Turkish Journal of Physics

http://journals.tubitak.gov.tr/physics/

Research Article

Turk J Phys
(2014) 38: 253 – 260

doi:10.3906/fiz-1402-4

Neutron-rich ^{208}Pb nucleus with delta excitation under compression

Mohammed Hassen Eid ABU-SEI'LEEK*
Department of Physics, Faculty of Sciences, Majmaah University, Zulfi, Saudi Arabia

Received: 10.02.2014 • **Accepted:** 14.05.2014 • **Published Online:** 11.06.2014 • **Printed:** 10.07.2014

Abstract: The spherical Hartree–Fock approximation is applied to a no-core shell model with a realistic effective baryon–baryon interaction. The ground state properties of a heavy spherical neutron-rich doubly magic ^{208}Pb nucleus under compression are investigated. It is found that the nucleus becomes more bound with the occurrence of Δ resonances. The creation of Δ increases as the compression is continuous. There is a considerable reduction in the compressibility when the Δ degree of freedom is activated. It is found that the Δ particle is the basic component of the ^{208}Pb nucleus besides nucleons at the ground state without any compression. When the nucleus is compressed to about 4.31 times the ordinary density, the Δ component is sharply increased to about 14.4% of all baryons in the system. It is found that there is a radial density distribution for Δ at the ground state of ^{208}Pb nucleus without any compression. The single particle energy levels are calculated and their behaviors are examined under compression too. A good agreement was obtained between the results of the effective Hamiltonian and the phenomenological shell model for the low lying single-particle spectra.

Key words: Nuclear structure, compressed finite nuclei, Δ-resonance, Hartree–Fock method, shell model, heavy spherical doubly magic ^{208}Pb nucleus

PACS: 21.10.Dr; 21.60.Cs; 21.60.Jz; 24.30.Gd; 27.80.+w

1. Introduction

Doubly magic nuclei have been the cornerstone of nuclear structure since the inception of the nuclear shell model [1]. The properties of the spherical doubly magic ^{208}Pb nucleus are the focus of nuclear structure studies [2].

The structure of nuclei at intermediate and high energies is importantly understood by the excitation of Δ isobars. Various probes are formed to provide an exciting challenge both theoretically and experimentally, especially in the search for constructive and coherent pion production [3,4]. Current interest in understanding the collision of both light and heavy ions [5–10] involves Δ excitation and its decay to nucleon and to pions. The heavy ion collision problem [11] and astrophysical phenomena such as supernova explosions or structure of neutron stars [12] are understood by investigating the delta formation in the nucleus, as a function of compression. The ground state properties of the spherical heaviest doubly magic ^{208}Pb nucleus and the formation of the Δ isobars at equilibrium under large radial compression are studied by the present work. The effect of Δ resonance on the ground state properties of closed shell nuclei is emphasized. By using a spherical mean field method, the calculations are performed. In other words, the constrained spherical Hartree–

*Correspondence: moh2hassen@yahoo.com

Fock (CSHF) approximations with a model space of the 8 space (9 major oscillator shells) and realistic effective Hamiltonian [13] with N–N, (N–Δ), and (Δ–Δ) interactions are used. By using the Brueckner G-matrix [14–19] and its adopted improvement [20–22], the effective baryon–baryon interaction is evaluated.

The results of the role of Δ's in finite nuclei have been investigated in a 6-model space (7 shells) [23–35]. A collection of nucleons and Δ-resonances is considered to form the nucleus. The effects of including the Δ-degrees of freedom on the Hartree–Fock energy, density distribution, and Δ-orbital occupations in the ground state and under large amplitude static compression at temperature T = 0 have also been examined. The selected nuclei were ^{16}O, ^{40}Ca, ^{56}Ni, ^{90}Zr, ^{100}Sn, and ^{132}Sn.

In the present work, the main goal extends these results to a new region of the periodic table. The previous effects with different adjusting parameters are considered in a large model space for the heavy doubly magic ^{208}Pb nucleus. The emphasis is on single particle energy levels for nucleons and deltas with large amplitude static compression in a model space consisting of 9 major oscillator shells with CSHF approximation. The calculation was done with the use of a realistic effective Hamiltonian with different potentials. The Brueckner G-matrices that are used are generated from coupled channels NN, NΔ, and πNN. This is done to give a good description of NN data up to 1 GeV. By using the same method reported before [36,37], the effective interactions of the nuclear shell model are calculated. It is a good tool to study highly compressed nuclei at densities accessible to relativistic heavy ion collisions.

The mean-field calculations demonstrated the effective Hamiltonian, H_{eff}, and the calculation procedures. Based on the study presented in the literature [38–40], the 2-body matrix elements are scaled in the N–N sector to an optimal value of $\hbar\omega'$, the oscillator energy for ^{208}Pb nucleus in the 9 major oscillator shells with the 10 delta orbits. By using adjusting parameters (λ_1, λ_2, and $\hbar\omega'$), it is possible to obtain such a fit to the equilibrium binding energy and r_{rms} radius. In this work, the adjusting parameters (λ_1, λ_2, and $\hbar\omega'$) are shown in the Table. This paper is organized as follows: section 2 contains the results and discussion and the conclusion is presented in section 3.

Table. Adjusting parameters of effective Hamiltonian for ^{208}Pb for the model space of 9 oscillator shells for which the calculations were performed. The binding energy (point mass r_{rms}) that was fitted was –1636.5 MeV (5.503 fm) for ^{208}Pb 44].

Nucleus ^{208}Pb	λ_1	λ_2	$\hbar\omega'$(MeV)
(9 shell)	0.997	1.001	7.345

2. Results and discussion

The total number of Δ's is zero in the ground state at equilibrium (zero constraint) in the case of the equal doubly magic ^{16}O, ^{40}Ca, ^{56}Ni, ^{90}Zr, and ^{100}Sn nuclei. Moreover, the number of Δ's is not zero in the ground state for the neutron-rich doubly magic ^{132}Sn nucleus. These results give us strong motivation to study the heavy neutron-rich doubly magic ^{208}Pb nucleus in a larger model space consisting of 9 major oscillator shells for nucleons and 10 orbits for Δ's, making a total of 47 baryons orbits.

Figure 1 displays the Hartree–Fock energies E_{HF} versus r_{rms} using RSC potential for ^{208}Pb. In this figure, there is virtually no difference in the results with and without Δ's at equilibrium. It is seen that without the Δ-degree of freedom in the system, E_{HF} increases steeply towards zero binding energy under compression. When the transition to Δ is allowed (dashed curve), the nucleus remains bound as density is increased to 4.31

of normal density. This shows about 1005.6 MeV and 627.3 MeV of excitation energy to achieve a 49% volume reduction in the nucleon-only results, and nucleons and Δ's results, respectively.

It appears from the above results that 627.3 MeV of excitation energy is enough to reduce the volume by 49% and the energy by 38% more. This suggests that the less dense outer part of the nucleus initially responds to the external load more readily than the inner part. The results show that there is a significant reduction in the static compression modulus for RSC static compressions, which is reduced by including the Δ excitations. The consequence of this reduction is the softening of the nuclear equation of state at larger compression. To see the role of Δ in determining the equation of state, Figure 1 shows the dependence of E_{HF} on the compression characterized by r_{rms}. It can be seen that near equilibrium (r_{rms} = 5.50 fm) all curves agree. Furthermore, the inclusion of Δ orbits tends to decrease E_{HF} for compressed nuclei. The role of the Δ's is less significant as r_{rms} approaches the ground state value.

Figure 1 shows that as the static load force increases the compression of the nucleus with nucleons only is less than that of the other nucleus with nucleons and Δ's.

In terms of relativistic heavy-ion collision, Figure 1 implies that the heavy nucleus can more easily penetrate when the Δ degree of freedom becomes explicit.

Figure 2 shows that the number of Δ's increases rapidly as volume decreases. It is interesting to note in Figure 2 that the numbers of Δ^0's and Δ^+'s are the same until r_{rms} = 3.90 fm. Beyond this radius, the increment in the number of Δ^0's is larger than the number of Δ^+'s as the compression is continuous. This is due to the fact that the number of neutrons is greater than the number of protons in the ^{208}Pb nucleus.

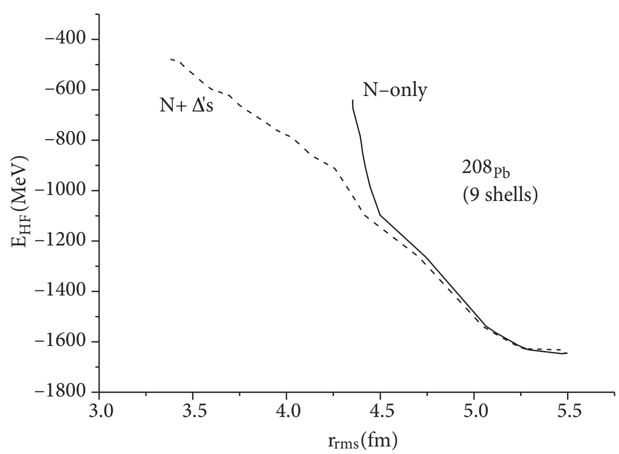

Figure 1. CSHF energy as a function of the point mass r_{rms} for ^{208}Pb evaluated in 9 major oscillator shells with 10 Δ-orbitals. The dashed curve corresponds to CSHF calculations including the Δ's, while the solid curve corresponds to CSHF with nucleons only.

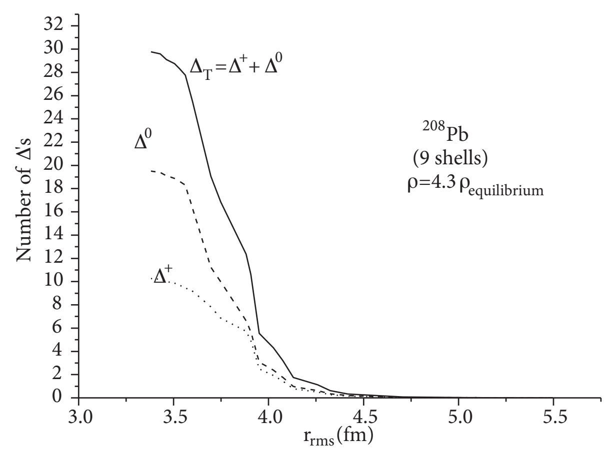

Figure 2. Number of Δ's as a function of r_{rms} for ^{208}Pb in 9 major shells model space. The upper curve is for the total number of Δ's. The dotted curve is for the number of Δ^+ and the dashed curve is for Δ^0.

Although there is a rapid rise in the Δ-population as compression increases, the change in the total number of Δ's is 0.144. It is interesting to note that there is a consistency of the amount of N–Δ mixing with the amount of excitation energy exhibited with compression. That is, when 0.144 Δ's are presented, the excitation energy is in the order of 0.144(M-m) $\approx$ 42.77 MeV. Thus, on the scale of the unperturbed single-particle energies, a substantial fraction of the compressive energy is delivered through the NΔ and Δ–Δ interactions to create more massive baryons in the lowest energy configuration of the nucleus. In other words, the number of Δ's can be increased to about 30 at r_{rms} = 3.38 fm, which corresponds to about 4.31 times the normal density.

Figure 2 shows that the number of created Δ's increases sharply when the ^{208}Pb nucleus is compressed to a volume of about 0.77 of its equilibrium size. However, at this nuclear density, which is 4.31 times the normal density, the percentage of nucleons converted to Δ is only about 14.4% in ^{208}Pb. This result is consistent with the information extracted from the data of relativistic heavy-ions collisions. In some heavy-ion collision experiments, the Δ's may constitute up to 10% of the nuclear constituents when the system is compressed [41,42].

The calculations of Figure 2 show that the number of Δ's is not zero at equilibrium (zero constraint) in the ^{208}Pb nucleus. This result is consistent with the previous finding for ^{132}Sn.

Therefore, the heavy nuclei are a composite of the Δ's besides the nucleons at equilibrium ground state without any compression.

Because of the limitations of the model space, our calculations for higher densities are more speculative. Nevertheless, they can give us some idea about how the Δ population can be increased as the nucleus is compressed to higher densities accessible to relativistic heavy-ion collisions.

The radial density distribution for ^{208}Pb at equilibrium state without any compression is displayed in Figure 3.

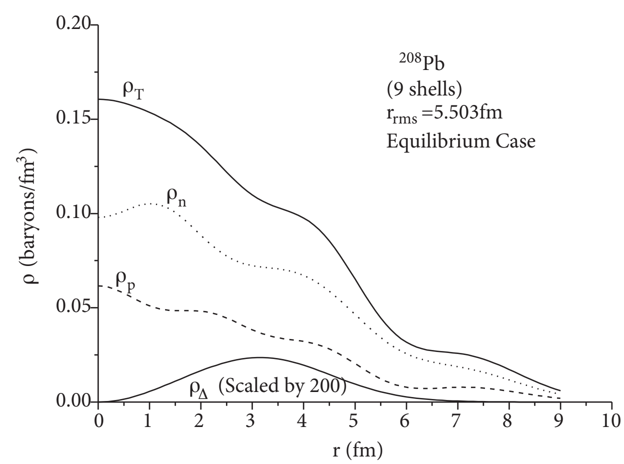

Figure 3. Total ρ_T, proton ρ_p (dashed line), neutron ρ_n (dotted line), and delta ρ_Δ (solid line) radial density distribution for ^{208}Pb at equilibrium state without any compression in a model space of 9 major oscillator shells.

It can be seen from this figure that there is a radial density distribution for Δ at the ground state of the ^{208}Pb nucleus without any compression. This emphasizes the important fact that the Δ particle is the basic component of heavy nuclei besides nucleons (protons and neutrons) at ground state without any constraints. Our results shown in this figure are consistent with the results of figure 2 in Zuo et al. [43] that shows many methods to find the radial density of ^{208}Pb.

Figure 4 shows the radial density distribution under compression. The neutron radial density is higher than the proton density at all values of r. This is due to Coulomb repulsion between the protons. The Δ-radial density distribution, under high compression (point mass r_{rms}= 5.05 fm), reaches a peak value of about 0.02 of the proton (or neutron) radial density at r = 3.1 fm. Δ-mixing with the nucleons in the $0p_{3/2}$, $0p_{1/2}$,

$0d_{5/2}$, $0d_{3/2}$, $0f_{7/2}$, $0f_{5/2}$, $1p_{3/2}$, $1p_{1/2}$, $0g_{9/2}$, and $0g_{7/2}$ orbitals occurs and this explains the shape of the Δ-radial distribution presented in Figure 4.

Figure 5 displays the radial density distributions of ^{208}Pb, which is evaluated of about 0.23 reduced volumes. In this case, the Δ-radial density distribution reaches a peak value of about 0.95 of the proton radial density.

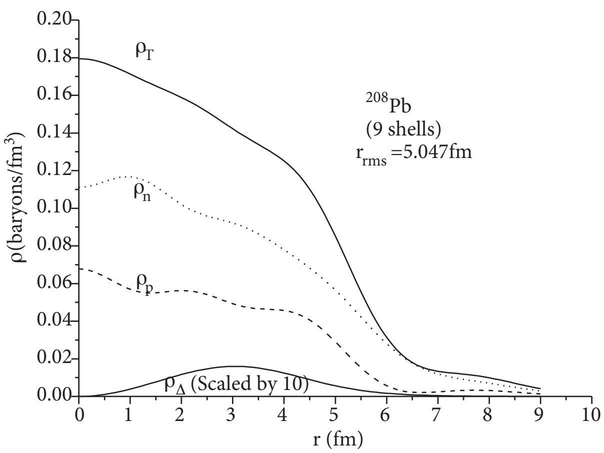

Figure 4. Total ρ_T, proton ρ_p (dashed line), neutron ρ_n (dotted line), and delta ρ_Δ (solid line) radial density distribution for ^{208}Pb at point mass $r_{rms} = 5.047$ fm in a model space of 9 major oscillator shells.

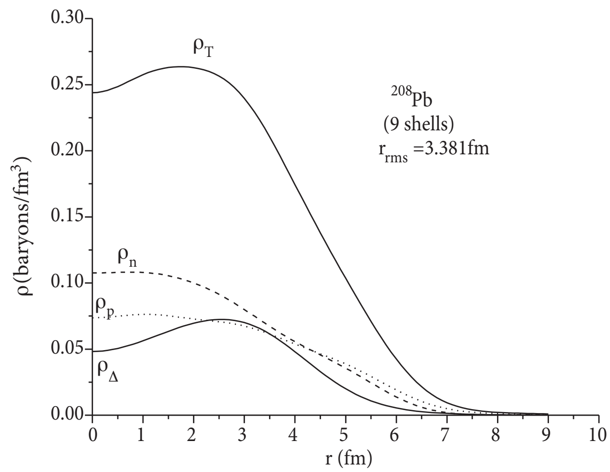

Figure 5. Total ρ_T, proton ρ_p (dashed line), neutron ρ_n (dotted line), and delta ρ_Δ (solid line) radial density distribution for ^{208}Pb at point mass $r_{rms} = 3.381$ fm in a model space of 9 major oscillator shells.

It can be seen from Figures 4 and 5 that as compression increases the total radial density increases and the radial density distribution of Δ's increases sharply, but the radial density of nucleons decreases sharply.

Figure 6 shows the total radial density for ^{208}Pb in 9 oscillator shells at large compression ($r_{rms} = 3.38$ fm) and at equilibrium (point mass radius $r_{rms} = 5.50$ fm). This figure shows that when the volume of the nucleus is decreased by 0.77 of the equilibrium volume the radial density is increased by 1.65 of its value at the equilibrium case.

Clearly, the density in the interior rises relative to the interior density at equilibrium as the nucleus is compressed. This is in contrast to the behavior of the radial density on the outer surface, where the radial density distribution is higher at equilibrium than the radial density when the static load is applied.

In Figure 7, the lowest neutron single particle energy levels as a function of r_{rms} are displayed. The orbits curved up as the load on the nucleus increased. This is because the kinetic energy of the baryons, which is a positive quantity, becomes more influential than the attractive mean field of the baryons.

The single particle energies increase linearly after $r_{rms} = 4.0$ fm; they intersect at higher compression especially for higher orbitals, and consequently the nucleus becomes unbound.

The behavior of single particle energy levels has good agreement with the orbital ordering of the standard shell model. The gap is very clear between the shells. The splitting of the levels in each shell is an indication that L–S coupling is strong enough in RSC potential, i.e. L–S coupling becomes stronger as the static load on the nucleus increases. When the nucleus is compressed, the splitting of the orbitals becomes clearer, especially in delta orbitals.

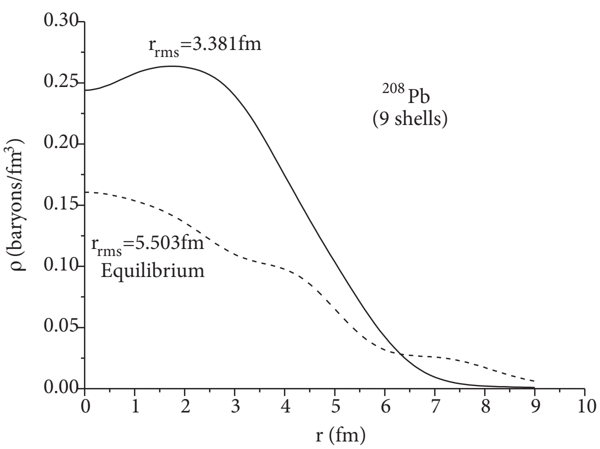

Figure 6. Total radial density distribution for ^{208}Pb at equilibrium (r_{rms} = 5.503 fm) (dashed line) and at r_{rms} = 3.381 fm (solid line).

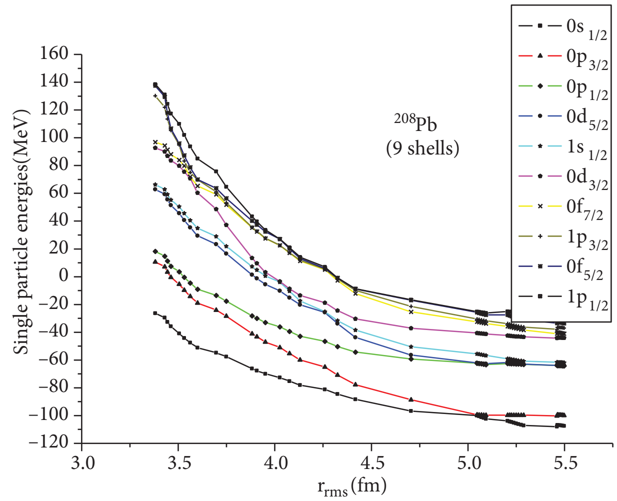

Figure 7. Single particle energy of lowest 10 neutron states for ^{208}Pb in 9 oscillator shells as a function of r_{rms}.

Further efforts will concentrate on extending this study to a large model space for heavier nuclei and alternative potentials in order to investigate the sensitivity of nuclear properties to the N–Δ and Δ–Δ interactions. One ultimate goal is to predict the behavior of nuclear systems at finite temperature with the inclusion of the Δ-degree of freedom.

3. Conclusion

The ground state properties of the heavy neutron-rich spherical ^{208}Pb nucleus have been investigated by using a realistic effective baryon–baryon Hamiltonian in the constrained Hartree–Fock approximation. It is found that the nucleus becomes more bound with the occurrence of Δ-resonances. The nuclear shell model is derived in this approach with single particle levels occupied by baryons that are a mixture of nucleons and deltas. As shown in ^{208}Pb, the results compare favorably with those of the phenomenological successful shell model.

It is found that the Δ particle is the basis component of the heavy nuclei besides nucleons at ground state without any compression. There is a considerable reduction in compressibility when the Δ degree of freedom is active. The creation of Δ's increases as the compression continues.

Finally, a large fraction of the excitation energy is required to compress the nucleus used to create mass in the form of Δ's.

Acknowledgment

I acknowledge support for this work from the Engineering and Applied Sciences, Research Center, Scientific Research Deanship, Majmaah University, under Award No. 10-2012.

References

[1] Dariusz, S. *Nucl. Phys. News* **2009**, *19*, 18–21.

[2] Haddad, S. *Rom. Journ. Phys.* **2007**, *52*, 751–757.

[3] Deutchman, P. A. *Aust. J. Phys.* **1999**, *52*, 955–971.

[4] Terbert, C. M.; Watts, D. P.; Aguar, P.; Ahrens J.; Annand J. R. M.; Arends, H. J.; Beck, R.; Bekrenev V.; Boillat B.; Braghieri A.; et al. *Phys. Rev. Lett.* **2008**, *100*, 132301–132306.

[5] Fernández de Córdoba, P.; Oset, E.; Vicente-Vacas, M. J. *Nucl. Phys. A* **1995**, *592*, 472–486.

[6] Badalá, A.; Barbera, R.; Bonasera, A.; Palmeri, A.; Pappalardo, G. S.; Riggi, F.; Russo, A. C.; Turrisi, R. *Phys. Rev. C* **1996**, *54*, R2138–R2142.

[7] Mosel, U.; Metag, V. *Nucl. Phys. News* **1993**, *3*, 25–27.

[8] Xiong, L.; Wu Z. G.; Ko, C. M.; Wu, J. Q. *Nucl. Phys. A* **1990**, *512*, 772–786.

[9] Hoffmann, M.; Mattiello, R.; Sorge, H.; Stöcker, H.; Greiner, W. *Phys. Rev. C* **1995**, *51*, 2095–2098.

[10] Yong-Zhong, X.; Yu-Ming, Z.; Pornrad, S.; Yu-Peng, Y.; Chinorat, K. *Chin. Phys. Lett.* **2009**, *26*, 022501–022507.

[11] Shlomo, S; Kolomietz, V. *Rep. Prog. Phys.* **2005**, *68*, 1–7.

[12] Frick, T.; Müther, H. *Phys. Rev. C* **2005**, *71*, 014313–014321.

[13] Brandow, B. H. *Rev. Mod. Phys.* **1967**, *39*, 771–828.

[14] Lee, T. *Phys. Rev. Lett.* **1983**, *50*, 1571–1578.

[15] Lee, T. *Phys. Rev. C* **1984**, *29*, 195–203.

[16] Lee, T.; Matsuyama, A. *Phys. Rev. C* **1985**, *32*, 516–530.

[17] Lee T.; Matsuyama A. *Phys. Rev. C* **1987**, *36*, 1459–1471.

[18] Matsuyama, A.; Lee, T. *Phys. Rev. C* **1986**, *34*, 1900–1910.

[19] Matsuyama, A.; Lee, T. *Nucl. Phys.A* **1991**, *526*, 547–559.

[20] Hasan, M. A.; Lee, T. S. H.; Vary, J. P. *Phys. Rev. C* **1999**, *61*, 014301–014308.

[21] Hasan, M. A.; Lee, T. S. H.; Vary, J. P. *Phys. Rev. C* **1997**, *56*, 3063–3069.

[22] Hasan, M. A.; Vary, J. P.; Lee, T. S. H. *Phys. Rev. C* **2001**, *64*, 024306-1-024306-5.

[23] Hasan, M. A.; Köhler, S. H.; Vary, J. P. *Phys. Rev. C* **1987**, *36*, 2180–2182.

[24] Hasan, M. A.; Köhler, S. H.; Vary, J. P. *Phys. Rev. C* **1987**, *36*, 2649–2660.

[25] Vary, J. P.; Hasan, M. A. *Phys. Rep.* **1994**, *242*, 139–148.

[26] Vary, J. P.; Hasan, M. A. *Nucl. Phys. A* **1994**, *570*, 355–362.

[27] Hasan, M. A. *Dirasat J.* **1995**, *22*, 777–804.

[28] Hasan, M. A.; Vary, J. P. *Phys. Rev. C* **1994**, *50*, 202–212.

[29] Hasan, M. A.; Vary, J. P. *Phys. Rev. C* **1996**, *54*, 3035–3042.

[30] Abu-Sei'leek, M. H.; Hasan, M. A. *Comm. Theort. Phys.* **2010**, *54*, 339–346.

[31] Abu-Sei'leek M. H. *Pramana- J. Phys.* **2011**, *76*, 573–589.

[32] Abu-Sei'leek, M. H. *International Journal of Pure and Applied Physics* **2011**, *7*, 73–87.

[33] Abu-Sei'leek, M. H., *Comm. Theort. Phys.* **2011**, *55*, 115–122.

[34] Abu-Sei'leek, M. H. *Turk. J. Phys.* **2011**, *35*, 273–280.

[35] Abu-Sei'leek, M. H. *Nucl. Phys. Rev.* **2010**, *27*, 399–410.

[36] Abu-Sei'leek, M. H. *Nucl. Phys. Rev.* **2011**, *28*, 416–422.

[37] Abu-Sei'leek, M. H. *J. Phys. Soc. Jpn.* **2011**, *80*, 104201-1-104201-7.

[38] Bozzolo, G.; Vary, *J. Phys. Rev. Lett.* **1984**, *53*, 903–906.

[39] Bozzolo, G.; Vary *Phys. Rev. C* **1985**, *31*, 1909–1919.

[40] Saraceno, M.; Bozzolo, G.; Miller, H. G. *Phys. Rev. C* **1988**, *37*, 1267–1280.

[41] Pasyuk, E.; Morris, C.; Ullmann, J.; Zumbro, J.; Kwok, L.; Mathews, J.; Tan, Y.; *Acta. Phys. Pol. B* **1998**, *29*, 2335–2338.

[42] Morris, C.; Zumbro, J.; McGill, J.; Seestrom, S.; Whitten, R.; Reidel, C.; Williams, A.; Braunstein, M.; Koler, M.; Kriss, B., *Phys. Lett. B* **1998**, *419*, 25–29.

[43] Zuo, W.; Dong, J.; Gu, J. *Phys. Rev. C* **2013**, *87*, 014303–014311.

[44] Sharama, M.; Lalazissis, G.; Köning, J.; Ring, P. *Phys. Rev. Lett.* **1995**, *74*, 3744– 3747.