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Full Length Research Paper

Effect of geometric parameters on free vibration analysis of thick fiber reinforced plastic (FRP) skew cross-ply laminate with circular cutout

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The present investigation deals with the free vibration analysis of a thick four-layered symmetric cross-ply skew laminated composite plate with a circular cutout and clamped boundary conditions. Three dimensional finite element models (FEMs) which use the elasticity theory for the determination of stiffness matrices are modeled in ANSYS software to evaluate first five natural frequencies of the laminate. The variations of the first five natural frequencies with respect to the thickness ratio for different d/l ratios and skew angles are presented. It is observed that, the natural frequency decreases with increase in length to thickness ratio(s).

Key words: Fiber reinforced plastic (FRP), finite element model (FEM), skew plate, cutout, natural frequencies.

INTRODUCTION

Composite materials are replacing metals in many structural applications such as aerospace, transportation, novel and pressure vessels due to their high specific strength and specific modulus. The dynamic responses of these structures differ from that made of isotropic materials due to orthotropic nature of individual layers of the laminate. Free vibration analysis is a part of dynamic analysis where the natural frequencies of the structure can be estimated. The following paragraph provides a brief review of various research contributions on free vibration analysis of Fiber reinforced plastic (FRP) composite structures.

The free vibrations of multi layered composite plates are studied by Noor (1973). An eight-noded hybrid-stress finite element is used by Ramakrishna et al. (1993) to analyse the dynamic behavior of laminated plates with cutouts. The three-dimensional theory of laminated plates and shells has been developed by Chao and Yeong-Chyuan (2000) for the free vibration frequencies of rectangular plates. Free vibration frequencies of symmetric and anti-symmetric laminated plates are studied by Kant and Swaminathan (2001), Panda and Singh (2007) and Viswanathan and Lee (2007) using higher order shear deformation theories. A higher order theory has been developed by Nayak et al. (2002) for the determination of the natural frequencies of isotropic, orthotropic and sandwich plates. A general analytical model applicable to the dynamic behavior of a thin-walled channel section composite is developed by Jaehong and Seung-Eock (2002). The layer-wise shear deformation theory has been extended by Shimpi and Ainapure (2002, 2004) for the free vibration of two-layered cross-ply laminated plates and beams. Shen et al. (2003) dealt with the vibration characteristics of the laminated plate. A combined theoretical and experimental investigation on free vibration of thin symmetrically laminated anisotropic plates is developed by Reaz et al. (2005). The effects of various parameters on the natural frequencies are presented by Chun-Sheng (2005) for the buckling and vibration behaviours of hybrid composite plates. Based on a higher order shear deformation theory, Ajay et al. (2006) presented the free vibration analysis of sandwich

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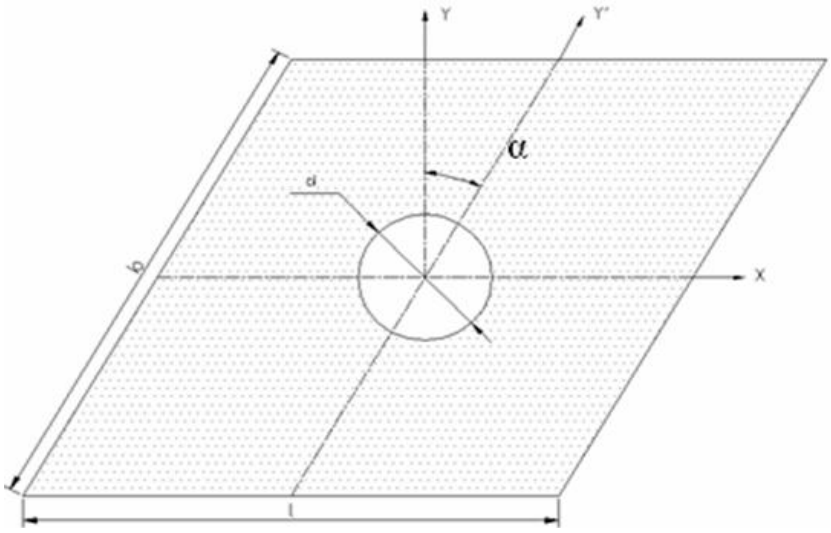


Figure 1. Skew plate with cutout.

skew laminates. The vibration analysis of angle-ply laminated beams subjected to different sets of boundary conditions is investigated by Metuin (2006), Bahar and Metin (2007), Li and Hua (2009), and Taehyo et al. (2006) studied linear static and dynamic analysis of laminated composite plates and shells using a 4-node quasi-conforming shell element.

The present investigation intends to apply three-dimensional finite element techniques for the free vibration analysis of clamped thick skew laminates with circular cutout. The lowest five natural frequencies are studied by varying the parameters of s, d/l ratios and skew angle.

PROBLEM STATEMENT

Geometric modeling

The geometry of the problem is shown in Figure 1. The sides of the plate 'l' and 'b' are taken equal to 2 m and the height of the plate is divided in to four layers of equal thickness (h/4), Where 'h' is the total thickness of the laminate. The skew angle (α) is varied from 0° to 50°. The circular hole is placed at the geometric centre of the plate. The diameter of the hole is varied as per the d/l ratio from 0.1 to 0.6, the length-to-thickness ratio is ranged from 5 to 80.

Finite element modeling

The element used for the present analysis is SOLID 95 of ANSYS, which is developed, based on three-dimensional elasticity theory and is defined by 20 nodes having three degrees of freedom at each node, translation in the node x, y and z directions. The model with finite element mesh is shown in Figure 2.

Boundary conditions

The sides of the skew laminate considered for the present analysis are clamped that is, all the three degrees of freedom of the nodes along the four sides of the skew plate are constrained.

Material properties

Each layer is unidirectional carbon fiber reinforced plastic possessing the following engineering constants as given in reference paper by Ramakrishna et al. (1993):

(i) Elastic modulus in the longitudinal direction of the fiber, E_L = 175 GPa,
(ii) Elastic modulus in the transverse direction of the fiber, E_T = 7 GPa,
(iii) Shear modulus in the longitudinal plane of the fiber, G_{LT} = 3.5 GPa,
(iv) Shear modulus in the transverse plane of the fiber, G_{TT} = 1.4 GPa,
(v) Poisons ratio, $v_{LT} = v_{TT}$ = 0.25,
(vi) Mass density, $\rho = 1.6 \times 10^3$ kg/m^3,
(vii) The stacking sequence of the laminate is taken as 0°/90°/90°/0°.

Validation of finite element model (FEM)

The 3-D FEM is validated with the results available in the literature and found good agreement (Table 1).

RESULTS AND DISCUSSION

Effect of thickness ratio(s) at different d/l

The variation of first five natural frequencies with respect to the thickness ratio(s) for different d/l ratio is shown in Tables 2 to 7. Frequency is found to be decreasing with increase in the thickness ratio and increasing with mode number for all d/l ratios. Increase in thickness ratio decreases both mass and stiffness of the structure since thickness is reduced to increase the value of (s).

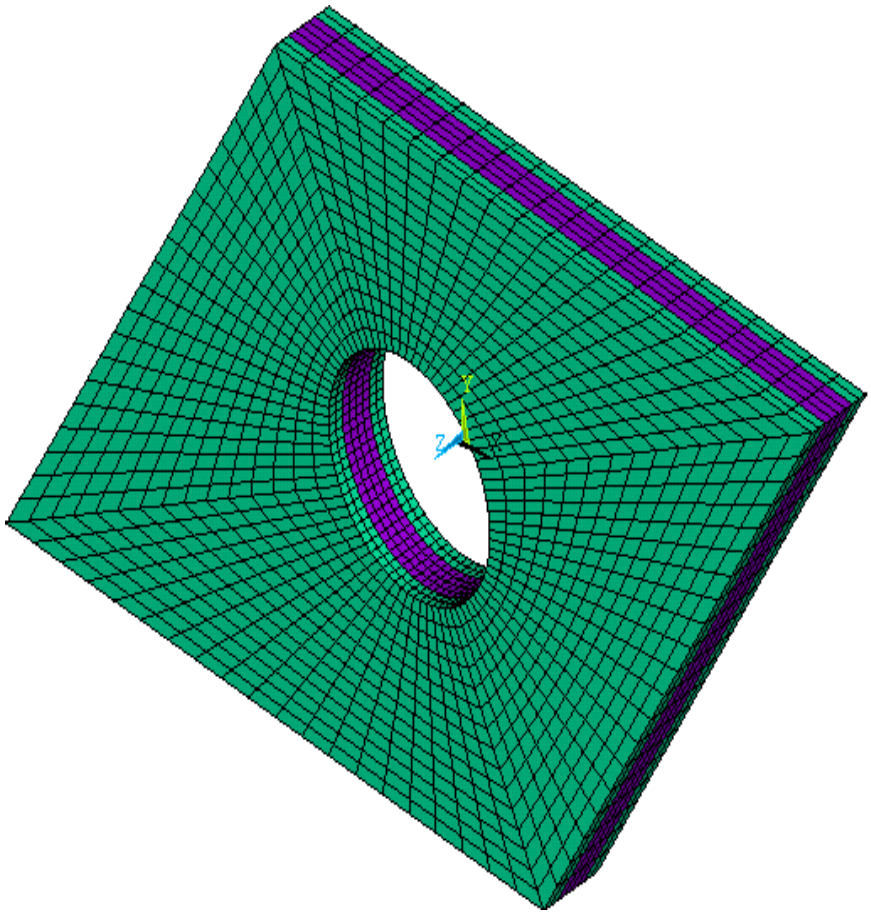


Figure 2. Finite Element mesh on the skew plate.

Table 1. Validation of the 3-D finite element results* (Ω_c,Ω_d of Rectangular laminated composite plate).

S_2	$[0/90\ldots.]^I_6$	$[45/-45]^{II}$
a/h	5	5
Present	3.4015	8.4325
Chao and Yeong-Chyuan (2000)	3.3495	8.2872
Noor (1973)	3.4250	-

*For composite i : E_1=10E_2, G_{12} = 0.6E_2, G_{23} = 0.5E_2, v_{12}= v_{23} = 0.25, For composite ii: E_1 = 15E_2, G_{12} = 0.429E_2, G_{23} = 0.343E_2, v_{12} = 0.4, v_{23} = 0.456. Where the normalized frequency, $\Omega_c = 10\omega h(\rho_m/E_2)^{1/2}$ and $\Omega_d = \omega a^2(\rho_m/E_2 h^2)^{1/2}$.

Reduction in frequency with increase in (s) for the present problem is due to the higher rate of decrease of stiffness than mass.

Effect of thickness ratio(s) at different α

The variation of first five natural frequencies with respect to the thickness ratio (s) for different values of (α) is shown in Tables 8 to 12. Frequency is found to be decreasing with increase in the thickness ratio and increasing with mode number for all values of (α). The reason for the variation of frequency is similar to that given previously.

Effect of d/l ratio and skew angle

The modal graph for the natural frequencies with respect to d/l ratio for Mode1 is plotted from the data provided in Tables 2 to 7. Frequency is found to be increasing with increase in the d/l ratio as shown in the Figure 3.

The modal graph for the natural frequencies with respect to (α) for Mode1 is plotted from the data provided in Tables 8 to 12. Frequency is found to be increasing

Table 2. Variation of natural frequency in Hz with respect to thickness ratio (d/l = 0.1, α = 30°, n = 4)

S	d/l = 0.1				
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
5	403.234	581.604	672.79	766.229	928.627
10	327.38	492.04	573.52	659.02	807.42
20	230.328	362.373	442.612	506.402	618.542
40	138.14	220.152	300.619	328.674	394.542
80	73.9727	118.031	174.136	182.471	219.373

Table 3. Variation of natural frequency in Hz with respect to thickness ratio (d/l = 0.2, α = 30°, n = 4).

S	d/l=0.2				
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
5	423.377	554.931	640.191	771.573	909.042
10	342.22	467.05	543.27	663.07	789.36
20	238.43	344.085	413.431	509.499	607.414
40	141.636	210.914	276.34	332.553	388.987
80	75.5267	113.972	159.901	185.748	215.994

Table 4. Variation of natural frequency in Hz with respect to thickness ratio (d/l = 0.3, α = 30°, n = 4).

S	d/l=0.3				
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
5	461.453	539.398	633.793	770.833	870.999
10	375.13	448.94	533.95	661.53	754.45
20	262.265	325.615	399.51	507.869	581.303
40	155.77	198.979	259.606	334.074	375.291
80	83.0032	107.981	147.584	189.161	208.89

Table 5. Variation of natural frequency in Hz with respect to thickness ratio (d/l = 0.4, α = 30°, n = 4).

S	d/l = 0.4				
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
5	517.153	551.091	676.108	777.503	850.316
10	426.43	457.58	571.57	666.02	732.43
20	302.445	328.239	428.334	510.086	560.558
40	181.102	198.632	276.703	336.431	363.203
80	96.8253	107.391	155.839	192.118	203.524

with increase in the d/l ratio as shown in the Figure 4. In these cases the increase in frequency may be attributed to reduction in mass of the structure. It states that, increase in skew angle reduces the mass of the plate that may cause increase in natural frequency.

Mode shapes

The mode shapes for first five natural frequencies are shown in Figures 5 to 9 for s = 10, d\l = 0.3 and α = 0°.

Conclusions

Free vibration analysis of a thick fiber reinforced plastic (FRP) skew cross-ply laminated composite plate with a circular cutout has been studied in the present work. The variations of the first five natural frequencies with respect to the length to thickness ratio for different d/l ratio and

Table 6. Variation of natural frequency in Hz with respect to thickness ratio (d/l = 0.5, α = 30°, n = 4).

S	d/l = 0.5				
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
5	583.404	590.617	782.474	842.399	885.893
10	489.04	494.97	672.91	728.11	765.43
20	353.226	357.599	514.126	561.46	589.096
40	214.018	216.759	336.275	367.879	390.766
80	115.2	116.928	189.863	207.97	223.562

Table 7. Variation of natural frequency in Hz with respect to thickness ratio (d/l = 0.6, α = 30°, n = 4).

S	d/l = 0.6				
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
5	650.608	650.881	971.048	994.988	1033.1
10	553.26	553.4	853.72	871.73	915.83
20	408.049	408.112	668.936	679.405	732.653
40	251.679	251.683	443.738	447.884	500.856
80	136.721	136.736	252.021	253.821	289.172

Table 8. Variation of natural frequency in Hz with respect to thickness ratio (α = 0°, d/l = 0.3, n = 4).

S	α = 0°				
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
5	393.301	508.889	515.475	695.055	776.843
10	318.325	416.83	435.499	589.392	662.418
20	225.206	292.158	338.316	451.754	491.955
40	134.918	172.917	227.786	301.458	302.328
80	71.997	92.111	131.481	162.06	173.766

Table 9. Variation of natural frequency in Hz with respect to thickness ratio (α = 10°, d/l = 0.3, n = 4).

S	A = 10°				
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
5	399.752	505.714	532.817	702.304	786.393
10	323.438	417.779	446.643	596.234	672.052
20	228.384	295.673	343.311	456.496	501.875
40	136.675	175.627	230.219	301.875	312.236
80	72.917	93.713	132.699	165.382	176.369

Table 10. Variation of natural frequency in Hz with respect to thickness ratio (α = 20°, d/l = 0.3, n = 4).

S	α=20°				
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
5	420.741	514.066	570.32	725.731	816.294
10	340.503	426.357	477.751	618.472	701.654
20	239.223	306.358	360.908	473.159	531.366
40	142.704	184.051	238.943	311.651	335.524
80	76.086	98.779	137.077	174.611	186.394

Table 11. Variation of natural frequency in Hz with respect to thickness ratio (α = 40°, d/l = 0.3, n = 4).

S	α = 40°				
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
5	531.679	588.8	741.145	850.176	961.189
10	438.378	494.025	632.365	737.408	840.102
20	307.627	359.822	475.685	571.877	657.509
40	182.42	222.791	305.616	377.551	435.955
80	96.2628	122.53	172.057	214.623	247.114

Table 12. Variation of natural frequency in Hz with respect to thickness ratio (α = 50°, d/l = 0.3, n = 4).

S	α = 50°				
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
5	649.5	678.684	939.715	995.441	1065.67
10	551.095	579.887	820.613	877.482	944.884
20	397.492	427.414	632.657	693.266	757.954
40	238.751	266.305	410.807	464.177	520.742
80	128.006	147.621	231.259	265.627	304.117

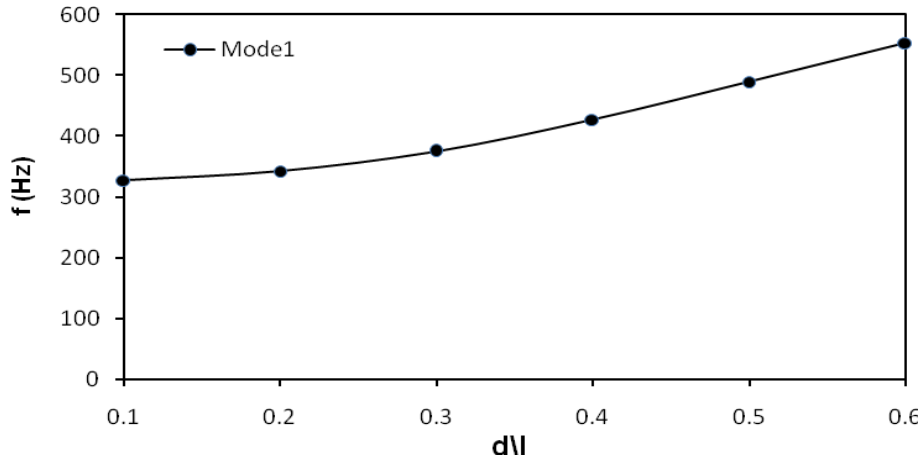


Figure 3. Variation of natural frequency with respect to d/l ratio (s = 10, α = 30°, n = 4).

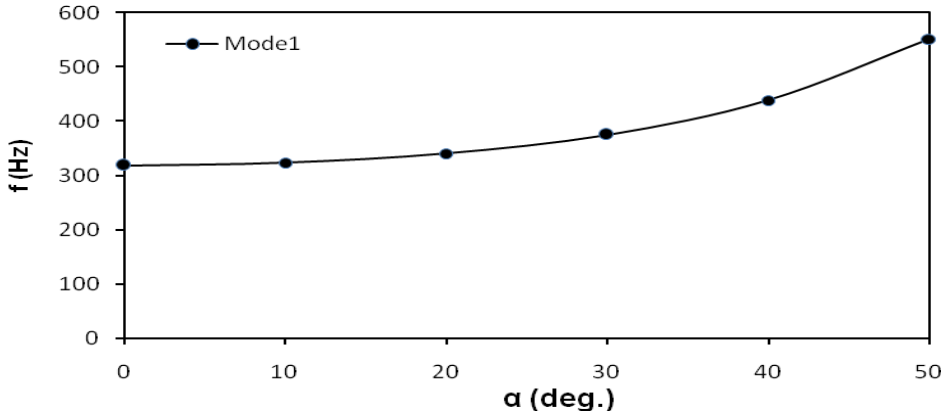


Figure 4. Variation of natural frequency with respect to skew angle (s = 10, d/l = 0.3, n = 4).

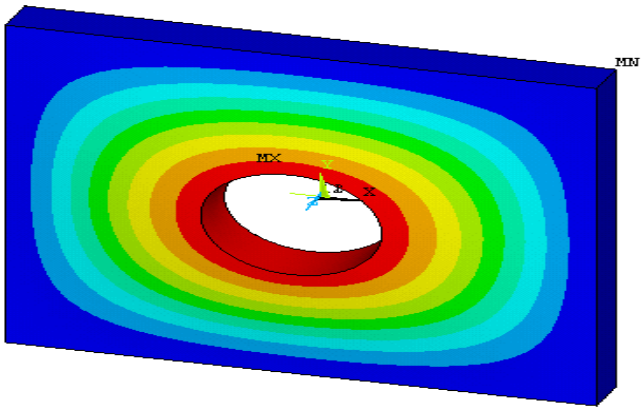


Figure 5. First mode shape for α = 0°, d/l = 0.3, n = 4, s = 10, Natural frequency for the first mode = 318.325 Hz.

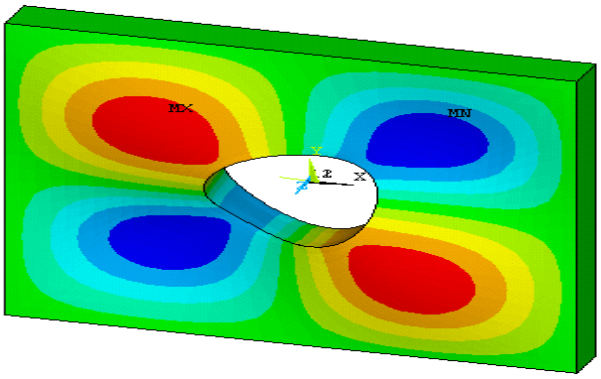


Figure 8. Fourth mode shape for α = 0°, d/l = 0.3, n = 4, s = 10, Natural frequency for the fourth mode = 589.392 Hz.

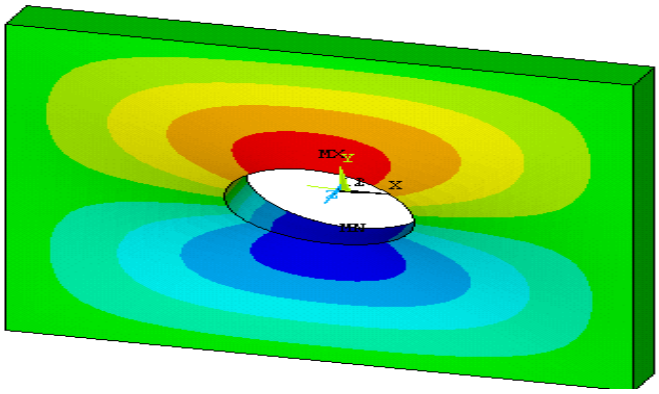


Figure 6. Second mode shape for α = 0°, d/l = 0.3, n = 4, s = 10, Natural frequency for the second mode = 416.83 Hz.

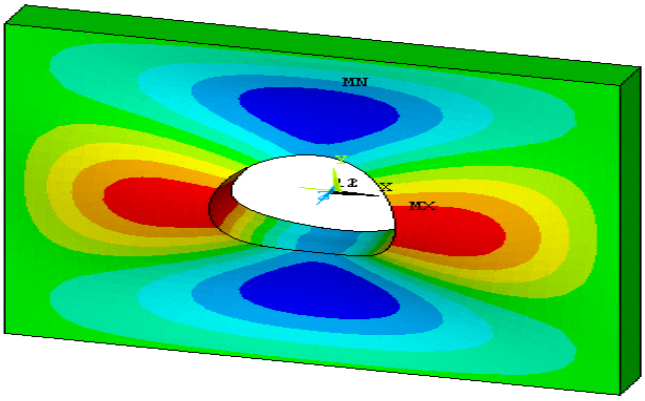


Figure 9. Fifth mode shape for α = 0°, d/l = 0.3, n = 4, s = 10. Natural frequency for the fifth mode = 662.418 Hz.

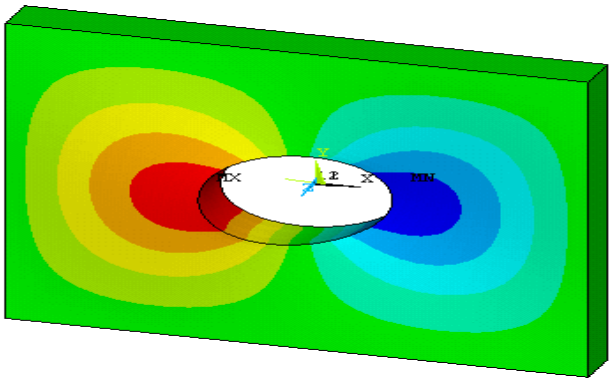


Figure 7. Third mode shape for α = 0°, d/l = 0.3, n = 4, s = 10, Natural frequency for the third mode = 435.499 Hz.

skew angle α are presented. The natural frequency decreases with increase in thickness ratio. Frequency is found to be increasing with increase d/l ratio and skew angle. The present analysis is useful for the design of skew plates with cutouts for dynamic response.

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