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Full Length Research Paper

Damage detection in beam-like structure based on wavelet packet

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Modern and efficient methods of structural damage detection focus on signal processing and have drawn researchers' attention in recent years. These methods mainly are based on Wavelet Packet Transforms (WPT). The WPT is an extension of the Wavelet Transform, which provides a complete level-by-level decomposition of signal. The wavelet packets are alternative bases formed by the linear combinations of the usual wavelet functions. Therefore, the WPT enables the extraction of features from the signals that combine the stationary and non-stationary characteristics with an arbitrary time-frequency resolution. In this study, a damage detection index called Wavelet Packet Energy Rate Index (WPERI) is proposed for the diagnosing of damage in beam-like structures with two different damage scenarios. For this purpose, the measured vibration signals of an experimental model of the beam were decomposed into the wavelet packet components and the wavelet energy rate index which were computed to indicate the structural damages. It will be shown that, the advantage of this method is applying minimum number of sensors which therefore makes this method practical and economical in comparison with other methods, for example Continues Wavelet Transform and Discrete Wavelet Transform. Also, in spite of low signal to noise ratio, the damage location and intensity will be identified correctly.

Key words: Wavelet packet transform, damage, signal, sensor.

INTRODUCTION

During the service life of all structures such as beams, large-scale frames, long-span bridges and high-rise buildings, local damage of their key positions may have been continually accumulated, and finally results in sudden failure of structures. Therefore damage identification of the structures is very vital to protect against disastrous collapses. A classification system of damage identification commonly defines four levels of damage assessment: (1) the presence of damage; (2) the location of the damage; (3) quantification of the severity of the damage; and (4) prediction of the remaining serviceability of the structure (Doebling and Farrar, 1998). The effect of damage on the structural behaviour is the variation of its stiffness that has great influences on dynamic properties of structure. This subject is very significant in variation of natural frequencies and modal shapes and these differences make it possible to explore the changes of damages. There are various methods for examination and distinction of these differences, but each of them has advantages and disadvantages. One of the methods is based on the Fourier analysis, that transforms the signal from a time-based or space-based domain to a frequency-based one. Unfortunately, the time or space information may be lost during performing this transformation and it is sometimes impossible to determine when or where a particular event takes place (Ren and Roeck, 2002).

Zhong and Oyadiji (2007) presented a new approach

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for crack detection in the beam-like structures when crack is relatively small. This approach is based on finding the difference between two sets of detail coefficients obtained by the use of the Stationary Wavelet Transform (SWT) of two sets of mode shape data of the beam-like structure. In spite of detection of crack, this method has a disadvantage, because the method can use only stationary signal. Yan et al. (2006) suggested intelligent damage diagnosis and its application prospects in structural damage detection based on wavelet. Also, the development trends of structural damage detection are also put forward in their method. Xiang et al. (2006) studied the model-based forward and inverse problems in the diagnosis of structural crack location and size by using the finite element method of a B-Spline Wavelet on the Interval (FEM BSWI). Zhu and Law (2005) presented a new method for crack identification of bridge under a moving load based on wavelet analysis. Maosen and Pizhong (2008) proposed a new technique, integrated wavelet transform with taking synergistic advantages of the Stationary Wavelet Transform (SWT) and the Continuous Wavelet Transform (CWT) that, this technique improved the robustness of abnormality analysis of mode shapes in damage detection. Rucka and Wilde (2006) presented a method for estimating the damage location in beam and plate structures. In addition, Zheng et al. (2006) suggested a damage detection method based on a continuous wavelet transform and applied to analyze flexural wave in a cracked beam.

Most of the crack identification methods, which were mentioned earlier, apply continuous wavelet transforms based on the modal analysis and a large number of sensors, so in practice this analysis can be very hard and in some cases impossible. The present study aims to devise a method of identifying the crack in a more applicable and practical way. Hence, Wavelet Packet Energy Rate Index is suggested, and the advantages of the suggested method are compared with other ones. Also, it will be shown that this index is sensitive to increased depth of crack and this method used a low number of sensors for detection of characteristics of crack.

The theory of the wavelet packet transform

The Wavelet Packet Transform (WPT) is an extension of the WT, which provides a complete level-by-level decomposition of signal. This transform is formed by the linear combination of the wavelet. So, the wavelet packet transform can indicate the permanent and temporary features of a signal with the desired frequency-time separation (Han et al., 2005). Wavelet packets consist of a set of linearly combined usual wavelet functions. The wavelet packets inherit the properties such as orthonormality and time-frequency localization from their corresponding wavelet functions. A wavelet packet $\psi^{i}_{j,k}(t)$ is a function with three indices where integers i, j and k are the modulation, scale and translation parameters, respectively:

$$\psi^{i}_{j,k}(t) = 2^{j/2}.\psi^{j}(2^{j}t - k) \tag{1}$$

The wavelet function is derived from the following rebound relation:

$$\psi^{2j}(t) = \sqrt{2}\sum_{k=-\infty}^{\infty} h(k)\psi^{i}(2t-k), \tag{2}$$

$$\psi^{2j+1}(t) = \sqrt{2}\sum_{k=-\infty}^{\infty} g(k)\psi^{i}(2t-k) \tag{3}$$

The first wavelet function is known as the mother function:

$$\psi^{1}(t) = \psi(t) \tag{4}$$

The discrete filters *h(k)* and *g(k)* are the quadrate mirror filters associated with the scaling function and the mother wavelet function. There are quite a few mother wavelets reported in the literature. Most of these mother wavelets are developed to satisfy some key properties such as the invisibility and orthogonally. Choosing the best wavelet function to find the location of the damage is very important. There have been different wavelets such as bior6.8, sym6, Gaus4, Gaus6, and db5.

In the current research, the mentioned wavelets are discussed and as the db5 functions are better than the other wavelets, it is proposed in this paper as the main function (Han et al., 2005). The transform of packet wavelet includes a complete decomposition of each surface, so it analyzes in an area with a high frequency. The rebound relation between the components of ith and *i+1th* surfaces are as follows:

$$f^{i}_{j}(t) = f^{2i-1}_{j+1}(t) + f^{2i}_{j+1}(t) \tag{5}$$

$$f^{2j-1}_{j+1}(t) = Hf^{i}_{j}(t) \tag{6}$$

$$f^{2i}_{j+1}(t) = Gf^{i}_{j}(t) \tag{7}$$

The amount of *H, G* is gained as follows:

$$H\{.\} = \sum_{k=-\infty}^{\infty} h(k-2t) \tag{8}$$

$$G\{.\} = \sum_{k=-\infty}^{\infty} g(k-2t) \tag{9}$$

After decomposition in "*j*" surface, the function of *f(t)* is indicated as follows:

$$f(t) = \sum_{i=1}^{2^j} f_j^i(t) \tag{10}$$

The $f_j^i(t)$ functions of linear combination of wavelet packet functions are gained as follows:

$$f_j^i(t) = \sum_{k=-\infty}^{\infty} c_{j,k}^i(t).\psi_{j,k}^i(t) \tag{11}$$

where the wavelet packet coefficients $c_{j,k}^i(t)$ can be obtained from:

$$c_{j,k}^i = \int_{-\infty}^{+\infty} f(t)\psi_{j,k}^i(t).dt \tag{12}$$

Providing that the wavelet packet functions are orthogonal:

$$\psi_{j,k}^m(t).\psi_{j,k}^n(t) = 0 \tag{13}$$

Each component in the WPT tree can be viewed as the output of a filter tuned to a particular basis function; thus, the whole tree can be regarded as a filter bank. At the top of WPT tree (lower level), the WPT yields a good resolution in the time domain but a poor resolution in the frequency domain. At the bottom of the WPT tree (higher level), the WPT results in a good resolution in the frequency domain yet a poor resolution in the time domain.

In this study, the wavelet packet energy index is proposed to identify the locations and severity of damage. To do that, the signal energy E_{f_i} at *j* level is first defined as:

$$E_{f_j} = \int_{-\infty}^{\infty} f^2(t)dt = \sum_{m=1}^{2^j}\sum_{n=1}^{2^j}\int_{-\infty}^{\infty} f_j^m(t) f_j^n(t).dt \tag{14}$$

Substituting Equation (14) into (17) and using the orthogonal condition Equation (16) yields:

$$E_{f_j} = \sum_{i=1}^{2^j} E_{f_j^i} \tag{15}$$

Finally:

$$E_{f_j} = \int_{-\infty}^{\infty} f_j^i(t)^2 dt \tag{16}$$

Equation (15) shows that, the overall energy of the signal can be decomposed to wavelet packet energy components in different frequency bands. Finally, the WPERI is proposed as follows, to identify the location and intensity of the crack (Han et al., 2005)

$$\Delta(E_{f_j}) = \sum_{i=1}^{2^j} \frac{\left|(E_{f_j^i})_b - (E_{f_j^i})_a\right|}{(E_{f_j^i})_a} \tag{17}$$

Where, $(E_{f_j^i})_a$ is the component signal energy $E_{f_j^i}$ at *j* level without damage, $(E_{f_j^i})_b$ is the component signal energy $E_{f_j^i}$ at *j* level with some damage.

It is postulated that structural damage would affect the wavelet packet component energies and subsequently would alter this damage indicator. It is desirable to select the WPERI, that is sensitive to the changes in the signal characteristics (Zeng et al., 2005).

Damage identification procedures

A damage identification procedure based on the proposed WPERI is described here. Two assumptions were adopted in this study:

(1) The reliable undamaged and damaged structural models are available;
(2) The structure is excited by the same impulse load and acts at the same location.

Vibration signals measured from the structure by sensors are first processed using the WPT. The level of wavelet packet decomposition is determined through a trial and error sensitivity analysis, using the undamaged and damaged structural models. Then the wavelet packet energy rates of signals are calculated (Benffey, 1993). If *n* stands for the total number of all sensors distributing in structure, a total of *n* WPERIs can be obtained, after performing the wavelet packet decomposition. When the mean values and the standard deviations of these WPERIs are expressed as μ_{WPERI} and σ_{WPERI}, the one-side $(1-\alpha)$ upper confidence limit for the WPERI

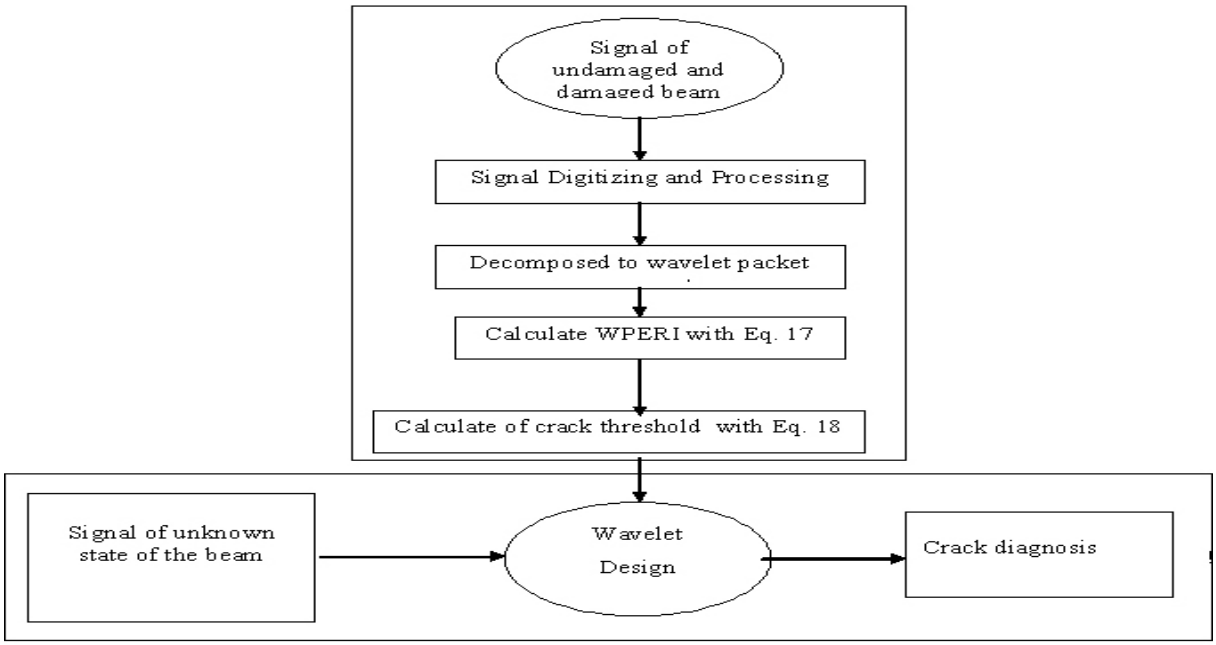


Figure 1. Flowchart of damage identification processing.

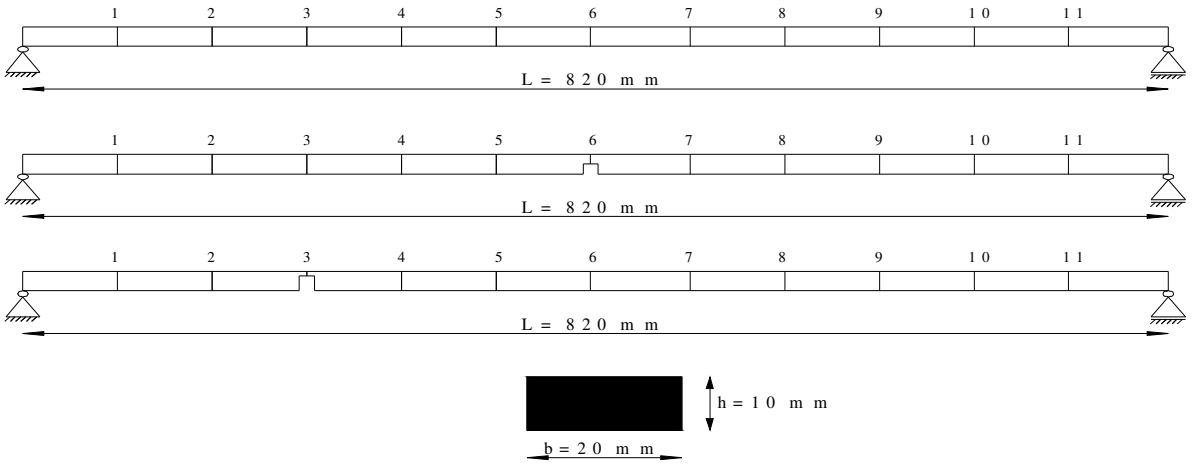


Figure 2. The experimental beam with its cross-section and crack positions.

can be obtained from;

$$UL^{\alpha}_{WPERI} = \mu_{WPERI} + Z_{\alpha}\left(\frac{\sigma_{WPERI}}{\sqrt{n}}\right) \quad (18)$$

Where Z_{α} is the value of a standard normal distribution with zero mean and unit variance, such that the cumulative probability is $100(1-\alpha)$. This limit can be considered as a threshold value for alarming of possible abnormality in the damage indicator WPERI. One special advantage of this damage identification is that, the setting of the threshold value is based on the statistical properties of the damage indicator which was measured with sensors. Any indicator that exceeds the threshold would cause damage alarming. The location of sensors whose WPERI values exceed the threshold will indicate where possible damage occurs (Ang and Tang, 1975). The flowchart in Figure 1 illustrates the considered processes schematically.

THE EXPERIMENTAL SETUP

Three Box-section aluminium beams with span length of 820 mm, as shown in Figure 2, are used to evaluate the proposed damage assessment index. The properties of the beams are as follows:

Table 1. Characteristics of local and intensity damage in all of the undamaged and damaged beams.

Specimen	Location of damage	Number of elements	Intensity of damage (percent of cutting relative to the height of the beam)
D0	Safe	-	-
D1	1/2 L	6	20
D2	1/2 L	6	30
D3	1/4 L	3	30
D4	1/4 L	3	40

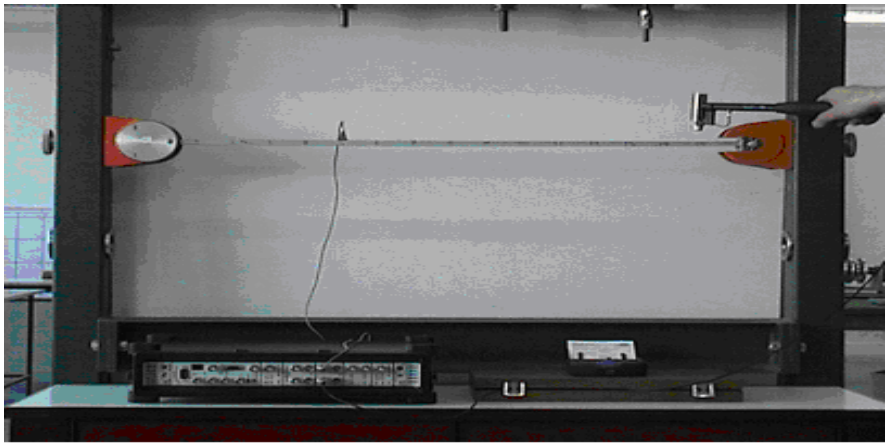


Figure 3. Beam dynamic measurements in the laboratory.

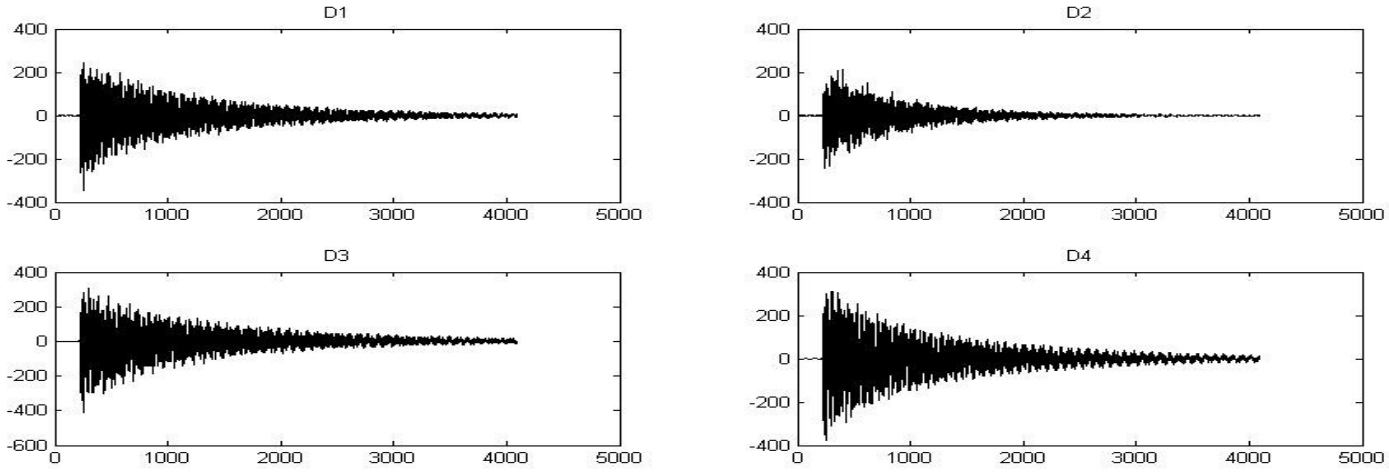


Figure 4. Acceleration–time history for two specimens.

mass density $\rho = 2700kg/m^3$, elastic modulus $E = 70GN/m^2$, cross section area $A = 200mm^2$, and the inertia moment of cross section $I_x = 1666.67mm^4, I_y = 6666.67mm^4$. Undamaged beam and two damaged ones with different locations and intensity of damage were considered. All beams were divided into 11 segments as shown in Figure 2. Also, intensity of all damages was calculated based on percent of element cutting (crack) relative to the height of the beam as shown in Table 1. Dynamic tests and measurements have been carried out on all the beams in the laboratory as shown in Figure 3. The excitation is provided by an impact hammer applied at node 11th. An accelerometer is used to measure the dynamic responses. The sampling frequency for all signals is 3000 Hz. The acceleration responses of beams at the same node (node 11th) are shown in Figure 4. It is shown that damage cannot be seen in the acceleration-time responses.

It should be mentioned that, applying one sensor for identification

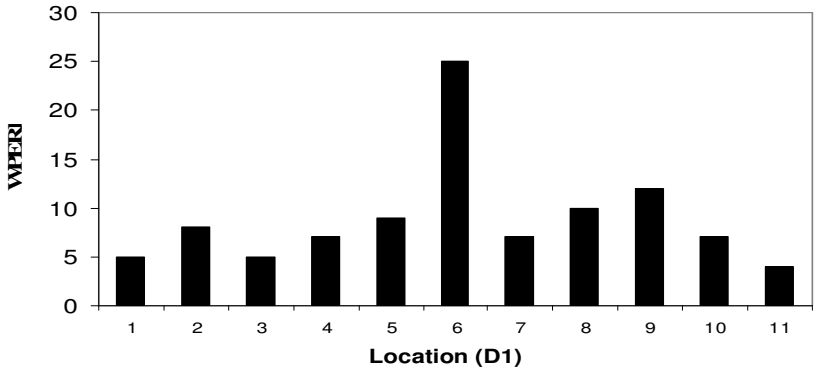


Figure 5. Histogram of wavelet packet energy rate index for specimen D1.

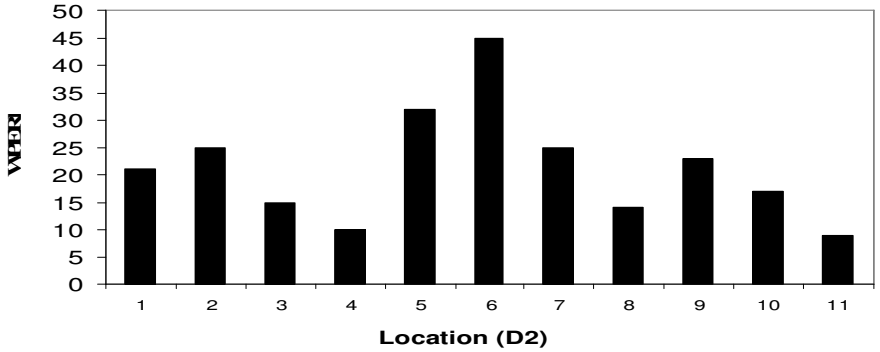


Figure 6. Histogram of wavelet packet energy rate index for specimen D2.

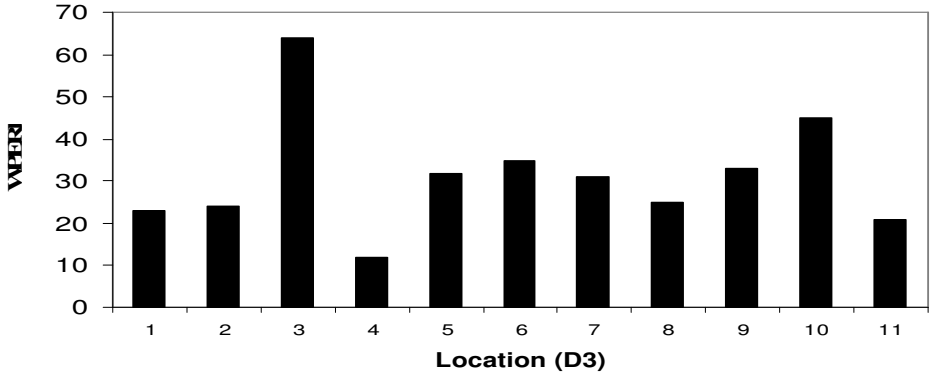


Figure 7. Histogram of wavelet packet energy rate index for specimen D3.

of crack is one of the main advantage of this method. That is, the beam exited in the 11th point and the response is extracted in 1st point. Then, the beam was exited in the 11th point and the sensor moved to the 2nd point and this processing continues. Therefore, the location of load is fixed but the position of sensor (one sensor) changes in the length of beam.

RESULTS AND DISCUSSION

For the tested beams, the decomposition level is chosen to be 5 where a total of 32 component energies are generated because when scale and level of decomposition are less than 4, the identification of damage will be impossible. After decomposing the signals, the WPERIs $\Delta(E_{f_j})$ of every node are calculated using Equation (4). The histograms of results are shown in Figures 5 to 8. As it can be seen in the stated figures, the suggested index is in maximum amount in the cracked position but in other positions, the index has low

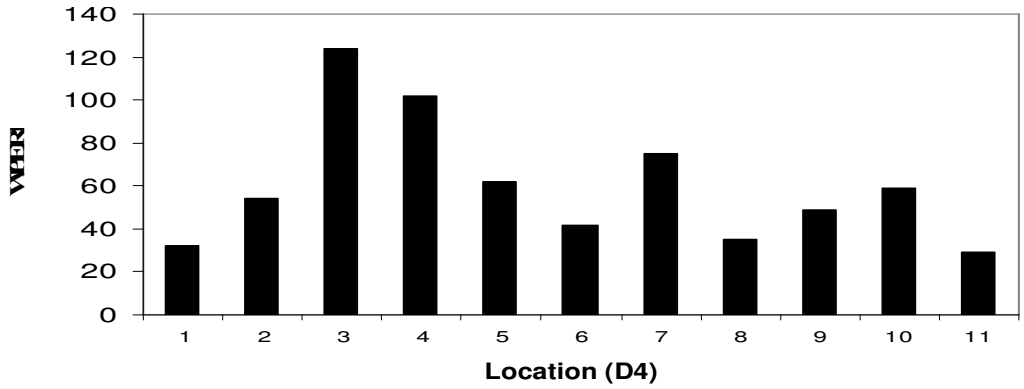


Figure 8. Histogram of wavelet packet energy rate index for specimen D4.

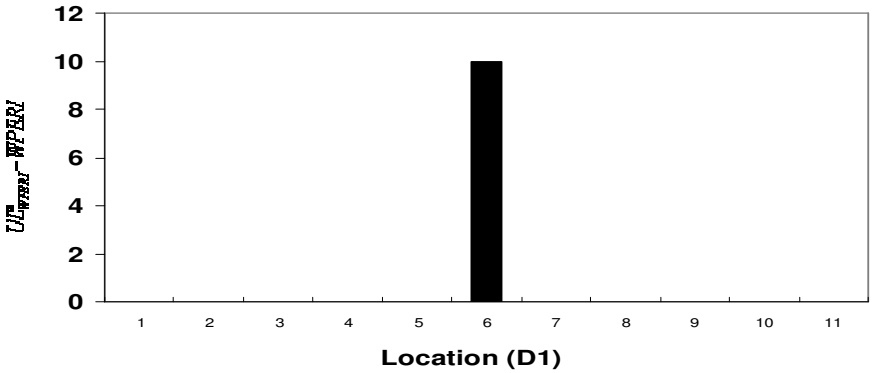


Figure 9. Histogram after considering crack threshold for specimen D1.

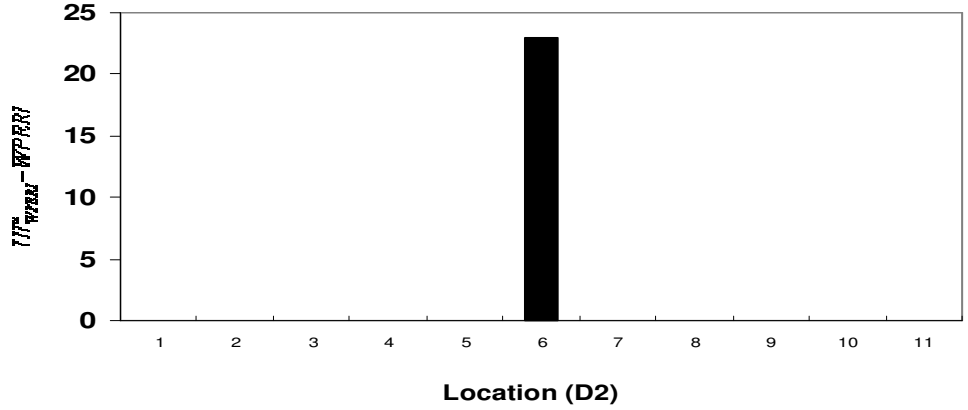


Figure 10. Histogram after considering crack threshold specimen D2.

amounts. So, it can be used to identify the position of the cracks. Furthermore, the present method not only identifies the crack position but also is sensitive to the crack depth.

With assuming $\alpha = 0.02$, the one-sided 98% confidence upper limit UL^{α}_{WPERI} for the WPERIs can be calculated from Equation (5). For every damaged beam, the histogram can be drawn when the UL^{α}_{WPERI} value is subtracted from the WPERI values. The histograms of result are shown in Figures 9 to 12. As it can be seen in the figures, the crack threshold is clearly visible and in crack-free positions, this index is zero. For instance in Figure 12, wavelet packet energy rate index in 6st nod shows the exact position of the crack.

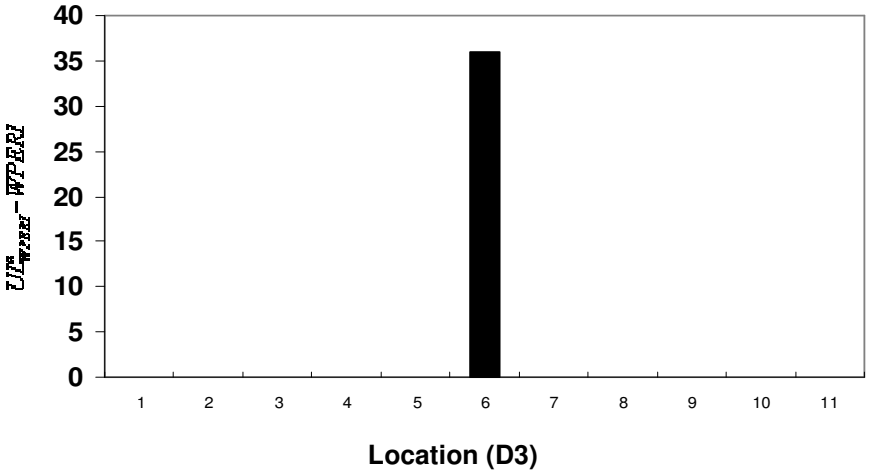


Figure 11. Histogram after considering crack threshold specimen D3.

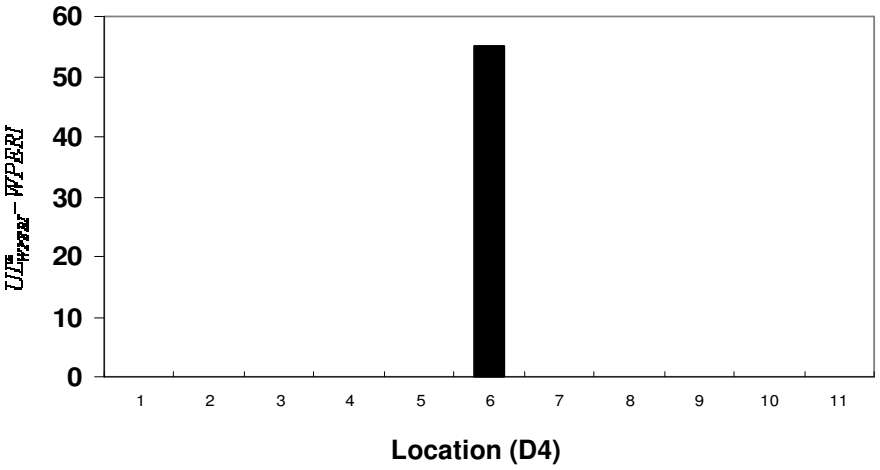


Figure 12. Histogram after considering crack threshold specimen D4.

CONCLUSION

Based on the analysis results of the simulated beam, it is demonstrated that the proposed WPT-based energy rate index is a good candidate index, that is sensitive to local structural damage. These calculations are rather straightforward and not time-consuming; hence, on-line implementation is possible if the reference information is available. The selection of scale and the level of suitable decomposition are very important in wavelet packet analysis.

As a result, if the scale and level of decomposition is less than 4, the identification of damage will be impossible. Also, increase in the depth of the crack raises the suggested index based on the wavelet packet, but does not have a linear relationship with high damage in the structure. In addition, the last and main point which can be concluded from this research is a possibility of using minimum sensor for crack detection in comparing with other methods. Therefore, this method is practicable and applicable.

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