International Journal of Physical Sciences Vol. 8(5), pp. 179-181, 9 February, 2013
Available online at http://www.academicjournals.org/IJPS
DOI: 10.5897/IJPS12.669
ISSN 1992 - 1950 ©2013 Academic Journals

Commentary

A comment on “Robust linear optimization under new distance measure”

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Accepted 4 February, 2013

Zhang et al. (2012) proposed a new robust counterpart for linear optimization, and shown the effectiveness of the new model by numerical results of AFIRO and ADLITTLE. In this comment, it is shown that the numerical results in their paper are not true, conversely, the model of Bertimas and Sim (2004) has better optimality than that of Zhang et al. (2012) if the same probability bounds that the i-th constraint violated is maintained.

Key words: Robust optimization, probability bounds, Netlib.

INTRODUCTION

Recently, Zhang et al. (2012) proposed a new robust counterpart to solving linear optimization problems with uncertain data, and claimed to reduce the conservatism of the solution compared to the well-known model of Bertsimas and Sim (2004). This claim was illustrated by numerical results for AFIRO and ADLITTLE, which are test problems of the Netlib (Dongarra et al., 2003), a collection of mathematical software, papers and databases. However, it is pointed out that the comparison given by Zhang et al. (2012) for the performance of the two methods is not valid.

Consider the nominal linear optimization problem as follows:

$$\begin{aligned} &\max c'x \\ &s.t. Ax \le b \\ &\quad l \le x \le u \end{aligned} \tag{1}$$

In Equation 1, assume that data uncertainty only affects the elements in matrix A. Consider a particular row i of the matrix A and let J_i represent the set of coefficients in row i that are subject to uncertainty. Ben-Tal and Nemirovski (2000) modeled each entry $a_{ij}, j \in J_i$ as a symmetric and bounded random variable $\tilde{a}_{ij}, j \in J_i$ that takes values in $[a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}_{ij}]$. By defining random variable $\eta_{ij} = (\tilde{a}_{ij} - a_{ij}) / \hat{a}_{ij}$, associated with the uncertain data $\tilde{a}_{ij}$, Bertsimas and Sim (2004) proposed a robust linear optimization model and shown that the probability of i-th constraint is violated at most

$$\Pr\left(\sum_j a_{ij} x_j^* > b_i\right) \le \exp\left(-\frac{\Gamma_i^2}{2|J_i|}\right) \tag{2}$$

Where x^* is assumed to be the optimal solution of the method, and the parameter $\Gamma_i \in [0, |J_i|]$ is introduced to adjust the robustness against the level of conservatism of the solution.

Let $X \subseteq R^n$ be compact set and $x, y \in X$, Wu and Yang (2002) and Zhang and Chen (2004) proposed a distance measure to research AFCM clustering problems as follows:

$$dist(x, y) = \sqrt{1 - \exp(-\beta \| x - y \|^2)} \tag{3}$$

Where $\|\cdot\|$ denotes the norm.

For every row i, Zhang et al. (2012) introduced a parameter β_i, the role is to adjust the robustness of the

Table 1. Comparison of different models under different disturbance values (AFIRO, $\Gamma_i = 0.5, \beta_i = \Gamma_i \sqrt{\mu_i^*}$).

$\hat{a}_{23} = \hat{a}_{36}$	Bertsimas and Sim (2004) model		Zhang et al. (2012) model	
	Optimal value	**% change**	**Optimal value**	**% change**
0.2	-448.4359	3.51	-438.6455	5.62
0.4	-432.1186	7.02	-415.8014	10.53
0.6	-415.8014	10.53	-395.6448	14.87
0.8	-399.4841	14.04	-377.7278	18.73
1.0	-383.1669	17.55	-361.6968	22.17
1.2	-366.8496	21.07	-347.2689	25.28
1.4	-350.5323	24.58	-334.2151	28.09
1.6	-334.2151	28.09	-322.3480	30.64
1.8	-317.8978	31.60	-311.5128	32.97
2.0	-301.5806	35.11	-301.5806	35.11

Table 2. Comparison of different models under different adjustment factors (AFIRO, $\hat{a}_{23} = \hat{a}_{36} = 0.6, \beta_i = \Gamma_i \sqrt{\mu_i^*}$).

Γ_i	Bertsimas and Sim (2004) model		Zhang et al. (2012) model	
	Optimal value	**% change**	**Optimal value**	**% change**
0.2	-454.1724	4.21	-433.8362	6.65
0.4	-425.5917	8.43	-407.4438	12.33
0.6	-406.0110	12.64	-384.6502	17.24
0.8	-386.4303	16.85	-364.7665	21.51
1.0	-366.8496	21.07	-347.2689	25.28
1.2	-361.6968	22.17	-331.7521	28.62
1.4	-357.2980	23.12	-317.8978	31.60
1.6	-353.4991	23.94	-305.4525	34.28
1.8	-350.1852	24.65	-48.6359	89.54
2.0	-347.2689	25.28	-48.6359	89.54

proposed method against the level of conservatism of the solution, and used the distance function of Equation 3 to set up a new robust counterpart of linear optimization as follows:

$$\begin{aligned} &\max c' x \\ &s.t. \sum_j a_{ij} x_j + \beta_i \sum_{j \in J_i} \sqrt{1 - \exp(-\| \tilde{a}_{ij} - a_{ij} \|^2)}\, y_j \leq b_i, \forall i \\ &-y_j \leq x_j \leq y_j, \forall j \in J_i \\ &l \leq x \leq u \\ &y \geq 0 \end{aligned} \tag{4}$$

This counterpart is the same as the model of Berstimas and Sim (2004), only with a different formation of the first constraint. Zhang et al. (2012) have shown that the probability of the i-th constraint is violated at most

$$\Pr(\sum_j a_{ij} x_j^* > b_i) \leq \exp(-\frac{\beta_i^2}{2\mu_i^* \, | J_i |}) \tag{5}$$

Where

$$\mu_i^* = \max\{\frac{\hat{a}_{ij}^2}{1 - \exp(-\hat{a}_{ij}^2)}\}.$$

Zhang et al. (2012) claimed to reduce the conservatism of the solution compared to the model of Bertsimas and Sim (2004) by illustrating numerical results for AFIRO and ADLITTLE. In their paper, Tables 1 and 3 show the comparison of different models under different disturbance values when $\Gamma_i = \beta_i = 0.5$, while Tables 2 and 4 show the comparison of different models under different adjustment factors when $\hat{a}_{23} = \hat{a}_{36}$ and $\hat{a}_{161} = \hat{a}_{176} = \hat{a}_{177} = 0.6$, respectively, then they claimed that their model obtained the optimal value under the influence of the uncertain parameters which have a smaller rate of change. Unfortunately, their comparisons are meaningless since they have given these results under the condition that $\Gamma_i = \beta_i$. In fact, it is meaningful to

Table 3. Comparison of different models under different disturbance values (ADLITTLE, $\Gamma_i = 0.5, \beta_i = \Gamma_i \sqrt{\mu_i^*}$).

$\hat{a}_{161} = \hat{a}_{176} = \hat{a}_{177}$	Bertsimas and Sim (2004) model		Zhang et al. (2004) model	
	Optimal value	**% change**	**Optimal value**	**% change**
0.0001	225495.5	0.0002	225495.9	0.0004
0.001	225500.0	0.0022	225504.4	0.0042
0..01	225545.5	0.0224	225588.5	0.0415
0.1	226021.8	0.2336	226366.8	0.3866
0.3	227332.3	0.8148	230050.4	2.0202
0.5	244221.4	8.3046	254412.8	12.8241
0.7	259360.5	15.0183	278566.0	23.5353
0.9	273173.2	21.1438	298384.5	32.3242

Table 4. Comparison of different models under different adjustment factors (ADLITTLE, $\hat{a}_{161} = \hat{a}_{176} = \hat{a}_{177} = 0.6, \beta_i = \Gamma_i \sqrt{\mu_i^*}$).

Γ_i	Bertsimas and Sim (2004) model		Zhang et al. (2004) model	
	Optimal value	**% change**	**Optimal value**	**% change**
0.2	226863.1	0.6067	228149.1	1.1770
0.4	242671.7	7.6173	251849.1	11.6872
0.6	260820.5	15.6658	280747.3	24.5027
0.8	277016.9	22.8484	303567.8	34.6228
1.0	291121.3	29.1032	322595.6	43.0611
1.5	319379.2	41.6347	357570.1	58.5712

compare the optimal values only if the probability bounds of Equations 2 and 5 are the same, that is $\beta_i = \Gamma_i \sqrt{\mu_i^*}$.

Here, the comparisons of the methods of Bertimas and Sim (2004) and Zhang et al. (2012) was given under the condition that $\beta_i = \Gamma_i \sqrt{\mu_i^*}$, any other parameters defined as in Zhang et al. (2012).

Tables 1 and 2 compare the results of AFIRO, while Tables 3 and 4 compare results of ADLITTLE.

Conclusion

All comparisons pointed out that the model of Bertimas and Sim (2004) has better optimality than that of Zhang et al. (2012) if maintaining the same probability bounds that the i -th constraint is violated.

ACKNOWLEDGEMENTS

The author is grateful to the referee for careful reading and useful comments. This work is supported by the Ministry of Education of Humanities and Social Science Project of China (Grant No. 10YJC630360).

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