

## Full Length Research Paper

# Computing multi-soliton solutions to Caudrey-Dodd-Gibbon equation by Hirota's method

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**In this paper, we make use of Hirota's bilinear method to obtain multi-soliton solutions to Caudrey-Dodd-Gibbon equation by using Hirota's bilinear method. This equation is the first transformed to its potential version and then the Cole-Hopf transformation is applied to it to obtain an equation that is transformed to a bilinear form. One and two soliton solutions are formally derived. Other solutions are obtained via limiting process. These solutions are illustrated graphically. A comparison with other methods to solve the mentioned equation is given in later in this paper.**

**Key words:** Caudrey-Dodd-Gibbon equation, Sawada-Kotera equation, nonlinear partial differential equation, fifth order Korteweg-de Vries equation, nonlinear evolution equation, one soliton solution, two soliton solution, travelling wave solution, Mathematica 8, Maple 15.

## INTRODUCTION

In the present paper we make use of the Hirota bilinear method to find multi-soliton solutions to Caudrey-Dodd-Gibbon equation (CDGE). This equation reads:

$$u_t + u_{xxxxx} + 30uu_{xxx} + 30u_x u_{xx} + 180u^2 u_x = 0. \quad (1)$$

Equation 1 is also called Sawada-Kotera equation (Salas, 2008). In a recent work (Jiang and Bi, 2010), the authors studied the bilinear form associated with Equation 1. Our aim is to obtain multi-soliton solutions to this equation starting from its bilinear form. As it was remarked (Hereman and Zhuang, 1994), in the early seventies, Hirota (1971, 2004) developed an ingenious method for obtaining the exact multi-soliton solutions of the Korteweg-de Vries (KdV) equation and derived an explicit expression for its  $N$ -soliton solutions. An elegant formulation of this method requires the use of bilinear operators, therefore it is called Hirota's bilinear method. Over the last two decades this method has been shown

to be applicable to a large class of nonlinear evolution equations, including difference-differential and integro-differential equations (Matsuno, 1984).

Hirota's bilinear method, which is usually applied to completely integrable equations, is well suited for partially integrable equations as well. One can indeed conjecture that all completely integrable nonlinear evolution equations can be put into bilinear form and will admit  $N$ -soliton solutions for any value of integer  $N$ . However, the mere fact that an equation admits a bilinear representation, does not guarantee the existence of multi-soliton solutions of any order. In fact, even the existence of two-soliton solutions is nontrivial for a completely general bilinear equation. It is widely believed that every nonlinear partial differential equation that has a four-soliton solution is completely integrable (linearizable by the Inverse Scattering Method (IST)). Furthermore, there are indications (Newell, 1985, Newell and Zeng, 1986) that the conditions for the existence of multi-soliton solutions are also necessary and sufficient for the partial differential equation to pass the Painlevé test. In that regard, Hirota's method incorporates yet another integrability test.

Hirota's bilinear method has been studied and used

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extensively. The fundamental idea behind the method is to use some dependent variable transformation to put the nonlinear partial differential equation in a form where the new dependent variable appears bilinearly. Once the bilinear form of the equation is found, one introduces a formal perturbation expansion to construct its solution step by step. If soliton solutions exist, this expansion will always truncate, and the then finite series will lead to an exact solution. The drawback of Hirota's method is that it requires a great deal of elementary algebra and calculus. These straightforward calculations can easily be performed with any symbolic manipulation program, such as Mathematica 8 or Maple 15.

### HIROTA'S OPERATORS BILINEAR FORM TO CAUDREY-DODD-GIBBON EQUATION

Here, we make use of Hirota's approach (Hirota, 1971, 2004) to solve nonlinear partial differential equation. For any two smooth functions  $f = f(t, x)$  and  $g = g(t, x)$ , we define for  $m, n = 1, 2, 3, \dots$  Hirota operators as:

$$D_t^m D_x^n (f \cdot g) = \frac{\partial^m}{\partial s^m} \frac{\partial^n}{\partial y^n} \left[ f(t+s, x+y) g(t-s, x-y) \right] \Big|_{s=0, y=0} \quad (2)$$

$$D_t^m (f \cdot g) = \frac{\partial^m}{\partial s^m} \left[ f(t+s, x) g(t-s, x) \right] \Big|_{s=0} \quad (3)$$

$$D_x^n (f \cdot g) = \frac{\partial^n}{\partial y^n} \left[ f(t, x+y) g(t, x-y) \right] \Big|_{y=0} \quad (4)$$

In particular,

$$D_{tx} (f \cdot f) = D_{xt} (f \cdot f) = D_t^1 D_x^1 (f \cdot f) = 2(\mathcal{H}_{tx} - f_t f_x). \quad (5)$$

$$D_{xx} (f \cdot f) = D_x^2 (f \cdot f) = -2(f_x^2 - \mathcal{H}_{xx}). \quad (6)$$

$$D_{xxxx} (f \cdot f) = D_x^4 (f \cdot f) = 6f_{xx}^2 g_{xx} - 8f_x f_{xxx} + 2\mathcal{H}_{xxxx}. \quad (7)$$

$$D_{xxxxx} (f \cdot f) = D_x^5 (f \cdot f) = -20f_{xxx}^2 - 12f_x f_{xxxx} + 30f_{xx} f_{xxx} + 2\mathcal{H}_{xxxxx}. \quad (8)$$

From Equation 5 to 8, it follows that:

$$f_{tx} = f_{xt} = \frac{2f_x f_x + D_{tx}(f \cdot f)}{2f}, \quad (9)$$

$$f_{xx} = \frac{2f_x^2 + D_x^2(f \cdot f)}{2f}, \quad (10)$$

$$f_{xxxx} = \frac{8f_x f_{xxx} - 6f_{xx}^2 + D_x^4(f \cdot f)}{2f}, \quad (11)$$

$$f_{xxxxx} = \frac{20f_{xxx}^2 - 30f_{xx} f_{xxxx} + 12f_x f_{xxxxx} + D_x^5(f \cdot f)}{2f}. \quad (12)$$

Introducing the potential  $w$  defined by:

$$u = w_x, \quad (13)$$

Equation 1 may be written in the form:

$$w_{tx} + w_{xxxxx} + 30w_x w_{xxx} + 30w_{xx} w_{xxx} + 180w_x^2 w_{xx} = 0. \quad (14)$$

Integrating Equation 14 once with respect to  $x$  and taking the constant of integration equal to zero, the following partial differential equation is obtained:

$$w_t + 60w_x^3 + 30w_x w_{xxx} + w_{xxxxx} = 0. \quad (15)$$

We shall call Equation 15 the potential Caudrey-Dodd-Gibbon equation associated with Equation 1. We seek solutions for Equation 15 in the form:

$$w = w(x, t) = A \partial_x \log(f(x, t)), A = \text{const.} \quad (16)$$

Equation 16 defines a Cole-Hopf transformation. Inserting Equation 16 into Equation 15 gives a partial differential equation in the new unknown function  $f = f(x, t)$ . Arranging its left hand side as a polynomial in the variable  $f$ , this equation may be written as follows:

$$\begin{aligned} & (f_{tx} + f_{xxxxx}) f^5 \\ & + (15(2A-1)f_{xx} f_{xxxx} - f_x (f_t + 6f_{xxxx}) - 10f_{xxx}^2) f^4 + \\ & 30(A-1)((2A-1)f_{xx}^3 - 4f_x f_{xx} f_{xxx} - f_x^2 f_{xxx}) f^3 - \\ & 30f_x^2 (A-1)((6A-9)f_{xx}^2 - 4f_x f_{xxx}) f^2 + \\ & 180f_x^4 f_{xx} (A-2)(A-1) f - \\ & 60f_x^6 (A-2)(A-1) = 0. \end{aligned} \quad (17)$$

It is clear that it is convenient to choose  $A = 1$  in order to simplify Equation 17. For this value of  $A$ , this equation reduces to

$$(f_{tx} + f_{xxxxx}) f + 15f_{xx} f_{xxxx} - f_x (f_t + 6f_{xxxx}) - 10f_{xxx}^2 = 0. \quad (18)$$

Now, we substitute Equations 9, 10, 11 and 12 into 18 to obtain:

$$D_{tx}(f \cdot f) + D_x^6(f \cdot f) = 0. \quad (19)$$

We will say that Equation 19 is the bilinear form associated with Equation 1. We also may say that Equation 19 is the bilinear form of Equation 1. It is clear

that if  $f$  is the solution to Equation 19 (which is equivalent to Equation 15), then function  $u$  defined by:

$$u = w_x = \partial_{xx} \log(f(x, t)) \quad (20)$$

is a solution to Equation 1.

## ONE SOLITON SOLUTIONS TO CAUDREY-DODD-GIBBON EQUATION

For one-soliton solutions, we set:

$$\begin{cases} f = f(x, t) = 1 + a \exp(\xi) + b \exp(-\xi), \\ \xi = kx - wt, \\ k \neq 0, a, b, k, w = \text{const.} \end{cases} \quad (21)$$

Inserting ansatz (Equation 21) into Equation 15 (which is equivalent to bilinear form (Equation 19)) we get the following polynomial equation in the variable  $\phi = \exp(kx - wt)$ :

$$b(k^5 - w) + 4b(16k^5 - w)\phi + (k^5 - w)\phi^2 = 0, \quad (22)$$

from where

$$\begin{cases} f = f(x, t) = \Delta + a \exp(\xi) + b \exp(-\xi) + c \exp(\eta) + d \exp(-\eta) + R \exp(\xi + \eta), \\ \xi = kx - wt, \eta = \mu x - \lambda t, \Delta \in \{0, 1\}, k\lambda \neq 0, a, b, c, d, R, k, w, \mu, \lambda = \text{const.} \end{cases} \quad (27)$$

Inserting ansatz (Equation 27) into Equation 15, we get a polynomial equation in the variables  $\phi = \exp(kx - wt)$  and  $\psi = \exp(\lambda x - \mu t)$ . Equating to zero the coefficients of  $\phi^i \psi^j$  ( $i, j = 0, 1, 2, 3, \dots$ ) yields an algebraic system in the unknowns  $a, b, c, d, R, k, w, \mu, \lambda$ . Solving it with the aid of a computer gives the following solutions.

$$\Delta = 1, \quad a = a, \quad b = 0, \quad c = c, \quad d = 0,$$

$$R = ac \frac{(k - \mu)^2(k^2 - k\mu + \mu^2)}{(k + \mu)^2(k^2 + k\mu + \mu^2)}, \quad w = k^5, \quad \lambda = \mu^5:$$

$$\begin{cases} u_5(x, t) = \partial_{x,x} \log \left( a \exp(\xi) + b \exp(-\xi) + \frac{abk^2(\mu^2 + 3k^2)}{d\mu^2(3\mu^2 + k^2)} \exp(\eta) + d \exp(-\eta) \right) \\ \xi = kx - k(k^4 + 10k^2\mu^2 + 5\mu^4)t, \quad \eta = \mu x - \mu(5k^4 + 10k^2\mu^2 + \mu^4)t. \end{cases} \quad (29)$$

Solution 29 gives us interesting solutions for some special choices of the parameters as follows

$$\begin{cases} b(k^5 - w) = 0, \\ b(16k^5 - w) = 0, \\ k^5 - w = 0. \end{cases} \quad (23)$$

Solving Equation 23 gives  $a = a, b = 0, w = k^5$ :

$$\begin{aligned} u_1(x, t) &= \partial_{x,x} \log(1 + a \exp(kx - k^5t)), \\ u_1(x, t) &= \frac{ak^2 \exp(kx - k^5t)}{(1 + a \exp(kx - k^5t))^2}. \end{aligned} \quad (24)$$

In particular, for  $a = \pm \exp(2C)$ ,

$$u_2(x, t) = \frac{k^2}{4} \text{sech}^2 \left( \frac{1}{2} kx - \frac{1}{2} k^5t + C \right), \quad (25)$$

Solution given by Equation 25 is shown in Figure 1.

$$u_3(x, t) = -\frac{k^2}{4} \text{csch}^2 \left( \frac{1}{2} kx - \frac{1}{2} k^5t + C \right). \quad (26)$$

## TWO-SOLITON SOLUTIONS TO CDGE

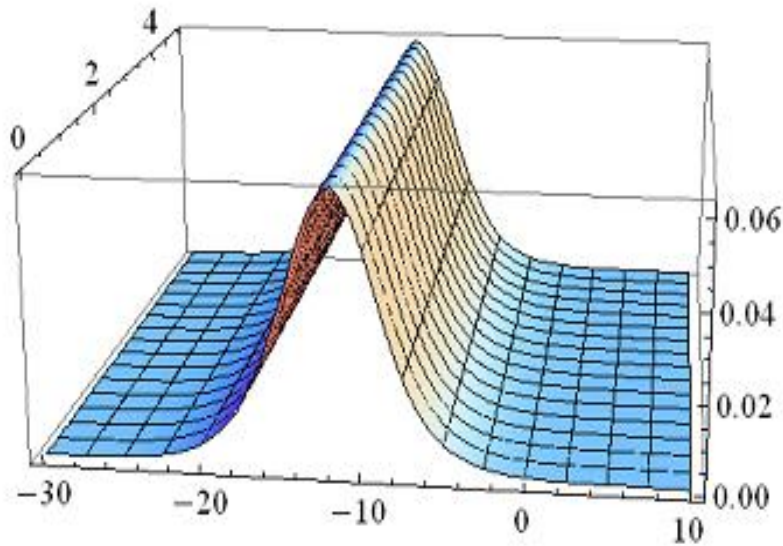
For two-soliton solutions, we set:

$$\begin{aligned} u_4(x, t) &= \partial_{x,x} \log(1 + a \exp(kx - k^5t) + c \exp(\mu x - \mu^5t) + \\ &ac \frac{(k - \mu)^2(k^2 - k\mu + \mu^2)}{(k + \mu)^2(k^2 + k\mu + \mu^2)} \exp((k + \mu)x - (k^5 + \mu^5)t)). \end{aligned} \quad (28)$$

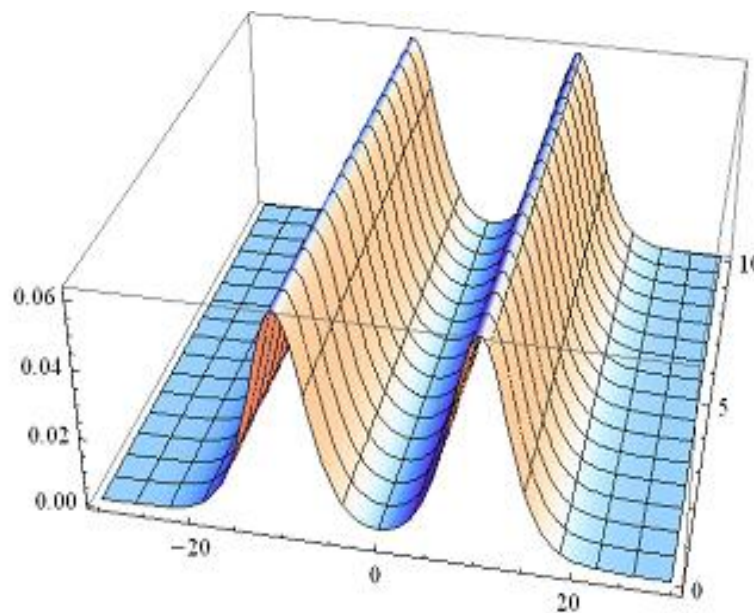
$$\begin{aligned} \Delta &= 0, \quad a = a, \quad b = b, \quad c = \frac{abk^2(\mu^2 + 3k^2)}{d\mu^2(3\mu^2 + k^2)}, \\ d &= d, \quad w = k(k^4 + 10k^2\mu^2 + 5\mu^4), \\ \lambda &= \mu(5k^4 + 10k^2\mu^2 + \mu^4), \quad R = 0: \end{aligned}$$

$$a = b = \frac{1}{2}, \quad d = \frac{k}{2\mu} \sqrt{\frac{3k^2 + \mu^2}{k^2 + 3\mu^2}} \quad (30)$$

$$\begin{cases} u_6(x, t) = \frac{-4dk\mu \sinh \xi \sinh \eta + 2d(k^2 + \mu^2) \cosh \xi \cosh \eta + k^2 + 4d^2\mu^2}{(\cosh \xi + 2d \cosh \eta)^2}, \\ \xi = kx - k(k^4 + 10k^2\mu^2 + 5\mu^4)t, \quad \eta = \mu x - \mu(5k^4 + 10k^2\mu^2 + \mu^4)t. \end{cases}$$



**Figure 1.** Graph of  $u_2(x,t)$  for  $C = 0, k = 0.5$ ,  $-30 \leq x \leq 10$  and  $0 \leq t \leq 5$ .

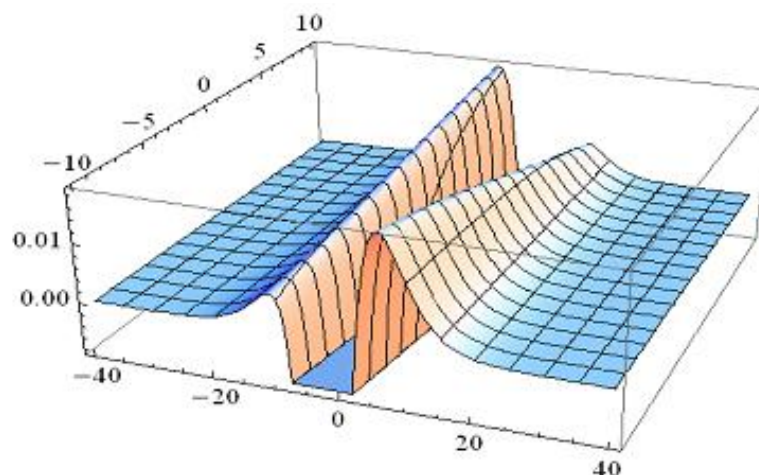


**Figure 2.** Graph of  $u_6(x,t)$  for  $k = 0.5$ ,  $\mu = 0.1$ ,  $-30 \leq x \leq 30$  and  $0 \leq t \leq 10$ .

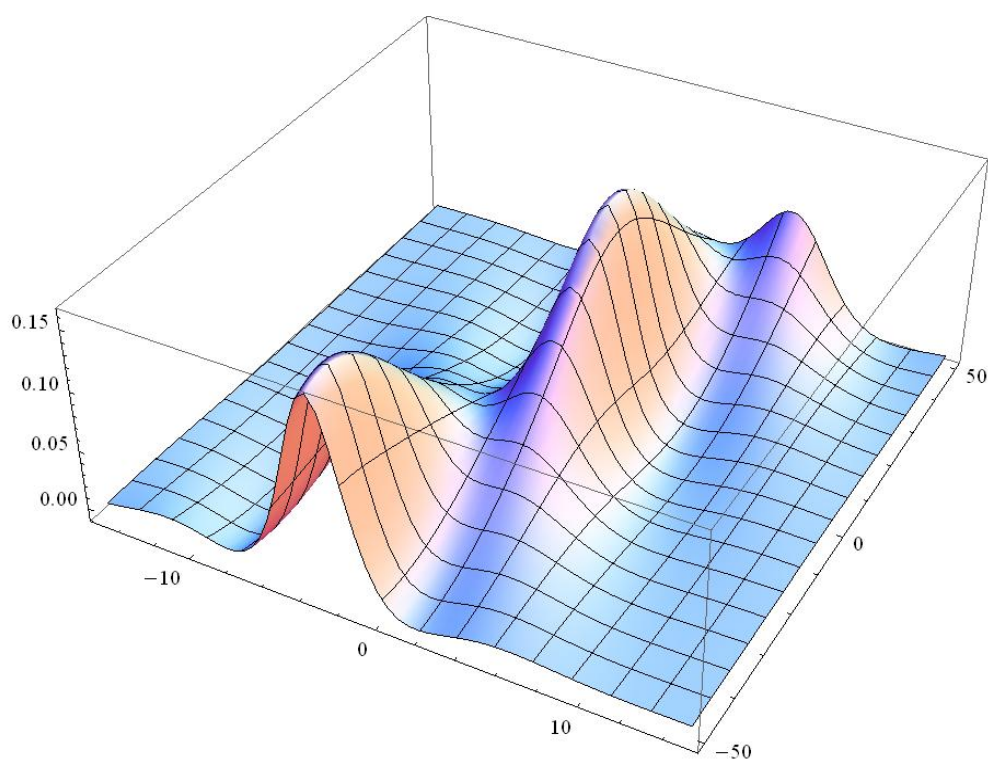
Figure 2 shows the solution of Equation 30.

$$a = \frac{1}{2}, \quad b = -\frac{1}{2}, \quad d = -\frac{k}{2\mu} \sqrt{\frac{3k^2 + \mu^2}{k^2 + 3\mu^2}}$$

$$\begin{cases} u_7(x,t) = \frac{4dk\mu \cosh \xi \cosh \eta - 2d(k^2 + \mu^2) \sinh \xi \sinh \eta - (k^2 + 4d^2\mu^2)}{(\sinh \xi - 2d \sinh \eta)^2}, \\ \xi = kx - k(k^4 + 10k^2\mu^2 + 5\mu^4)t, \quad \eta = \mu x - \mu(5k^4 + 10k^2\mu^2 + \mu^4)t. \end{cases} \quad (31)$$



**Figure 3.** Graph of  $u_7(x, t)$  for  $k = 0.2$ ,  $\mu = 0.5$ ,  $-40 \leq x \leq 40$  and  $-10 \leq t \leq 10$ .



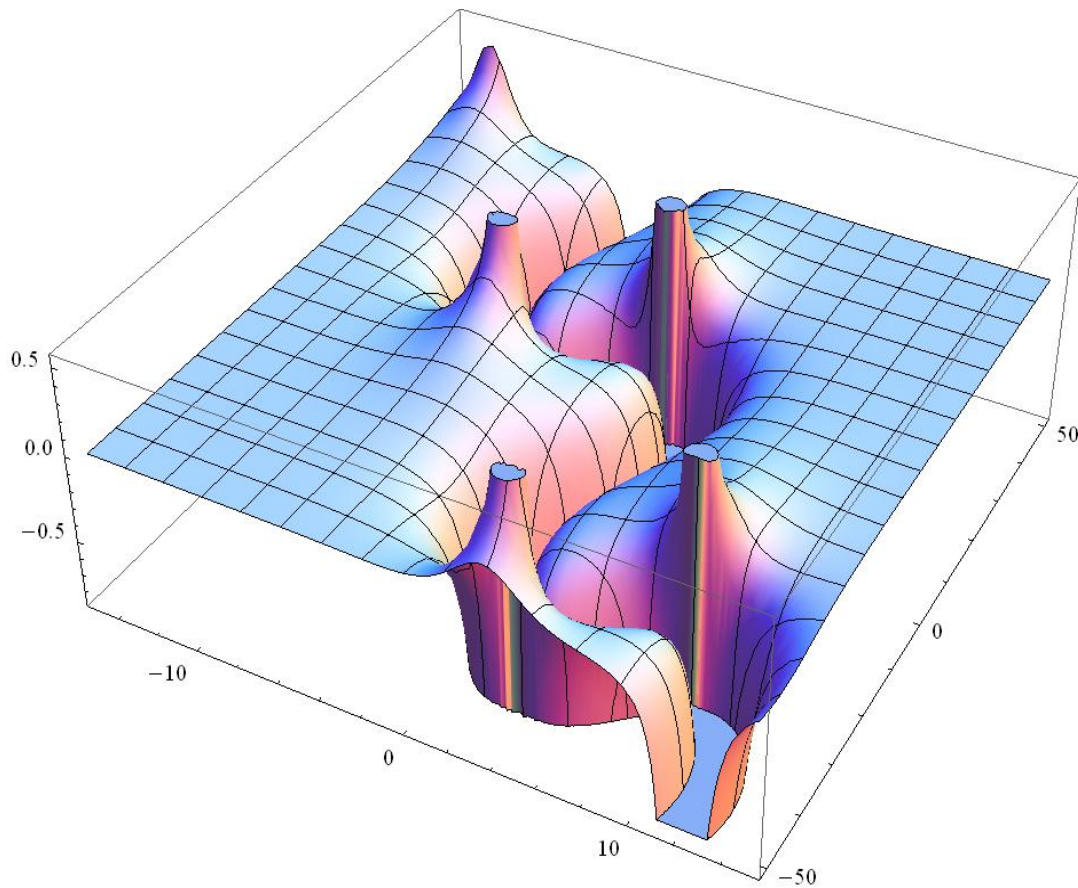
**Figure 4.** Graph of  $u_8(x, t)$  for  $k = 5/16$ ,  $m = 0.5$ ,  $-15 \leq x \leq 15$  and  $-50 \leq t \leq 50$ .

Figure 3 shows the solution of Equation 31.

$$a = b = \frac{1}{2}, \quad d = \pm \frac{k}{2m} \sqrt{\frac{m^2 - 3k^2}{k^2 - 3m^2}}, \quad \mu = \sqrt{-1}m$$

$$\begin{cases} u_8(x, t) = \frac{4dkm \sinh \xi \cos \eta - 2d(k^2 - m^2) \cosh \xi \sin \eta + (k^2 - 4d^2m^2)}{(\cosh \xi - 2d \sin \eta)^2}, \\ \xi = kx - k(k^4 - 10k^2m^2 + 5m^4)t, \quad \eta = mx - m(5k^4 - 10k^2m^2 + m^4)t. \end{cases} \quad (32)$$

Figure 4 shows the solution of Equation 32.



**Figure 5.** Graph of  $u_9(x, t)$  for  $k = 0.58$ ,  $m = 0.32$ ,  $-15 \leq x \leq 15$  and  $-50 \leq t \leq 50$ .

$$a = \frac{1}{2}, \quad b = -\frac{1}{2}, \quad d = \pm \frac{k}{2m} \sqrt{\frac{m^2 - 3k^2}{3m^2 - k^2}}, \quad \mu = m\sqrt{-1}$$

$$\begin{cases} u_9(x, t) = \frac{-4dkm \cosh \xi \sin \eta - 2d(k^2 - m^2) \sinh \xi \cos \eta - (k^2 + 4d^2m^2)}{(\sinh \xi - 2d \cos \eta)^2}, \\ \xi = kx - k(k^4 - 10k^2m^2 + 5m^4)t, \quad \eta = mx - m(5k^4 - 10k^2m^2 + m^4)t. \end{cases} \quad (33)$$

Figure 5 shows the solution of Equation 33.

$$u_{10}(x, t) = \lim_{k \rightarrow -\mu} u_6(x, t) = -\frac{\mu^2(1 + 4\mu^2x^2 + 2\cosh(2\mu(x - 16\mu^4t)))}{(3\cosh(\mu(x - 16\mu^4t)) - 2\mu x \sinh(\mu(x - 16\mu^4t)))^2}. \quad (34)$$

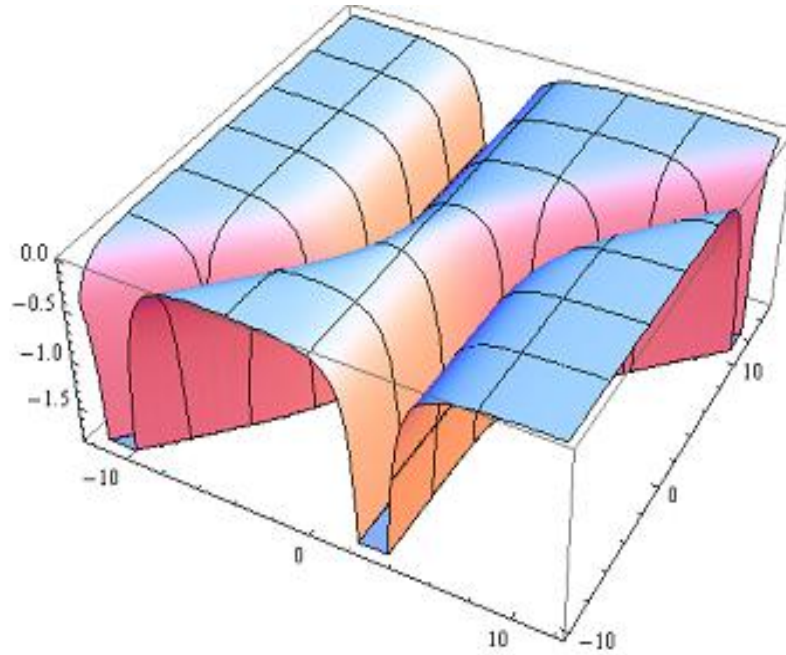
Figure 6 shows the solution of Equation 34.

Letting  $k \rightarrow 0$  in Equation 31 gives:

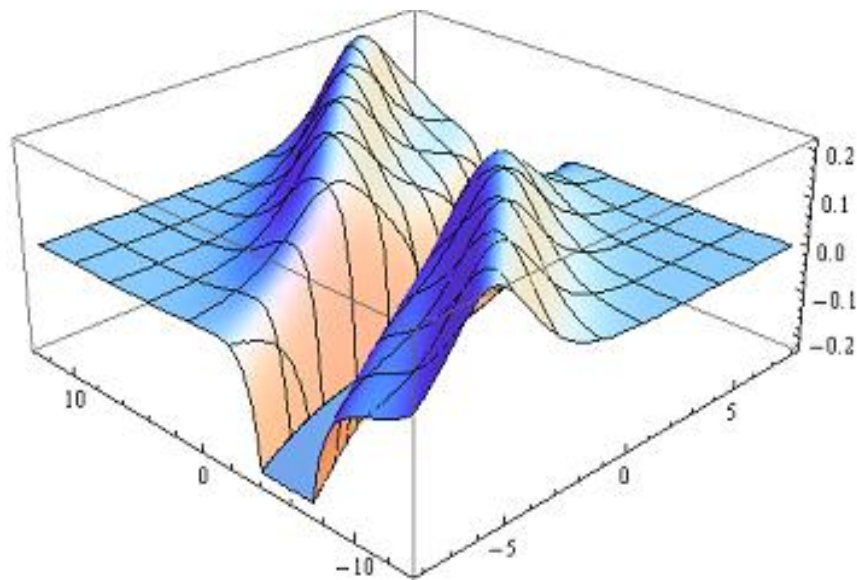
$$\begin{cases} u_{11}(x, t) = \lim_{k \rightarrow 0} u_7(x, t) = \frac{2\mu^2(-4 - 2\sqrt{3} \cosh \eta + \sqrt{3}\xi \sinh \eta)}{-1 + 6\xi^2 + \cosh(2\eta) + 4\sqrt{3}\xi \sinh \eta}, \\ \xi = \mu(x - 5\mu^4t), \quad \eta = \mu(x - \mu^4t). \end{cases} \quad (35)$$

Letting  $k \rightarrow -\mu$  in Equation 30 gives:

Figure 7 shows the solution to Equation 35.



**Figure 6.** Graph of  $u_{10}(x, t)$  for  $\mu = 0.5$ ,  $-12 \leq x \leq 12$  and  $-10 \leq t \leq 15$ .



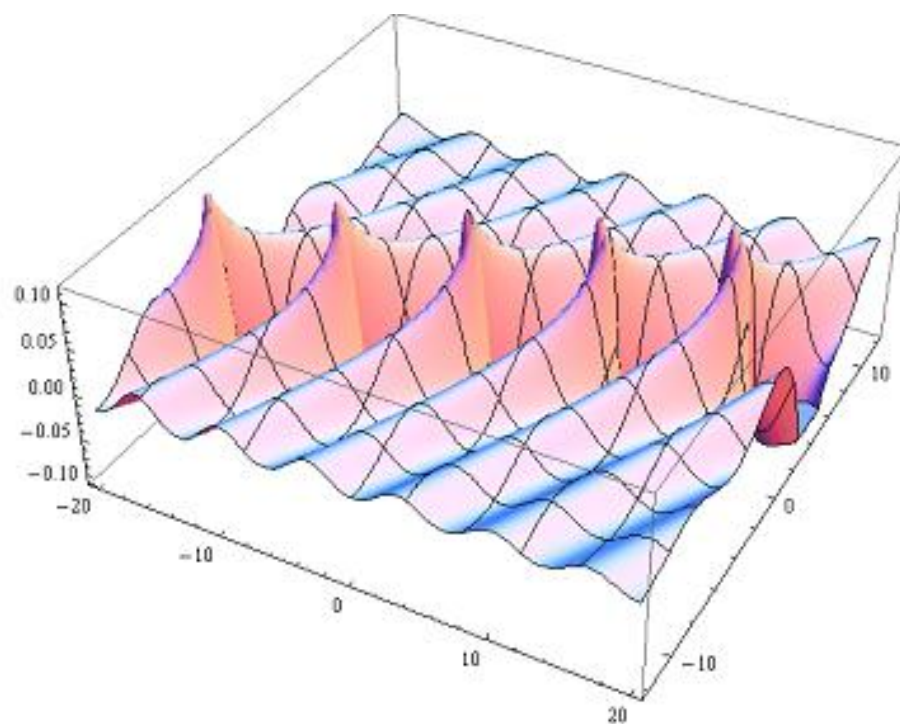
**Figure 7.** Graph of  $u_{11}(x, t)$  for  $\mu = 0.5$ ,  $-12 \leq x \leq 12$  and  $-10 \leq t \leq 15$ .

Letting  $k \rightarrow 0$  in Equation 33 gives:

$$\begin{cases} u_{12}(x, t) = \lim_{k \rightarrow 0} u_9(x, t) = \frac{2m^2(-4 + \sqrt{3}\xi \cos \eta - 2\sqrt{3} \sin \eta)}{1 + 6\xi^2 + \cos(2\eta) - 4\sqrt{3}\xi \cos \eta}, \\ \xi = m(x - 5m^4t), \eta = m(x - m^4t). \end{cases} \quad (36)$$

Figure 8 shows the solution to Equation 36. Letting  $\mu \rightarrow 0$  in Equation 35 gives:

$$u_{13}(x, t) = \lim_{\mu \rightarrow 0} u_{11}(x, t) = -\frac{1}{x^2}. \quad (37)$$



**Figure 8.** Graph of  $u_{12}(x, t)$  for  $m = 0.95$ ,  $-20 \leq x \leq 20$  and  $-12 \leq t \leq 12$ .

which is a rational solution.

## DISCUSSION

We obtained one and two soliton solutions to Caudrey-Dodd-Gibbon equation. The key idea consists of applying Hirota's bilinear method to potential Equation 15 via the Cole-Hopf transformation (Equation 16). In a recent works Ma et al. (2010) and Ma and Fan (2011) published other techniques to construct specific or general  $N$ -soliton solution for a given nonlinear partial differential equation. Let us comment about one of these methods (the so called multiple exp-function method) (Ma et al, 2010). In view of this method, one soliton solutions to Equation 15 have the form:

$$w(x, t) = \frac{a_0 + a_1 \exp(\eta)}{1 + b_1 \exp(\eta)}, \quad (38)$$

$$\eta = \eta(x, t) = kx - \omega t.$$

where  $a_0$ ,  $a_1$  and  $b_1$  are some constants to be determined. Inserting ansatz (Equation 38) into Equation 15 and simplifying it, we obtain the following polynomial equation in the variable  $\zeta = \exp(\eta)$ :

$$\begin{aligned} & k^5 - \omega + \\ & 2(-15a_0b_1k^4 + 15a_1k^4 - 13b_1k^5 - 2b_1\omega)\zeta + \\ & 6(20a_0b_1^2k^4 - 20a_1b_1k^4 + 10a_0^2b_1^2k^3 - 20a_0a_1b_1k^3 + 10a_1^2k^3 + 11b_1^2k^5 - b_1^2\omega)\zeta^2 + \\ & 2b_1^2(-15a_0b_1k^4 + 15a_1k^4 - 13b_1k^5 - 2b_1\omega)\zeta^3 + \\ & b_1^4(k^5 - \omega)\zeta^4 = 0. \end{aligned} \quad (39)$$

Equating the coefficients of different powers of  $\zeta$  to zero gives an algebraic system. Solving it with either Mathematica 8 or Maple 15, we obtain:

$$w = k^5, a_1 = b_1(a_0 + k). \quad (40)$$

Thus, the following is a one-soliton solution (or one wave solution in Ma's terminology) to Equation 15.

$$w(x, t) = \frac{a_0 + b_1(a_0 + k)\exp(\eta)}{1 + b_1\exp(\eta)}, \quad \eta = kx - k^5t. \quad (41)$$

In view of Equation 13, one-soliton solution to Caudrey-Dodd-Gibbon Equation 1 is:

$$u(x, t) = \frac{b_1k^2 \exp(kx - k^5t)}{(1 + b_1 \exp(kx - k^5t))^2}. \quad (42)$$

Observe that solutions 24 and 42 are the same. We conclude that Hirota's method and multiple exp method give the same result for one-soliton solutions.

Let us examine two soliton solutions. In view of Ma's method, the two solution solutions to Equation 15 are of the form:

$$w(x, t) = \frac{k_1 \eta_1 + k_2 \eta_2 + R(k_1 + k_2) \eta_1 \eta_2}{\Delta + \eta_1 + \eta_2 + R \eta_1 \eta_2}, \Delta \in \{0, 1\}, \quad (43)$$

$$\eta_1 = \exp(k_1 x - \omega_1 t), \quad \eta_2 = \exp(k_2 x - \omega_2 t).$$

Let  $\Delta = 1$ ; inserting ansatz (Equation 43) into Equation 15 and simplifying them, we obtain the following polynomial equation with respect to the variables  $\zeta_1 = \exp(\eta_1)$  and  $\zeta_2 = \exp(\eta_2)$ :

$$\begin{aligned} & c_1^2 c_2 k_2 R (k_2^5 - \omega_2) \zeta_1^2 \zeta_2 + \\ & c_1 c_2^2 k_1 R (k_1^5 - \omega_1) \zeta_1 \zeta_2^2 + \\ & \left( c_1 c_2 (k_1 + k_2) (k_1^5 + 5k_2 k_1^4 + 10k_2^2 k_1^3 + 10k_2^3 k_1^2 + 5k_2^4 k_1 + k_2^5 - \omega_1 - \omega_2) R + \right. \\ & \left. c_1 c_2 (k_1 - k_2) (k_1^5 - 5k_2 k_1^4 + 10k_2^2 k_1^3 - 10k_2^3 k_1^2 + 5k_2^4 k_1 - k_2^5 - \omega_1 + \omega_2) \right) \zeta_1 \zeta_2 + \\ & c_1 k_1 (k_1^5 - \omega_1) \zeta_1 + c_2 k_2 (k_2^5 - \omega_2) \zeta_2 = 0. \end{aligned} \quad (44)$$

Equating the coefficients of  $\zeta_1^2 \zeta_2$ ,  $\zeta_1 \zeta_2^2$ ,  $\zeta_1 \zeta_2$ ,  $\zeta_1$  and  $\zeta_2$  to zero, gives a system of algebraic equations. Solving it we obtain the following nontrivial solution:

$$w_1 = k_1^5, w_2 = k_2^5, R = \frac{(k_1 - k_2)^2 (k_1^2 - k_1 k_2 + k_2^2)}{(k_1 + k_2)^2 (k_1^2 + k_1 k_2 + k_2^2)}. \quad (45)$$

We see that solution 45 coincides with solution 28.

On the other hand, if we take  $\Delta = 0$ , we obtain solution 29. We conclude that Ma's method does not give any new solutions when compared with Hirota's method.

## Conclusions

We successfully applied Hirota's bilinear method to obtain one and two soliton solutions to Caudrey-Dodd-Gibbon equation. Application of Hirota's bilinear method to other nonlinear equations may be found in Salas (2010, 2011a, b, c) works. Other ideas and techniques to construct specific or general  $N$ -soliton solution may be found in the works (Ma and Fan, 2011) and (Sawada and Kotera, 1974). A review of other methods based on projective Riccati equations to find exact solutions to nonlinear partial differential equations is studied in the references (Salas and Castillo, 2011; Salas, 2011a, b, c). We think that the solutions given here are new in the open

literature. Finally, the existence of three soliton solutions to Caudrey-Dodd-Gibbon equation remains an open question.

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