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Full Length Research Paper

Synchronization and anti-synchronization of two identical hyperchaotic systems based on active backstepping design

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This paper presents an active backstepping design method for synchronization and anti-synchronization of two identical hyperchaotic Chen systems. The proposed control method, combining backstepping design and active control approach, extends the application of backstepping technique in chaos control. Based on this method, different combinations of controllers can be designed to meet the needs of different applications. Numerical simulations are shown to verify the results.

Key words: Synchronization, anti-synchronization, hyperchaotic system, active backstepping design.

INTRODUCTION

Since the pioneering work of Pecora and Corroll, chaos synchronization has become an active research subject in the nonlinear field for its many potential applications in chemical reactions, biological systems, information processing, power converters, secure communications, etc. The excitement is well comprehended in the academic community as its potential implications and applications are bountiful. Another interesting phenomenon discovered was the anti-synchronization (AS), which is noticeable in periodic oscillators. It is a well-known fact that the first observation of synchronization between two oscillators by Huygens in the seventeenth century was, in fact, an AS between two pendulum clocks. Recent re-investigation of Huygens experiment by Blekhman (Bennett et al., 2002) shows that either synchronization or AS can appear depending on the initial conditions of the coupled pendula. Here, AS can also be interpreted as anti-phase synchronization (APS) (Al-Sawalha and Noorani, 2009; Ruihong et al., 2009). In other words, there is no difference between AS and APS for oscillators with identical amplitudes. So far, there exist many types of synchronization such as complete synchronization (CS) (Agiza, 2004), phase synchronization (PS) (Junge and Parlitz, 2000; Li and Chen, 2004), anti-synchronization (Al-sawalha et al., 2010), partially synchronization (Femat and Solis-Perales, 1999), generalized synchronization (GS) (Kacarev and Parlitz, 1996), lag synchronization (Shahverdiev et al., 2002), Q-S synchronization (Yan et al., 2005), projective synchronization (Hu and Xu, 2008; Hu et al., 2008), etc. In this paper we will consider the synchronization and anti-synchronization of two identical hyperchaotic Chen systems. The active backstepping design (Wang et al., 2007; Yu and Li, 2010; Yu and Zhang, 2004; Tan et al., 2003; Ge et al., 2000; Wang and Ge, 2001) is employed to realize the synchronization and anti-synchronization of hyperchaotic systems.

SYSTEM DESCRIPTION

The hyperchaotic Chen system is given by:

$$\begin{cases} \dot{x} = a(y-x) + w \\ \dot{y} = dx - xz + cy \\ \dot{z} = xy - bz \\ \dot{w} = yz + rw \end{cases} \tag{1}$$

where x, y, z and w are state variables, and a, b, c, d and r are real constants.

SYNCHRONIZATION OF TWO IDENTICAL HYPERCHAOTIC CHEN SYSTEMS VIA ACTIVE BACKSTEPPING

Here, we develop a design procedure via active backstepping

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which can realize the synchronization and anti-synchronization of two identical hyperchaotic systems by giving the corresponding update laws. To verify the effectiveness of the proposed method, two identical hyperchaotic Chen systems as the master and slave system for detailed description is used. Suppose the master system is defined as the following form:

$$\begin{cases} \dot{x}_1 = a(y_1 - x_1) + w_1 \\ \dot{y}_1 = dx_1 - x_1 z_1 + cy_1 \\ \dot{z}_1 = x_1 y_1 - bz_1 \\ \dot{w}_1 = y_1 z_1 + rw_1 \end{cases} \quad (2)$$

and the slave system is taken as follows:

$$\begin{cases} \dot{x}_2 = a(y_2 - x_2) + w_2 + u_1 \\ \dot{y}_2 = dx_2 - x_2 z_2 + cy_2 + u_2 \\ \dot{z}_2 = x_2 y_2 - bz_2 + u_3 \\ \dot{w}_2 = y_2 z_2 + rw_2 + u_4 \end{cases} \quad (3)$$

where $u_i(t)(i = 1,2,3,4)$ are controllers to be designed by the following backstepping method.

Define $e_1 = x_2 - x_1$, $e_2 = y_2 - y_1$, $e_3 = z_2 - z_1$ and $e_4 = w_2 - w_1$. Subtracting Equation 3 from Equation 2, we have:

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + e_4 + u_1 \\ \dot{e}_2 &= de_1 - x_2 z_2 - x_1 z_1 + ce_2 + u_2 \\ \dot{e}_3 &= x_2 y_2 + x_1 y_1 - be_3 + u_3 \\ \dot{e}_4 &= y_2 z_2 + y_1 z_1 + re_4 + u_4 \end{aligned} \quad (4)$$

Our goal is to find the controller functions which can make the systems of Equations 2 and 3 realize the synchronization by active backstepping design, that is:

$\mathrm{Lim}_{t\to\infty}\|e_i(t)\| = 0\,, i = 1, 2, 3, 4$.

The backstepping design procedure includes four steps as follow:

Step 1

Let $\mathrm{k}_1 = \mathrm{e}_1$, its derivative is given as:

$$\dot{k}_1 = \dot{e}_1 = a(e_2 - e_1) + e_4 + u_1 \quad (5)$$

Choosing $e_2 = \alpha_1(k_1)$ is regarded as a virtual controller. For the design of $\alpha_1(k_1)$ to stabilize $k_1 -$ subsystem in Equation 5, we select the first Lyapunov function:

$$v_1 = \frac{1}{2}{k_1}^2 \quad (6)$$

The derivative of v_1 is given as:

$$\dot{v}_1 = k_1(a\alpha_1(k_1) - ak_1 + \alpha_3(k_1, k_2, k_3) + u_1$$

If we set $\alpha_1(k_1) = 0, \alpha_3(k_1, k_2, k_3) = 0$, and $u_1 = -ak_1$ (7)

Then $\dot{v}_1 = -2a{k_1}^2 < 0$ is negative definite.

This implies that the k_1- subsystem (Equation 5) is asymptotically stable. Since the virtual control function α_1 is estimative, the error between e_2 and α_1 is given as:

$$k_2 = e_2 - \alpha_1(\mathrm{k}_1) \quad (8)$$

Studying the following (k_1, k_2)-subsystem.

$$\begin{aligned} \dot{k}_1 &= -2ak_1 + ak_2 + k_4 \\ \dot{k}_2 &= -c\mathrm{k}_2 \end{aligned} \quad (9)$$

Step 2

In order to stabilize the (k_1, k_2)-subsystem (Equation 9), we choose the second Lyapunov function defined by:

$$v_2 = v_1 + \frac{1}{2}{k_2}^2 \quad (10)$$

Its derivative is given as:

$$\dot{v}_2 = -2a{k_1}^2 + k_2(dk_1 + ck_2 - x_1 z_1 - x_2 z_2 + u_2) \quad (11)$$

Let $u_2 = x_1 z_1 + x_2 z_2 - dk_1 - 2ck_2,$ then $\dot{v}_2 = -2a{k_1}^2 - c{k_2}^2 < 0$ makes subsystem (Equation 9) asymptotically stable. Similarly, we assume that $k_3 = e_3 - \alpha_2(k_1, k_2)$, then we have:

$$\begin{cases} \dot{k}_1 = -2ak_1 + ak_2 + k_4 \\ \dot{k}_2 = -c\mathrm{k}_2 \\ \dot{k}_3 = -(b+1)k_3 \end{cases} \quad (12)$$

Step 3

To stabilize system (Equation 12), we consider the following Lyapunov function:

$$v_3 = v_2 + \frac{1}{2}{k_3}^2 \quad (13)$$

Taking its derivative, we have:

$$\dot{v}_3 = -2a{k_1}^2 - c{k_2}^2 + k_3(-be_3 + x_1 y_1 + x_2 y_2 + u_3) \quad (14)$$

Let $u_3 = -x_1 y_1 - x_2 y_2 - k_3$ and $\alpha_2(k_1, k_2) = 0$, then $\dot{v}_3 = -2a{k_1}^2 - c{k_2}^2 - k^2{}_3 < 0$ is negative definite and makes the (k_1, k_2, k_3) system (Equation 12) asymptotically stable. Similarly, we assume that $k_4 = e_4 - \alpha_3(k_1, k_2, k_3)$, then we obtain:

$$\begin{aligned} \dot{k}_1 &= -2ak_1 + ak_2 + k_4 \\ \dot{k}_2 &= -c\mathrm{k}_2 \\ \dot{k}_3 &= -(b+1)k_3 \\ \dot{k}_4 &= (r-1)e_4 \end{aligned} \quad (15)$$

Step 4

In order to stabilize system (Equation 15), the following Lyapunov function can be chosen as:

$$v_4 = v_3 + \frac{1}{2}{k_4}^2 \quad (16)$$

Its derivative is given as:

$$\dot{v}_4 = -2ak_1{}^2 - ck_2{}^2 - k_3{}^2 + k_4(r\alpha_3(k_1, k_2, k_3) + y_1 z_1 + y_2 z_2 + u_4 \quad (17)$$

Let $u_4 = -y_1 z_1 - y_2 z_2 - k_4$ and $\alpha_3 = 0$, so $\dot{v}_4 = -2ak_1{}^2 - ck_2{}^2 - k^2{}_3 - k^2{}_4 < 0$, is negative definite and the (k_1, k_2, k_3, k_4) system (Equation 15) is asymptotically stable.

Simulation and results

In this simulation, the parameters are chosen as $\mathrm{a} = 35$, $\mathrm{b} = 3$, $\mathrm{c} = 12$, $\mathrm{d} = 7$, $\mathrm{r} = 0.5$, so that, the systems exhibits a hyperchaotic behavior. The initial values of the master and slave systems are $x_1(0) = -20$, $y_1(0) = 0$, $z_1(0) = 0$, $w_1(0) = 15$ and $x_2(0) = 5$, $y_2(0) = 7$, $z_2(0) = 9$, $w_2(0) = 11$. Figure 1 displays the time response of states x_1, y_1, z_1, w_1 for the master system (Equation 2) and the states x_2, y_2, z_2, w_2 for the slave system (Equation 3). Figure 2 displays the synchronization errors of systems (Equation 2) and (Equation 3). It can be seen from the figures that the synchronization errors converge to zero rapidly.

ANTI-SYNCHRONIZATION OF TWO IDENTICAL HYPERCHAOTIC CHEN SYSTEMS VIA ACTIVE BACKSTEPPING

In order to determine control functions and to realize the anti-synchronization via active backstepping between the systems in Equations 2 and 3, we add Equation 2 to Equation 3 to get:

$$\begin{aligned}
\dot{e}_1 &= a(e_2 - e_1) + e_4 + u_1 \\
\dot{e}_2 &= de_1 - x_2 z_2 + x_1 z_1 + ce_2 + u_2 \\
\dot{e}_3 &= x_2 y_2 - x_1 y_1 - be_3 + u_3 \\
\dot{e}_4 &= y_2 z_2 - y_1 z_1 + re_4 + u_4
\end{aligned} \quad (18)$$

where $e_1 = x_2 + x_1$, $e_2 = y_2 + y_1$, $e_3 = z_2 + z_1$ and $e_4 = w_2 + w_1$. Our goal is to find proper control functions, such that system Equation 3 globally becomes anti-synchronizes system Equation 2, that is, $\mathrm{Lim}_{t\to\infty}\|e_i(t)\| = 0$, $\mathrm{i} = 1, 2, 3, 4$.

Now, as the previous part, we design the controllers of Equation 3 according to active backstepping method.

Step 1

Let $\mathrm{k}_1 = \mathrm{e}_1$, its derivative is given as:

$$\dot{k}_1 = \dot{e}_1 = a(e_2 - e_1) + e_4 + u_1 \quad (19)$$

Let $e_2 = \alpha_1(k_1)$, it is regarded as a virtual controller. For the design of $\alpha_1(k_1)$ to stabilize k_1 –subsystem (Equation 19), we choose the first Lyapunov function:

$$v_1 = \frac{1}{2}k_1{}^2 \quad (20)$$

Its derivative is given by:

$$\dot{v}_1 = k_1(a\alpha_1(k_1) - ak_1 + \alpha_3(k_1, k_2, k_3) + u_1 \quad (21)$$

If we set $\alpha_1(k_1) = 0$, $\alpha_3(k_1, k_2, k_3) = 0$ and $u_1 = -ak_1$. Then $\dot{v}_1 = -2ak_1{}^2 < 0$ is negative definite. This implies that the k_1- subsystem (Equation 19) is asymptotically stable. Since the virtual control function α_1 is estimative, choosing:

$$k_2 = e_2 - \alpha_1(\mathrm{k}_1) \quad (22)$$

Study the following (k_1, k_2)-subsystem:

$$\begin{cases} \dot{k}_1 = -2ak_1 + ak_2 + k_4 \\ \dot{k}_2 = -c\mathrm{k}_2 \end{cases} \quad (23)$$

Step 2

To stabilize the (k_1, k_2)-subsystem (Equation 23), the following Lyapunov function is chosen:

$$v_2 = v_1 + \frac{1}{2}k_2{}^2 \quad (24)$$

Its derivative is given as:

$$\dot{v}_2 = -2ak_1{}^2 + k_2(dk_1 + ck_2 + x_1 z_1 - x_2 z_2 + u_2) \quad (25)$$

If $u_2 = -x_1 z_1 + x_2 z_2 - dk_1 - 2ck_2$, then, $\dot{v}_2 = -2ak_1{}^2 - ck_2{}^2 < 0$ makes subsystem (Equation 23) asymptotically stable. Similarly, assume that $k_3 = e_3 - \alpha_2(k_1, k_2)$ then,

$$\begin{aligned}
\dot{k}_1 &= -2ak_1 + ak_2 + k_4 \\
\dot{k}_2 &= -c\mathrm{k}_2 \\
\dot{k}_3 &= -(b+1)k_3
\end{aligned} \quad (26)$$

Step 3

For stabilization of system (Equation 26), let the Lyapunov function candidate V_3 be such that:

$$v_3 = v_2 + \frac{1}{2}k_3{}^2 \quad (27)$$

Taking the time derivative of V_3, gives:

$$\dot{v}_3 = -2ak_1{}^2 - ck_2{}^2 + k_3(-be_3 + x_1 y_1 + x_2 y_2 + u_3) \quad (28)$$

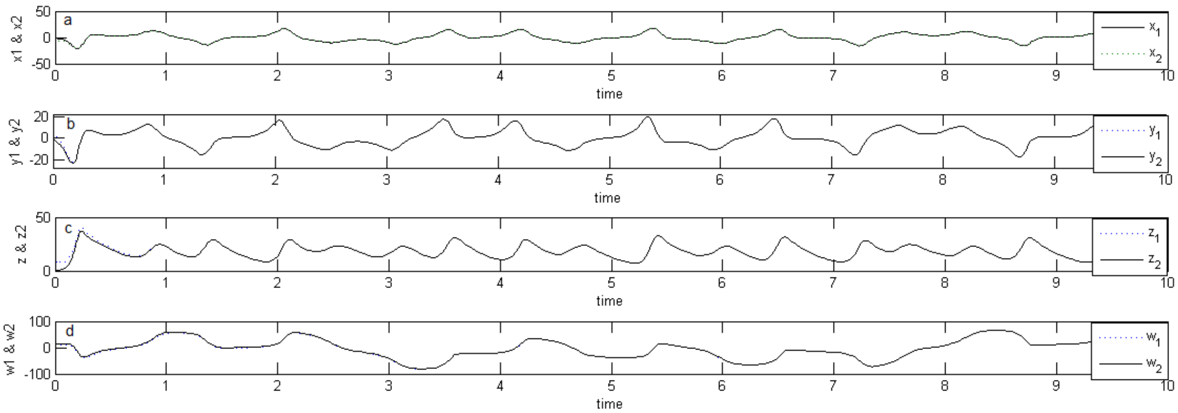

Figure 1. The time response of states for master system (2) and the slave system (3) for synchronization via active backstepping (a) signals x_1 and x_2; (b) signals y_1 and y_2; (c) signals z_1 and z_2;(d) signsla $w_1$and w_2.

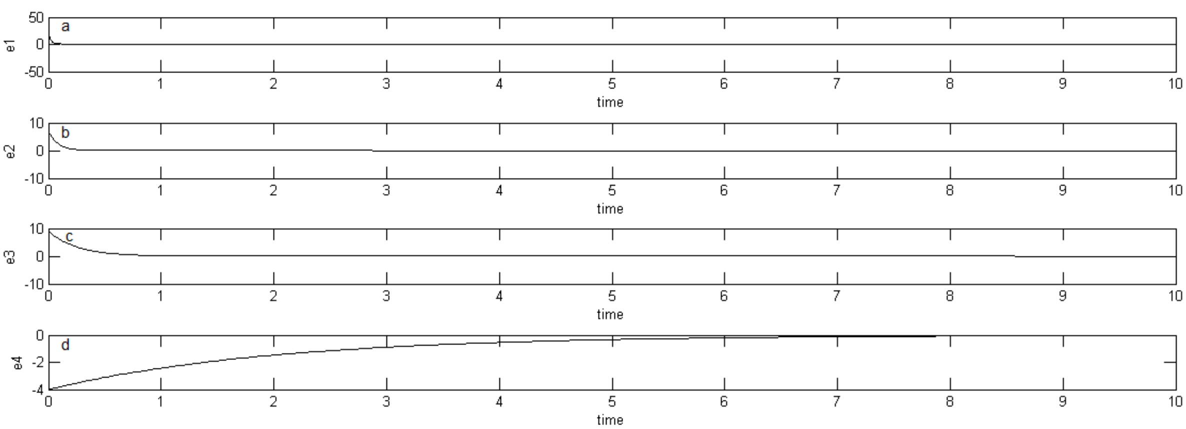

Figure 2. Dynamics of synchronization errors for two identical hyperchaotic Chen systems (master system (2) and slave system (3)) (a) $e_1 = x_2 - x_1$ (b) $e_2 = y_2 - y_1$; (c) $e_3 = z_2 - z_1$; (d) $e_4 = w_2 - w_1$.

Select $u_3 = +x_1y_1 - x_2y_2 - k_3$ and $\alpha_2(k_1,k_2) = 0$, so $\dot{v}_3 = -2ak_1{}^2 - ck_2{}^2 - k^2{}_3 < 0$ is negative definite and the (k_1,k_2,k_3) system (Equation 26) is asymptotically stable. Now, choosing $k_4 = e_4 - \alpha_3(k_1,k_2,k_3)$, cause the following equation:

$$\dot{k}_1 = -2ak_1 + ak_2 + k_4$$

$$\dot{k}_2 = -ck_2 \tag{29}$$

$$\dot{k}_3 = -(b+1)k_3$$

$$\dot{k}_4 = (r-1)e_4$$

Step 4

To stabilize system (Equation 15), the following Lyapunov is selected:

$$v_4 = v_3 + \frac{1}{2}k_4{}^2 \tag{30}$$

The time derivative of V_4 is given as:

$$\dot{v}_4 = -2ak_1{}^2 - ck_2{}^2 - k_3{}^2 + k_4(r\alpha_3(k_1,k_2,k_3)y_1z_1 + -y_2z_2 + u_4 \tag{31}$$

If $u_4 = +y_1z_1 - y_2z_2 - k_4$ and $\alpha_3 = 0$, so $\dot{v}_4 = -2ak_1{}^2 - ck_2{}^2 - k^2{}_3 - k^2{}_4 < 0$, is negative definite and makes the (k_1,k_2,k_3,k_4) system (15) asymptotically stable.

Simulation and results

To verify and demonstrate the effectiveness of the proposed method, we discuss the simulation result for the anti-synchronization between two identical hyperchaotic Chen systems. The parameters and initial values are chosen as Equation 3. Figure 3 displays the time response

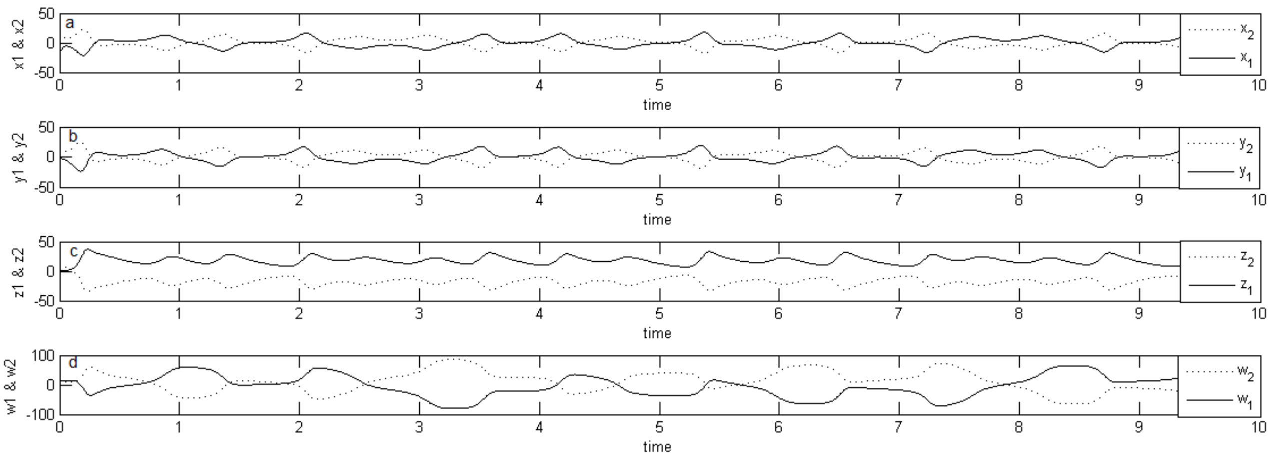

Figure 3. The time response of states for master system (2) and the slave system (3)) for anti-synchronization via active backstepping (a) signals x_1 and x_2; (b) signals y_1 and y_2; (c) signals z_1 and z_2; (d) signals w_1 and w_2.

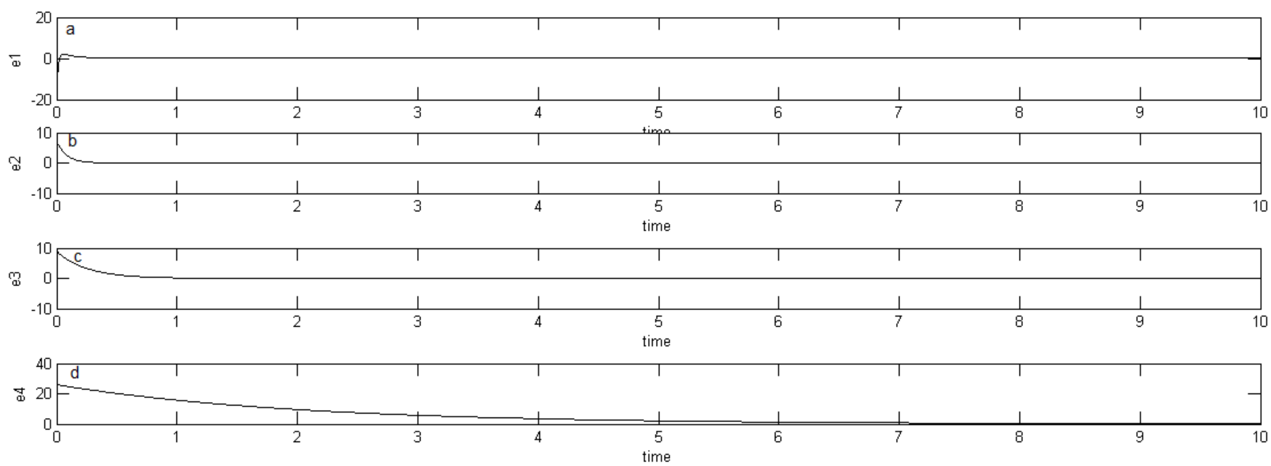

Figure 4. Dynamics of anti-synchronization errors for two identical hyperchaotic Chen systems (master system (2) and slave system (3)) (a) $e_1 = x_2 + x_1$ (b) $e_2 = y_2 + y_1$; (c) $e_3 = z_2 + z_1$; (d) $e_4 = w_2 + w_1$.

of states x_1, y_1, z_1, w_1 for the master system (Equation 2) and the states x_2, y_2, z_2, w_2 for the slave system (Equation 3). Figure 4 displays the synchronization errors of systems (Equations 2 and 3). The simulation results indicate that the proposed active backstepping controller works well in both phenomenon synchronization and anti-synchronization.

CONCLUSION

This paper mainly presents the synchronization and anti-synchronization between two identical hyperchaotic systems based on an active backstepping design. We have proposed this control scheme for hyperchaos synchronization and anti-synchronization by using the Lyapunov stability theory. Finally, numerical simulations were provided to show the effectiveness of our method.

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