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Decoherence in two Coupled Qubits

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Abstract

We have developed quantitative description of quantum coherent oscillations in the system of two coupled qubits in the presence of weak decoherence that in general can be correlated between the two qubits. It is shown that in the experimentally realized scheme of excitation of the oscillations, their waveform is not very sensitive to the magnitude of decoherence correlations. Modification of this scheme into potentially useful probe of the degree of decoherence correlations at the two qubits is suggested.

Key Words: decoherence, coupled qubits, density matrix

1. Introduction

Despite the large number of successful demonstrations of quantum coherent oscillations in individual [1, 2, 3, 4, 5, 6] and coupled [7, 8] Josephson-junction qubits, quantitative understanding of the details of these oscillations is so far quite limited. One of the most important open problems is decoherence, many aspects of which still remain to be understood. The purpose of this work is to develop theoretical description of decoherence in dynamics of a system of two coupled qubits. The approach we are using is based on the evolution equation for the density matrix in the Markovian approximation that is standard for description of weak decoherence. Although there are indications from the single-qubit experiments that more sophisticated approaches are needed for quantitatively accurate description of decoherence, the result we obtain within the simple scheme can be useful as the benchmark for more elaborate models.

Motivation for studying decoherence in coupled qubits is provided by the recent experiment [7] with two coupled charge qubits, where it was found that the decoherence rate for quantum coherent oscillations in two qubits at the optimal bias point is with good accuracy factor-of-4 larger than the decoherence rate in effectively decoupled qubits. An interesting question for theory is whether this factor-of-4 increase of the decoherence rate is a numerical coincidence, or it reflects some basic property of the decoherence mechanisms in charge qubits. As will become clear from the discussion below, the theory developed in this work favors "numerical coincidence" point of view. Other aspects of decoherence in coupled qubits has been studied before numerically in [9, 10, 11].

In general, it is well understood that decoherence rates of different states of two coupled qubits can be quite different if the random forces created by the qubit environments responsible for decoherence are completely or partially correlated at the two qubits. Most importantly, in the case of complete correlation, the qubit system should have a "decoherence-free subspace" (DFS) spanned by the states $|01\rangle$, $|10\rangle$ [12, 13, 14], since completely correlated external environments can not distinguish these states. In contrast, the subspace spanned by $|00\rangle$ and $|11\rangle$ experiences decoherence that is made stronger by the correlations between environmental forces acting on the two qubits. So the role of the quantitative theory in description of decoherence in the dynamics of coupled qubits is to see to what extent subspaces with different decoherence rates participate in the qubit oscillations for different methods of their excitation.

2. The model and Environmental Correlations

The Hamiltonian of the system of two qubits coupled directly by interaction between the basis-forming degrees of freedom (i.e., electrostatic interaction through finite coupling capacitance for charge qubits, or magnetic interaction for flux qubits) is:

$$H_0 = \sum_{j=1,2} (\varepsilon_j \sigma_z^{(j)} + \Delta_j \sigma_+^{(j)} + \Delta_j^* \sigma_-^{(j)}) + \nu \sigma_z^{(1)} \sigma_z^{(2)} \,, \tag{1}$$

where σ's denote the Pauli matrices, ν is the qubit interaction energy, Δ_j is the tunnel amplitude and ε_j is the bias of the jth qubit. Four energy levels of the Hamiltonian (1) are shown schematically in Fig. 1 as functions of the common bias $\bar{\varepsilon} \equiv \varepsilon_1 = \varepsilon_2$ of the two qubits. In this work, we will consider quantum coherent oscillations in the qubits biased at the "co-resonance" point [7], where $\varepsilon_1 = \varepsilon_2 = 0$. Such bias conditions are optimal for the oscillations.

It can be shown explicitly that the occupation probabilities of the qubit basis states (that are of interest for us) are insensitive to the phases of the qubit tunnel amplitudes Δ_j, so without the loss of generality we will assume that Δ_j's are real. The Hamiltonian (1) at the co-resonance reduces then to

$$H_0 = \sum_{j=1,2} \Delta_j \sigma_x^{(j)} + \nu \sigma_z^{(1)} \sigma_z^{(2)} \,. \tag{2}$$

In the basis composed of eigenstates of the $\sigma_x^{(j)}$ operators, the Hamiltonian (2) can be diagonalized easily. Eigenenergies and eigenstates are:

$$\begin{aligned}
E_1 = \Omega, \quad & |\psi_1\rangle = \frac{1}{2}[(\gamma+\eta)(|00\rangle + |11\rangle) + (\gamma-\eta)(|01\rangle + |10\rangle)], \\
E_2 = -\Omega, \quad & |\psi_2\rangle = \frac{1}{2}[(\eta-\gamma)(|00\rangle + |11\rangle) + (\gamma+\eta)(|01\rangle + |10\rangle)], \\
E_3 = \epsilon, \quad & |\psi_3\rangle = \frac{1}{2}[(\alpha+\beta)(|00\rangle - |11\rangle) + (\alpha-\beta)(|10\rangle - |01\rangle)], \\
E_4 = -\epsilon, \quad & |\psi_4\rangle = \frac{1}{2}[(\beta-\alpha)(|00\rangle - |11\rangle) + (\alpha+\beta)(|10\rangle - |01\rangle)],
\end{aligned} \tag{3}$$

where

$$\Omega = (\Delta^2 + \nu^2)^{1/2} \,, \quad \epsilon = (\delta^2 + \nu^2)^{1/2} \,, \quad \alpha, \beta = \frac{1}{\sqrt{2}}(1 \pm \frac{\delta}{\epsilon})^{1/2} \,, \quad \eta, \gamma = \frac{1}{\sqrt{2}}(1 \pm \frac{\Delta}{\Omega})^{1/2} \,,$$

and $\Delta \equiv \Delta_1 + \Delta_2$, $\delta \equiv \Delta_1 - \Delta_2$. The states $|kl\rangle$ with $\{k, l\} = \{0, 1\}$ in Eqs. (3) are the eigenstates of the operators $\sigma_z^{(1,2)}$ in the natural notations: $\sigma_z^{(1)}|kl\rangle = (-1)^k |kl\rangle$ and $\sigma_z^{(2)}|kl\rangle = (-1)^l |kl\rangle$.

We assume that external environments responsible for the decoherence couple to the basis-forming degrees of freedom of the qubits. The interaction Hamiltonian is then:

$$H_i = \sum_{j=1,2} \xi_j(t) \sigma_z^{(j)} \,. \tag{4}$$

The random forces $\xi_{1,2}(t)$ acting on the qubits are in general correlated. To describe the weakly dissipative dynamics of the system in the basis of states (3) induced by the interaction (4) with the reservoirs, we use the standard equation for the evolution of the qubit density matrix ρ in the interaction representation (see, e.g., [15]):

$$\dot{\rho} = -\int_{-\infty}^{t} d\tau \langle [H_i(t), [H_i(\tau), \rho]] \rangle \,, \tag{5}$$

where angled brackets denote averaging over the reservoirs. Proceeding in the standard way, we keep in Eqn. (5) only terms that do not oscillate as functions of time with large frequencies on the order of eigenenergies of the system, and therefore lead to changes in ρ that accumulate over time. Equations for the matrix

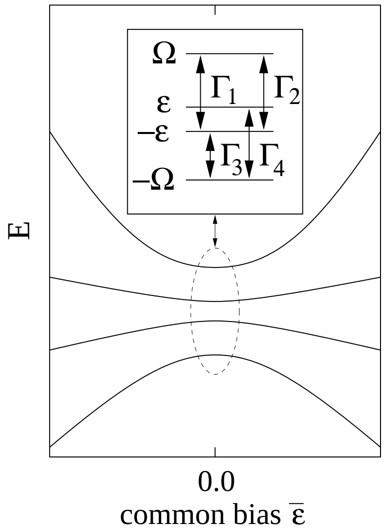

Figure 1. Schematic structure of the energy levels of the two coupled qubits as functions of the common bias of the qubits. The inset shows the diagram of the decoherence-induced transitions between the levels at "co-resonance" point where the bias vanishes.

elements ρ_{nm}, $n, m = 1, ..., 4$, of ρ in the basis (3) are transformed then as follows:

$$\begin{aligned}\dot{\rho}_{nm} \;=\; \sum_{j,j'=1,2} \big[& -\rho_{nm}(\sigma^{(j)}_{mm} - \sigma^{(j)}_{nn})(\tilde{F}^*_{jj'}(0)\sigma^{(j')}_{mm} - \tilde{F}_{jj'}(0)\sigma^{(j')}_{nn}) \\ & -\rho_{nm}\sum_k (\sigma^{(j)}_{nk}\sigma^{(j')}_{kn}\tilde{F}_{jj'}(\epsilon_n-\epsilon_k) + \sigma^{(j')}_{mk}\sigma^{(j)}_{km}\tilde{F}^*_{jj'}(\epsilon_m-\epsilon_k)) \\ & +\delta_{nm}\sum_k \rho_{kk}(\sigma^{(j')}_{nk}\sigma^{(j)}_{kn}\tilde{F}_{jj'}(\epsilon_k-\epsilon_n) + \sigma^{(j)}_{nk}\sigma^{(j')}_{kn}\tilde{F}^*_{jj'}(\epsilon_k-\epsilon_n)) \\ & +\sum_{(k,l)} \rho_{kl}\sigma^{(j')}_{nk}\sigma^{(j)}_{lm}(\tilde{F}_{jj'}(\epsilon_l-\epsilon_m) + \tilde{F}^*_{j'j}(\epsilon_l-\epsilon_m))\big]. \end{aligned} \tag{6}$$

Here $\sigma^{(j)}_{nm}$ denote the matrix elements $\langle n|\sigma^{(j)}_z|m\rangle$, the last sum is taken over the pairs (k, l) of states that satisfy the "resonance" condition:

$$\epsilon_k - \epsilon_l = \epsilon_n - \epsilon_m\,, \quad (k,l) \neq (n,m)\,,$$

and

$$\tilde{F}_{jj'}(\omega) = \int_0^\infty dt \langle \xi_j(t)\xi_{j'}(0)\rangle e^{i\omega t}\,.$$

The first term in Eqn. (6) represents "pure dephasing" that exists when the system operators that couple it to the environment have non-vanishing average values in the eigenstates. As one can see explicitly from Eqs. (3), the average values $\sigma^{(j)}_{nn}$ of $\sigma^{(j)}_z$ are vanishing in all states, so that there is no pure dephasing term in the evolution of the density matrix in our case. The fact that $\sigma^{(j)}_{nn}$ are vanishing can be related to the vanishing slope of the system energies with respect to variations of the bias in the vicinity of the co-resonance point – see Fig. 1. Since all coefficients in the eigenfunctions (3) are real, the matrix elements $\sigma^{(j)}_{nm}$ are also real. For real $\sigma^{(j)}_{nm}$, imaginary parts of the noise correlators $\tilde{F}_{j'j}$ in the second term on the right-hand-side of Eqn. (6) represent the decoherence-induced shifts of the system energy levels. These shifts do not affect decoherence and we will neglect them in our discussion. With these simplifications, Eqn. (6) takes the form

$$\dot{\rho}_{nm} \;=\; \sum_{j,j'=1,2} \big[-(\rho_{nm}/2)\sum_k (\sigma^{(j)}_{nk}\sigma^{(j')}_{kn}\mathrm{Re}F_{jj'}(\epsilon_n-\epsilon_k) + \sigma^{(j')}_{mk}\sigma^{(j)}_{km}\mathrm{Re}F_{jj'}(\epsilon_m-\epsilon_k))$$

$$+\delta_{nm}\sum_k \rho_{kk}\sigma_{nk}^{(j')}\sigma_{kn}^{(j)}\mathrm{Re}F_{jj'}(\epsilon_k-\epsilon_n)+\sum_{(k,l)}\rho_{kl}\sigma_{nk}^{(j')}\sigma_{lm}^{(j)}F_{jj'}(\epsilon_l-\epsilon_m)\big], \tag{7}$$

where

$$F_{jj'}(\omega)=\int_{-\infty}^{\infty} dt\langle \xi_j(t)\xi_{j'}(0)\rangle e^{i\omega t}\,. \tag{8}$$

The function F_{12} characterizes correlations between the environmental forces acting on the two qubits. For instance, if the two qubits interact with different environments and ξ_1, ξ_2 are uncorrelated, $F_{12}=0$, whereas $F_{12}=F_{11}=F_{22}$, if the qubits are acted upon by the force produced by one environment coupled equally to the two qubits. While the correlators F_{11} and F_{22} are necessarily real, F_{12} can be imaginary, and $F_{21}^{*}=F_{12}$. Non-vanishing imaginary part of F_{12} corresponds to the non-vanishing commutator $[\xi_1,\xi_2]$ and implies that the two qubits are coupled to the two non-commuting variables of the same reservoir. While this is probably not very likely for qubits with the basis-forming degrees of freedom of the same nature (which in a typical situation should be coupled to the same set of environmental degrees of freedom), the non-vanishing $\mathrm{Im}F_{12}$ should be typical if the qubits have different basis-forming variables. Using the spectral decomposition of the correlators $F_{jj'}(\omega)$ and Swartz inequality, one can prove (similarly to what is done in a different context of linear quantum measurements [16]) that for arbitrary stationary reservoirs the correlators satisfy the inequality that imposes the constraint on $F_{12}(\omega)$:

$$F_{11}(\omega)F_{22}(\omega)\geq |F_{12}(\omega)|^2\,. \tag{9}$$

If the reservoirs are in equilibrium at temperature T, the correlators satisfy also the standard detailed balance relations:

$$F_{jj'}(-\omega)=e^{-\frac{\omega}{T}}F_{j'j}(\omega)\,. \tag{10}$$

Equation (7) with the noise correlators (8) govern weakly dissipative time evolution of the two coupled qubits in a generic situation. Below we use them to determine decoherence properties of quantum coherent oscillations of the qubits. Before doing this, however, we would like to briefly discuss applicability of our approach to realistic Josephson-junction qubits. As we saw above, one of the main features of Eqn. (7) is that the pure dephasing terms disappear at the co-resonance point and remaining decoherence is related to transitions between the energy eigenstates. This implies that within the approach based on Eqn. (7), the decoherence rates are on the order of half of the transition rates, whereas experiments with charge qubits (see, e.g., [4]) indicate that decoherence rates are larger than the transition rates even at the optimum bias point when the pure-dephasing terms should disappear. Apparently, this is related to the low-frequency charge noise [4, 17] that is coupled to qubit strongly enough for the lowest-order perturbation theory in coupling (5) to be insufficient. This implies that the theory presented in this work might be only qualitatively correct for realistic charge qubits, for which one should develop more accurate non-perturbative description of the low-frequency noise to achieve quantitative agreement with experiments. Our simple perturbative approach, however, should be applicable to flux qubits, where the low-frequency noise should not be as strong as in the charge qubits.

3. Quantum Coherent Oscillations in Coupled Qubits

One of the most direct ways of excitation of quantum coherent oscillations in individual or coupled qubits that will be discussed in this work is based on the abrupt variation of the bias conditions [1, 7]. If the qubits are initially localized in one of their basis states, e.g. $|00\rangle$, and abrupt variation of the bias brings them to the co-resonance point, the probabilities for the qubits to be in other basis states start oscillating with time. In the simplest detection scheme (realized, for instance, in experiment [7]) the probability for each qubit to be in the state $|1\rangle$ is measured independently of the state of the other qubit. Quantitatively, these probabilities p_j are obtained from the projection operators P_j:

$$p_j=\mathrm{Tr}\{\rho P_j\}\,,\quad P_1=\sum_{k=1,2}|1k\rangle\langle k1|\,,\quad P_2=\sum_{k=1,2}|k1\rangle\langle 1k|\,.$$

Finding explicitly the matrix elements of P_j from the wavefunctions (3), one gets:

$$p_1(t) = \frac{1}{2} + (\alpha\eta + \beta\gamma)\mathrm{Re}[e^{-i\omega_- t}(\rho_{42}(t) - \rho_{13}(t))] + (\alpha\gamma - \beta\eta)\mathrm{Re}[e^{-i\omega_+ t}(\rho_{14}(t) + \rho_{32}(t))] , \qquad (11)$$

$$p_2(t) = \frac{1}{2} - (\alpha\gamma + \beta\eta)\mathrm{Re}[e^{-i\omega_- t}(\rho_{13}(t) + \rho_{42}(t))] + (\alpha\eta - \beta\gamma)\mathrm{Re}[e^{-i\omega_+ t}(\rho_{14}(t) - \rho_{32}(t))] , \qquad (12)$$

where $\omega_\pm \equiv \Omega \pm \epsilon$, and as in the Eqn. (7), the matrix elements of the density matrix are taken in the interaction representation. Equations (12) and (7) show that the waveform of the coherent oscillations in coupled qubits is determined by the time evolution of the two pairs of the matrix elements of ρ:

$$\dot\rho_{13} = -\Gamma_{13}\rho_{13} + u_-\rho_{42} , \quad \dot\rho_{42} = -\Gamma_{42}\rho_{42} + u_+\rho_{13} , \qquad (13)$$

$$\dot\rho_{14} = -\Gamma_{14}\rho_{14} + v_-\rho_{32} , \quad \dot\rho_{32} = -\Gamma_{32}\rho_{32} + v_+\rho_{14} . \qquad (14)$$

The decoherence rates in these equations are determined by the rates of transitions between different energy eigenstates:

$$\Gamma_{13} = \frac{1}{2}\big(\Gamma_1^{(+)} + \Gamma_2^{(-)} + \Gamma_2^{(+)} + \Gamma_4^{(+)}\big) , \quad \Gamma_{14} = \frac{1}{2}\big(\Gamma_1^{(-)} + \Gamma_1^{(+)} + \Gamma_2^{(+)} + \Gamma_3^{(+)}\big) ,$$
$$\Gamma_{32} = \frac{1}{2}\big(\Gamma_2^{(-)} + \Gamma_3^{(-)} + \Gamma_4^{(-)} + \Gamma_4^{(+)}\big) , \quad \Gamma_{42} = \frac{1}{2}\big(\Gamma_1^{(-)} + \Gamma_3^{(-)} + \Gamma_3^{(+)} + \Gamma_4^{(-)}\big) . \qquad (15)$$

where labeling of the transitions is indicated in the inset in Fig. 1. Transition rates are:

$$\Gamma_1^{(\pm)} = \mathrm{Re}\sum_{j,j'} F_{jj'}(\pm\omega_+)\sigma_{14}^{(j)}\sigma_{41}^{(j')} , \quad \Gamma_2^{(\pm)} = \mathrm{Re}\sum_{j,j'} F_{jj'}(\pm\omega_-)\sigma_{13}^{(j)}\sigma_{31}^{(j')} ,$$
$$\Gamma_3^{(\pm)} = \mathrm{Re}\sum_{j,j'} F_{jj'}(\pm\omega_-)\sigma_{24}^{(j)}\sigma_{42}^{(j')} , \quad \Gamma_4^{(\pm)} = \mathrm{Re}\sum_{j,j'} F_{jj'}(\pm\omega_+)\sigma_{23}^{(j)}\sigma_{32}^{(j')} . \qquad (16)$$

The superscripts $\pm$ refer here to transitions in the direction of decreasing (+) or increasing (-) energy. Finding matrix elements σ_{nm} from the wavefunctions (3) we see explicitly that transitions between the states 1 and 2, as well as 3 and 4 are suppressed, since the corresponding matrix elements are zero, and that the rates (16) are:

$$\Gamma_1 = \frac{1}{2}F_{11}(1 - \frac{\delta\Delta + \nu^2}{\epsilon\Omega}) + \frac{1}{2}F_{22}(1 + \frac{\delta\Delta - \nu^2}{\epsilon\Omega}) + \mathrm{Re}F_{12}(\frac{\nu}{\Omega} - \frac{\nu}{\epsilon}) ,$$
$$\Gamma_2 = \frac{1}{2}F_{11}(1 + \frac{\delta\Delta + \nu^2}{\epsilon\Omega}) + \frac{1}{2}F_{22}(1 - \frac{\delta\Delta - \nu^2}{\epsilon\Omega}) + \mathrm{Re}F_{12}(\frac{\nu}{\Omega} + \frac{\nu}{\epsilon}) ,$$
$$\Gamma_3 = \frac{1}{2}F_{11}(1 + \frac{\delta\Delta + \nu^2}{\epsilon\Omega}) + \frac{1}{2}F_{22}(1 - \frac{\delta\Delta - \nu^2}{\epsilon\Omega}) - \mathrm{Re}F_{12}(\frac{\nu}{\Omega} + \frac{\nu}{\epsilon}) , \qquad (17)$$
$$\Gamma_4 = \frac{1}{2}F_{11}(1 - \frac{\delta\Delta + \nu^2}{\epsilon\Omega}) + \frac{1}{2}F_{22}(1 + \frac{\delta\Delta - \nu^2}{\epsilon\Omega}) - \mathrm{Re}F_{12}(\frac{\nu}{\Omega} - \frac{\nu}{\epsilon}) .$$

The transfer "rates" u, v in Eqs. (13) and (14) are:

$$u_\pm = \sum_{j,j'=1,2} \sigma_{14}^{(j')}\sigma_{32}^{(j)}F_{jj'}(\pm\omega_+) , \quad v_\pm = \sum_{j,j'=1,2} \sigma_{13}^{(j')}\sigma_{42}^{(j)}F_{jj'}(\pm\omega_-) . \qquad (18)$$

Explicitly:

$$u = \frac{1}{2}F_{11}(1 - \frac{\delta\Delta + \nu^2}{\epsilon\Omega}) - \frac{1}{2}F_{22}(1 + \frac{\delta\Delta - \nu^2}{\epsilon\Omega}) + i\mathrm{Im}F_{12}(\frac{\nu}{\Omega} - \frac{\nu}{\epsilon}) ,$$
$$v = -\frac{1}{2}F_{11}(1 + \frac{\delta\Delta + \nu^2}{\epsilon\Omega}) + \frac{1}{2}F_{22}(1 - \frac{\delta\Delta - \nu^2}{\epsilon\Omega}) - i\mathrm{Im}F_{12}(\frac{\nu}{\Omega} + \frac{\nu}{\epsilon}) . \qquad (19)$$

Equations (17) and (19) do not show the frequency dependence of noise correlators, which is the same, respectively, as in the Eqs. (16) and (18).

Each pair, (13) and (14), of coupled equations can be solved directly by diagonalization of the matrix of the evolution coefficients with a non-orthogonal transformation. In this way we obtain for the pair of equations (13):

$$\rho_{13}(t) = \frac{1}{u_+u_- + c^2}\left[\rho_{13}(0)(u_+u_- e^{-\gamma_+ t} + c^2 e^{-\gamma_- t}) + cu_-\rho_{42}(0)(e^{-\gamma_+ t} - e^{-\gamma_- t})\right],$$
$$\rho_{42}(t) = \frac{1}{u_+u_- + c^2}\left[\rho_{42}(0)(u_+u_- e^{-\gamma_- t} + c^2 e^{-\gamma_+ t}) + cu_+\rho_{13}(0)(e^{-\gamma_+ t} - e^{-\gamma_- t})\right]. \tag{20}$$

where

$$\gamma_\pm \equiv (\Gamma_{13}+\Gamma_{42})/2 \pm \left[(\Gamma_{13}-\Gamma_{42})^2/4 + u_+u_-\right]^{1/2}, \quad c \equiv (\Gamma_{13}-\Gamma_{42})/2 - \left[(\Gamma_{13}-\Gamma_{42})^2/4 + u_+u_-\right]^{1/2}, \tag{21}$$

and $\rho_{13}(0)$, $\rho_{42}(0)$ are the initial values of the density matrix elements that depend on preparation of the initial state. If, as in the experiment [7], the qubits are abruptly driven to co-resonance maintaining the state $|00\rangle$, these initial values are:

$$\rho_{13}(0) = \frac{1}{4}(\gamma+\eta)(\alpha+\beta), \quad \rho_{32}(0) = \frac{1}{4}(\gamma-\eta)(\alpha+\beta),$$
$$\rho_{14}(0) = \frac{1}{4}(\gamma+\eta)(\beta-\alpha), \quad \rho_{42}(0) = \frac{1}{4}(\gamma-\eta)(\alpha-\beta). \tag{22}$$

Another type of initial conditions that will be discussed in this work is starting the oscillations from the state $|10\rangle$. In this case:

$$\rho_{13}(0) = \frac{1}{4}(\gamma-\eta)(\alpha-\beta), \quad \rho_{32}(0) = \frac{1}{4}(\gamma+\eta)(\alpha-\beta),$$
$$\rho_{14}(0) = \frac{1}{4}(\gamma-\eta)(\alpha+\beta), \quad \rho_{42}(0) = \frac{1}{4}(\gamma+\eta)(\alpha+\beta). \tag{23}$$

Equations (22) and (23) follow directly from the wavefunctions (3): $\rho_{nm}(0) = \langle n|i\rangle\langle i|m\rangle$, where $|i\rangle$ is the initial state.

Solution of the other pair (14) of coupled equation is given by the same Eqs. (20) and (21) with obvious substitutions: $u_\pm \to v_\pm$, $\Gamma_{13} \to \Gamma_{14}$, $\Gamma_{42} \to \Gamma_{32}$. In this work, we are mostly interested in the low-temperature regime $T \ll \epsilon, \Omega$, when transitions up in energy can be neglected. In this regime, $u_-, v_- \to 0$, and equations for the evolution of the density matrix elements are simplified. For instance, for $u_- \to 0$, $c \simeq u_+u_-/(\Gamma_{13}-\Gamma_{42})$, and Eqs. (20) are reduced to:

$$\rho_{13}(t) = \rho_{13}(0)e^{-\Gamma_{13}t}, \quad \rho_{42}(t) = \rho_{42}(0)e^{-\Gamma_{42}t} + \frac{u_+}{\Gamma_{13}-\Gamma_{42}}\rho_{13}(0)(e^{-\Gamma_{13}t} - e^{-\Gamma_{42}t}), \tag{24}$$

where now $\Gamma_{13} = (\Gamma_1+\Gamma_2+\Gamma_4)/2$ and $\Gamma_{42} = \Gamma_3/2$.

Time evolution (24) of the density matrix elements together with the rates (17) and (19), and initial conditions (22) and (23) determines the shape of the coherent oscillations in two coupled qubits. In the next Section, we discuss this shape in several specific situations.

4. Results and Conclusions

The shape of coherent oscillations determined in the previous Section depends on the large number of parameters: temperature, degree of asymmetry of qubit tunnel energies and couplings to the environments, frequency dependence of the decoherence, and strength and nature of the correlations between the two reservoirs. We analyze some of these dependencies below.

4.1. Experimentally-motivated case

We start by considering the situation that is close to the experimentally realized case of oscillations in coupled charge qubits [7]. As we argued above, the correlations between environments in this case should be real: $\mathrm{Im}F_{12} = 0$. The oscillations are excited by driving the system to co-resonance in the initial state $|00\rangle$. Equations of the previous Section give in this case the following expression for the shape of the oscillations:

$$p_1(t) = \frac{1}{2} - \frac{1}{8}\big[Ae^{-\Gamma_{42}t} + B(e^{-\Gamma_{13}t} - \frac{u_+}{\Gamma_{13}-\Gamma_{42}}(e^{-\Gamma_{13}t} - e^{-\Gamma_{42}t}))\big]\cos\omega_- t$$
$$-\frac{1}{8}\big[Ce^{-\Gamma_{32}t} + D(e^{-\Gamma_{14}t} + \frac{v_+}{\Gamma_{14}-\Gamma_{32}}(e^{-\Gamma_{14}t} - e^{-\Gamma_{32}t}))\big]\cos\omega_+ t\,, \qquad (25)$$

where

$$A = 1+\frac{\delta\Delta+\nu^2}{\epsilon\Omega} - \frac{\nu}{\Omega} - \frac{\nu}{\epsilon}\,, \quad B = 1+\frac{\delta\Delta+\nu^2}{\epsilon\Omega} + \frac{\nu}{\Omega} + \frac{\nu}{\epsilon}\,, \quad C = 1-\frac{\delta\Delta+\nu^2}{\epsilon\Omega} - \frac{\nu}{\Omega} + \frac{\nu}{\epsilon}\,, \quad D = 1-\frac{\delta\Delta+\nu^2}{\epsilon\Omega} + \frac{\nu}{\Omega} - \frac{\nu}{\epsilon}\,.$$

Equation for $p_2(t)$ is the same with signs in front of δ, u_+ and v_+ reversed. As a simplifying assumption we take $F_{11} = F_{22} \equiv F$. The decoherence rates in Eqn. (25) then are:

$$\Gamma_{13} = F(\omega_+)(1-\frac{\nu^2}{\epsilon\Omega}) + \frac{1}{2}F(\omega_-)(1+\frac{\nu^2}{\epsilon\Omega}) + \frac{1}{2}F_c(\omega_-)(\frac{\nu}{\Omega}+\frac{\nu}{\epsilon})\,, \quad u_+ = -F(\omega_+)\frac{\delta\Delta}{\epsilon\Omega}\,,$$
$$\Gamma_{14} = F(\omega_-)(1+\frac{\nu^2}{\epsilon\Omega}) + \frac{1}{2}F(\omega_+)(1-\frac{\nu^2}{\epsilon\Omega}) + \frac{1}{2}F_c(\omega_+)(\frac{\nu}{\Omega}-\frac{\nu}{\epsilon})\,, \quad v_+ = -F(\omega_-)\frac{\delta\Delta}{\epsilon\Omega}\,, \qquad (26)$$
$$\Gamma_{32} = \frac{1}{2}F(\omega_+)(1-\frac{\nu^2}{\epsilon\Omega}) + \frac{1}{2}F_c(\omega_+)(\frac{\nu}{\epsilon}-\frac{\nu}{\Omega})\,, \quad \Gamma_{42} = \frac{1}{2}F(\omega_-)(1+\frac{\nu^2}{\epsilon\Omega}) - \frac{1}{2}F_c(\omega_-)(\frac{\nu}{\epsilon}+\frac{\nu}{\Omega})\,,$$

where $F_c(\omega) \equiv \mathrm{Re}F_{12}(\omega)$. To enable comparison of these rates to those of individual qubits, we note that the rate of decoherence of oscillations in a single qubit with vanishing bias is equal to $F(\Delta_j)/2$ for the jth qubit.

The functions $p_{1,2}(t)$ for the qubit parameters, δ, ν, and F, close to those in experiment [7] are plotted in Fig. 2 under additional assumption that the decoherence is the same at two frequencies, ω_+ and ω_-. (The decoherence strength F was taken from the data for the single-qubit regime in [7].) The curves are plotted for the two situations, when decoherence is completely uncorrelated ($F_{12} = 0$) and completely correlated ($F_{12} = F$) between the two qubits. One can see that the difference between the two regimes is not very big numerically, with correlations between the two reservoirs leading to the effective decoherence rate that is increased in comparison with the uncorrelated regime by roughly $30 \div 50\%$, although the description with a single decoherence rate is not quite appropriate quantitatively – see Eqs. (25) and (26).

The increase of the effective decoherence rate by correlations illustrated in Fig. 2 can be related to the fact that the initial qubit state, $|00\rangle$, belongs to the subspace where the correlations increase the decoherence rate, despite the mixing of this subspace with the DFS where the decoherence rate is decreased in the eigenstates (3) of the coupled qubit system. (Here and below we use the term "DFS" for the subspace spanned by the $|01\rangle$ and $|10\rangle$ states, although for interacting qubits it, strictly speaking, does not fully have the properties of real DFS.) This implies that increase of decoherence rate by correlations should be to a large extent insensitive to qubit parameters. This conclusion is supported by the case of identical qubits ($\delta = 0$), when $u_+ = v_+ = 0$ and Eq. (25) is reduced to a very simple form:

$$p_1(t) = \frac{1}{2} - \frac{1}{4}\big[(1+\frac{\nu}{\Omega})e^{-\Gamma_{13}t}\cos\omega_- t + (1-\frac{\nu}{\Omega})e^{-\Gamma_{32}t}\cos\omega_+ t\big]\,, \qquad (27)$$

and $p_2(t) = p_1(t)$. One can see from Eqs. (26) that both decoherence rates relevant for Eq. (27), Γ_{13} and Γ_{32} increase with increasing correlation strength F_c. Equation (27) shows also that the description of the oscillation decay with a single decoherence rate can be quite inaccurate: for weak interaction, $\nu < \Omega$ the amplitudes of the two (high- and low-frequency) components of the oscillations are nearly the same while their decoherence rates can be very different.

4.2. Excitation into the DFS

Now we discuss decoherence properties of the oscillations in coupled qubits in the case when they start with the initial qubit state $|10\rangle$. We note that in the case of experiment similar to [7], such an initial

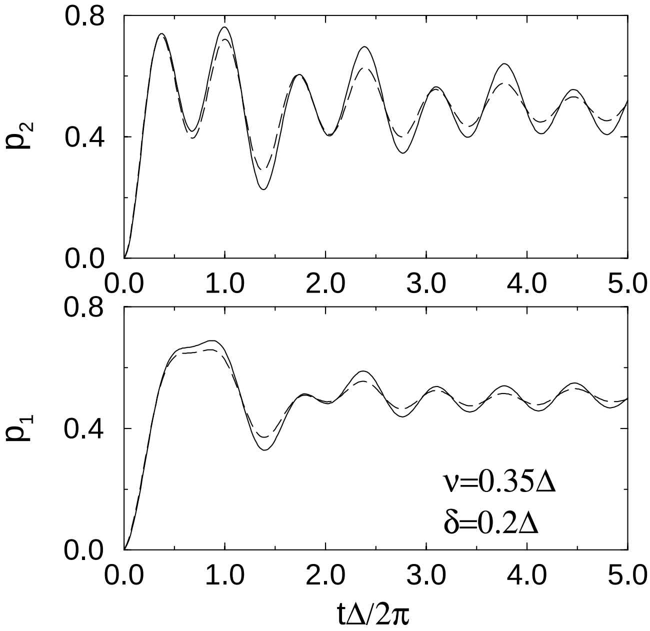

Figure 2. Probabilities p_j to find jth qubit in the state $|1\rangle$ in the process of quantum coherent oscillations starting with the state $|00\rangle$ of two coupled qubits. The decoherence strength is $F = 0.08\Delta$. Solid and dashed lines correspond, respectively, to the decoherence that is uncorrelated ($F_c = 0$) and completely correlated ($F_c = F$) between the two qubits.

condition would require separate gate control of the two qubits, since the bias change bringing them into co-resonance is different in this state for the two qubits. Since the state $|10\rangle$ belongs to the DFS in the case of completely correlated noise, one can expect that oscillations with these initial conditions will be more sensitive to the degree of inter-qubit decoherence correlations than oscillations with $|00\rangle$ initial condition, and that the effective decoherence rate will decrease with correlation strength. All this indeed can be seen from Eqs. (24) with the initial conditions (23) that correspond to the $|10\rangle$ state. Under the same assumptions as were used in Eqn. (25), we get for the now different $p_1(t)$ and $p_2(t)$:

$$p_j(t) = \frac{1}{2} - \frac{(-1)^j}{8}\Big\{\big[A_j e^{-\Gamma_{42}t} + B_j(e^{-\Gamma_{13}t} + \frac{(-1)^j u_+}{\Gamma_{13}-\Gamma_{42}}(e^{-\Gamma_{13}t} - e^{-\Gamma_{42}t}))\big]\cos\omega_- t$$
$$+\big[C_j e^{-\Gamma_{32}t} + D_j(e^{-\Gamma_{14}t} - \frac{(-1)^j v_+}{\Gamma_{14}-\Gamma_{32}}(e^{-\Gamma_{14}t} - e^{-\Gamma_{32}t}))\big]\cos\omega_+ t\Big\}\,, \quad j = 1, 2\,, \tag{28}$$

where

$$A_1 = 1 + \frac{\delta\Delta + \nu^2}{\epsilon\Omega} + \frac{\nu}{\Omega} + \frac{\nu}{\epsilon}\,, \quad B_1 = 1 + \frac{\delta\Delta + \nu^2}{\epsilon\Omega} - \frac{\nu}{\Omega} - \frac{\nu}{\epsilon}\,,$$
$$C_1 = 1 - \frac{\delta\Delta + \nu^2}{\epsilon\Omega} + \frac{\nu}{\Omega} - \frac{\nu}{\epsilon}\,, \quad D_1 = 1 - \frac{\delta\Delta + \nu^2}{\epsilon\Omega} - \frac{\nu}{\Omega} + \frac{\nu}{\epsilon}\,,$$

and the amplitudes A_2, B_2, C_2, D_2 are given by the same expressions with $\delta \to -\delta$.

For identical qubits Eqn. (28) reduces to:

$$p_j(t) = \frac{1}{2} - \frac{(-1)^j}{4}\big[(1 + \frac{\nu}{\Omega})e^{-\Gamma_{42}t}\cos\omega_- t + (1 - \frac{\nu}{\Omega})e^{-\Gamma_{14}t}\cos\omega_+ t\big]\,. \tag{29}$$

This expression and Eqs. (26) show that in contrast to Eqn. (27), the decoherence rate of the low-frequency component that has larger amplitude is strongly suppressed by the non-vanishing inter-qubit noise correlations F_c: $\Gamma_{42} = \frac{1}{2}(1 + \nu/\Omega)[F(\omega_-) - F_c(\omega_-)]$. This means that the shape of the coherent oscillations in coupled qubit starting with the state $|10\rangle$ should indeed be more sensitive to the strength of these correlations than the shape of the oscillations starting with the $|00\rangle$ state. As one can see from Fig. 3 which shows the shape (28) of the $|10\rangle$ oscillations for the same set of experimentally realized parameters as in Fig. 3, this conclusion also remains valid in the case of not fully symmetric qubits. Even in this case there is a pronounced weakly decaying component of the oscillations if the decoherence is completely correlated between the two qubits. For partial correlations, the effective decoherence rate is reduced.

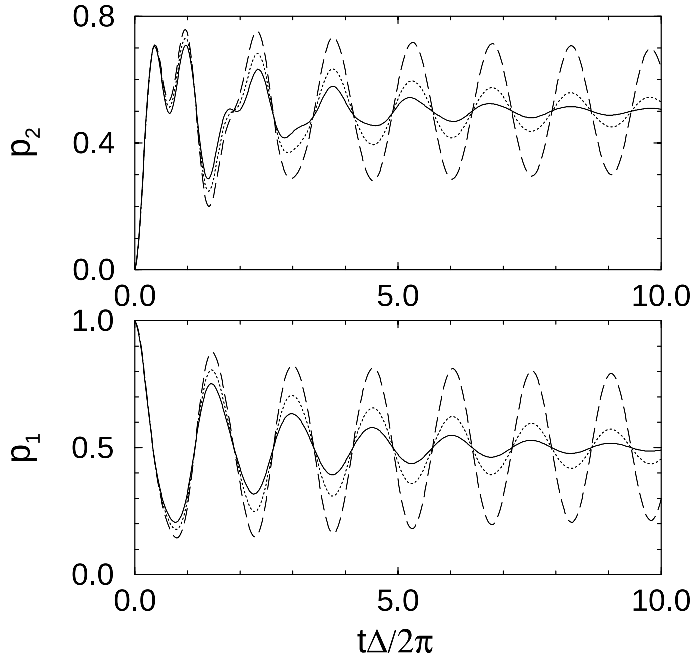

Figure 3. Probabilities p_j to find jth qubit in the state $|1\rangle$ in the process of quantum coherent oscillations starting with the state $|10\rangle$ of two coupled qubits. Qubit parameters are the same as in Fig. 2. Solid, dotted, and dashed lines correspond, respectively, to the decoherence that is uncorrelated ($F_c = 0$), partially ($F_c = 0.5F$), and completely ($F_c = F$) correlated between the two qubits.

In summary, we have developed quantitative description of weakly dissipative dynamics of two coupled qubits based on the standard Markovian evolution equation for the density matrix. This description shows that decoherence properties of currently realized oscillations in coupled qubits are not very sensitive to inter-qubit correlations of decoherence, while relatively simple modification of the excitation scheme for the oscillations should make them sensitive to these correlations.

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References

[1] Y. Nakamura, Yu.A. Pashkin, and J.S. Tsai, *Nature,* **398**, (1999), 786.

[2] J.R. Friedman, V. Patel, W. Chen, S.K. Tolpygo, and J.E. Lukens, *Nature*, **406**, (2000), 43.

[3] C.H. van der Wal, A.C.J. ter Haar, F.K. Wilhelm, R.N. Schouten, C. Harmans, T.P. Orlando, S. Lloyd, and J.E. Mooij, *Science*, **290**, (2000), 773; I. Chiorescu, Y. Nakamura, C.J.P.M. Harmans, and J.E. Mooij, *Science*, **299**, (2003), 1869.

[4] D. Vion, A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve, and M.H. Devoret, *Science*, **296**, (2002), 886.

[5] Y. Yu, S.Y. Han, X. Chu, S.I. Chu, and Z. Wang, *Science,* **296**, (2002), 889.

[6] J.M. Martinis, S. Nam, J. Aumentado, and C. Urbina, *Phys. Rev. Lett.,* **89**, (2002), 117901.

[7] Yu. A. Pashkin, T. Yamamoto, O. Astafiev, Y. Nakamura, D.V. Averin, and J.S. Tsai, *Nature*, **421**, (2003), 823.

[8] A.J. Berkley, H.Xu, R.C. Ramos, M.A. Gubrud, F.W. Strauch, P.R. Johnson, J.R. Anderson, A.J. Dragt, C.J. Lobb, and F.C. Wellstood, *Science,* **300**, (2003), 1548.

[9] M. Governale, M. Grifoni and G. Schön, *Chem. Phys.,* **268**, (2001) 273.

[10] M. Thorwart and P. Hänggi, *Phys. Rev. A,* **65**, (2002), 012309.

[11] M.J. Storcz and F.K. Wilhelm, *Phys. Rev. A,* **67**, (2003), 042319.

[12] L.-M. Duan and G.-C. Guo, *Phys. Rev. Lett.,* **79**, (1997), 1953.

[13] P. Zanardi and M. Rasetti, *Phys. Rev. Lett.,* **79**, (1997), 3306.

[14] D.A. Lidar, I.L. Chuang, K.B. Whaley, *Phys. Rev. Lett.,* **81**, (1998), 2594.

[15] K. Blum, *Density matrix theory and applications*, (Plenum, NY, 1981).

[16] D.V. Averin, in: "Quantum noise in mesoscopic physics", Ed. by Yu.V. Nazarov, (Kluwer, 2003), p. 229; cond-mat/0301524.

[17] Y. Nakamura, Yu.A. Pashkin, T. Yamamoto, and J.S. Tsai, *Phys. Rev. Lett.,* **88**, (2002), 047901.