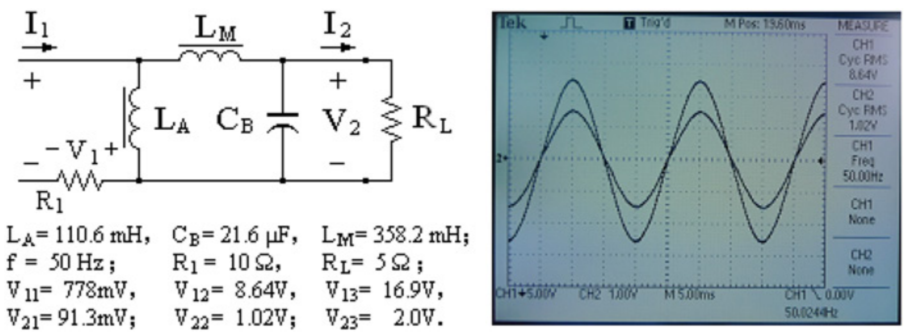

Fig. 7 *Experimental circuit diagram, data and a wave form of a unity-coupled current type mutual capacitor*

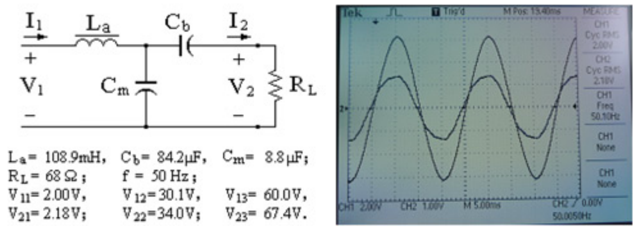

Fig. 8 *Experimental circuit diagram, data and a wave form of a unity-coupled voltage type mutual capacitor*

parameter values, experimental conditions, measurement results and one of its waveform records are shown in Fig. 8, where the records of (V_{11}, V_{21}), (V_{12}, V_{22}) and (V_{13}, V_{23}), respectively, represent three pairs of the waveform data (V_1:V_2) measured at low, middle and high sections of operating range of the mutual capacitor (the photo displays the first pair). The voltage ratios calculated from the waveforms are lined in turn as $V_1/V_2 = 0.9174$, 0.8853, 0.8902, whereas the ratio calculated by applying (19) yields $n_v = V_1/V_2 = 0.9050$. This unity-coupled mutual capacitor has a relative error of $|\delta_v| \simeq 1.73\%$, higher than expected although also reasonable, mostly because of the reason n_v being close to one [Note: it is sensitive for errors when n_v is in the vicinity of 1, see the note following (18).].

3.3 *Characteristics of waveform conversion of the unity-coupled mutual capacitor*

The voltage type unity-coupled mutual capacitor in Fig. 8 has its voltages and currents on both ports, all shaped like sine wave when it has a linear load R across its output. However in Fig. 9*a*, V_1 is a square-wave voltage source from an inverter and V_2 has a load as actually is an inductive rectifier. Thus, the output has the current, i_{II} (see V_{II} in Fig. 9*b*), measured as an approximate square-wave, whereas its input has a current, i_I (see V_I in Fig. 9*b*), displayed as an approximate sine-wave (although both are not good enough). This illustrates that the mutual capacitor actually functions well of a waveform separation or waveform conversion between the output square-wave current and the input quasi-sinusoidal current or being termed the backward harmonic current filtering. The waveforms in Fig. 9*c* were measured for a contrast between its input voltage V_1 and its output voltage V_2 and V_1 being shaped approximately like a square-wave, whereas V_2 approximately like a sine wave, which proves that this mutual capacitor has a good ability of forward harmonic voltage filtering [Note: waveforms, both squared and sinusoidal, in photos showing distorted or non-ideal originate mostly from the weak output capability of the square-wave voltage source (see V_1 in Fig. 9*c*) or the source V_1 has too large an internal impedance of some non-linearity. The author had another experiment similar to this but in high frequency resulting with satisfactory waveforms.].

4 Conclusions

The mutual capacitor, or the mutual-capacitance transformer, being a new circuit element or component, will enrich the applications of the network components and also cause the principle of duality into practical use since it makes a dual of the mutual inductor. Being a new ac transformer it can do most of the performances which the mutual inductor, or the conventional inductance transformer, does in diverse aspects; and it can also function something that the inductance transformer cannot. Featuring distinctly, the mutual capacitor and the mutual inductor complement each other. A mutual inductor or an inductance transformer is safer for use in a case of relatively small power and low voltage, and it has a smaller size, costs less and operates reliably, whereas at high-voltage circumstances, particularly for huge power use, it has a series of problems – such as in heat cooling, and it costs too much for maintenances. The mutual capacitor or the capacitance transformer, compared with its inductive partner however, has higher transformation accuracy (for its ease to be unity-coupled), higher efficiency (for its less iron-and-copper power loss), much better cooling performances (thanks to its separated and assembled structure), much lower use of materials (for its less uses of copper, iron, chemicals

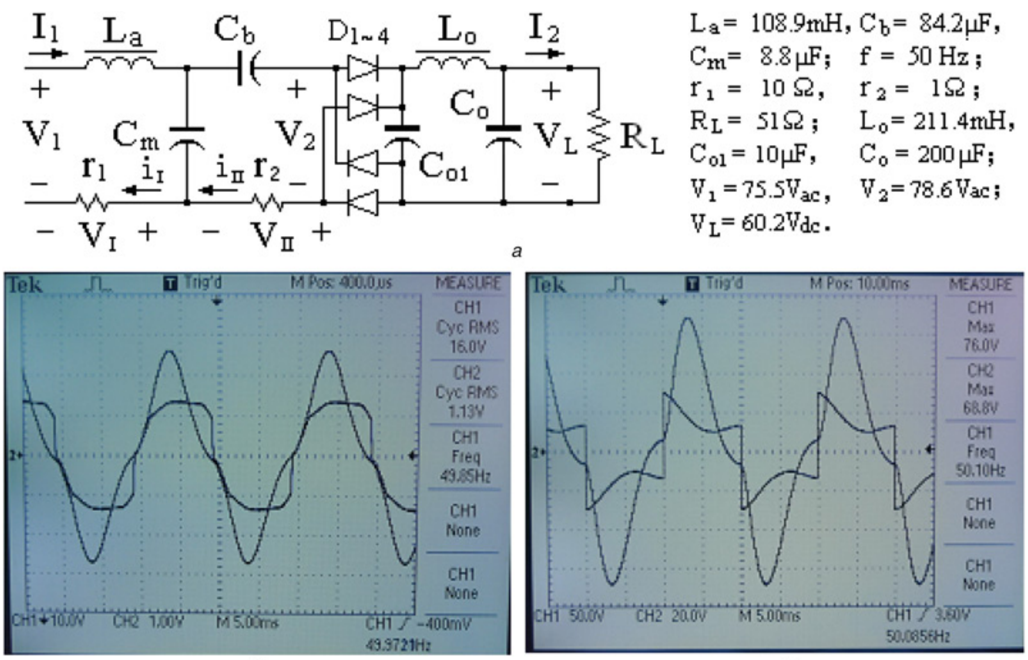

Fig. 9 *Experiment on waveform conversions with a voltage type mutual capacitor*
a Circuit diagram and data
b Current i_I against i_{II} ($V_I = 16.0$ V, $V_{II} = 1.13$ V)
c Voltage V_1 against V_2 ($V_{1max} = 76.0$ V, $V_{2max} = 68.8$ V)

and equipment for cooling) and much less maintenance jobs needed; it also can raise power factor of the grid, an advantage in huge power applications and concurrently improves the waveform quality of electricity, although it is not suited to be made as a small transformer, especially for use at low frequency, because it must be set a constant LC value satisfying (14) or (17). A power mutual capacitor must operate at a fixed frequency of high stability; and the manufacture of big power mutual capacitors must be based on the development of the manufacturing level of non-polar capacitors and so forth. The mutual capacitor will be good for use in the following aspects: (i) current/voltage transformations for measurement or test, especially of high voltage or high current; (ii) high-power and high-voltage applications of voltage/current transformers or constant-current generators for power transmission, concurrently with waveform separation; (iii) applications of power deliveries and waveform conversions/separations between switching devices in power electronics (much lower switching-transition power-losses especially at high frequencies); and (iv) IC transformers constructed of IC capacitors and IC NICs [3, 6]. The author believes it may have a long way to go for the mutual-capacitance transformer to function as main transformers at power substations, but we can expect almost any day when it plays a role as rectifier transformer such as for electroplating, smelting, locomotive powering or laboratory use etc., as well as acts as the gate-guard transformers of power consumers to keep harmonics generated by all the users' domestic apparatuses off the grid.

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