International Journal of the Physical Sciences Vol. 6 (21), pp. 5012-5021, 30 September, 2011
Available online at http://www.academicjournals.org/IJPS
DOI: 10.5897/IJPS11.694
ISSN 1992 - 1950 ©2011 Academic Journals

Full Length Research Paper

Search for critical failure surface in slope stability analysis by gravitational search algorithm

Mohammad Khajehzadeh[1]*, Mohd Raihan Taha[1], Ahmed El-Shafie[1] and Mahdiyeh Eslami[2]

[1]Civil and Structural Engineering Department, University Kebangsaan Malaysia, Bangi, Malaysia.
[2]Electrical Engineering Department, Science and Research Branch, Islamic Azad University (SRBIAU), Hesarak, Tehran, I. R. Iran.

Accepted 6 July, 2011

Slope stability analysis of any natural or artificial slope aims at determining the factor of safety of the slip surface that possesses the lowest factor of safety. The search for the minimum factor of safety based on limit equilibrium methods is a complex optimization problem where there are several local minima. In this paper, a newly developed heuristic global optimization algorithm, called gravitational search algorithm (GSA), is introduced and applied in slope stability analysis. The safety factors of the general slip surfaces are calculated using a concise algorithm of Morgenstern and Price method, which satisfies both force and moment equilibrium. The reliability and efficiency of the proposed algorithm are examined by considering a number of published cases. The results indicate that, the proposed method could provide solutions of high quality, accuracy and efficiency, and could predict the critical failure mechanisms of earth slope and outperforms the other methods in the literatures.

Key words: Slope stability, minimum factor of safety, gravitational search algorithm.

INTRODUCTION

The slope stability analysis is a problem of significant concern in many engineering fields, particularly, mining engineering, hydraulic engineering and geotechnical engineering. Slope stability analysis involves attention of a wide variety of variables such as external forces, pore water pressures, soil strength parameters, topographic and geologic conditions, etc. A quantitative assessment of the stability of a slope is clearly important when a judgment is needed about whether the slope is stable or not. For slope stability analysis a lot of solution methods have been developed over the years. Such methods include limit equilibrium methods, limit analysis method, rigid element method, finite element method, distinct element methods and probabilistic analysis approaches (Cheng and Lau, 2008).

The generally adopted approach of slope stability analysis is the limit equilibrium method of slices. Most problems in slope stability are highly statically indeterminate, and to render it determinate some simplifying assumptions are made in order to determine a unique factor of safety. Based on different assumptions, various methods of slices have been proposed such as Bishop (1955), Morgenstern and Price (1965), Spencer (1967), Janbu (1973), etc. A number of these methods are applicable to a circular slip surface and satisfy only overall moment equilibrium such as Bishop's method, while others are applicable to any shape of slip surface and satisfy both moment and force equilibrium such as Morgenstern and Price and Spencer method. In the method of slices, the potential sliding mass is subdivided into a number of slices. Then a critical slip surface of predetermined shape is searched, for which the factor of safety is minimized.

Several optimization methods have been employed to automate the search for the critical failure surface and associated factor of safety. Attempts to use this approach include Baker (1980), Greco (1996), Malkawi et al. (2001), Pham and Fredlund (2003), Cheng (2003), Zolfaghari et al. (2005), Cheng et al. (2008), Kahatadeniya et al. (2009), Khajehzadeh et al. (2010), and others. Be different with other heuristic optimization algorithm based on swarm behaviors, such as genetic algorithm and particle swarm optimization, gravitational search algorithm (GSA) is a newly developed heuristic optimization method based on the law of gravity and

*Corresponding author. E-mail: mohammad.khajehzadeh@ gmail.com.

mass interactions (Rashedi et al., 2009). GSA is characterized as a simple concept that is both easy to implement and computationally efficient. The method has confirmed higher performance in solving various nonlinear functions, when compared with some well-know search methods (Rashedi et al., 2009).

In this paper, GSA combined with a new approach of Morgenstern and Price method is proposed to search for the critical slip surface of earth slope. Traditionally, nonlinear equations that arise from the Morgenstern and Price (MP) method are solved using a numerical method like the Newton-Raphson method. Zhu et al. (2005) introduced a concise algorithm for computing the factor of safety using the MP method. In this approach, the two equilibrium equations used in the MP method are re-derived to obtain two expressions for the factor of safety (FS) and the scaling factor (λ). Generally speaking, the major advantages of the proposed methodology for the slope stability analysis may be expressed as follows: (1) the method can easily be dealt with the complex soil profiles, variable soil properties and loading conditions; (2) the minimum factor of safety and critical failure surface are found automatically without the need for a trial and error search; (3) the slip surface can be of any shape; (4) the method is derived from both moment and force equilibrium, and the factor of safety derived from both moment and force equilibrium is more reliable than the one derived either from the moment or force equilibrium; (5) the numerical methods are not required to solve the equations of the MP method; (6) the gravitational search algorithm can perform well even for non-convex functions with many local minima, and is very suitable for the present study. Based on these advantages, a computer program called stability analysis of slopes using gravitational search algorithm (SAS-GSA) was developed by MATLAB. The program searches for the most critical slip surface and calculates its associated minimum factor of safety.

GRAVITATIONAL SEARCH ALGORITHM

Gravitational search algorithm (GSA) is a newly developed stochastic search algorithm based on the law of gravity and mass interactions (Rashedi et al., 2009). In this approach, the search agents are a collection of masses which interact with each other based on the Newtonian gravity and the laws of motion, in which the method is completely different from other well-known population-based optimization method inspired by the swarm behaviors. In GSA, agents are considered as objects and their performance are measured by their masses. All of the objects attract each other by the gravity force, while this force causes a global movement of all objects towards the objects with heavier masses (Rashedi et al., 2009). The heavy masses correspond to good solutions of the problem. In other words, each mass presents a solution, and the algorithm is navigated by properly adjusting the gravitational and inertia masses. By lapse of time, the masses will be attracted by the heaviest mass which it presents an optimum solution in the search space.

To describe the GSA, consider a system with N agents (masses), the position of the agent i is defined by:

$$X_i = (x_i^1, \ldots, x_i^d, \ldots, x_i^n) \text{ for } i = 1, 2, \ldots, N \quad (1)$$

where x_i^d presents the position of the agent i in the dimension d and n is the search space dimension.

After evaluating the current population fitness, the mass of each agent is calculated as follows:

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^{N} m_j(t)} \quad (2)$$

where

$$m_i(t) = \frac{\mathrm{fit}_i(t) - \mathrm{worst}(t)}{\mathrm{best}(t) - \mathrm{worst}(t)} \quad (3)$$

where $\mathrm{fit}_i(t)$ represent the fitness value of the agent i at time t. Best(t) and worst(t) are the best and worst fitness of all agents, respectively and are defined as follows:

$$\begin{aligned} \mathrm{best}(t) &= \min_{j \in \{1,\ldots,N\}} \mathrm{fit}_j(t) \\ \mathrm{worst}(t) &= \max_{j \in \{1,\ldots,N\}} \mathrm{fit}_j(t) \end{aligned} \quad (4)$$

To evaluate the acceleration of an agent, total forces from a set of heavier masses applied on it should be considered based on a combination of the law of gravity according to:

$$F_i^d(t) = \sum_{j \in kbest, j \neq i} rand_j G(t) \frac{M_j(t) \times M_i(t)}{R_{i,j}(t) + \varepsilon} (x_j^d(t) - x_i^d(t)) \quad (5)$$

where $rand_j$ is a random number in the interval [0, 1], $G(t)$ is the gravitational constant at time t, M_i and M_j are masses of agents i and j, ε is a small value and $R_{i,j}(t)$ is the Euclidean distance between two agents, i and j. *kbest* is the set of first K agents with the best fitness value and biggest mass, which is a function of time, initialized to K_0 at the beginning and decreased with time. Here K_0 is set to N (total number of agents) and is decreased linearly to 1.

By the law of motion, the acceleration of the agent i at time t, and in direction d, $a_i^d(t)$ is given as follows:

$$a_i^d(t)=\frac{F_i^d(t)}{M_i(t)}=\sum_{j\in kbest, j\neq i} rand_j G(t)\frac{M_j(t)}{\left\|X_i(t),X_j(t)\right\|_2+\varepsilon}\left(x_j^d(t)-x_i^d(t)\right) \quad (6)$$

Finally, the searching strategy on this concept can be described by following equations:

$$v_i^d(t+1)=rand_i\times v_i^d(t)+a_i^d(t) \quad (7)$$

$$x_i^d(t+1)=x_i^d(t)+v_i^d(t+1) \quad (8)$$

where x_i^d, v_i^d and a_i^d represents the position, velocity and acceleration of *i*th agent in *d*th dimension, respectively. $rand_i$ is a uniform random variable in the interval [0, 1]. This random number is applied to give a randomized characteristic to the search.

It must be pointed out that the gravitational constant $G(t)$ is important in determining the performance of GSA and is defined as a function of time t:

$$G(t)=G_0\times\exp\left(-\beta\times\frac{t}{t_{\max}}\right) \quad (9)$$

where G_0 is the initial value, β is a constant, t is the current iterations and t_{max} is the maximum number of iteration. The parameters of maximum iteration t_{max}, population size N, initial gravitational constant G_0 and constant β control the performance of GSA (N, G_0, β and t_{max}).

According to the earlier description, the whole workflow of the gravitational search algorithm is as follows:

Step 1: Define the problem space and set the boundaries, that is equality and inequality constraints.
Step 2: Initialize an array of masses with random positions.
Step 3: Check if the current position is inside the problem space or not. If not, adjust the positions so as to be inside the problem space.
Step 4: Evaluate the fitness value of agents.
Step 5: Update $G(t)$, $best(t)$, $worst(t)$ and $M_i(t)$ for i = 1,2,. . .,N.
Step 6: Calculation of the total force in different directions and acceleration for each agent based on Equations 5 and 6.
Step 7: Update the velocities according to Equation 7.
Step 8: Move each agent to the new position according to Equation 8 and return to Step 3.
Step 9: Repeat Step 4 to Step 8 until a stopping criteria is satisfied.

In (Rashedi et al., 2009), GSA has been compared with some well known heuristic search methods. The high performance of GSA has been confirmed in solving various nonlinear functions. As an excellent optimization algorithm, GSA has the potential to solve a broad range of optimization problems. In this paper, the method is applied for slope stability analysis.

SLOPE STABILITY ANALYSIS MODEL

The generally adopted approach to evaluating the factor of safety and deterministic analysis of slopes is the limit equilibrium method of slices. In this study, a concise algorithm of Morgenstern and Price (1965) method is used for calculation of the safety factor (Zhu et al., 2005). The MP method is one of the popular methods among limit equilibrium methods. In this method, both moment and force equilibrium will be satisfied simultaneously for any shape of failure surfaces. The essence of the method is to divide the sliding mass into a finite number of vertical slices. The MP method assumes the inclination of the resultant inter-slice force varying symmetrically across the slide mass. Thus, the relationship between the normal and shear inter-slice force may be expressed as:

$$T=f(x).\lambda.E \quad (10)$$

where T is the shear inter-slice force, E is the normal inter-slice force, λ is a scaling factor to be evaluated in solving for the safety factor and $f(x)$ is the assumed inter-slice force function with respect to x. Several functions may be used as $f(x)$ such as constant function, trapezoidal function, sine function and half-sine function (Fredlund and Krahn, 1977). In order to evaluate the factor of safety (FS), one should consider the forces acting on a typical vertical slice of a natural slope with general-shaped slip surface as shown in Figure 1.

Referring to Figure 1, W_i = weight of the slice; N'_i = effective normal force at the base of the slice; S_i = mobilized shear strength at the base of the slice ($S_i=\frac{c_i' l_i+N_i'\tan\varphi'}{FS}$); c_i' = effective cohesion at the base of the slice; φ_i' = effective angle of internal friction at the base of the slice; l_i= length at the base of the slice; U_i = pore water pressure at the base of the slice; Q_i = external surcharge of the slice; α_i = inclination of the slice base; b_i = width of the slice; h_i = average height of the slice; K_h = horizontal seismic coefficient; h_a = height of center of the slice; β_i = inclination of the slice top; δ_i = inclination of surcharge load.

Considering the force equilibrium of the slice i and resolving perpendicular to the slip surface, we have:

$$N_i'=(W_i+\lambda f_{i-1}E_{i-1}-\lambda f_i E_i+Q_i\cos\delta_i)\cos\alpha_i+(-k_h W_i+E_i-E_{i-1}+Q_i\sin\delta_i)\sin\alpha_i-U_i \quad (11)$$

and resolving parallel to the slip surface, we have:

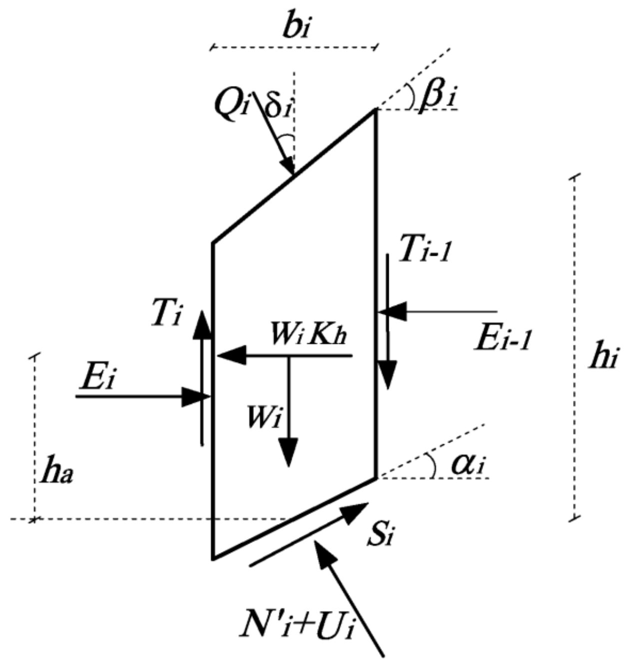

Figure 1. Forces acting on a typical slice of a natural slope.

$$(N_i' \tan\phi_i' + c_i' l_i)/FS = (W_i + \lambda f_{i-1} E_{i-1} - \lambda f_i E_i + Q_i \cos\delta_i)\sin\alpha_i \\ - (-k_h W_i + E_i - E_{i-1} + Q_i \sin\delta_i)\cos\alpha_i \tag{12}$$

Substituting Equation 11 into Equation 12 yields

$$E_i[(\sin\alpha_i - \lambda f_i \cos\alpha_i)\tan\phi_i' + (\cos\alpha_i + \lambda f_i \sin\alpha_i)FS] = \\ E_{i-1}[(\sin\alpha_i - \lambda f_{i-1}\cos\alpha_i)\tan\phi_i' + (\cos\alpha_i + \lambda f_{i-1}\sin\alpha_i)FS] + T_i FS - R_i \tag{13}$$

in which

$$T_i = W_i \sin\alpha_i + k_h W_i \cos\alpha_i - Q_i \sin(\delta_i - \alpha_i) \tag{14}$$

$$R_i = [W_i \cos\alpha_i - K_h W_i \sin\alpha_i + Q_i \cos(\delta_i - \alpha_i) - U_i] \times \tan\phi_i + c'_i l_i \tag{15}$$

Equation 13 is rearranged in the form as follows:

$$E_i \Phi_i = \psi_{i-1} E_{i-1} \Phi_{i-1} + T_i FS - R_i \tag{16}$$

in which

$$\Phi_i = (\sin\alpha_i - \lambda f_i \cos\alpha_i)\tan\phi_i + (\cos\alpha_i + \lambda f_i \sin\alpha_i)FS \tag{17}$$

$$\Phi_{i-1} = (\sin\alpha_{i-1} - \lambda f_{i-1}\cos\alpha_{i-1})\tan\phi_{i-1} + (\cos\alpha_{i-1} + \lambda f_{i-1}\sin\alpha_{i-1})FS \tag{18}$$

$$\psi_{i-1} = [(\sin\alpha_i - \lambda f_{i-1}\cos\alpha_i)\tan\phi_i + (\cos\alpha_i + \lambda f_{i-1}\sin\alpha_i)FS]/\Phi_{i-1} \tag{19}$$

With the condition $E_0 = 0$ and $E_n = 0$ (where E_0 and E_n are the inter-slice forces at the boundaries), from Equation 16 the force equilibrium equation is derived in the form of an expression for the factor of safety in the form as follows:

$$FS = \frac{\sum_{i=1}^{n-1}\left(R_i \prod_{j=i}^{n-1} \psi_j\right) + R_n}{\sum_{i=1}^{n-1}\left(T_i \prod_{j=i}^{n-1} \psi_j\right) + T_n} \tag{20}$$

Consider the summation of moments about the center point of the base of the *i*th slice.

$$E_i\left[h_L - \frac{b_i}{2}\tan\alpha_i\right] = E_{i-1}\left[h_R + \frac{b_i}{2}\tan\alpha_i\right] - \lambda\frac{b_i}{2}(f_i E_i + f_{i-1}E_{i-1}) + k_h W_i \frac{h_i}{2} - Q_i \sin\delta_i h_i \tag{21}$$

The moment equilibrium equation is derived in the form of an explicit expression for the scaling factor λ as follows:

$$\lambda = \frac{\sum_{i=1}^{n}[b_i(E_i + E_{i-1})\tan\alpha_i + K_h W_i h_i + 2Q_i \sin\delta_i h_i]}{\sum_{i=1}^{n}[b_i(f_i E_i + f_{i-1}E_{i-1})]} \tag{22}$$

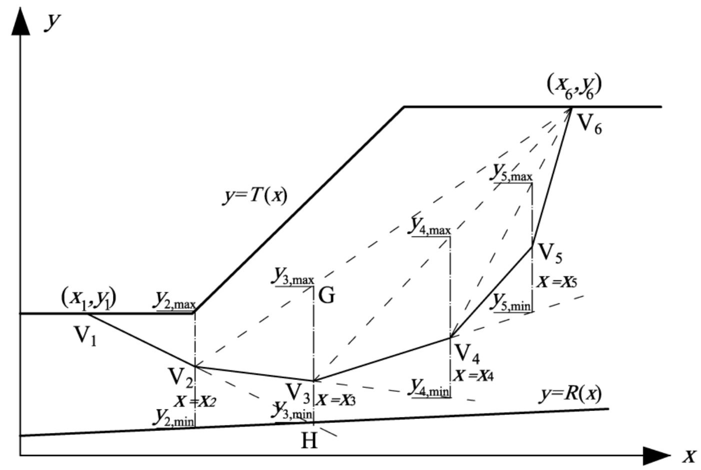

Figure 2. Procedure for generating acceptable failure surface.

To solve for the factor of safety, first specify the form of the inter-slice function $f(x)$ and assume the initial value for FS and λ. The appropriate choice for initial values of FS and λ are 1 and 0, respectively (Zhu et al., 2005). Then, FS is obtained by an iterative procedure. After that, the values of E_i and λ are calculated based on the Equations 16 and 22. Finally, the factor of safety is recalculated with these computed values of scaling factor. The iterative procedure is completed when the difference between computed values of FS and λ is within an acceptable tolerance.

GENERATION OF GENERAL SLIP SURFACE

To find the position of critical slip surface, a trial failure surface generation algorithm is required. In this study, the procedure for the generation of potential failure surfaces proposed by Cheng (2003) is used. Consider the Cartesian coordinate system xy as shown in Figure 2. The mathematical function $y=T(\mathbf{x})$ describes the geometry of the slope and bedrock line may be presented by another mathematical function $y=R(\boldsymbol{x})$.

The essential of slice method requires the failure soil mass to be divided into n vertical slices and the slip surface is represented by $n + 1$ vertices $[V_1, V_2,...,V_{n+1}]$ with coordinates (x_1, y_1), (x_2, y_2), . . . , (x_{n+1}, y_{n+1}) as follows:

$$V = [x_1, y_1, x_2, y_2, \ldots, x_n, y_n, x_{n+1}, y_{n+1}] \tag{23}$$

To satisfy the requirements of kinematic acceptability, these slices, defined by any two adjacent nodal points, are further assumed to be concave upward. The concave upward requirement can be formulated as:

$$\alpha_1 \le \alpha_2 \le \ldots \le \alpha_i \le \ldots \le \alpha_n \tag{24}$$

where α_i is the inclination of the base of the slice i. In order to reduce the number of variables, the horizontal distance between x_i and x_{n+1} can simply be subdivided into n equal segments using the equation:

$$x_i = x_1 + \frac{x_{n+1} - x_1}{n} \times (i-1) \qquad for\ i = 2,3,\ldots,n \tag{25}$$

Moreover, y_1 and y_n are related to the slope geometry and as a result, a specific slip surface can be identified mathematically by an n-element vector:

$$\boldsymbol{V}=[x_1, x_{n+1}, y_2, y_3, \ldots, y_n] \tag{26}$$

The upper and lower bounds to the y-coordinates ($y_{i,\max}$ and $y_{i,\min}$) can be obtained by utilizing the geometry of the slope and the bedrock line, and may be calculated as follows:

$$\begin{cases} y_{i,\min} = R(x_i) \\ y_{i,\max} = y_1(x_i) \end{cases} \tag{27}$$

The solution domains for variables x_1 and x_{n+1} can easily be defined by engineering experience or sufficiently wide domains can be specified.

After the values of x_1 to x_{n+1} are generated by even division between x_1 and x_{n+1}, $y_{2\min}$ and $y_{2\max}$ can be determined by geometry, $y_1(\boldsymbol{x})$ and bed rock line, $R(\boldsymbol{x})$, of the slope. Then, y_2 is randomly generated in the range of $[y_{2\min}, y_{2\max}]$. The line between points V_{n+1} and V_2 is intersected with line $x=x_3$ at point G with y-ordinate y_G, and the line passing through points V_1 and V_2 is extended to intersect with line $x=x_3$ at point H and y-ordinate y_H is received. After that, $y_{3\min}$ and $y_{3\max}$ can be determined using the following equation:

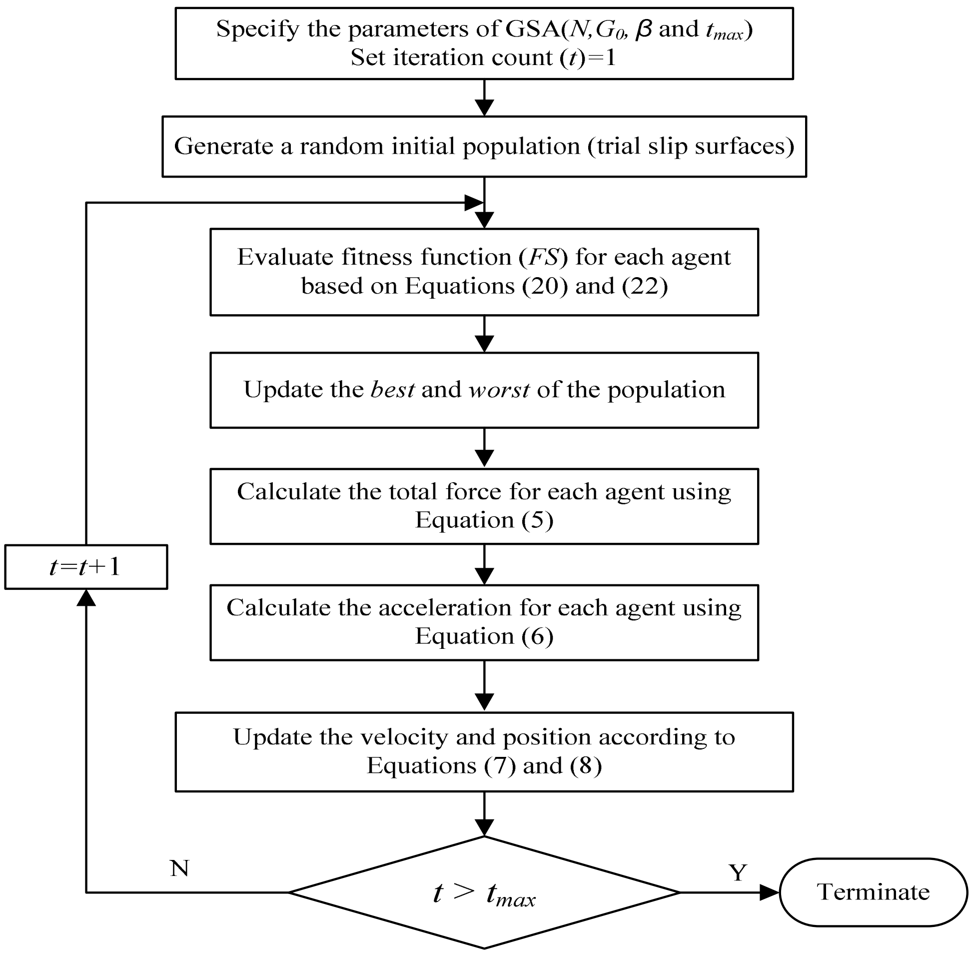

Figure 3. Flowchart showing the application of GSA for slope stability analysis.

$$\begin{cases} y_{3,\min} = \max\{y_H, R(x_3)\} \\ y_{3,\max} = \min\{y_G, y_1(x_3)\} \end{cases} \quad (28)$$

$y_{4\min}$, $y_{4\max}$, . . . , $y_{n\ \min}$, $y_{n\ \max}$ can be obtained in away similar to the procedures described earlier.

NUMERICAL STUDIES

In this part, the validity and effectiveness of the proposed GSA algorithm are investigated by solving three numerical examples from the literature. These examples include problems with homogeneous and inhomogeneous soils. One of the problems also considers the effects of an earthquake and ground water. The implementation procedure of the proposed GSA for slope stability analysis is presented as a flowchart in Figure 3. In each example, the sliding masses are divided into 40 slices and all vertical slices are assumed to have the same width. The computed factors of safety using GSA are compared with those obtained from other methods. In the following examples, proper fine tuning of GSA's parameters is evaluated by several experimental studies examining the effect of each parameter on the final solution and convergence of the algorithm. As a result, parameters of GSA are set as follows: population size is 50; maximum iteration number is 1000; G_0 = 100 and β = 20. The optimization procedure was terminated when the maximum number of iterations is reached.

Example 1

Slope in homogeneous soil

The first example is a slope in homogeneous soil and it is taken from the study by Zolfaghari et al. (2005). For this case the effective friction angle of soil (φ') is 20°, the effective cohesion intercept (c') is 15 kPa and unit weight

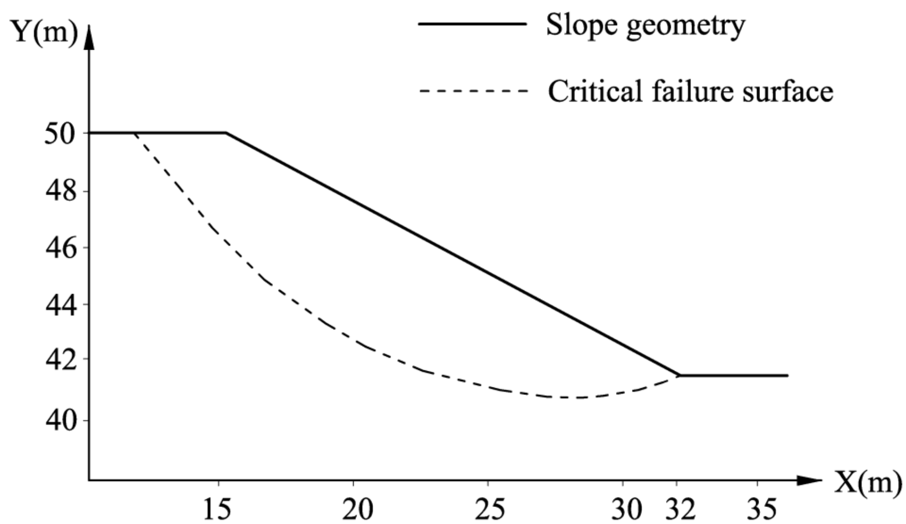

Figure 4. Geometry of slope for Example 1.

Table 1. Minimum safety factor for Example 1.

Optimization method	Minimum factor of safety
Genetic algorithm (Zolfaghari et al., 2005)	1.75
Simulated annealing (Cheng et al., 2007)	1.7267
Genetic algorithm (Cheng et al., 2007)	1.7297
Particle swarm optimization (Cheng et al., 2007)	1.7284
Simple harmony search (Cheng et al., 2007)	1.7264
Modified harmony search (Cheng et al., 2007)	1.7279
Tabu search (Cheng et al., 2007)	1.7415
Ant colony optimization (Cheng et al., 2007)	1.7647
Gravitational search algorithm (present study)	1.7184

is set to 17.64 kN/m^3. Figure 4 depicts the geometry of the slope.

Zolfaghari et al. (2005) solved the problem and employed simple genetic algorithm combined with the Morgenstern and Price method by considering the inclinations of the inter slice forces as constant (f (x) =1) to search for the minimum factor of safety. Cheng et al. (2007) solved the problem using the Spencer method, which is equivalent to the Morgenstern and Price method with f (x) =1, together with the different heuristic global optimization methods include simulated annealing (SA), genetic algorithm (GA), particle swarm optimization (PSO), simple harmony search (SHS), modified harmony search (MHS), tabu search (TS) and ant colony optimization (ACO) for finding the minimum factor of safety.

This case is solved using the proposed methodology and the comparison of the results with those obtained by different methods is summarized in Table 1. The first column of the table shows the optimization method applied for the solution and second column shows the minimum factor of safety related to the critical failure surface. From analyzing the results of Table 1, it can be observed that the factor of safety obtained by GSA is 1.7184, which is lower than those calculated using other methods. As mentioned before, in the slope stability problems the objective function of the safety factor is a complex optimization problem with the presence of several local minima points within the solution domain. Therefore, application of GSA as an effective global optimization method could provide a better solution for the problem and found the lower factor of safety and a more critical failure surface as compared to the other methods. The critical slip surface found by the GSA is shown in Figure 4. Moreover Table 1 shows that, SHM, SA, MHM and PSO algorithms could provide a reasonable solution near optimum for the homogenous slope. However, TS and ACO give the larger factors of safety and are not efficient methods for the solution. Table 1 also shows that, the results of genetic algorithm

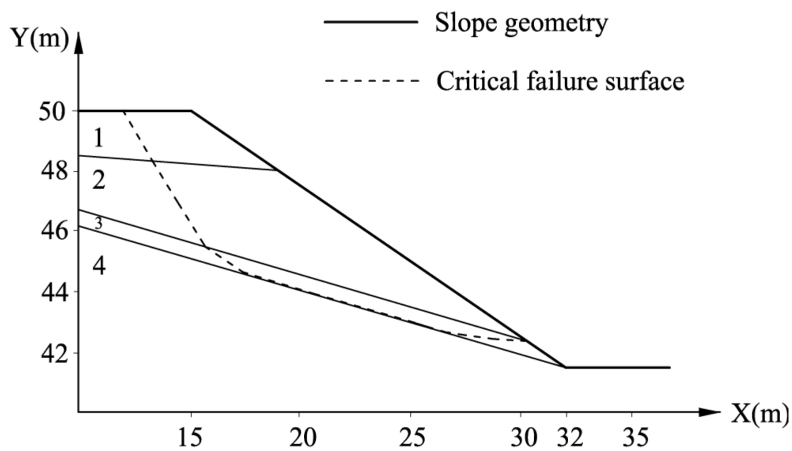

Figure 5. Geometry of slope for Example 2.

Table 2. Soil parameters for Examples 2 and 3.

Layer of soil	Unit weight (kN/m^3)	Effective cohesion intercept (kPa)	Effective friction angle (º)
Layer 1	19.0	15.0	20.0
Layer 2	19.0	17.0	21.0
Layer 3	19.0	5.0	10.0
Layer 4	19.0	35.0	28.0

Table 3. Minimum safety factor for Example 2.

Optimization method	Minimum factor of safety
Genetic algorithm (Zolfaghari et al., 2005)	1.24
Simulated annealing (Cheng et al., 2007)	1.2813
Genetic algorithm (Cheng et al., 2007)	1.1440
Particle swarm optimization (Cheng et al., 2007)	1.1095
Simple harmony search (Cheng et al., 2007)	1.2068
Modified harmony search (Cheng et al., 2007)	1.1385
Tabu Search (Cheng et al., 2007)	1.4650
Ant colony optimization (Cheng et al., 2007)	1.5817
Gravitational search algorithm (present study)	1.0785

from the study of Zolfaghari et al. (2005) and Cheng et al. (2007) are almost different and it may be caused by the differences between the slip surface generation methods applied in each study.

Example 2

Slope in layered soil

The second example is also abstracted from the work by Zolfaghari et al. (2005) which is a slope in multilayered soil. Geometrical feature of the slope is shown in Figure 5 and the geotechnical parameters of the various layers are presented in Table 2.

To solve this problem, Zolfaghari et al. (2005) used genetic algorithm combined with the Morgenstern and Price method with the assumption of $f(x) = 1$. Similarly, Cheng et al. (2007) applied the Spencer method to formulate the safety factor and different heuristic global optimization methods (SA, GA, PSO, SHM, MHM, TS and ACO), for the solution.

This problem is also solved using the proposed method. Table 3 compares the minimum factor of safety

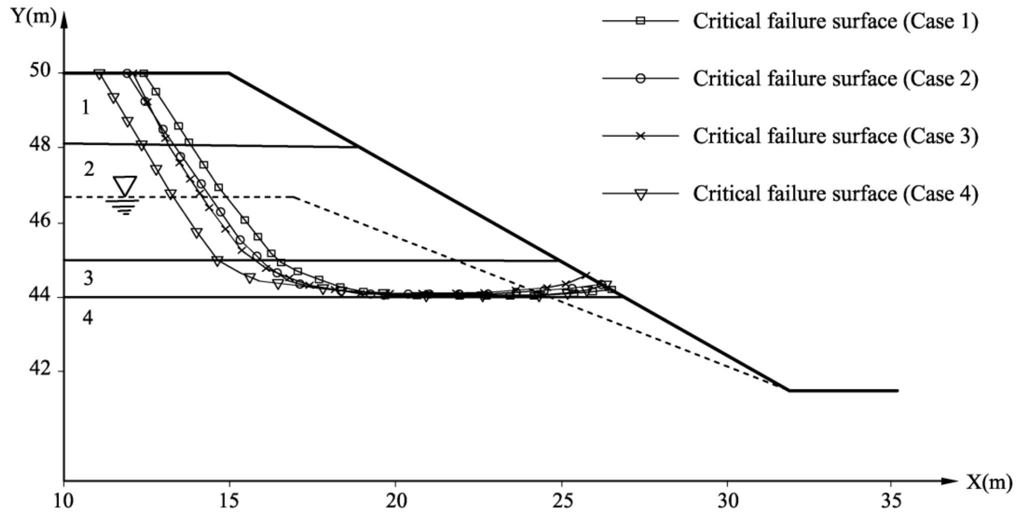

Figure 6. Geometry of slope for Example 3.

Table 4. Minimum safety factor for Example 3.

Optimization method	Loading case			
	Case 1	Case 2	Case 3	Case 4
Genetic algorithm (Zolfaghari et al., 2005)	1.48	1.36	1.37	0.98
Simulated annealing (Cheng et al., 2007)	1.3961	1.2837	1.1334	1.0081
Genetic algorithm (Cheng et al., 2007)	1.3733	1.2324	1.0675	0.9631
Particle swarm optimization (Cheng et al., 2007)	1.3372	1.2100	1.0474	0.9451
Simple harmony search (Cheng et al., 2007)	1.3729	1.2326	1.0733	0.9570
Modified harmony search (Cheng et al., 2007)	1.3501	1.2247	1.0578	0.9411
Tabu Search (Cheng et al., 2007)	1.4802	1.3426	1.1858	1.0848
Ant colony optimization (Cheng et al., 2007)	1.5749	1.4488	1.3028	1.1372
Ant colony optimization (Kahatadeniya et al., 2009)	1.501	1.377	1.091	0.846
Gravitational search algorithm (present study)	1.3268	1.2038	1.0451	0.8435

obtained from the current study with the previously reported outcomes. From the results of this table, it can be seen that, the factor of safety achieved by GSA is 1.0785 which is smaller and therefore, better than the other optimization methods. Moreover, Figure 5 depicts the critical failure surface found by the proposed method. Similar to the first example, the ACO and TS methods did not reach a good result for the stratified slope.

Example 3

Slope subjected to water pressure and earthquake

The third example is also taken from the study of Zolfaghari et al. (2005) where an inhomogeneous slope comprises of four layers of different soils as well as a water table as shown in Figure 6. Except for the boundary between the first and second layers where slight inclination of the boundary exists, the other layer boundaries are horizontal. In addition to the ground water, the pseudo-static coefficient for horizontal earthquake loading of 0.1 is considered. Again, the geotechnical parameters are tabulated in Table 2. The problem is solved with four different loading cases (Zolfaghari et al., 2005), no earthquake and no water pressure (case 1), no earthquake and water pressure (case 2), earthquake and no water pressure (case 3) and earthquake and water pressure (case 4).

This example is also solved by Zolfaghari et al. (2005) and Cheng et al. (2007) with the same methodology which was discussed in Example 1. Moreover, Kahatadeniya et al. (2009) applied the Morgenstern and Price method with the assumption of $f(x)=1$ together with ant colony optimization (ACO) algorithm for the minimization of the safety factor. The results obtained by the previous researchers and the present study are summarized in Table 4. It is obvious that the result obtained in the current study is comparable with the results found in the literatures. Besides, the factor of safety obtained by GSA is found to be smaller than the others for all cases. Figure 6 presents the critical slip surfaces found by the proposed method for each loading case. Table 4 also shows that, the results of the GA from the study of Zolfaghari et al. (2005) and Cheng et al. (2007) or the results of the ACO from the study of Kahatadeniya et al. (2009) and Cheng et al. (2007) are almost different. It is maybe due to the differences between the slip surface generation method applied in each study or differences between the algorithms' parameters selection.

Conclusions

This study presents an effective method based on gravitational search algorithm (GSA) to determine the minimum factor of safety and its associated failure surface in slope stability analysis. The proposed method is classifiable as direct search algorithms and does not require any continuity or derivatives of the objective function (safety factor). The present algorithm employs the simple form of Morgenstern and Price method for evaluation of the safety factor and locating the general-shaped critical failure surface. The general-shaped potential slip surfaces are generated using the straight line technique. To use the GSA for slope stability analysis, a computer program is developed in MATLAB. The efficiency and accuracy of the presented method were investigated through the three numerical examples of slope stability analysis, including simple and complicated slopes, which were solved by different optimization algorithms in the previous literature. The comparison of the results of case studies in terms of the minimum factor of safety demonstrated that the proposed method found much lower factor of safety and more critical slip surface and significantly outperforms the other methods in the literatures. In addition, the results show that GSA can be successfully employed to analyze the slope stability problems especially to the complicated slopes containing complex slope geometry, layered soil profile and complex loadings.

ACKNOWLEDGEMENT

The research reported herein was supported by the Ministry of Higher Education, Malaysia (Grant No. UKM-AP-PLW-04-2009/2). This support is gratefully acknowledged.

REFERENCES

Baker R (1980). Determination of the critical slip surface in slope stability computations. Int. J. Numer. Anal. Met., 4(4): 333-359.
Bishop AW (1955). The use of the slip circle in the stability analysis of earth slopes. Geotechinque, 5(1): 7-17.
Cheng YM (2003). Location of critical failure surface and some further studies on slope stability analysis. Comput. Geotech., 30(3): 255-267.
Cheng YM, Li L, Chi SC (2007). Performance studies on six heuristic global optimization methods in the location of critical slip surface. Comput. Geotech., 34(6): 462-484.
Cheng YM, Lau CK (2008). Slope stability analysis and stabilization: new methods and insight. Taylor & Francis, London.
Cheng YM, Liang L, Chi SC, Wei WB (2008). Determination of the critical slip surface using artificial fish swarms algorithm. J. Geotech. Geoenviron., 134(2): 244-251.
Fredlund D, Krahn J (1977). Comparison of slope stability methods of analysis. Can. Geotech. J., 14(3): 429-439.
Greco VR (1996). Efficient Monte Carlo technique for locating critical slip surface. J. Geotech. Eng., 122 (7): 517-525.
Janbu N (1973). Slope stability computations. Embankment Dam Engineering. John Wiley, New York.
Kahatadeniya KS, Nanakorn P, Neaupane KM (2009). Determination of the critical failure surface for slope stability analysis using ant colony optimization. Eng. Geol., 108 (1-2): 133-141.
Khajehzadeh M, Taha MR, El-Shafie A (2010). Modified particle swarm optimization for probabilistic slope stability analysis. Int. J. Phys. Sci., 5(15): 2248-2258.
Malkawi A, Hassan WF, Sarma SK (2001). Global search method for locating general slip surface using Monte Carlo techniques. J. Geotech. Geoenviron, 127: 688-698.
Morgenstern NR, Price VE (1965). The analysis of the stability of general slip surfaces. Geotechinque, 15(1): 79-93.
Pham HTV, Fredlund DG (2003). The application of dynamic programming to slope stability analysis. Can. Geotech. J., 40 (4): 830-847.
Rashedi E, Nezamabadi-pour H, Saryazdi S (2009). GSA:agravitationalsearch algorithm. Inform. Sciences, 179(13): 2232-2248.
Spencer E (1967). A method of analysis of the stability of embankments assuming parallel inter-slice forces. Geotechnique, 17(1): 11-26.
Zhu DY, Lee CF, Qian QH, Chen GR (2005). A concise algorithm for computing the factor of safety using the Morgenstern-Price method. Can. Geotech. J., 42(1): 272-278.
Zolfaghari AR, Heath AC, McCombie PF (2005). Simple genetic algorithm search for critical non-circular failure surface in slope stability analysis. Comput. Geotech., 32(3): 139-152.