JOURNAL OF ADVANCED TRANSPORTATION
J. Adv. Transp. 2014; **48**:304–317
Published online 6 March 2012 in Wiley Online Library (wileyonlinelibrary.com). DOI: 10.1002/atr.215

A macro model for traffic flow on road networks with varying road conditions

Tie-Qiao Tang[1]*, Lou Caccetta[2], Yong-Hong Wu[2], Hai-Jun Huang[1] and Xiao-Bao Yang[3]

[1]*Beihang University, Beijing, China*
[2]*Curtin University, Perth WA, Australia*
[3]*Beijing Jiaotong University, Beijing, China*

SUMMARY

In this paper, we develop a macro traffic flow model with consideration of varying road conditions. Our analytical and numerical results illustrate that good road condition can enhance the speed and flow of uniform traffic flow whereas bad road condition will reduce the speed and flow. The numerical results also show that good road condition can smooth shock wave and improve the stability of traffic flow whereas bad road condition will lead to steeper shock wave and reduce the stability of traffic flow. Our results are also qualitatively accordant with empirical results, which implies that the proposed model can qualitatively describe the effects of road conditions on traffic flow. These results can guide traffic engineers to improve the road quality in traffic engineering. Copyright © 2012 John Wiley & Sons, Ltd.

KEY WORDS: road condition; macro model; traffic flow

1. INTRODUCTION

To date, many traffic flow models have been developed to study various complex traffic phenomena from different perspectives. For more details, the reader is referred to the review papers [1–5]. The first simple traffic flow model was developed independently by Lighthill and Whitham [6] and Richards [7] (called the LWR model). The LWR model can reproduce the formation, propagation, and evolution of shock wave, so the model was later extended to study the multi-class traffic flow [8]. However, the LWR model and its extension cannot reproduce the non-equilibrium traffic flow as the speed in the model cannot deviate from the equilibrium speed. To conquer this drawback, Payne [9] proposed the first high-order model (called the Payne model); Michalopoulos *et al*. [10] considered the impacts of the geometry of road structure on traffic flow and proposed a high-order model with consideration of the geometry of road structure; Zhang [11,12] constructed two high-order models based on the Payne model; Jiang *et al*. [13] developed a speed-gradient (SG) model from the full velocity difference (FVD) model [14], and Chang and Zhu [15] extended the model to the multi-lane traffic system and constructed a multi-lane traffic flow model; Gupta and Katiyar [16] established a dynamic model for multi-class traffic flow on the basis of the Payne model, and Gundaliya *et al*. [17] extended the work and developed a grid-based model to explore multi-class traffic flow; Tang *et al*. [18] further extended the SG model [13] and developed a two-lane traffic flow model with consideration of lane-changing; on the basis of the traffic flow model [15], Bie *et al*. [19] developed a lane-based method to evaluate the capacity of multi-lane traffic roundabout.

Besides the above macro models, there are many car-following models. One of the important car-following models is the optimal velocity (OV) model [20], which is simple but can describe many

*Correspondence to: Tie-Qiao Tang, Beihang University, Beijing, China. E-mail: tieqiaotang@buaa.edu.cn

complex traffic phenomena. On the basis of the OV model, many extended OV models were developed. For example, Helbing and Tilch [21] proposed a generalized force (GF) model on the basis of their experimental data, and Jiang *et al*. [14] extended the GF model to develop an FVD model; Nagatani [22] considered the impacts of the leading vehicle's headway and developed an extended OV model, and Ge *et al*. [23] further extended this work and proposed an extended OV model with multi-headways; Zhao and Gao [24] found that the FVD model may produce collision under some specific conditions and thus proposed a car-following model with consideration of the leading vehicle's acceleration; Wang *et al*. [25] extended the FVD model and constructed an extended FVD model with consideration of multi-velocity differences, and Peng and Sun [26] further extended this work.

However, the abovementioned models cannot be used to directly explore various complex traffic phenomena that resulted from different road conditions as they do not explicitly consider these factors. In order to study the impacts of road conditions on traffic flow, Delitala and Tosin [27] used the discrete theory to construct a discrete velocity mathematical model for vehicular traffic, and Bellouquid and Delitala [28] extended the work to develop a traffic flow model with road conditions from the asymptotic limit of the discrete model [29]. In addition, Li *et al*. [30] presented a car-following model with consideration of the driving resistance. The models [27,28,30] can be used to explore some complex traffic phenomena resulted from the road conditions from different perspectives, especially that the model [28] can be used to directly study different road conditions from the mathematical perspective, but these models cannot completely describe various complex impacts of road conditions on traffic flow as they still have some limitations. For instance, the work [28] focused on studying the effects of road conditions on traffic flow from the mathematical perspective but did not carry out numerical tests to further testify whether the model can qualitatively reproduce the effects of road conditions on traffic flow (e.g., the impacts of road conditions on the fundamental diagram, the formation, evolution and propagation of shock and rarefaction waves, the stability of traffic flow and other traffic phenomena); in addition, the authors only derived the anticipation term of the proposed model for the worst and best road conditions; and furthermore, the work under the worst road condition is limited to only the case in which the equilibrium speed is zero. The car-following model [30] only explored the effects of the driving resistance on the car-following behavior; in addition, the model has a fatal drawback, that is, different road conditions are assumed to have the same qualitative effects on traffic flow with differences only in the value of some parameters.

The aforementioned work focuses on studying the traffic phenomena mainly from the analytical and numerical perspectives; it is thus necessary to further explore whether the results obtained from the aforementioned models are quantitatively accordant with the real traffic phenomena. In order to study the real traffic phenomena, Sarvi *et al*. [31] used empirical data to undertake a comprehensive investigation of traffic behavior and characteristics in the freeway ramp merging under congested traffic conditions; Ngoduy [32,33] applied observed data to explore the flow–density relationship, and Castillo [34] developed three traffic flow models for the flow–density relationship and used empirical data to calibrate the four main parameters of the three models. However, little effort has been made to use empirical data to study the effects of road conditions on the flow–density relationship and other traffic phenomena.

In order to be able to simulate traffic flow on road networks with varying road conditions, it is necessary for one to construct a robust model that takes into account road conditions. Thus, in this paper, on the basis of the models [28,30], we develop a model for road networks with varying road conditions and then apply the model to explore various complex traffic phenomena under different road conditions. In comparison with the work by Bellouquid and Delitala [28], our work has three new contributions. Firstly, explicit formulae are given in our model to consider a wider range of road conditions varying from the worst to the best road conditions, whereas explicit formulae for the effects of road conditions in the cited work are given only for the extreme (the worst and the best) road conditions, and also the term for the worst road conditions is limited to only the case where the equilibrium speed is zero. Secondly, our model is validated on its capability of qualitatively reproducing the effects of road conditions on traffic flow, whereas the capability of the model in the cited work is not demonstrated. Thirdly, many interesting results are established to show the effects of road conditions on uniform flow, evolutions of traffic waves, and small perturbation. The rest of this paper is organized as follows. The new model is developed in Section 2; the impacts of road conditions on

DOI: 10.1002/atr

uniform flow, traffic waves, and small perturbation are studied in Section 3; a case study is used to study the effects of road conditions of the fundamental diagram in Section 4; some conclusions are given in Section 5.

2. A MACRO MODEL WITH CONSIDERATION OF ROAD CONDITION

The simplest macro model is the Lighthill–Whitham–Richards (LWR) model [6,7], which can be expressed by

$$\rho_t + (\rho v_{\mathrm{e}}(\rho))_x = 0 \tag{1}$$

where $\rho, v_{\mathrm{e}}(\rho)$ are respectively the density and equilibrium speed. Equation (1) can simulate the formation and evolution of shock wave, but it cannot be used to study non-equilibrium traffic flow as the speed in the model cannot deviate from $v_{\mathrm{e}}(\rho)$. To conquer this drawback, many density-gradient (DG) and SG models were developed to explore non-equilibrium traffic flow. The classical DG model is the Payne model [9]:

$$\begin{cases} \rho_t + (\rho v)_x = 0 \\ v_t + v v_x = \dfrac{v_{\mathrm{e}} - v}{\tau} - \dfrac{\upsilon}{\rho\tau}\rho_x \end{cases} \tag{2}$$

where v is the speed, τ is the relaxation time, and $\upsilon = -0.5v'_{\mathrm{e}}(\rho)$ is the anticipation coefficient. The typical SG model [13] can be expressed by

$$\begin{cases} \rho_t + (\rho v)_x = 0 \\ v_t + v v_x = \dfrac{v_{\mathrm{e}} - v}{\tau} + c_0 v_x \end{cases} \tag{3}$$

where c_0 is the propagation speed of small perturbation.

Equations (1)–(3) and their extensions cannot describe the impacts of road conditions on traffic flow as they do not explicitly consider these factors. In order to study the impacts of road conditions, Delitala and Tosin [27] developed a discrete velocity mathematical model for vehicular traffic, and Bellouquid and Delitala [28] extended the work to propose a traffic flow model with consideration of road conditions from the mathematical perspective. To explicitly display the effects of road conditions on traffic flow, Bellouquid and Delitala [28] considered two extreme cases (the worst and the best road conditions) and deduced the explicit control equations for the two cases. For the worst road condition, Bellouquid and Delitala's control equations can be expressed as follows:

$$\begin{cases} \rho_t + (\rho v)_x = 0 \\ v_t + v v_x = v/\tau \end{cases} \tag{4}$$

whereas for the best road condition, the control equations can be written as follows:

$$\begin{cases} \rho_t + (\rho v)_x = 0 \\ v_t + v v_x + \left(2 - 3\rho + \dfrac{\zeta}{2}\dfrac{\rho}{1-\rho}\right)\dfrac{\partial\rho}{\partial x} = \dfrac{v - (1-\rho)}{\tau} \end{cases} \tag{5}$$

where ζ is a dimensionless parameter. Equations (4) and (5) have explicitly considered road conditions, so the model [28] can be used to directly study the effects of road conditions on traffic flow. However, the explicit control equations consider only two extreme cases; also model (4) implies that the equilibrium speed under the worst road condition is zero, but in fact, the equilibrium speed is not zero under any road conditions. Hence, the model [28] cannot completely describe the complex traffic phenomena under certain varying road conditions. Furthermore, the authors did not use numerical tests to testify whether their model can qualitatively reproduce the impacts of road conditions on traffic flow.

To further describe the effects of road conditions on traffic flow, we should analyze the driving behavior under different road conditions. Quantitatively, the impacts of road conditions on driving behavior are complex and related to many factors (e.g., the traffic density, the driver's individual properties), so we need to use empirical data to quantify and calibrate them if we are to quantitatively study the effects of road conditions. However, the main purpose of this paper was to propose a macro traffic flow model to explore the qualitative effects of road conditions on traffic flow, so we focus only on the quantitative behavior. Qualitatively, bad road condition has negative effects on the driving behavior (i.e., bad road condition will motivate drivers to decelerate), and good road condition has positive influences on the driving behavior (i.e., good road condition will motivate drivers to speed up), which shows that road conditions should be explicitly considered when we study traffic flow. As the SG model [13] is simple and widely used to study traffic flow (this model is cited 91 times by the science citation index journals), we here propose a model with consideration of road conditions on the basis of the SG model.[1] As this paper mainly focuses on proposing a model with consideration of road conditions to study the qualitative effects of road conditions on traffic flow, we neither construct similar models corresponding to other types of basic traffic flow models nor further compare the results obtained by different models with consideration of road conditions.

In the existing traffic flow models, the effects of each traffic factor on driving behavior are reflected by some terms of the dynamic equations, so we can explore the impacts of road conditions on traffic flow by adding an additional term into the dynamic equation of the SG model [13] (we here call the addition term as the friction effect). In the real traffic system, the friction effects resulted from road conditions are complex and related to many factors (the traffic density, road conditions, etc.). For the purpose of analyzing the qualitative effects of the friction term, we can define the friction effect from qualitative perspective. Of course, for the exact friction impacts of road conditions, we need to use empirical data to calibrate it. So, we here define the friction effect as $\mu_r a_r$ for simplicity, where $a_r \geq 0$ is the adjustable term that the driver can use to adjust his or her acceleration under different road conditions, and μ_r is the adjustment coefficient. As $\mu_r a_r$ is complex and often related to many factors (e.g., the traffic density, road conditions) in the real traffic system, we should further study it when we analyze the proposed model and carry out the numerical tests. On the other hand, road conditions also influence the equilibrium speed $v_e(\rho)$ of the SG model [13] if the model is to be used to study the effects of road conditions, so we should consider the impacts of road conditions on the equilibrium speed of the proposed model with consideration of road conditions.

On the basis of the above discussions, we can establish the following traffic flow model with consideration of road conditions on the basis of the SG model [13],

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \dfrac{\partial \rho v}{\partial x} = 0 \\ \dfrac{\partial v}{\partial t} + v\dfrac{\partial v}{\partial x} = \dfrac{v_{r,e}(\rho) - v}{\tau} + c_{r,0}\dfrac{\partial v}{\partial x} + \mu_r a_r \end{cases} \tag{6}$$

where $v_{r,e}(\rho)$ is the equilibrium speed and $c_{r,0}$ is the propagation speed of perturbation. It is noticed that Equation (6) can also be obtained from a kinetic discrete model by an asymptotic analysis similar to that used in the References [27,29] to explore the effects of road conditions; but here, we do not develop Equation (6) from the kinetic discrete theory because we can directly obtain it from the qualitative perspective based on the SG model [13].

As $v_{r,e}(\rho)$, $c_{r,0}$, μ_r, a_r are complex and related to the concrete road conditions, we should define them on the basis of the concrete road conditions and apply empirical data to calibrate them. But, the main aim of this paper was to propose a model with consideration of road conditions to study the qualitative effects of road conditions on traffic flow. The magnitudes of the parameters $c_{r,0}$, μ_r, a_r will not influence

[1]Note: we can also develop some similar models with consideration of road conditions on the basis of other traffic flow models and obtain some similar results.

DOI: 10.1002/atr

the qualitative results, so we can here define them from the qualitative perspectives. For simplicity, we define the parameter a_{r} as a constant. Qualitatively, when the density is relatively low (i.e., the traffic density is less than a critical value), there is enough room that the drivers can adjust their accelerations on the basis of their current traffic states (in other word, the external traffic factor will influence the driving behavior), so road conditions will influence the driving behavior at this time; but the room that the drivers adjust their accelerations decreases when the traffic density increases, so the effects of road conditions on driving behavior will also decrease when the traffic density increases. When the traffic density is relatively high (i.e., the traffic density is larger than the critical value), there is little room that the drivers can adjust their accelerations on the basis of their current traffic states, and the room will gradually become small and finally disappear when the traffic density increases to the jam density (i.e., the effects of the external traffic factor on the driving behavior will gradually become weak and finally disappear), so the effects of road conditions on the driving behavior will gradually become weak and finally disappear when the traffic density increases to the jam density. Thus, we can define μ_{r} as follows:

$$\mu_{\mathrm{r}} = \begin{cases} \mu_0 g(r)\left(1 - \dfrac{\rho}{\rho_{\mathrm{cr}}}\right), & \text{if } \rho \leq \rho_{\mathrm{cr}} \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

where μ_0 is a constant, $g(r)$ is a continuous function within the range $[-1,1]$ that can reflect the road conditions (here r is the road conditions variable), and ρ_{cr} is a critical value. Note that the function $g(r)$ has the following properties:

(1) $g(r)$ is a continuous function of the road conditions;
(2) the road is bad when $g(r) < 0$ and good when $g(r) > 0$;
(3) the road is in the worst road condition when $g(r) = -1$ and in the best road condition when $g(r) = 1$.

The parameter $c_{\mathrm{r},0}$ in Equation (6) is related to the road conditions but has little qualitative effects on the results, so we can for simplicity define it as follows:

$$c_{\mathrm{r},0} = \begin{cases} 20, & \text{under good road condition} \\ 9, & \text{under bad road condition} \end{cases} \tag{8}$$

Note that $v_{\mathrm{r,e}}(\rho)$, $c_{\mathrm{r},0}$, μ_{r}, a_{r} are defined from the qualitative perspectives in this paper. In other word, the definition of the equilibrium speed $v_{\mathrm{r,e}}(\rho)$ and the values of the parameters $c_{\mathrm{r},0}$, μ_{r}, a_{r} are just assumptions; that is, there are no empirical evidences for the chosen forms and the forms of $v_{\mathrm{r,e}}(\rho)$, $c_{\mathrm{r},0}$, μ_{r}, a_{r} are also not derived from any concrete assumptions. As for the exact definition of the equilibrium speed $v_{\mathrm{r,e}}(\rho)$ and the exact values of the parameters $c_{\mathrm{r},0}$, μ_{r}, a_{r}, we should use empirical data to further define and calibrate them in the future.

Equation (6) has explicitly considered road conditions, so it can be used to explore some qualitative complex traffic phenomena under varying road conditions.

3. NUMERICAL TESTS

In this section, we first study the effects of road conditions on uniform flow. Setting ρ_0, v_0 as the density and speed of uniform flow and substituting them into Equation (6), we obtain

$$v_0 = v_{\mathrm{r,e}}(\rho_0) + \tau \mu_{\mathrm{r}} a_{\mathrm{r}} \tag{9}$$

DOI: 10.1002/atr

Using the model [13] to study the equilibrium flow, we get

$$v_0 = v_e(\rho_0) \tag{10}$$

where $v_e(\rho_0)$ is the equilibrium speed in Equation (5).

To explore the impacts of road conditions, we should define $v_{r,e}(\rho)$. Qualitatively, bad road condition may reduce the equilibrium speed whereas good condition can enhance the equilibrium speed, which implies that the equilibrium speed is related to road conditions. However, the quantitative relationships between the equilibrium speed and road conditions are very complex and related to many factors (e.g., the traffic density, the driver's individual properties), and we should use empirical data to calibrate it in the future. For the purpose of qualitative analysis, we can for simplicity define $v_{r,e}(\rho)$ as follows:

$$v_{r,e}(\rho) = \frac{v_{r,f}}{v_f} v_e(\rho) + \omega_r \hat{v}_{r,0} \tag{11}$$

where $v_{r,f}$, v_f are respectively the free flow speeds with and with no consideration of road conditions, $\hat{v}_{r,0} > 0$ is the adjustment speed, and ω_r is the adjustment coefficient. Likely, good road condition can enhance the free flow speed whereas bad road condition will reduce the free flow speed. For simplicity, we here define $v_{r,f}$ as follows:

$$v_{r,f} = v_f + \eta_r \hat{v}_{r,f} \tag{12}$$

where $\hat{v}_{r,f} > 0$ is the adjustment speed and η_r is the adjustment coefficient. Notice that the parameters μ_r, ω_r, η_r have the same qualitative properties, so we here define $\mu_r = \omega_r = \eta_r$ for simplicity.

With the aforementioned discussions, we can obtain the following proposition:

Proposition

In comparison with the model [13], bad road condition will reduce the speed and flow of uniform traffic flow, and good road condition can increase the speed and flow, which shows that improving the road quality can enhance the speed and flow.

To further describe the quantitative impacts of road conditions on uniform traffic flow, we should define $v_e(\rho)$. Again for the purpose of qualitative analysis, we can for simplicity define $v_e(\rho)$ as follows [35]:

$$v_e(\rho) = v_f\left(1\Big/\left(1 + \exp\left\{\frac{\rho/\rho_j - 0.25}{0.06}\right\}\right) - 3.72 \times 10^{-6}\right) \tag{13}$$

where ρ_j is the jam density.

The parameters have no qualitative impacts on the results, so we define them as follows in order to compare our results with those of the SG model [13]:

$$\begin{aligned} &v_f = 30\,\mathrm{m/s},\ \tau = 10\,\mathrm{s},\ a_r = 2\,\mathrm{m/s^2},\ \hat{v}_{r,0} = 2\,\mathrm{m/s},\ \mu_0 = 0.1 \\ &\hat{v}_{r,f} = 5\,\mathrm{m/s},\ \rho_j = 0.2\,\mathrm{veh/m},\ \rho_{cr} = 0.08\,\mathrm{veh/m} \end{aligned} \tag{14}$$

Applying the above parameters, we can obtain the speed–density and flow–density curves of the uniform traffic flow under the best and worst road conditions (see Figure 1). From Figure 1, we can obtain the following findings:

(a) The good road condition can enhance the speed and flow of the uniform traffic flow whereas the bad road condition will reduce the speed and flow, which is completely accordant with the analytical results.

DOI: 10.1002/atr

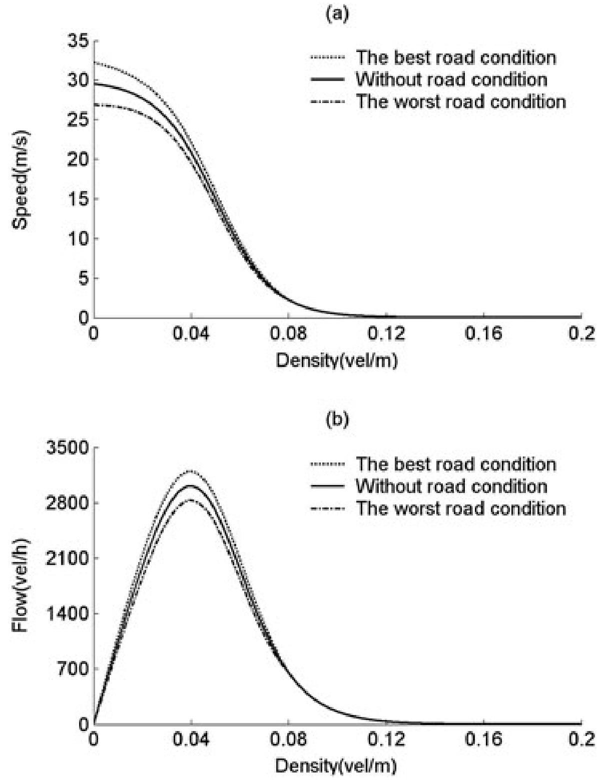

Figure 1. Speed–density and flow–density curves of uniform flow under different road conditions.

(b) Under other good road conditions, the speed and flow can also be enhanced, and the speed–density and flow–density curves are between those under the best road condition and those with no consideration of road conditions; under other bad road conditions, the speed and flow can also be reduced, and the speed–density and flow–density curves are between those under the worst road condition and those with no consideration of road conditions.

Next, we explore the impacts of road conditions on traffic waves and small perturbation. To compare with the model [13], we use the upwind scheme to discretize Equation (6), that is,

$$\rho_i^{j+1} = \rho_i^j + \frac{\Delta\Delta t}{\Delta x}\rho_i^j\left(v_i^j - v_{i+1}^j\right) + \frac{\Delta t}{\Delta x}v_i^j\left(\rho_{i-1}^j - \rho_i^j\right) \tag{15}$$

If $v_i^j < c_{\mathrm{r}0}$:

$$v_i^{j+1} = v_i^j + \frac{\Delta t}{\Delta x}\left(c_{\mathrm{r}0} - v_i^j\right)\left(v_{i+1}^j - v_i^j\right) + \frac{\Delta t}{\tau}\left(v_{\mathrm{r,e}}\left(\rho_i^j\right) - v_i^j\right) + \Delta t \mu_{\mathrm{r}}\sigma_{\mathrm{r}}a_{\mathrm{r}} \tag{16a}$$

otherwise

$$v_i^{j+1} = v_i^j + \frac{\Delta t}{\Delta x}\left(c_{\mathrm{r}0} - v_i^j\right)\left(v_i^j - v_{i-1}^j\right) + \frac{\Delta t}{\tau}\left(v_{\mathrm{r,e}}\left(\rho_i^j\right) - v_i^j\right) + \Delta t \mu_{\mathrm{r}}\sigma_{\mathrm{r}}a_{\mathrm{r}} \tag{16b}$$

where i, j are respectively the space and time indexes; $\Delta x, \Delta t$ are respectively the lengths of the space step and time step.

The initial conditions of shock and rarefaction waves are as follows:

$$\rho_{\mathrm{up}}^1 = 0.04\,\mathrm{veh/m},\ \ \rho_{\mathrm{down}}^1 = 0.18\,\mathrm{veh/m},\ \ \rho_{\mathrm{up}}^2 = 0.18\,\mathrm{veh/m},\ \ \rho_{\mathrm{down}}^2 = 0.04\mathrm{veh/m} \tag{17}$$

DOI: 10.1002/atr

where $\rho^i_{\mathrm{up}}, \rho^i_{\mathrm{down}}$ are respectively the upstream and downstream densities of shock and rarefaction waves. The initial speeds are as follows:

$$v^i_{\mathrm{up}} = v_{\mathrm{r,e}}\left(\rho^i_{\mathrm{up}}\right), v^i_{\mathrm{down}} = v_{\mathrm{r,e}}\left(\rho^i_{\mathrm{down}}\right), i = 1, 2 \tag{18}$$

Here, we use the free boundary condition and define $v_{\mathrm{e}}(\rho)$ as follows [36]:

$$v_{\mathrm{e}}(\rho) = v_{\mathrm{f}}\left[1 - \exp\left(1 - \exp\left(\frac{c_{\mathrm{m}}}{v_{\mathrm{f}}}\left(\frac{\rho_{\mathrm{j}}}{\rho} - 1\right)\right)\right)\right] \tag{19}$$

where c_{m} is the speed of the kinematic wave under the jam density. The road length used for the simulation is 20 km, and the discontinuous point is its mid point. In the numerical tests, the parameters c_{m}, Δx, Δt are chosen as $c_{\mathrm{m}} = 11$ m/s, $\Delta x = 200$ m, $\Delta t = 1$ s. Other parameters are the same as those for Figure 1. Thus, we can obtain the evolutions of Equation (18) (see Figures 2 and 3). From these figures, we can conclude the following results:

(a) The new model can qualitatively reproduce shock and rarefaction waves under different road conditions (see Figures 2 and 3).
(b) In comparison with the model [13], the shock wave is a little smoother under the good road condition and smoothest under the best road condition, and the shock wave is a little steeper under the bad road condition and steepest under the worst road condition (see Figures 2(a) and 3(a)); the road conditions have no qualitative effects on the rarefaction wave (see Figures 2(b) and 3(b));

For the evolution of perturbation, in order to compare with the model [13], we here use the following small perturbation [37]:

$$\rho(x,0) = \rho_0 + \Delta\rho\left\{\cosh^{-2}\left(\frac{160}{L}\left(x - \frac{5L}{16}\right)\right) - \frac{1}{4}\cosh^{-2}\left(\frac{40}{L}\left(x - \frac{11L}{32}\right)\right)\right\} \tag{20}$$

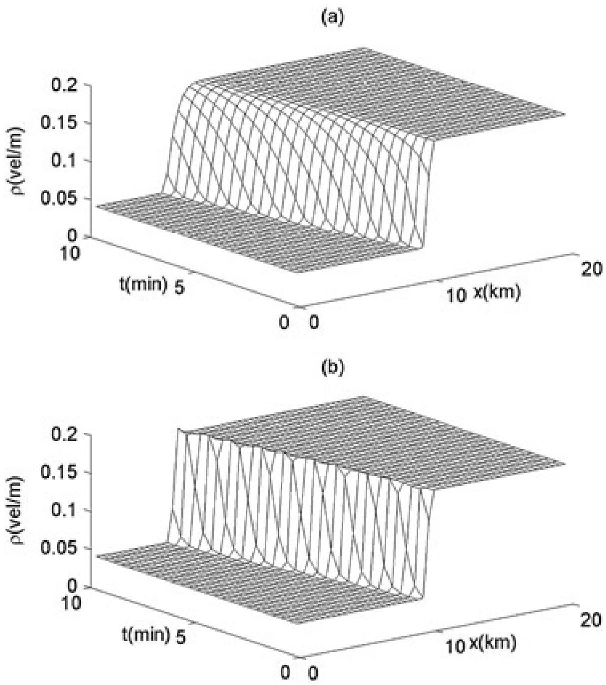

Figure 2. Shock waves under different road conditions. (a) The best road condition; (b) the worst road condition.

DOI: 10.1002/atr

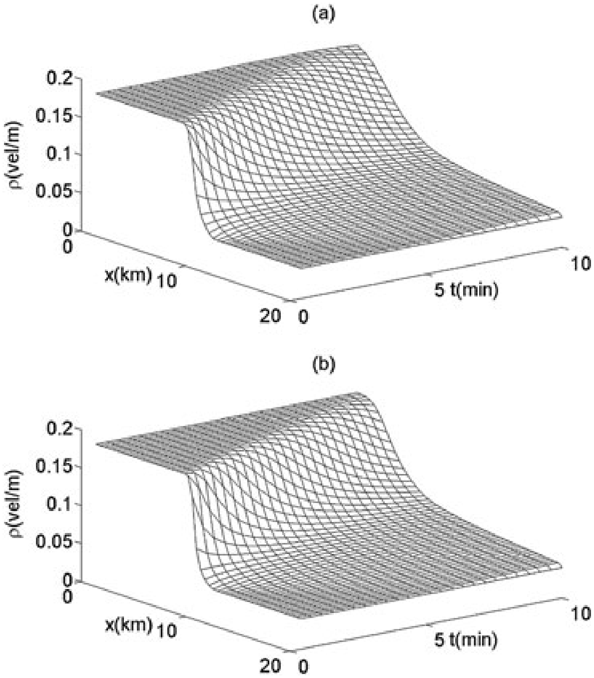

Figure 3. Rarefaction waves under different road conditions. (a) The best road condition; (b) the worst road condition.

where the first and second terms are respectively the initial density and perturbation; $L = 32.2$ km is the road length. In the following numerical tests, we use the periodic boundary condition, that is,

$$\rho(0, t) = \rho(L, t), v(0, t) = v(L, t) \tag{21}$$

To compare with the model [13], we define $v_e(\rho)$ by Equation (13) and set $v(x, 0) = v_{r,e}(\rho(x, 0))$, $\Delta\rho = 0.01$ veh/m, $\Delta x = 100$ m, $\Delta t = 1$ s. Other parameters are the same as those for Figures 2 and 3. Using the above conditions, we obtain the evolutions of Equation (20) under different road conditions (see Figures 4 and 5). From the two figures, we can conclude the following results:

(1) The small perturbation (20) will quickly evolve into uniform flow when the initial density is very low or high (see Figures 4(a, b, d, e) and 5(a, e)) but produce stop-and-go traffic when the initial density is moderate (see Figures 4(c) and 5(b–d)).
(2) In comparison with the model [13], good road condition can improve the stability of traffic flow, and the stability is the best under the best road condition; bad road condition will reduce the stability of traffic flow, and the stability is the worst under the worst road condition (see Figures 4 and 5). When the initial density lies in the unstable region, the stop-and-go traffic under a good condition is more serious than the one under the best road condition, but it is weaker than the one with no consideration of road conditions; the stop-and-go traffic under a bad road condition is weaker than the one under the worst road condition, but it is more serious than the one with no consideration of road conditions (see Figures 4(c) and 5(c)). The above results show that good road condition has positive effects on the stability of traffic flow whereas bad road condition has negative impacts on the stability of traffic flow.

4. CASE STUDY

In this section, we apply observed data to testify whether Figure 1 and the above proposition are qualitatively accordant with the real traffic. Because the road conditions include the lane width, the

DOI: 10.1002/atr

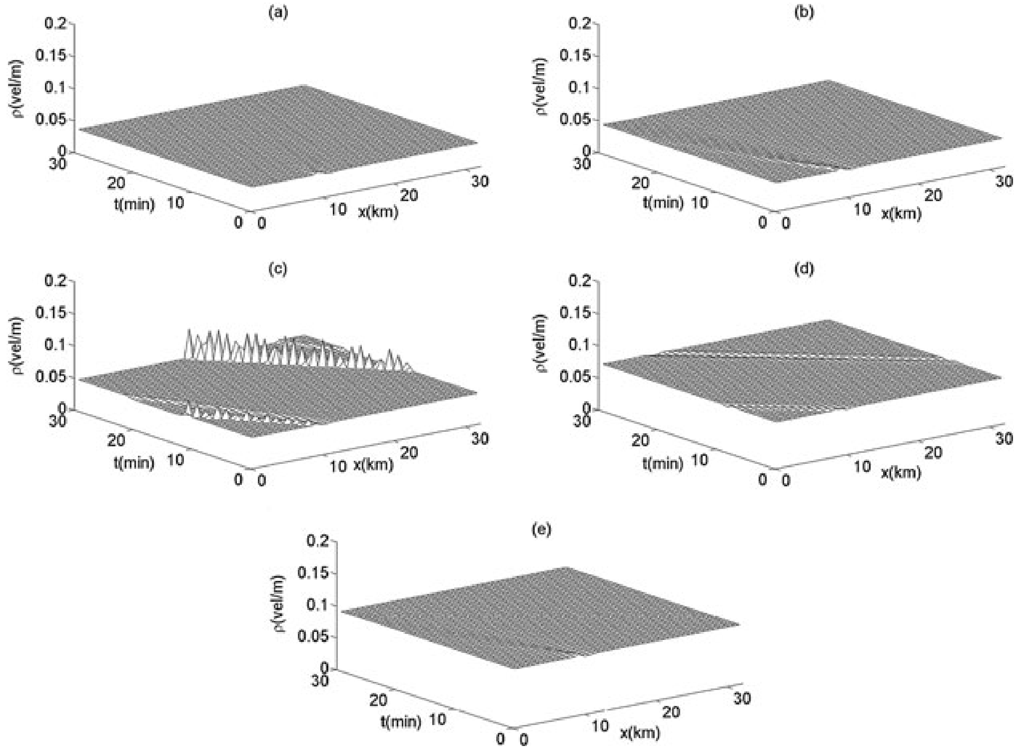

Figure 4. Evolution of small perturbation (21) under the best road condition. (a) $\rho_0 = 0.035$; (b) $\rho_0 = 0.042$; (c) $\rho_0 = 0.046$; (d) $\rho_0 = 0.07$; (e) $\rho_0 = 0.09$.

quality of road, the number of lanes, and other factors, we should classify these factors if we need to obtain the quantitative effects of each factor of road conditions on traffic flow. In this section, we use the observed data to study the effects of one of the road conditions (i.e., the lane width) on the fundamental diagram. A lane segment with width of 3.25 m is chosen in Shanghai, and a lane segment with width of 3.75 m is chosen in Beijing. In the observed tests, we used electronic loop detectors and a digital video camera to collect data for calculation of the flow and density on the road segments from March to June of 2006. The traffic data of every 5 minutes are categorized into a group. The units of the density and flow used in the observed results are respectively PCU/km and PCU/h (PCU is passenger car units) as the vehicles on the road consist of different types of vehicles (e.g., cars, buses) in the real traffic system. We assume that the traffic is homogeneous in the above figures, so we define the units of the density and flow respectively as veh/m and veh/h. In addition, the speed unit is often defined as km/h in the empirical data whereas it is often defined as m/s in the simulation results. Here, we set the road condition of the lane width 3.25 m as the bad road condition and the one with the lane width 3.75 m as the good road condition.

From our surveys, we obtain the fundamental diagram under the two road conditions (see Figure 6). From Figure 6, we can find that the results of Figure 1 and the above proposition are qualitatively accordant with the real traffic, which further implies that the proposed model can qualitatively reproduce some complex traffic phenomena. We should address here that, quantitatively, the results in Figures 1 and 6 have the following differences:

(1) The speed difference under different conditions becomes smaller when the traffic density is relatively high (see Figure 1(a)); but in Figure 6(a), it seems that the speed difference under the two road conditions are more apparent when the traffic density is relatively high.
(2) The flow difference under different road conditions turns smaller beyond the critical density (see Figure 1(b)) whereas it is not exactly the case in Figure 6(b).

The above differences are due to the fact that the results of Figure 1 are obtained from analytical deduction whereas the results of Figure 6 are obtained from empirical data. Generally, it is very

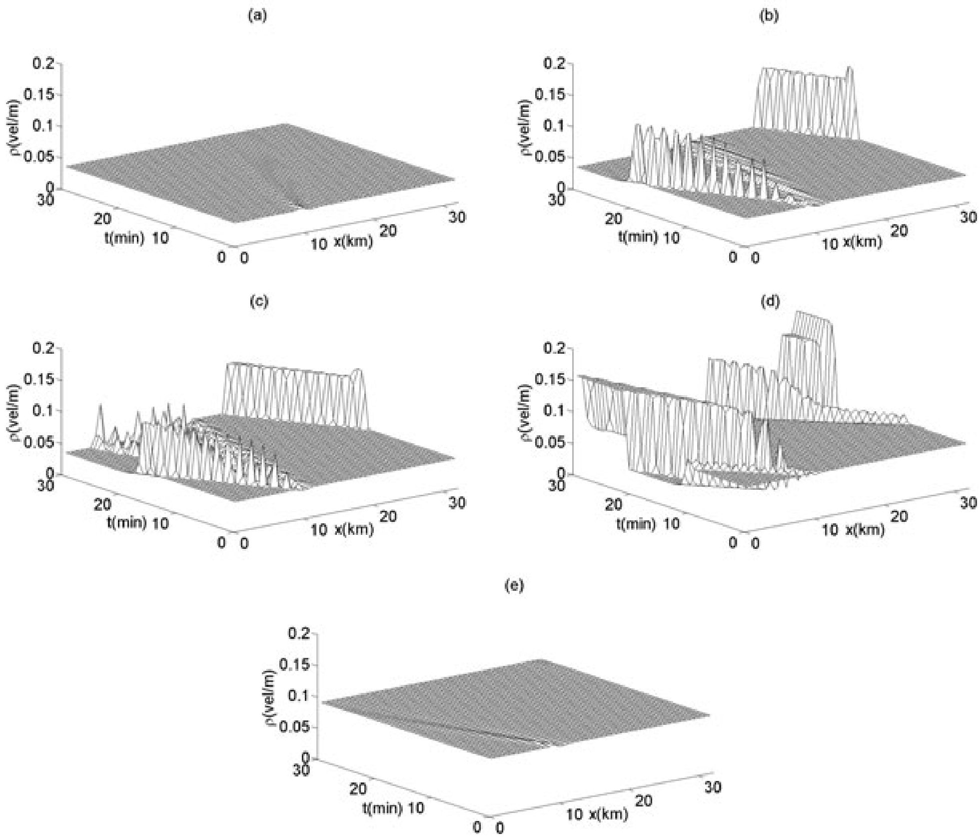

Figure 5. Evolution of small perturbation (21) under the worst road condition. (a) $\rho_0 = 0.035$; (b) $\rho_0 = 0.042$; (c) $\rho_0 = 0.046$; (d) $\rho_0 = 0.07$; (e) $\rho_0 = 0.09$.

difficult to have empirical data (Figure 6) completely accordant with analytical data (Figure 1) as empirical data often contain some errors.

5. CONCLUSIONS

Much traffic flow models have been developed to describe various complex traffic phenomena, but the existing models cannot completely reproduce the impacts of road conditions on traffic flow. In this paper, we propose a new macro model with consideration of varying road conditions on the basis of the effects of road conditions on the dynamic property of traffic flow. The analytical results show that good road condition can enhance the speed and flow of uniform traffic flow whereas bad road condition will reduce its speed and flow, which shows that improving the road quality can enhance the speed and flow. We should also address here that most literatures in traffic flow consider only ideal conditions, which correspond to the best road condition. The numerical results show that our model can qualitatively describe the effects of road conditions on uniform flow, shock and rarefaction waves, and small perturbation. Finally, the empirical data are qualitatively accordant with the analytical and numerical results, which implies that the proposed model can qualitatively describe the effects of road conditions on traffic flow. From the theoretical perspective, the proposed model provides an analytical method for studying the effects of road conditions on traffic flow. From the application perspective, the proposed model can be used for the study of traffic in real networks consisting of roads with different road conditions; also the results obtained can be used to guide the traffic engineers to improve the road quality as the results show that improving the road quality can enhance the speed, flow, stability of traffic flow, and running efficiency of the traffic system.

DOI: 10.1002/atr

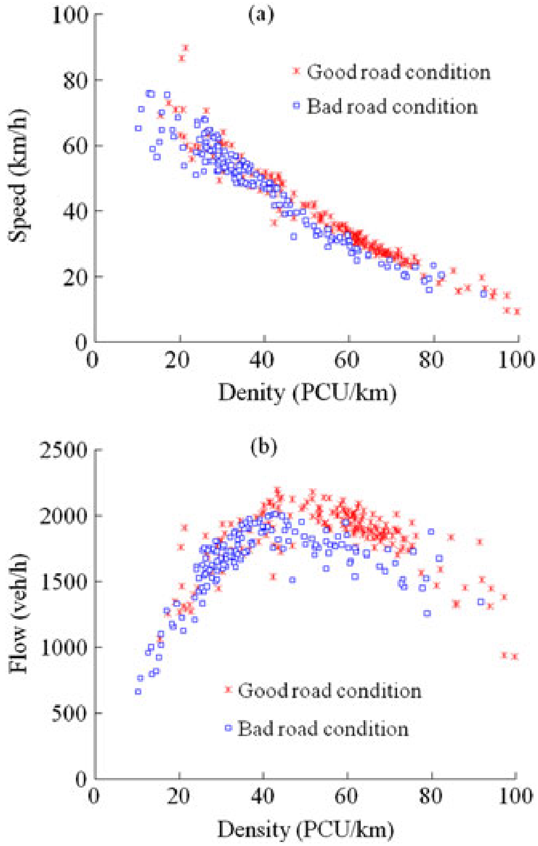

Figure 6. The fundamental diagram under the good and bad road conditions, where (a) is the speed–density relationship and (b) is the flow–density relationship.

We should also address here that this paper still has the following limitations:

(1) In this paper, we explore neither the combined effects of different road conditions on traffic flow nor the quantitative effects of various road conditions on traffic flow. But, we should address that the proposed model can be used to simulate the traffic phenomena for road networks with varying road conditions from the best condition to the worst condition. In fact, road often consists of segments with some in good and some in bad road conditions, so we should in future use empirical data to explore the combined effects of the mixed road conditions on traffic flow.
(2) The macro model with consideration of road conditions is directly proposed from the qualitative perspective based on the SG model [13], and the proposed model is not obtained from the kinetic discrete theory. Hence, we neither use the method [27,29] to further explore the activity that identifies the qualities of the driver–vehicle micro-system nor study the effects of road conditions on the kinetic properties of traffic flow. Thus, we should in the future apply the similar method [27,29] to further explore the effects of road conditions on the kinetic properties of traffic flow.
(3) The value of each parameter is defined either from the qualitative perspectives or based on the SG model [13], and we do not further explore the quantitative impacts of the parameters (especially the main parameters) on the numerical results. So, we should apply empirical data to further study the quantitative phenomena (including the sensitivity analysis) that the proposed model can reproduce.
(4) We use numerical tests to compare the proposed model only with the SG model [13] and have not further compared with the model [28], so we shall use empirical data to further compare with the model [28] from the mathematical and numerical perspectives in the future.
(5) We do not further compare the proposed model with other models that can be used to study the effects of road conditions, so the results and transferability may be limited. In order to better

DOI: 10.1002/atr

describe the effects of road conditions, we should compare the proposed model with other models that can be used to describe the effects of road conditions and then propose a more exact model by incorporating the physical structures of road conditions into the structure of the models and finally use empirical data to further study various quantitative effects of road conditions on traffic flow.

ACKNOWLEDGEMENT

This study has been supported by the Program for New Century Excellent Talents in University, China (NCET-08-0038), the National Natural Science Foundation of China (70971007 and 70821061), and the WA Centre of Excellence in Industrial Optimization. The authors would like to thank the anonymous referees for their helpful comments and valuable suggestions, which have improved the content and composition substantially.

REFERENCES

1. Chowdhury D, Santen L, Schreckenberg A. Statistics physics of vehicular traffic and some related systems. *Physics Reports* 2000; **329**:199–329.
2. Helbing D. Traffic and related self-driven many-particle systems. *Reviews of Modern Physics* 2001; **73**:1067–1141.
3. Bellomo N, Delitala M, Coscia V. On the mathematical theory of vehicular traffic flow I—fluid dynamic and kinematic modeling. *Mathematical Models & Methods in Applied Science* 2002; **12**:1801–1843.
4. Klar A, Wegener R. *Traffic flow: models and numerics, in Modeling and Computational Methods for Kinematic Equations*. Birkhäuser: Boston, 2004; 219–258.
5. Bandara JMSJ, Wong SC. Special issue: some recent advances in transportation modeling. *Journal of Advanced Transportation* 2005; **39**:241–245.
6. Lighthill MJ, Whitham GB. On kinematic waves: II. A theory of traffic flow on long crowed roads. *Proceeding Royal Society London* 1955; **229**:317–345.
7. Richards PI. Shock waves on the highway. *Operational Research* 1956; **4**:42–51.
8. Wong GCK, Wong SC. A multi-class traffic flow model-an extension of LWR model with heterogeneous drivers. *Transportation Research Part A* 2002; **36**:827–841.
9. Payne HJ. Models of freeway traffic and control. In *Mathematical Models of Public System*, Bekey GA (ed), Simulation Councils, inc.: La Jolla, California, 1971; **1**:51–61.
10. Michalopoulos PG, Yi P, Lyrintzis AS. Development of an improved high order continuum traffic flow model. *Transportation Research Part B* 1993; **27**:125–132.
11. Zhang HM. A theory of nonequilibrium traffic flow. *Transportation research Part B* 1998; **32**:485–498.
12. Zhang HM. A non-equilibrium traffic model devoid of gas-like behavior. *Transportation research Part B* 2002; **36**:275–290.
13. Jiang R, Wu QS, Zhu ZJ. A new continuum model for traffic flow and numerical tests. *Transportation Research Part B* 2002; **36**:405–419.
14. Jiang R, Wu QS, Zhu ZJ. Full velocity difference model for car-following theory. *Physical Review E* 2001; **64**:017101.
15. Chang G, Zhu Z. A macroscopic traffic model for highway work zones: formulations and numerical results. *Journal of Advanced Transportation* 2006; **40**:265–287.
16. Gupta AK, Katiyar VK. A new multi-class continuum model for traffic flow. *Transportmetrica* 2007; **3**:73–85.
17. Gundaliya PJ, Mathew TV, Dhingra SL. Heterogeneous traffic flow modelling for an arterial using grid based approach. *Journal of Advanced Transportation* 2008; **42**:467–491.
18. Tang TQ, Wong SC, Huang HJ, Zhang P. Macroscopic modeling of lane-changing for two-lane traffic flow. *Journal of Advanced Transportation* 2009; **43**:245–273.
19. Bie J, Lo HK, Wong SC. Capacity evaluation of multi-lane traffic roundabout. *Journal of Advanced Transportation* 2010; **44**:245–255.
20. Bando M, Hasebe K, Nakayama A, Shibata A, Sugiyama Y. Dynamical model of traffic congestion and numerical simulation. *Physical Review E* 1995; **51**:1035–1042.
21. Helbing D, Tilch B. Generalized force model of traffic flow. *Physical Review E* 1998; **58**:133–138.
22. Nagatani T. Stabilization and enhancement of traffic flow by next-nearest-neighbor interaction. *Physical Review E* 1999; **60**:6395–6401.
23. Ge HX, Dai SQ, Dong LY, Xue Y. Stabilization effect of traffic flow in an extended car-following model based on intelligent transportation system application. *Physical Review E* 2004; **70**:066134.
24. Zhao XM, Gao ZY. A new car-following model: full velocity and acceleration difference model. *The European Physical Journal B* 2005; **47**:145–150.
25. Wang T, Gao ZY, Zhao XM. Multiple velocity difference model and its stability analysis. *Acta Phys Sin.* 2006; **55**:634–640.
26. Peng GH, Sun DH. A dynamical model of car-following with the consideration of multiple information of preceding cars. *Physics Letters A* 2010; **374**:1694–1698.

27. Delitala M, Tosin A. Mathematical modelling of vehicular traffic: a discrete kinetic theory approach. *Mathematical Models & Methods in Applied Science* 2007; **17**:901–932.
28. Bellouquid A, Delitala M. Asymptotic limits of a discrete kinematic theory model of vehicular traffic. *Applied Mathematics Letters* 2011; **24**:672–678.
29. Bellomo N, Bellouquid A. On the modelling of vehicular traffic and crowds by the kinetic theory of active particles. In *Mathematical Modelling of Collective Behaviour in Socio-economics and Life Sciences*, Naldi G, Pareschi L, Toscani G (Eds). Birkhäuser: Boston, 2010; 273–296.
30. Li CY, Tang TQ, Huang HJ, Shang HY. A new car-following model with consideration of driving resistance. *Chinese Physics Letters* 2011; **28**:038902.
31. Sarvi M, Kuwahara M, Ceder A. Observing freeway ramp merging phenomena in congested traffic. *Journal of Advanced Transportation* 2007; **41**:145–170.
32. Ngoduy D. Multiclass first order modelling of traffic networks using discontinuous flow–density relationships. *Transportmetrica* 2010; **6**:121–141.
33. Ngoduy D. Multiclass first-order traffic model using stochastic fundamental diagrams. *Transportmetrica* 2011; **7**:111–125.
34. Castillo JM. Three new models for the flow–density relationship: derivation and testing for freeway and urban data. *Transportmetrica* 2011; in press.
35. Kerner BS, Konhäuster P. Cluster effect in initially homogeneous traffic flow. *Physical Review E* 1993; **48**: R2335–R2338.
36. Del Castillo JM, Benitez FG. On functional form of the speed–density relationship—I: general theory, II: empirical investigation. *Transportation Research Part B* 1995; **29**:373–406.
37. Herrmann M, Kerner BS. Local cluster effect in different traffic flow models. *Physica A* 1998; **255**:163–188.

DOI: 10.1002/atr