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Some Bulk Viscous String Cosmological Models

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Abstract

Five dimensional String cosmological models in the presence of variable bulk viscous coefficient, constant bulk viscous coefficient and in the absence of bulk viscosity are constructed and some physical and kinematical behaviors of these models are discussed.

Key Words: Higher dimension, String cosmology, Bulk viscous coefficient.

1. Introduction

In recent years cosmologists have been interested in constructing string cosmological models of the universe. The concept of string theory is developed to describe events at the early stages of the universe. Kibble [1], Zeldovich [2] and Vilenkin [3] believed that strings may be one source of density perturbations, and that are required for the formation of large scale structures in the universe. Therefore it is a subject of considerable interest of cosmologists to study cosmic strings in the framework of general relativity. The general relativistic formalism of cosmic strings are given by Letelier [4] and Stachel [5]. They considered the energy momentum tensor for string distribution in the form

$$T_{ij} = \rho u_i u_j - \lambda \omega_i \omega_j, \tag{1}$$

where

$$u_i u^i = -\omega_i \omega^i = -1 \tag{2}$$

and

$$u^i \omega_i = 0, \tag{3}$$

where ρ is the energy density for a cloud of strings with particles attached to them, λ the string tension density, the unit time like vector u^i is the flow vector and the unit space like vector ω^i specifies the direction of the strings. The particle density associated with the configuration is given by

$$\rho_p = \rho - \lambda. \tag{4}$$

Now days, it is conjectured that material distribution behaves like a viscous fluid during an early phase of the evolution of the universe when galaxies were formed [6]. Misner [7] studied the effect of viscosity in the evolution of the universe. Mohanty and Pradhan [8] constructed Robertson-Walker cosmological model with bulk viscosity with equation of state $p = (\gamma - 1)\,\rho$ where$0 \leq \gamma \leq 2$. Murphy [9] constructed isotropic homogeneous spatially flat cosmological models with bulk viscous fluid alone because the shear viscosity can not exist due to the assumption of isotropy. Bali and Jain [10, 11] investigated some expanding and shearing viscous fluid cosmological models in which coefficient of shear viscosity is proportional to the rate of expansion on the model. Further, they showed that in free gravitational field the model yields "degenerate and non degenerate Petrov Type-1 universe." Roy and Prakash [12, 13] investigated viscous fluid cosmological models of Petrov type–I D and non degenerate Petrov Type I in which coefficients of shear and bulk viscosities are constants. Bali and Dave [14] constructed cosmological models in the presence and absence of bulk viscous fluid in which coefficient of bulk viscosity is constant.

Recently, considerable interest have been evinced in theories of more than four dimensions, in which the extra dimensions are compacted to small size in the course of evolution of the universe [15]. The cosmological study in higher dimensional space time are necessitated, even made urgent, by the growing belief that the nature of space time in the universe are higher than four. Chatterjee [16] studied massive strings in higher dimensional homogeneous space time. Krori et al. [17] discussed Bianchi type-1 higher dimensional cosmologies and concluded that, physically, strings will be like geometric string, and matter and strings coexist throughout the evolution of the universe. They mentioned that cosmic strings with some specific orientation do not occur in Bianchi type-V cosmology. Rahaman et al. [18] discussed some string cosmological models in a higher dimensional spherically symmetric space time based on Lyra's geometry. Venkateswarlu [19] constructed higher dimensional string cosmological models in scale covariant theory of gravitation. Recently Mohanty et al. [20] and Mohanty and Mahanta [21] constructed various higher dimensional string cosmological models and studied their geometrical and physical behaviors.

In this paper we have constructed and studied higher dimensional bulk viscous string cosmological models in general relativity. The behaviors of the models in presence of variable bulk viscous coefficient, constant bulk viscous coefficient and in the absence of bulk viscous coefficient are discussed.

2. Metric and Field Equations

We consider the five dimensional line element of the form

$$ds^2 = -dt^2 + R^2\left(dx^2 + dy^2 + dz^2\right) + A^2 d\psi^2, \tag{5}$$

where R and A are functions of "t," only. The fifth co-ordinate is taken to be space-like and the coordinates are co-moving, where

$$u^0 = 1, \;\; u^1 = u^2 = u^3 = u^4 = 0. \tag{6}$$

Without loss of generality, we choose

$$\omega^i = \left(0, 0, 0, 0, A^{-1}\right). \tag{7}$$

Further, we consider energy momentum tensor for cloud of string dust with bulk viscous coefficient as [22, 23]

$$T_i^j = \rho\, u_i\, u^j - \lambda\, \omega_i\, \omega^j - \xi\, u^l_{;l}\left(g_i^j + u_i\, u^j\right), \tag{8}$$

where ξ is the coefficient of bulk viscosity, ρ is the proper energy density for a cloud of strings with particles attached to them, λ is the string tension density, $\theta = u^l_{;l}$ is the expansion scalar, ω^i represent the direction of the string satisfying

$$u_i\, u^i = -\omega_i\, \omega^i = -1, \tag{9}$$

and

$$u^i \omega_i = 0. \tag{10}$$

Einstein's field equations are given by

$$R_{ij} - \frac{1}{2} R\, g_{ij} = -8\,\pi\, T_{ij}. \tag{11}$$

Field equations (11) for the metric (5) yield

$$3\left(\frac{R'}{R}\right)^2 + 3\frac{R'A'}{RA} = 8\pi\rho \tag{12}$$

$$2\frac{R''}{R} + \left(\frac{R'}{R}\right)^2 + 2\frac{R'A'}{RA} + \frac{A''}{A} = 8\pi\xi\left(3\frac{R'}{R} + \frac{A'}{A}\right) \tag{13}$$

$$3\frac{R''}{R} + 3\left(\frac{R'}{R}\right)^2 = 8\pi\left[\lambda + \left(3\frac{R'}{R} + \frac{A'}{A}\right)\right] \tag{14}$$

3. Solutions of the Field Equations

In this Section we intend to solve the field equations explicitly for cosmic strings with variable bulk viscous coefficient, constant bulk viscous coefficient, and in the absence of bulk viscous coefficient.

3.1. Variable bulk viscous coefficient

In this case there are five unknowns, viz. λ, ρ, R, A and ξ involved in three field equations (12)–(14). Hence to get a determinate solution, we take different physical conditions found in the literature [22, 23]:

$$\rho = \lambda \text{ (Geometric string)} \tag{15a}$$

$$\rho = (1+\omega)\,\lambda \text{ (p-string)} \tag{15b}$$

$$\rho + \lambda = 0 \text{ (Mohanty et al., 2007)} \tag{15c}$$

Case I: Geometric string ($\rho = \lambda$)

In this case, due to paucity of one equation, an additional constraint relating these parameters is required to obtain explicit exact solutions of the system of field equations. Therefore we consider the power law [24–29]

$$A = R^n, \tag{16}$$

where n is a constant. Using equations (15a) and (16) in field equations (12)–(14), we get

$$\frac{R''}{R'} - \left(\frac{n^2+4n+1}{1-n}\right)\frac{R'}{R} = 0. \tag{17}$$

Equation (17) yields

$$R = \left[\frac{n\,(n+5)}{n-1}\,(kt+k_1)\right]^{\frac{n-1}{n(n+5)}}, \tag{18}$$

$$A = \left[\frac{n\,(n+5)}{n-1}\,(kt+k_1)\right]^{\frac{n-1}{(n+5)}}, \tag{19}$$

where k and k_1 are non-zero constants of integration.

Using equations (18) and (19) in equation (13), we get

$$\xi = -\frac{k}{8\pi}\frac{(n+1)\,(n+2)}{n\,(n+3)\,(n+5)}\frac{1}{(kt+k_1)}. \tag{20}$$

In this case, the geometry of the string cosmological model with variable bulk viscous coefficient is described by the metric

$$ds^2 = -dt^2 + \left[\frac{n\,(n+5)}{n-1}\,(kt+k_1)\right]^{\frac{2(n-1)}{n(n+5)}} \left[dx^2 + dy^2 + dz^2\right] + \left[\frac{n\,(n+5)}{n-1}\,(kt+k_1)\right]^{\frac{2(n-1)}{(n+5)}} d\psi^2 \quad . \tag{21}$$

The rest energy density ρ, the scalar of expansion θ, the shear σ, the spatial volume V and the deceleration parameter q for model (21) are obtained as

$$\rho(=\lambda) = \frac{1}{8\pi}\frac{3k^2(n+1)}{\left[\frac{n(n+5)}{n-1}(kt+k_1)\right]^2} \tag{22}$$

$$\theta = \frac{k(n+3)}{\frac{n(n+5)}{n-1}(kt+k_1)} \tag{23}$$

$$\sigma^2 = \tfrac{1}{2}\left[\tfrac{4}{9} + 3\left\{\frac{k}{\frac{n(n+5)}{n-1}(kt+k_1)} + \tfrac{1}{3}\right\}^2 + \left\{\frac{nk}{\frac{n(n+5)}{n-1}(kt+k_1)} + \tfrac{1}{3}\right\}^2\right] \tag{24}$$

$$V = \left[\frac{n(n+5)}{n-1}(kt+k_1)\right]^{\frac{(n-1)(n+3)}{n(n+5)}} \tag{25}$$

$$q = \frac{2(n+2)}{(n-1)(n+3)}. \tag{26}$$

The energy density, string tension density, expansion scalar, shear become infinite for $t = -\frac{k_1}{k}$, which indicates that the evolution of the universe starts with singularity at this value of t. Moreover, we observe that the three spatial coordinates expand indefinitely but the extra dimension contracts as $t \to \infty$ in the interval -5< n <-1. The scalar of expansion is finite at t=0 and $\theta \to 0$ when $t \to \infty$. Since $\lim\limits_{t\to\infty} \frac{\sigma^2}{\theta^2} \neq 0$, the model does not approach isotropy for large values of t. The spatial volume is finite when $t \to 0$ and it becomes infinite when $t \to \infty$. The deceleration parameter q becomes negative when $n \in (-\infty,-2)\cup(-2,1)$. Therefore the model is inflationary.

Case II: Takabayasi string (p-string)

In this case, using equations (15b) and (16) in field equations (12)–(14) we get

$$R = \left[\frac{\omega(n+3)+n}{\omega+1}(at+a_1)\right]^{\frac{(\omega+1)}{\omega(n+3)+n}}, \tag{27}$$

$$A = \left[\frac{\omega(n+3)+n}{\omega+1}(at+a_1)\right]^{\frac{n(\omega+1)}{\omega(n+3)+n}}, \tag{28}$$

$$\xi = \frac{1}{8\pi}\frac{a[(2n-1)-\omega(n+1)]}{(n+3)(\omega(n+3)+n)(at+a_1)}. \tag{29}$$

Hence the geometry of the model is described by the metric

$$\begin{aligned} ds^2 \quad &= \quad -dt^2 + \left[\frac{\omega(n+3)+n}{\omega+1}(at+a_1)\right]^{\frac{2(\omega+1)}{\omega(n+3)+n}} \left[dx^2+dy^2+dz^2\right] \\ &\quad + \left[\frac{\omega(n+3)+n}{\omega+1}(at+a_1)\right]^{\frac{2n(\omega+1)}{\omega(n+3)+n}} d\psi^2. \end{aligned} \tag{30}$$

The physical and kinematical quantities have the following expressions:

$$\rho = (1+\omega)\lambda = \frac{1}{8\pi}\frac{3a^2(n+1)(\omega+1)^2}{[(\omega(n+3)+n)(at+a_1)]^2} \tag{31}$$

$$\rho_p = \frac{1}{8\pi}\frac{3a^2(n+1)(\omega+1)\omega}{[(\omega(n+3)+n)(at+a_1)]^2} \tag{32}$$

$$\theta = \frac{a(n+3)(\omega+1)}{(\omega(n+3)+n)(at+a_1)} \tag{33}$$

$$\sigma^2 = \frac{1}{2}\left[\frac{4}{9} + 3\left\{\frac{\omega+1}{\omega(n+3)+n)(at+a_1)} + \frac{1}{3}\right\}^2 + \left\{\frac{an(\omega+1)}{\omega(n+3)+n)(at+a_1)} + \frac{1}{3}\right\}^2\right] \tag{34}$$

$$V = \left[\frac{\omega(n+3)+n}{\omega+1}(at+a_1)\right]^{\frac{(\omega+1)(n+3)}{\omega(n+3)+n}} \tag{35}$$

$$q = -\frac{3}{\omega+1}. \tag{36}$$

The reality condition $\rho > 0$ is satisfied when $n+1 > 0$. At early era $(t \to 0)$, we have $\lambda > 0$, $\rho > 0$, $\rho_p > 0$. Also, we get

$$\frac{\rho_p}{|\lambda|} = \omega, \tag{37}$$

which indicates that strings and particles coexist. From equation (37) we observe that the particles dominate over the strings for $\omega > 1$, and string dominate over the particles for $\omega < 1$. Moreover, the strings and particles contribute gravitational field equally for $\omega = 1$. As time increases, the scale factor A gradually decreases for -3/2 $< n < 0$, while other scale factor R increases. The extra dimension becomes insignificant as time proceeds after the creation of the universe and we are left with real four dimensional world. The spatial volume is finite when $t \to 0$ and it becomes infinite when $t \to \infty$. The deceleration parameter is negative. So the model is inflationary.

Case III: Special string ($\rho + \lambda = 0$)

In this case, using equations (15c) and (16) in field equations (12)-(14), we get

$$R = \left[\frac{(n+2)(n-3)}{n-1}(b_1 t + b_2)\right]^{\frac{(n-1)}{(n+2)(n-3)}}, \tag{38}$$

$$A = \left[\frac{(n+2)(n-3)}{n-1}(b_1 t + b_2)\right]^{\frac{n(n-1)}{(n+2)(n-3)}}, \tag{39}$$

$$\xi = \frac{1}{8\pi}\frac{9b_1(n+1)]}{(n^2-9)(n+2)(b_1 t + b_2)}, \tag{40}$$

where $b_1(\neq 0)$ and b_2 are constants of integration. The geometry of the model is described by the metric

$$\begin{aligned} ds^2 &= -dt^2 + \left[\frac{(n+2)(n-3)}{n-1}(b_1 t + b_2)\right]^{\frac{2(n-1)}{(n+2)(n-3)}} \left[dx^2 + dy^2 + dz^2\right] \\ &\quad + \left[\frac{(n+2)(n-3)}{(n-1)}(b_1 t + b_2)\right]^{\frac{2n(n-1)}{(n+2)(n-3)}} d\psi^2. \end{aligned} \tag{41}$$

In this case physical and kinematical quantities have the expressions

$$\rho(=-\lambda) = \frac{1}{8\pi}\frac{3b_1^2(n+1)}{\left[\frac{(n+2)(n-3)}{n-1}(b_1 t + b_2)\right]} \tag{42}$$

$$\rho_p = \frac{1}{8\pi}\frac{6b_1^2(n+1)(n-1)}{(n+2)(n-3)(b_1t+b_2)} \tag{43}$$

$$\theta = \frac{b_1(n+3)(n-1)}{(n+2)(n-3)(b_1t+b_2)} \tag{44}$$

$$\sigma^2 = \frac{1}{2}\left[\frac{4}{9} + 3\left\{\frac{b_1(n-1)}{(n+2)(n+3)(b_1t+b_2)} + \frac{1}{3}\right\}^2 + \left\{\frac{nb_1(n-1)}{(n+2)(n-3)(b_1t+b_2)} + \frac{1}{3}\right\}^2\right] \tag{45}$$

$$V = \left[\frac{(n+2)(n-3)}{n-1}(b_1t+b_2)\right]^{\frac{(n-1)(n+3)}{(n+2)(n-3)}} \tag{46}$$

$$q = -\frac{3(n+1)b_1}{n-1}. \tag{47}$$

The energy conditions $\rho > 0$ and $\rho_p > 0$ are satisfied for $n \in (-1,\ 1) \cup (3, \infty)$. We have

$$\frac{\rho_p}{|\lambda|} = 2, \tag{48}$$

which indicates that particles dominate over strings.

The expansion scalar θ is always positive for $n \in$ (-2,1) $\cup$ (3,∞) and $b_1 > 0$. Therefore the model is expanding. The three spatial co-ordinates expand indefinitely as $t \to \infty$, but the extra dimension contracts as $t \to \infty$ when $n \in$ (-2,0). The spatial volume is finite as $t \to 0$; and it becomes infinite as $t \to \infty$, when $n \in (-\infty, -3) \cup (-2, 1) \cup (3, \infty)$. The deceleration parameter is negative for $n \in (-\infty, -1) \cup (1, \infty)$. So the model is inflationary. The model is not inflationary when $n \in (-1, 1)$.

3.2. Constant bulk viscous coefficient

In this case, field equations (12)–(14) constitute a system of three equations with four unknown parameters, viz. λ, ρ, R and A. Using equation (16), we obtain solution for equations (12)–(14) as

$$R = \left[\left(\frac{n^2+2n+3}{n+2}\right)\left(\frac{c(n+2)}{8\pi\xi(n+3)}\right)e^{8\pi\xi\frac{(n+3)}{n+2}t} + c_1\right]^{\frac{n+2}{n^2+2n+3}} \tag{49}$$

$$A = \left[\left(\frac{n^2+2n+3}{n+2}\right)\left(\frac{c(n+2)}{8\pi\xi(n+3)}\right)e^{8\pi\xi\frac{(n+3)}{n+2}t} + c_1\right]^{\frac{n(n+2)}{n^2+2n+3}}. \tag{50}$$

The geometry of the string model in this case is described by the metric

$$\begin{aligned} ds^2 &= -dt^2 + \left[\frac{n^2+2n+3}{n+2}\left(\frac{c(n+2)}{8\pi\xi(n+3)}e^{8\pi\xi\frac{n+3}{n+2}t} + c_1\right)\right]^{\frac{2(n+2)}{n^2+2n+3}}\left[dx^2+dy^2+dz^2\right] \\ &+ \left[\frac{n^2+2n+3}{n+2}\left(\frac{c(n+2)}{8\pi\xi(n+3)}e^{8\pi\xi\frac{n+3}{n+2}t} + c_1\right)\right]^{\frac{2n(n+2)}{n^2+2n+3}} d\psi^2 \end{aligned} \tag{51}$$

The physical and kinematical parameters for this model are obtained as

$$8\pi\rho = \frac{3(n+1)c^2 e^{16\pi\xi\left(\frac{n+3}{n+2}\right)t}}{\left[\left(\frac{n^2+2n+3}{n+2}\right)\left(\frac{c(n+2)}{8\pi\xi(n+3)}e^{8\pi\xi\left(\frac{n+3}{n+2}\right)t} + c_1\right)\right]^2} \tag{52}$$

$$\begin{aligned} 8\pi\lambda &= \frac{c\left(8\pi\xi\left(\frac{n+3}{n+2}\right) - 24\pi\xi c - 8\pi\xi n c\right)e^{8\pi\xi\left(\frac{n+3}{n+2}\right)t}}{\left[\left(\frac{n^2+2n+3}{n+2}\right)\left(\frac{c(n+2)}{8\pi\xi(n+3)}e^{8\pi\xi\left(\frac{n+3}{n+2}\right)t} + c_1\right)\right]} \\ &- \frac{c^2\left(\frac{n^2+2n+3}{n+2}\right)e^{16\pi\xi\left(\frac{n+3}{n+2}\right)t}}{\left[\left(\frac{n^2+2n+3}{n+2}\right)\left(\frac{c(n+2)}{8\pi\xi(n+3)}e^{8\pi\xi\left(\frac{n+3}{n+2}\right)t} + c_1\right)\right]^2} \end{aligned} \tag{53}$$

$$\rho_p = \frac{1}{8\pi}\frac{\left[3(n+1)+\frac{n^2+2n+3}{n+2}\right]c^2 e^{16\pi\,\xi\left(\frac{n+3}{n+2}\right)t}}{\left[\left(\frac{n^2+2n+3}{n+2}\right)\left(\frac{c(n+2)}{8\pi\xi(n+3)}e^{8\pi\,\xi\left(\frac{n+3}{n+2}\right)t}+c_1\right)\right]^2} \tag{54}$$

$$\theta = \frac{(n+3)ce^{8\pi\,\xi\left(\frac{n+3}{n+2}\right)t}}{\left(\frac{n^2+2n+3}{n+2}\right)\left(\frac{(n+2)}{8\pi\xi(n+3)}e^{8\pi\,\xi\left(\frac{n+3}{n+2}\right)t}+c_1\right)} \tag{55}$$

$$\begin{aligned}\sigma^2 \quad &= \quad \frac{1}{2}\left[\frac{4}{9}+3\left\{\frac{(n+2)\,ce^{8\pi\,\xi\left(\frac{n+3}{n+2}\right)t}}{(n^2+2n+3)\left(\frac{c(n+2)}{8\pi\xi(n+3)}e^{8\pi\,\xi\left(\frac{n+3}{n+2}\right)t}+c_1\right)}+\frac{1}{3}\right\}^2\right.\\ &\quad\left.+\left\{\frac{(n+2)\,nce^{8\pi\,\xi\left(\frac{n+3}{n+2}\right)t}}{(n^2+2n+3)\left(\frac{c(n+2)}{8\pi\xi(n+3)}e^{8\pi\,\xi\left(\frac{n+3}{n+2}\right)t}+c_1\right)}+\frac{1}{3}\right\}^2\right]\end{aligned} \tag{56}$$

$$V = \left[\frac{(n^2+2n+3)}{n+2}\left(\frac{c(n+2)}{8\pi\xi(n+3)}e^{8\pi\,\xi\left(\frac{n+3}{n+2}\right)t}+c_1\right)\right]^{\frac{(n+2)(n+3)}{n^2+2n+3}} \tag{57}$$

and

$$q = -\left[\frac{c_1 8\pi\xi}{(n+2)ce^{8\pi\,\xi\left(\frac{n+3}{n+2}\right)t}}+1\right]. \tag{58}$$

The reality condition $\rho > 0$ is satisfied when $n+1>0$. The expansion scalar θ is always positive for $n \in (-\infty,-3)\cup(-2,\infty)$. The scalar of expansion θ is finite at $t=0$ and $\theta = \frac{8\pi\xi(n+3)^2}{n^2+2n+3}$ when $t\to\infty$. Therefore the model is expanding. The ratio σ^2/θ^2 tends to a finite limit as $t\to\infty$. So the model is highly anisotropic for large t. The spatial volume V tends to infinite as $t\to\infty$ for $n\in(-\infty,-3)\cup(-2,\infty)$. The proper energy density ρ, the string tension density$\lambda$and the particle density $\rho_p$are finite for any value of t, i.e. they are in damped motion. The deceleration parameter q is negative for $n\in(-2,\infty)$. Therefore the model is inflationary.

3.3. String with out bulk viscous fluid

The field equations (12)–(14) in the absence of bulk viscous fluid ($\xi=0$) reduce to

$$3\left(\frac{R'}{R}\right)^2+3\frac{R'A'}{RA}=8\pi\rho, \tag{59}$$

$$2\frac{R''}{R}+\left(\frac{R'}{R}\right)^2+2\frac{R'A'}{RA}+\frac{A''}{A}=0, \tag{60}$$

$$3\frac{R''}{R}+3\left(\frac{R'}{R}\right)^2=8\pi\lambda. \tag{61}$$

In order to overcome the paucity of field equations for determining four unknowns from three equations, we consider (16). Equations (59)–(61) yield the solution

$$R=\left[\left(\frac{n^2+2n+3}{n+2}\right)(dt+d_1)\right]^{\frac{n+2}{n^2+2n+3}}, \tag{62}$$

$$A=\left[\left(\frac{n^2+2n+3}{n+2}\right)(dt+d_1)\right]^{\frac{n(n+2)}{n^2+2n+3}}, \tag{63}$$

where $d(\neq 0)$ and d_2 are constants of integration. The geometry of the string cosmological model in this case is described by the metric

$$
\begin{aligned}
ds^2 &= -dt^2 + \left[\left(\frac{n^2+2n+3}{n+2}\right)(dt+d_1)\right]^{\frac{2(n+2)}{n^2+2n+3}} \left[dx^2+dy^2+dz^2\right] \\
&\quad + \left[\left(\frac{n^2+2n+3}{n+2}\right)(dt+d_1)\right]^{\frac{2n(n+2)}{n^2+2n+3}} d\psi^2
\end{aligned}
\tag{64}
$$

The physical and kinematical parameters for this model have the following expressions:

$$8\pi\rho = \frac{3d^2(n+1)}{\left[\left(\frac{n^2+2n+3}{n+2}\right)(dt+d_1)\right]^2} \tag{65}$$

$$8\pi\lambda = \frac{3d^2(n+1)(1-n)(n+2)}{\left[(n^2+2n+3)(dt+d_1)\right]^2} \tag{66}$$

$$\rho_p = \frac{3d^2(n+1)(2n+1)(n+2)}{8\pi\left[(n^2+2n+3)(dt+d_1)\right]^2} \tag{67}$$

$$\theta = \frac{d(n+3)(n+2)}{(n^2+2n+3)(dt+d_1)} \tag{68}$$

$$\sigma^2 = \frac{\frac{1}{2}\left[\begin{array}{l} 4\left\{\left(\frac{n^2+2n+3}{n+2}\right)(dt+d_1)\right\}^2 + 3\left\{3d + \left(\frac{n^2+2n+3}{n+2}\right)(dt+d_1)\right\}^2 \\ + \left\{3nd + \left(\frac{n^2+2n+3}{n+2}\right)(dt+d_1)\right\}^2 \end{array}\right]}{d\left\{\left(\frac{n^2+2n+3}{n+2}\right)(dt+d_1)\right\}^2} \tag{69}$$

$$V = \left[\left(\frac{n^2+2n+3}{n+2}\right)(dt+d_1)\right]^{\frac{(n+3)(n+2)}{n^2+2n+3}}, \tag{70}$$

and

$$q = -\frac{-3(n+1)}{(n+2)(n+3)}. \tag{71}$$

It follows from these expressions that $\rho > 0$, $\lambda > 0$ and $\rho_p > 0$ for $n \in (-\frac{1}{2}, 1)$. Moreover ρ, λ, ρ_p, σ^2 and θ become infinite and volume $V = 0$ for $t = -\frac{d_1}{d}$. Subsequently for $n = 1, -1$ we get $\lambda = 0$ and $\rho = \rho_p$. In these cases the model represents a dust filled universe. At the early era $(t \to 0)$, we have

$$\frac{\rho_p}{|\lambda|} = \frac{2n+1}{1-n}. \tag{72}$$

From equation (72), we have

$$\frac{\rho_p}{|\lambda|} > 1 \text{ for } 0 < n < 1 \tag{73}$$

$$\frac{\rho_p}{|\lambda|} < 1 \text{ for } -\infty < n < 0 \text{ and } 1 < n < \infty \tag{74}$$

$$\frac{\rho_p}{|\lambda|} = 1 \text{ for } n = 0 \tag{75}$$

Therefore strings and particles coexist throughout the evolution. Moreover, equation (73) indicates that particles dominate over strings for $0 < n < 1$. From (74) it is clear that the strings dominate over the particles for $n \in (-\infty, 0) \cup (1, \infty)$. Also from (75), we observe that the strings and particles equally contribute to the gravitational field for $n = 0$. The scalar expansion θ is always positive for $n \in (-\infty, -3) \cup (-2, \infty)$ and d,

$d_1 > 0$. Therefore the model describes an expanding model in the interval $(-\infty, -3) \cup (-2, \infty)$. The scalar expansion θ is finite at $t = 0$ and $\theta \to 0$ as $t \to \infty$. The spatial volume tends to infinite as $t \to \infty$ when $n \in (-\infty, -3) \cup (-2, \infty)$. We observe that the three space coordinates expand and extra coordinate contracts as t increases for $n \in (-2, 0)$. The deceleration parameter is negative when $n \in (-\infty, -3) \cup (-1, \infty)$, vanishes for n = -1 and is positive for $n \in (-\infty, -3) \cup (-2, -1)$.

4. Conclusion

In the preceding sections we constructed five dimensional string cosmological models in the presence of variable bulk viscous coefficient, constant bulk viscous coefficient and in the absence of bulk viscous coefficient. In the case of variable bulk viscous coefficient, it is observed that at initial epoch the bulk viscous coefficient is constant and tends to zero as t tends to infinity. In case of constant bulk viscous coefficient the scalar expansion θ is finite at $t = 0$ and $\theta = \frac{8\pi\xi(n+3)(n+2)}{n^2+2n+3}$ when $t \to \infty$. In the absence of bulk viscosity $\theta \to 0$ when $t \to \infty$. Thus there is finite expansion due to bulk viscosity. The string tension density and particle density are finite throughout the evolution and are in damped motion in case of constant viscous coefficient whereas in case of variable viscous coefficient the string tension density and particle density gradually decrease to zero as $t \to \infty$.

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