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Original Article

Aspects Regarding the Compensation of a Triangulation Network in Terms of Planimetry

SĂLĂGEAN Tudor[1], Andra PORUŢIU[1*], Sergiu POPESCUL[2]

[1]University of Agricultural Sciences and Veterinary Medicine Cluj-Napoca, Faculty of Horticulture, Calea Mănăştur 3-5, 400372, Cluj-Napoca, Romania
[2]State Agrarian University of Moldova, Faculty of Cadastre and Law, St. Mirceşti 42, 2049, Chişinău, Republic of Moldavia

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Abstract

Using the measured directions in calculations, due to the inherent errors that occur in any measurement, conducts to obtaining multiple values for the same point. Therefore before the final calculations of the triangulation, are made compensations calculations, aiming to obtain for each point that is to be determined, only one and the most probable value of his coordinates.Triangulation networks of higher order are compensated by rigorous methods: indirect measurements method (variation of point coordinates) and conditional measurement method (variation of angles and directions).

Keywords: compensation, geodesy, triangulation network.

1. Introduction

The basic geodetic network of our country was conducted from 1950-1960 by the Military Topographic Directorate, conceived as a unified network structured on order.

The compensation was done on orders and groups.

The method used was that of conditional measurements.

For unitary representation in a given projection system, the areas that are to be made various engineering works, is necessary that the measurements to be linked to support points whose position should be determined rigorously [2].

* Corresponding author.
Tel: +40-264-596384
Fax: ++40-264-593792
e-mail: andra.porutiu@usamvcluj.com

Geodetic points are grouped in two categories namely: geodetic points for establishing the plane coordinates X and Y in a certain reference system and geodetic points for establishing the heights (quotas) from the surface of the reference ellipsoid. The first category of points form the geodetic planimetric network and the second category form the state leveling network [1].

The geodesic triangulation of higher order consists of points of I, II, III and IV order which are performed along parallels and meridians, forming the so-called primary network that connects networks of neighboring countries [3].

The triangulation of inferior order, also called the topographic triangulation, constitute the enrichment network and consists of points of order V. The topographic triangulation is used to supplement (thickening) the support networks in order to bind detailed measurements of geodetic points.

2. Material and Method

In triangulation networks are always measured a much higher number of dirrections than is necessary for determining the positions of new points. The direction which are measured in addition allow to discover errors in measurements and calculations to make judgments on the accuracy of field observations, to determine more precisely the coordinates of points.

Using the measured directions in calculations, due to the inherent errors that occur in any measurement, conducts to obtaining multiple values for the same point. Therefore before the final calculations of the triangulation, are made compensations calculations, aiming to obtain for each point that is to be determined, only one and the most probable value of his coordinates.

Triangulation networks of higher order are compensated by rigorous methods: indirect measurements method (variation of point coordinates) and conditional measurement method (variation of angles and directions).

Correction values determined by the indirect measurements method are applied to the coordinates of the points and the ones obtained through the conditional measurements method refers to the angles and sides and respond to the geometry of network conditions.

By applying both methods are obtained the same results, but the calculations volume needed for compensating is different depending on the geometric configuration of the network.

3. Results and Discusions

Verification of the network in terms of planimetric involves determining the plane coordinates of geodetic points using as known sizes the plane coordinates of other geodetic points (given sizes) and directions of the link between geodetic points (measured sizes).

As given sizes are considered the plane coordinates of points Călău Forest and Peana Hill, points that form the basis of initial network, marked with dual line in Fig. 1.

The coordinates of the triangulation network are given in Table 1.

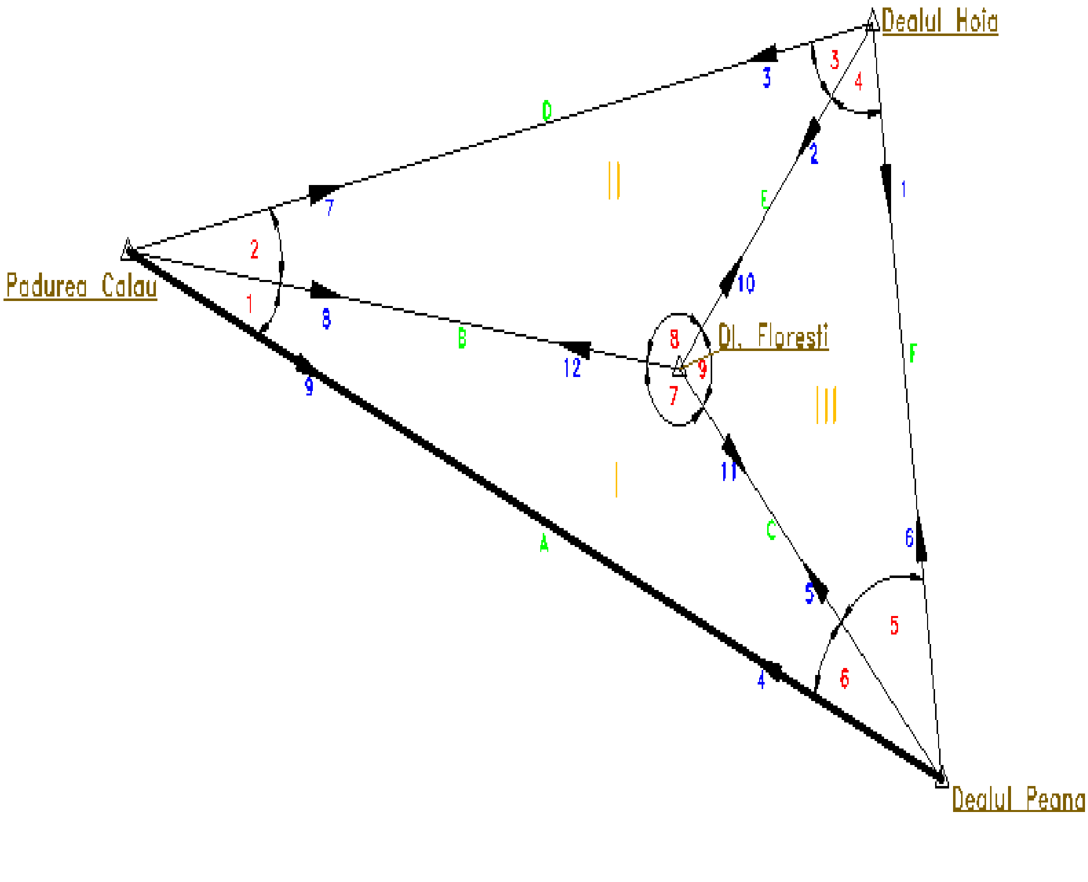

Figure 1. Triangulation network

Table 1. Coordinates of the triangulation network

Point	X [m]	Y [m]	Z [m]
Păd. Călău	583834.017	373673.324	636.30
Hoia	586465.374	388398.459	506.60
Dl. Peana	577752.860	389768.363	832.00
Dl. Florești	582482.559	384573.438	445.60

In the field phase were positioned the stations in each of the other four points and lectures were taken of the directions of each point, the lectures are presented in Table 2.

Knowing that the network teken into study has more known elements than required, we follow that we are dealing with an open network dependent. In such a network are formed following conditions:

- Geometrial conditions
- Conditions of figure,
- Conditions for the horizon tour
- Pole conditions.

Table 2. Measured dirrections

Station point	Tracked point	Number of visa	Measured dirrection [g c cc]
Hoia	Dl. Peana	1	190.0715
	Dl. Florești	2	248.7139
	Păd. Călău	3	288.7428
Dl. Peana	Păd. Călău	4	322.9980
	Dl. Florești	5	347.0183
	Dl. Hoia	6	390.0718
Păd. Călău	Dl. Hoia	7	88.74280
	Dl. Florești	8	107.8534
	Dl. Peana	9	122.9980
Dl. Florești	Dl. Hoia	10	48.71390
	Dl. Peana	11	147.0183
	Păd. Călău	12	307.8534

From the theory of conditional measurements results that the total geometric conditions that must be accomplished by a triangulation network, determines the appropriate number of equations conditions.

To solve the normal system of equations first are calculated the correlates coefficients, using the coefficients of the equations of errors.

Thus, these equations, form the equations system of errors, which takes the form:

$$\begin{cases} v_1 + v_6 + v_7 + w_1 = 0 \\ v_2 + v_3 + v_8 + w_2 = 0 \\ v_4 + v_5 + v_9 + w_3 = 0 \\ v_7 + v_8 + v_9 + w_4 = 0 \\ d_1 v_1 - d_2 v_2 + d_3 v_3 - d_4 v_4 + d_5 v_5 - d_6 v_6 + w_5 = 0 \end{cases}$$

Table 3. Normal system of equations of corrections

Crt. No.	a]	b]	c]	d]	e]	s]	Control	
1	1	0	0	0	-4.124011291	-3.124011291		
2	0	1	0	0	3.230587822	4.230587822		
3	0	1	0	0	-1.375077868	-0.375077868		
4	0	0	1	0	0.759654668	1.759654668		
5	0	0	1	0	-1.246065951	-0.246065951		
6	1	0	0	0	2.523399135	3.523399135		
7	1	0	0	1	0	2		
8	0	1	0	1	0	2		
9	0	0	1	1	0	2		
[ ]	3	3	3	3	-0.231513487	11.76848651	11.76848651	TRUE
[a	3	0	0	1	-1.600612156	2.399387844	2.399387844	TRUE
[b		3	0	1	1.855509954	5.855509954	5.855509954	TRUE
[c			3	1	0.486411284	3.513588716	3.513588716	TRUE
[d				3	0	6	6	TRUE
[e					37.83230471	37.60079122	37.60079122	TRUE

Using the triangular scheme Gauss Doolittle (Table 4) was solved the normal equations system of correlates and in the free terms column (column no. 6) are noted the un-closures "w" expressed in seconds.

Calculations in Gaussian-Dollitlle scheme are verified by two checks: one in the red line (optional control) and the second in the line containing the sum equation (compulsory control).

The corrections values $\mathbf{v_i}$ were determined in Table 5 using the correlates $\mathbf{k_i}$. In table 6 was made the control specific for conditional measurements: [VV] = **-** [kw]. Initially, from the coordinates, was calculated the orientation for the basis side $\Theta_{Pad.Calau-Dl.Peana}$, and using this orientation we calculate all the others orientations (Fig. 2).

Table 4. Gauss-Doolittle scheme

	a]	b]	c]	d]	e]	w]	s]	Control	
1	3	0	0	1	-1.600612156	-5.00000	-2.60061		
2	-1	0	0	-0.333333333	0.533537385	1.666666666	0.866870719	0.866870719	TRUE
3		3	0	1	1.855509954	0.33333	6.18884		
4		0	0	0	0	0	0		
5		3	0	1	1.855509954	0.3333	6.188843287		
6		-1	0	-0.333333333	-0.618503318	-0.111111111	-2.062947762	-2.062947762	TRUE
7			3	1	-0.486411284	-6.00000	-2.48641		
8			0	0	0	0	0		
9			0	0	0	0	0		
10			3	1	-0.486411284	-6.000000	-2.486411285		
11			-1	-0.333333333	0.162137095	2	0.828803762	0.828803762	TRUE
12				3	0	0	6.00000		
13				-0.333333333	0.533537385	1.666666666	0.866870719		
14				-0.333333333	-0.618503318	-0.111111111	-2.062947762		
15				-0.333333333	0.162137095	2	0.828803762		
16				2	0.077171162	3.555555556	5.632726718		
17				-1	-0.038585581	-1.777777778	-2.816363359	-2.816363359	TRUE
18					37.83230471	-1.975934667	35.62485656		
19					-0.853986425	-2.667686927	-1.38752381		
20					-1.147639063	-0.206167772	-3.827820107		
21					-0.078865312	-0.972822568	-0.403139502		
22					-0.002977694	-0.137193177	-0.217342034		
23					35.74883622	-5.959805112	29.7890311		
24					-1	0.166713262	-0.833286738	-0.833286738	TRUE

Table 5. Calculus of the correlates

	k1	k2	k3	k4	k5	Vi	v□v
Nr. Crt.	**2.35035126**	**0.380513018**	**2.62176724**	**-1.784210506**	**0.166713262**		
1	1	0	0	0	-4.124011291	1.662823883	2.764983267
2	0	1	0	0	3.230587822	0.919094854	0.84473535
3	0	1	0	0	-1.375077868	0.151269301	0.022882401
4	0	0	1	0	0.759654668	2.748411747	7.553767134
5	0	0	1	0	-1.246065951	2.41403152	5.827548178
6	1	0	0	0	2.523399135	2.771035362	7.678636977
7	1	0	0	1	0	0.566140754	0.320515353
8	0	1	0	1	0	-1.403697488	1.970366637
9	0	0	1	1	0	0.837556734	0.701501282
[ ]	**3**	**3**	**3**	**3**	**-0.231513487**	**10.66666667**	**27.68493658**

Table 6. Control of the conditional measurements

Nr. Crt.	ki	w	kw	
1	2.35035126	-5.00000	-11.7517563	
2	0.380513018	0.33333	0.126837673	
3	2.62176724	-6.00000	-15.73060344	
4	-1.784210506	0.00000	0	
5	0.166713262	-1.97593	-0.329414515	
		[kw]=	-27.68493658	
		[vv]=-[kw]=	-27.68493658	**TRUE**

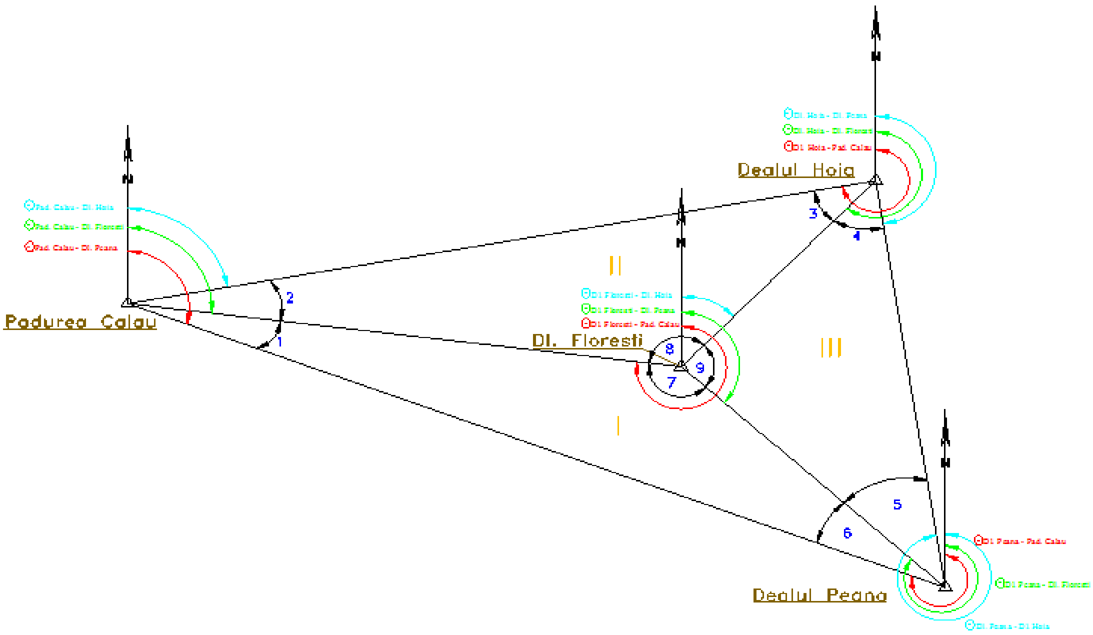

Figure 2. Orientation sketch

The calculation of the sides, it starts from a measured base or a base determined from the points coordinates of higher order. As base was chosen the side Păd. Călău - Dl. Peana.

$$D_{Pad.Calau-Dl.Peana} = \sqrt{\Delta X^2_{Pad.Calau-Dl.Peana} + \Delta Y^2_{Pad.Calau-Dl.Peana}} = 17205.54m$$

Using the base length and successively applying sine theorem, will obtain the length of the sides of triangles that form the studied network (Table 7).

Table 7. Length of the triangle sides

Triangle	Value of the compensated angle [g c cc]	Sinus	Points from the triangle	Known distance [m]	Calculated distance [m]
I Păd.Călău-Dl.Peana-Dl. Florești	15.1448	0.235655911	Păd. Călău	Păd. Călau - Dl. Peana	Păd. Călău - Dl. Florești
	24.0202	0.368420192	Dl. Florești		*10983.55*
	160.8350	0.577123939	Dl. Peana	*17205.54*	Dl. Florești - Dl. Peana
	200.0000				7025.51
II Păd.Călău-Dl.Hoia-Dl. Florești	19.1106	0.295700259	Păd. Călău	Păd. Călău – Dl. Florești	Dl. Florești – Dl. Hoia
	40.0287	0.588150105	Hoia		*5522.13*
	140.8607	0.800996600	Dl. Florești	*10983.55*	Păd. Călău – Dl. Hoia
	200.0000				*14958.40*
III Dl.Hoia-Dl.Peana-Dl. Florești	58.6422	0.796297260	Hoia	Dl. Florești – Dl. Hoia	Dl. Hoia - Dl. Peana
	43.0535	0.625898389	Dl. Peana		*8819.59*
	98.3043	0.999645290	Dl. Florești	*5522.13*	Dl. Florești - Dl. Peana
	200.0000				7025.51

The end of the compensation is given by a control obtained by calculating the compensated coordinates of the network points and compare them with those initially intended as fixed elements.

In Table 8 was carried out the calculation of the coordinates of the four points. The comparison with the original coordinates obtained after compensation is shown in Table 9.

Table 8. Coordinates calculation

Point	Initial coordinates		Final coordinates		Differences	
	X	Y	X	Y	X	Y
	[m]	[m]	[m]	[m]	[cm]	[cm]
Păd. Călău	583834.017	373673.324	583834.002	373673.335	1.5456	-1.0552
Dl. Hoia	586465.374	388398.459	586465.405	388398.468	-3.0601	-0.8904
Dl. Peana	577752.860	389768.363	577752.856	389768.378	0.4018	-1.4874
Dl. Florești	582482.559	384573.438	582482.574	384573.427	-1.5456	1.0552

Table 9. Comparison between the initial coordinates and the compensated coordinates

Point	Initial coordinates		Final coordinates		Differences	
	X	Y	X	Y	X	Y
	[m]	[m]	[m]	[m]	[cm]	[cm]
Păd. Călău	583834.017	373673.324	583834.002	373673.335	1.5456	-1.0552
Dl. Hoia	586465.374	388398.459	586465.405	388398.468	-3.0601	-0.8904
Dl. Peana	577752.860	389768.363	577752.856	389768.378	0.4018	-1.4874
Dl. Florești	582482.559	384573.438	582482.574	384573.427	-1.5456	1.0552

4. Conclusions

After the calculations it appears that the differences between the calculated coordinates and the initial ones are integrated in the tolerance of +/- 15 cm, so it can be concluded that the geodetic network is stable.

For the network taken into study, being a simple network with a relatively small number of initial points, as a method of compensation was used the conditional measurements method. In the case of this method it should be taken care when it is established the number of necessary and sufficient of condition equations that is determined from additional number of measurements.

Otherwise there may be some drawbacks as the omission of the condition equations or writing equations that constitute the consequence of other equations. In the case of omitting some equations, the final results of compensation will be inaccurate because they will verify the conditions omitted, resulting in an incomplete geometric network.

In order to process the measurements made in a geodesic triangulation network it is important in the first stage, to determine the number of geometric conditions, to be known the form of the geometric conditions and corresponding to them to write corrections equations.

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